

UNIVERSITY OF BOLOGNA
DEPARTMENT OF ELECTRICAL ENGINEERING
LABORATORY OF APPLIED SUPERCONDUCTIVITY

ANTONIO MORANDI

**CIRCUIT METHODS FOR
THREE DIMENSIONAL FIELD ANALYSIS
IN LARGE SCALE SUPERCONDUCTING SYSTEMS**

DOCTORAL THESIS, 2004

UNIVERSITÀ DI BOLOGNA
DIPARTIMENTO DI INGEGNERIA ELETTRICA
LABORATORIO DI SUPERCONDUTTIVITÀ APPLICATA

**CIRCUIT METHODS FOR
THREE DIMENSIONAL FIELD ANALYSIS
IN LARGE SCALE SUPERCONDUCTING SYSTEMS**

**TESI DI DOTTORATO DI
ANTONIO MORANDI
XVI CICLO**

Coordinatore

**Prof. Ing. Francesco
Negrini**

Tutori

**Dr. Massimo Fabbri
Prof. Francesco Negrini
Prof. Pier Luigi Ribani**

*to Claudia, Marco, Oriana, Luigi,
Gabriella, Christine and Natalie*

the authorities of my family

All what is written in this thesis has been inspired by three years of conversations and exchange with Prof. Pier Luigi Ribani and Dr. Massimo Fabbri. For this reason and for having welcome me very warmly I want to express them my true gratitude. The equal gratitude I want to express to Prof. Francesco Negrini for having me continuously encouraged and addressed.

My thankfulness goes also to all the friends of the cafe Miki&Max, for having shared with me little about superconductivity but lot of different and fundamental aspects of life. Finally, a special thanks goes to Angela, for her sweetness.

I have placed no deliberate errors in this thesis, but I have lived long enough to be quite familiar with my own imperfections

Contents

Introduction	1
1. Current distribution in superconducting domains subject to transport current and external magnetic field - The equivalent electric network	
Introduction	5
1.1. The mathematical formulation	6
1.2. The discretized problem and the equivalent electric network	9
1.3. Problems with lower dimensionality. The tree-cotree decomposition and the reduced solving system	27
1.4. A simple axis-symmetric problem	35
1.5. Numerical results and analytical benchmarks	40
1.6. Constitutive relation for non homogeneous superconductors	49
1.7. Linear reconstruction of current density	57
2. Magnetization distribution in superconducting domains subject to external magnetic field – The equivalent magnetic network	
Introduction	65
2.1. The mathematical formulation	66
2.2. The discretized problem and the equivalent magnetic network	72
2.3. The hysteresis model	91
2.4. Numerical analysis of the Pulsed Field Magnetization of YBCO rings	113
3. Current distributions in superconducting domains interacting with magnetizable bodies and voltage-driven normal conducting coils - The coupled equivalent electric and magnetic networks	
Introduction	121

3.1. The mathematical formulation	122
3.2. The discretized problem and the coupled equivalent electric and magnetic networks	124
3.3. The voltage characteristic of the system	140
3.4. Equivalent circuit of a magnetic shield type SFCL	145

4. Experimental testing of a magnetic shield type superconducting fault current limiter

Introduction	155
4.1. The superconducting fault current limiter prototype	156
4.2. The experimental set up	161
4.3. Experimental results	163
4.4. Numerical results	168

Conclusion	173
-------------------	-----

References	175
-------------------	-----

Introduction

Superconductivity is becoming a key technology in many fields of engineering. The use of superconducting materials brings remarkable improvements to the performance of existing electrical devices and allows the conception of new ones which are able to carry out functions which cannot not be achieved by using conventional materials. Superconductors greatly reduce the consumption of electrical power with respect to normal conductors. A DC current can flow in a superconducting wire without generating losses at all. Moreover, the magnetic properties of these materials are very peculiar. Superconducting materials can be used in *large scale* applications, i.e. in the field of large currents and power, as well as in signal processing, digital circuits, ultra sensitive magnetometers, detectors and microwave devices, which are referred to as *small scale* applications.

Concerning the large scale applications of superconductivity, very intense magnetic fields needed for particle accelerators, nuclear fusion or even material processing can only be conveniently produced by means of superconducting magnets. Magnetic energy which is permanently stored in superconducting coils with a DC current flowing through (SMES), can be exchanged with an external system by means of static electronic converters; nowadays small scale SMES (micro-SMES) are used for the local and short term control of power quality for critical loads. Bulk superconductors are used for superconducting levitation and flywheels and also for trapping high magnetic fields in cryo-permanent magnets. Superconducting fault current limiters (SFCL) are regarded as crucial components for modern electric power systems as they offer ideal performances: negligible impedance in normal conditions and passive (i.e. reliable) switch to high impedance state in case of fault. Moreover, combined used of new emerging superconducting materials (MgB_2) and technology of hydrogen for transmitting and storing energy produced by renewable sources, is foreseen as a possible future application of superconductivity. In all these applications the materials used are type II superconductors. As the number of applications of superconductors increases, the demand for robust and reliable modeling, both on the microscopic and the macroscopic scale, increases as well. Microscopic models aim to explain the fundamental physical properties; macroscopic models use simplified description of

these properties and are application oriented [1]. This thesis is a contribution to the three dimensional macroscopic modeling of type II superconductors for large scale applications.

Given a large scale superconducting device, it is possible, by stating an appropriate mathematical formulation and using an efficient numerical technique, to calculate the distribution of the *local quantities*, i.e. field vectors, inside the device and to deduce, on their base, some *integral quantities*, e.g. currents, voltages, power losses, forces, etc. This is what we usually call a *numerical model*, and it represents the only modeling opportunity when actual three dimensional geometries are considered [2]. In the majority of the large scale applications of superconductivity, the field problem belongs to the general class of *eddy currents* problems, which can be stated as: “find the vectors **B**, **H**, **E**, **J** which satisfy the Ampere and the Faraday’s law at any point of the considered domain and some specified condition on the boundary”. In some bounded sub-domains, which we can call sources, the current density is constrained to be equal to some known (determined elsewhere) distribution. In order to solve the problem the Ampere and the Faraday law should be supplemented by two constitutive relations: one linking vectors **B** and **H** and one linking vectors **E** and **J**. The *characterization*, i.e. the modification (with respect to the case of a normal conductor) of one or even both these two relations in order to correctly reproduce the peculiar macroscopic features of the superconductors, represents the main effort in their modeling. The most widely used characterization for the superconductors is the Bean critical state model [3,4] (or the Kim variant [5]), which assumes that no matter how small an electric field, if different from zero, it induces the circulation of the full critical current in the superconductor. The critical state model leads to hysteretic behavior of the B-H curve [6-9]. Though very simple and easy to implement in one or two dimensions, this multi-valued characteristic can hardly be applied to complex three-dimensional geometries, due to the fact that the consequent free boundary problem requires complicated “front-tracking” numerical techniques to be solved. By modifying the critical state model, assuming that at a given electric field the superconductor switches to the ohm law with its normal state resistivity, it is possible, by means of a variational approach, to deal with three dimensional cases [10-13]. A widely used alternative, which also allows full three dimensional problems to be numerically dealt with, is the smooth approximation of the critical state model by means of the E-J power law. This is the approach followed

in chapter 1 of this thesis. Not merely a convenient mathematical approximation, the E-J power law is regarded as a physical based constitutive relation for type II superconductors, because it gives account of the thermal activated flux creep and leads to hysteretic behavior of the B-H curve [14-16]. The possibility of using the hysteretic dependence of B on H directly as a constitutive relation and of considering the superconductor as a non conducting material is discussed in chapter 2, for the cases where no transport current is applied. The coupling of the E-J and the B-H based approaches is discussed in chapter 3, in order to study the interaction among superconducting and magnetizable domains [17-19].

Throughout this thesis, whatever the chosen constitutive relation, an integral formulation is followed to solve the field problem. Part of the reason why this route was chosen are general and applies to all the fields of science and engineering where numerical modeling is used: integral formulations enable good accuracy even with relatively coarse discretization [20-21] and permit the boundary conditions to be taken into account in a simpler way. There is however a further reason, more specific to electrical engineering, why the integral approach is preferable and is related to the fact that a large scale superconducting device often interacts with a power system rather than be insulated. We have stated before that by means of a numerical model both local and integral quantities can be calculated. The integral quantities are of great practical interest, because it is through them that the device “talks together” with the power system; nevertheless we stress that they never exclude the field problem. A numerical model focused on the field quantities allows analysis on the device scale and is very suitable as a design tool. If the detail of the field distribution is not important and only the convergence of the numerical solution with respect to the integral quantities is required, the numerical model becomes simpler and can be exported to a power system analysis tool to study the device/system reciprocal interaction [22,23]. The easiest way to export a numerical model of a device to a power system simulator, which is usually based on the circuit theory, is to express it directly in a circuit form. This mission is accomplished automatically if the numerical problem is solved by means of an integral formulation, since it can be easily identified as a circuit method [19,22-31]. The details of the integral formulation and the circuit analogy are fully worked out in chapters 1 and 2. In chapter 3 the numerical model of a magnetic shield type SFCL is arrived at in the circuit form, that is, an equivalent circuit of the device is determined by solving the

interior field problem at the desired level of accuracy. However, the more the accuracy of the field solution the greater the complexity of the equivalent circuit. In chapter 4 the results of the experimental testing of a magnetic shield type SFCL basic prototype, aimed at the validation of the equivalent circuit, are reported and compared with the numerical ones.