MODELING OF LIGHTNING-INDUCED VOLTAGES ON DISTRIBUTION NETWORKS FOR THE SOLUTION OF POWER QUALITY PROBLEMS, AND RELEVANT IMPLEMENTATION IN A TRANSIENT PROGRAM

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List of symbols

\( A_e \): incident vector potential;
\( A'_e \): vertical component of the incident vector potential;
\( B \): total magnetic field;
\( B'_e \): external incident magnetic field;
\( B_r \): radial component of the LEMP magnetic field;
\( B_s \): scattered magnetic field;
\( B'_y \): scattered induction magnetic field;
\( C' \): single-conductor line capacitance per-unit-length;
\( c \): vacuum light velocity;
\( C_{dyn} \): voltage-dependent dynamic capacitance in the corona phenomenon;
\( [C'_{ij}] \): multi-conductor line capacitance matrix per unit length;
\( d \): distance of the stroke location from the line;
\( d_l \): lateral attractive distance;
\( E \): total electric field;
\( E'_e \): external incident electric field;
\( E'_z \): vertical component of the incident electric field along the \( z \) axis;
\( E_{h_k}'' \): spatial-temporal discretized value of the horizontal component of the incident electric field along the line;
\( E'_x \): horizontal component of the incident electric field along the \( x \) axis;
\( E_r \): radial component of the LEMP electric field;
\( E_r \): Fourier-transform of the electric field radial component;
\( E_{r_p} \): Fourier-transform of the electric field radial component for an ideal ground;
\( E_s \): scattered electric field;
\( E_z \): vertical component of the LEMP electric field;
\( \text{erfc} \): complementary error function;
\( F_p \): number of annual insulation flashovers per km of distribution line;
\( G' \): single-conductor line conductance per-unit-length;
\( G_1, G_2 \): Bergeron equivalent generators;
\( H \): lightning channel height;
$h$: single-conductor line height;

$\hat{H}_{\phi}$: Fourier-transform of the magnetic field radial component for an ideal ground;

$i_0(t)$: current at the EMTP node;

$i_g$: current that flow through the connection between a line conductor and ground;

$L_r$: lightning return stroke peak value;

$I_0$: channel-base current peak value of the Heidler function;

$i(z,t)$: spatial-temporal lightning current distribution along the return stroke channel;

$i(x,t)$: single-conductor line current induced along the line;

$i_h^n$: spatial-temporal discretized value of the induced current along the line;

$[i_{xy}]$: vector of the induced currents diverted to ground in correspondence of the pole;

$[i_i(x,t)]$: multi-conductor line induced current vector along the line;

$k$: spatial discretization index;

$k_{max}$: maximum value of the spatial discretization index;

$k_1, k_2$: parameters related to the corona model;

$L_r'$: single-conductor line external inductance per-unit-length;

$[L_g']$: multi-conductor line external inductance matrix per unit length;

$n$: exponent of the Heidler function;

$n$: temporal discretization index;

$NC$: number of line conductors;

$N_g$: ground flash density;

$n_{max}$: maximum value of the temporal discretization index;

$O(\Delta x)$: remainder term, which approaches zero as the first power of the spatial increment;

$O(\Delta t^3)$: remainder term, which approaches zero as the third power of the temporal increment;

$P_i$: probability of lightning current peak to be within interval $i$;

$R$: distance of the electric dipole from the observation point;

$r$: projection of $R$ in the plane $xy$;

$R_0$: left line termination resistive load;

$r_g$: striking distances to ground;

$R_g$: pole grounding resistance;

$R_L$: right line termination resistive load;
$[R_{gp}]$: diagonal matrix of the phase-to-ground resistances of the poles when a flashover occurs;
$r_s$: striking distances to wire;
v: return stroke wave front velocity;
$v'(x,t)$: single-conductor line incident voltage along the line;
$v''(x,t)$: single-conductor line scattered voltage along the line;
$v(x,t)$: single-conductor line total voltage along the line;
$v_0(t)$: voltage at the EMTP node;
$v^i_k$: spatial-temporal discretized value of the scattered voltage along the line;
$v'_s(x,t)$: voltage drop due to the transient ground impedance;
$v_{th}(x,t)$: corona threshold voltage;
$[v'(x,t)]$: multi-conductor line incident voltage along the line;
$[v''(x,t)]$: multi-conductor line scattered voltage vector for along the line;
$[v(x,t)]$: multi-conductor line total voltage vector for along the line;
$Z_0$: vacuum impedance;
$Z_c$: single-conductor line characteristic impedance;
$Z_g$: ground impedances;
$Z_w$: wire impedances;
$\Delta t$: time integration step;
$\Delta x$: spatial integration step;
$\delta$: skin depth factor;
$\varepsilon_0$: vacuum permittivity constant;
$\varepsilon_{rg}$: relative ground permittivity;
$\phi$: total induced scalar potential;
$\phi^i$: incident scalar potential;
$\Gamma_0(t)$ is the EMTP termination type;
$\Gamma$: integro-differential operator for the representation of a single-conductor line transverse discontinuities;
$[IK]$: integro-differential matrix operators for the representation of a multi-conductor line transverse discontinuities;
η: correction factor of the Heidler function;
λ: decay constant of the MTLE model;
μ₀: vacuum permeability constant;
μ₉: ground permittivity;
ρ: correlation coefficient between lightning current amplitude and its front duration;
σ₉: ground conductivity;
τ₁: time constant of the wave front of the Heidler function;
τ₂: time constant of the wave decay of the Heidler function;
τ₉: ground time constant;
ω: angular frequency;
ξₑ: inverse Fourier-transform of the ground impedance;
⊗: convolution product.
Chapter 1. – Introduction

One of the main causes of a-periodic disturbances on distribution networks, which seriously affect the power quality, is certainly the lightning activity\(^1\), and in particular the so-called lightning ‘indirect’ activity\(^2\).

Due to the limited height of distribution lines of medium and low voltage distribution networks as compared to that of the structures in their vicinity, indirect lightning return strokes are more frequent events than direct strokes\(^3\), and for this reason we shall focus on such a type of lightning event.

The analysis of the distribution networks response against Lightning Electro Magnetic Pulse (LEMP), requires the availability of accurate models of LEMP-illuminated lines. These should be able to reproduce the real and complex configuration of distribution systems including the presence of shielding wires and their groundings, as well as that of surge arresters and distribution transformers.

In addition to the accurate modeling of the overhead lines, the development of models of the entire distribution networks is clearly necessary. This should allow, in principle, to optimize the number and location of protective devices and then to minimize the number of outages.

The first part of the research activity here reported has been dedicated to the development of a complex LEMP-illuminated line model based on a modification of the Agrawal et al. [1980] coupling model, able of predicting LEMP response of a multi-conductor overhead line with transverse discontinuities represented by groundings of shielding wire, or neutral conductor, and surge arresters on the phase conductors.

The model has been then implemented in a computer code using a numerical integration scheme based on the finite difference time domain (FDTD) technique. In particular a 2\(^{nd}\) order FDTD scheme has been selected to improve the numerical stability of the line model.

The developed line model has been validated by means of a series of results obtained during an experimental campaign carried out at the Swiss Federal Institute of Technology in Lausanne during the year 2000 by means of a NEMP simulator.

The natural extension of the developed LEMP-illuminated line model consists in a LEMP-illuminated distribution system. This can be accomplished, in principle, by appropriately rewriting, for each type of termination or model component, the boundary condition of each illuminated line forming the distribution system. An alternative approach has been used to develop such a complex distribution system model. The model is based on

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\(^1\) [e.g. Boonseng and Kinnares, 2001].

\(^2\) [e.g. Gunther and Mehta, 1995].

\(^3\) [Rusck, 1977; IEEE WG on lightning performance of distribution lines, 1997].
the contemporary use of the above mentioned illuminated line model, and the Electro Magnetic Transient Program (EMTP96). Other similar programs have been presented in the literature: they present limitations concerning the calculation of the LEMP and the type of phenomena that can be dealt (e.g. corona), which the present model is aimed at overcoming.

The developed complex system model has also been validated by means of experimental results provided by the lightning group of the University of São Paulo.

We have then made use of the Monte Carlo method and of the developed models, to assemble a statistical procedure aimed at assessing the lightning performance of distribution lines.

So far, several studies on the subject have been carried out without considering the real configuration of the distribution systems, i.e. without considering the presence of shielding wires, of their groundings, of the steady-state voltage superimposed to that induced by lightning and, more important, without any indication on the nature of the real number of faults. The procedure here proposed is aimed at overcoming the above mentioned limitations.

The structure of the thesis is the following:

Chapter 2. A résumé on the models proposed and applied in the literature to calculate lightning-induced overvoltages is given. The chapter is divided in three paragraphs to illustrate: a) the most used lightning current return stroke models, b) the relevant electromagnetic field appraisal and c) the coupling between the LEMP and the overhead multi-conductor line. Section c) also compares the simplified formula of Rusck [1958] with the Agrawal et al. [1980] coupling model.

Chapter 3. This chapter illustrates the models implementation. It gives a description of the illuminated-line model proposed to take into account line transverse discontinuities. It illustrates the numerical implementation of a FDTD 2nd order integration scheme, applied to the complex line model. The experimental validation of such model is then shown by means of the experimental results obtained with the EMP simulator of the Swiss Federal Institute of Technology. A sensitivity analysis on the effect of periodical groundings of the shielding wire, and on the presence of surge arresters on lightning induced overvoltages is illustrated and discussed.

In the same chapter is also described the illuminated distribution system model based on an interface of the developed line model with the Electro Magnetic Transient Program (EMTP96). A similar interface has been realized also concerning the Power System Blockset in Matlab Environment and a description of it is also given.
A comparison between theoretical and experimental results is presented. It is based on the experimental results obtained using a LEMP simulator on a reduced scale distribution system by the lightning research group at the University of São Paulo.

Chapter 4. The statistical procedure based on the developed models and on the Monte Carlo method is first illustrated and then compared, for simple line configuration, with the similar one proposed in the IEEE Std 1410-1997. A sensitivity analysis is then given and discussed which involves the main parameters such as the ground conductivity, the multi-conductor line configuration, the shielding wire height, its grounding resistance and grounding spacing and the presence of surge arresters. The possibility to take into account the steady-state line voltage and the change of the phase-to-ground coupling factor when a flashover occurs, is finally illustrated and its implications are discussed. The application of the statistical procedure to a typical Italian distribution line is finally presented in order to evaluate the number and type of flashovers that occur per 100 km per year.

Chapter 5 is devoted to the conclusions.

This thesis represents one of the results obtained in a framework of an international collaboration between the University of Bologna, the Swiss Federal Institute of Technology of Lausanne and the University of Rome ‘La Sapienza’.

I would like to thank chiefly prof. Carlo Alberto Nucci for his fundamental guide all along the three years of PhD and his constant and helpful support. Acknowledgements are also due to prof. Dino Zanobetti for precious advises during the layout of this thesis and to prof. Alberto Borghetti for important discussions on the statistical procedure and on the implementation of the developed models in the transient program.

I would also like to thank prof. Farhad Rachidi for his fundamental help in the experimental activity carried out at the Swiss Federal Institute of Technology in Lausanne and for very useful discussions.

Thanks are also due to prof. Alexandre Piantini for providing the experimental results of the induced surges on the reduced scale distribution system.

Acknowledgements are due to dr. Alberto Gutierrez who has developed the Mat-LIOV code and who has contributed in developing the Mat-LIOV interface with the Power system Blockset in Matlab environment.

Finally I would to thank Luigia for her comprehension and fundamental support all along the three years of PhD.
Chapter 2. – Evaluation of lightning-induced voltages on transmission lines

The estimation of lightning induced overvoltages has been the object of various studies since the early years of the past century.

The first studies reported by Wagner K.W. [1908], Bewley [1929], Norinder [1936] considered the overvoltage as being produced basically via electrostatic induction by a charged cloud. According to Wagner K.W. [1908], when the lightning discharge occurs, the charge bound to the line is released in form of traveling waves of voltage and current. Wagner did not consider the electromagnetic field radiated by the lightning return-stroke current.

In the early 1940’s Wagner C.F and McCann [1942], based on Schonland’s [1934] investigations of the nature of the lightning discharge, published a paper in which the overvoltage was considered mostly due to the return stroke phase; an assumption that was accepted in practically all the subsequent studies. Rachidi et al. [1994] have shown that for lightning particularly close to the distribution line (30 m or so), some important overvoltages can be produced also by leader phase preceding the return stroke one. However, in what follows, we shall be interested only in the voltages induced by the electromagnetic field change produced by the return-stroke phase.

In parallel with the theoretical studies, several measurement campaigns on lightning electromagnetic fields and many tests with voltages induced on experimental lines have been performed. For our purposes it is worth reminding that practically all the above

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2 All the models dealt with in this thesis can indeed be extended to take into account also the leader field in relatively simple manner. It is reminding that taking into account the leader field not change significantly the results reported here.

3 [Lin and Uman, 1973; Tiller et al., 1976; Weidman and Krider, 1978; Lin et al., 1979; McDonald et al., 1979; Yokoyama, 1980; Cooray and Lundquist, 1982; Krider and Guo, 1983; Cooray and Lundquist, 1985; Rakov and Uman, 1990; Rakov et al., 1998, Shostak et al., 2000; Willett and Krider, 2000; Rachidi et al., 2001].

4 [Norinder, 1936; Berger, 1955; Koga et al., 1981; Eriksson et al., 1982; Yokoyama et al., 1983; Darveniza and Uman, 1984; Master et al., 1984; Cooray and de la Rosa, 1986; Yokoyama et al., 1986; de la Rosa et al., 1988; Rubinstein et al., 1989; Yokoyama et al., 1989; Yacoub et al. 1991; Georgiadis et al., 1992; Michishita and Ishii, 1992; Barker et al., 1993; Fernandez et al., 1999a, 1999b; Mata et al., 2000; Uman et al., 2000].
mentioned studies, both theoretical and experimental, have dealt with single line configuration.

Nowadays the evaluation of lightning induced overvoltages is generally performed in the following way:

- The lightning return-stroke electromagnetic field change is calculated at a number of points along the line employing a lightning return-stroke current model, namely a model that describes the form of the return stroke current as a function of height and time along the vertical channel. To this purpose, the return stroke channel is generally considered as a straight vertical antenna (see Fig. 2.1);

- The electromagnetic field (LEMP – Lightning Electro Magnetic Pulse) is then evaluated and used to calculate the induced overvoltages making use of a coupling model which describes the interaction between the field and the line conductors.

Fig. 2.1. – Return Stroke channel.

The first point will be discussed in the next paragraph 2.1, while the second one will be dealt with in the two subsequent paragraphs devoted to the LEMP radiated by a return stroke 2.2, and to the coupling models for the evaluation of the induced overvoltages 2.3 respectively.
2.1. – Engineering return-stroke current models

A comprehensive review of the various models proposed in the literature to predict some of the observed properties of lightning return strokes is beyond the scope of this thesis; the interested reader is referred to two papers by Rakov and Uman [1998] and by Gomes and Cooray [2000]; here we shall limit the discussion to those models that are generally called ‘engineering return-stroke current models’.

A return stroke current model is a mathematical specification of the spatial-temporal distribution of the lightning current along the discharge channel (or the channel line charge density). Such a mathematical specification include the return stroke wavefront velocity, which is generally one of the model inputs [e.g. Uman and McLain, 1969], the charge distribution along the channel, and a number of adjustable parameters related, to a certain extent, to the discharge phenomenon [e.g. Gomes and Cooray, 2000] and which should be inferred by means of model comparison with experimental results [e.g. Nucci and Rachidi, 1989]. Outputs can be directly used for computation of electromagnetic fields.

In these models the lightning channel is in general assumed to be straight, vertical and perpendicular to the conducting ground plane, as shown in Fig. 2.1, where the geometry of the problem is also defined.

Now, for the problem we are dealing with, only those models in which the return stroke current \( i(z',t) \) can be simply related to the specified channel-base current \( i(0,t) \) are of interest from an engineering point of view, since it is only the channel-base current that can be measured directly and for which experimental data are available. We shall call these models, ‘engineering return stroke current models’, or simply ‘engineering models’ in accordance with Rakov [2001].

The most popular engineering model is probably still the Transmission Line (TL) model, proposed about 30 years ago by Uman and McLain [1969]. A number of other models have been subsequently proposed with the primary aim of calculating the return stroke electromagnetic field given a certain lightning current or, vice versa, inferring the lightning current characteristics from remote electromagnetic field measurements\(^5\). In addition and although not originally formulated as engineering models, we feel worth mentioning certain more physically plausible models, such as the models by Lin et al. [1980], by Master et al. [1981] and by Cooray [1993,1998a].

In the large majority of the return-stroke models above mentioned\(^6\), the channel-base current is viewed as the result of the flowing towards ground of charges contained in the leader channel and in the corona sheath around the channel during the return-stroke phase.

\(^5\) [Heidler, 1985b; Rakov and Dulzon, 1987; Nucci et al.,1988; Diendorfer and Uman, 1990; Thottappillil et al., 1991; Rakov and Uman, 1998; Gomes and Cooray, 2000].

\(^6\) [Lin et al., 1980; Master et al. 1981; Heidler, 1985b; Nucci et al., 1988; Diendorfer and Uman, 1990; Thottappillil et al., 1991; Cooray, 1993; Cooray, 1998].
They are the so-called traveling-current-source-type \cite{Rakov1998}, or discharge-type \cite{Gomes2000} models. These models have been conceived to describe the field change due to the return-stroke phase, the total return-stroke field being the sum of the return-stroke field change plus the final value of the field produced by the preceding leader phase.

The validation of the various return-stroke models is not straightforward since, for natural lightning, no experimental set of simultaneously-measured currents and fields is available. Thus, although a direct comparison between model predictions and experimental results is possible for artificially-initiated lightning\textsuperscript{7} only a qualitative comparison is possible for natural lightning. In particular, a return stroke model is to be considered adequate if, starting from a typical channel-base current, it reproduces the typical features of the observed fields at different distances. The characterisation of natural lightning electromagnetic fields is therefore of importance, and an exhaustive survey can be found in \cite{Uman1982, Uman1987, Rakov1995, Rakov1998}.

The validation only with the comparison among each other of some of the most popular return-stroke models can be found in several papers, e.g. \cite{Lin1980, Nucci1990, Thottappillil1994}, who used essentially data from natural lightning. \cite{Thottappillil1994} used data from triggered lightning to compare some engineering return-stroke models.

Not all of the above mentioned engineering return-stroke models have been used to calculate lightning-induced voltages. To the best of our knowledge, the most commonly adopted for such a purpose are nowadays:

- the Transmission Line (TL) model \cite{Uman1969};
- the Traveling Current Source (TCS) model \cite{Heidler1985};
- the Modified Transmission Line Exponential (MTLE) model \cite{Nucci1988, Rachidi1990};
- the Diendorfer-Uman (DU) model \cite{Diendorfer1990}.

From the review papers earlier mentioned (in particular from \cite{Nucci1990, Thottappillil1993b, Rakov1998} and \cite{Gomes2000}) we conclude that all the above models allow the reproduction of overall fields that are reasonable approximations of measured fields from natural and triggered lightning; \cite{Gomes2000} have shown that also the models by \cite{Cooray1993}, can predict with good approximation all typical features of experimentally observed fields.

\textsuperscript{7} [Saint-Privat-D'Allier Res. Group, 1982; Willett et al., 1988; Leteinturier et al., 1990; Thottappillil and Uman, 1993b; Rubinstein et al., 1995; Uman et al., 1995].
If one considers, however, that for lightning-induced voltage calculation it is the early time region of the field that plays the major role in the coupling mechanism [Nucci et al., 1993], it follows that the most adequate models are probably the MTL-type ones (see Tab. 2.1). Additionally, although the TL model does not allow for any net charge removal from the channel and does not reproduce realistic fields for late time calculations [Nucci et al., 1990], the early time field prediction of the TL model is very similar to that of the more physically reasonable MTLL and MTLE models and thus, for the problem of interest, it can be considered a useful and relatively simple engineering tool.

<table>
<thead>
<tr>
<th>Absolute Error =</th>
<th>(E_{calc} - E_{meas})/E_{meas}</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TL</td>
</tr>
<tr>
<td>Mean</td>
<td>0.17</td>
</tr>
<tr>
<td>St.Dev.</td>
<td>0.12</td>
</tr>
<tr>
<td>Min.</td>
<td>0.00</td>
</tr>
<tr>
<td>Max.</td>
<td>0.51</td>
</tr>
</tbody>
</table>

Tab. 2.1. – Summary of statistics on the absolute error of the model peak fields on the basis of triggered lightning simultaneously measured currents, velocities and fields (subsequent return strokes). With reasonable approximation the results of the MTL column apply to both MTLE and MTLL. Adapted from Thottappillil and Uman [1993b].

For the current at the channel base the following analytical expression [Heidler, 1985a] is adopted\(^8\):

\[
 i(0,t) = I_0 \left( \frac{t}{\tau_1} \right)^n \frac{t}{\eta} \cdot e^{-\frac{t}{\tau_1}} \tag{2.1}
\]

where

\[
 \eta = e^{\left( \frac{\tau_2}{\tau_1} \right) \left( \frac{\tau_1^2}{\tau_2} \right)^{\frac{1}{n}}} \tag{2.2}
\]

\(^8\) It is worth noting that so far it has been implicitly assumed that the channel base current is not affected by reflection of the charges flowing downward the channel during the return stroke phase. Heidler and Hopf [1994] have addressed this issue by extending the TCS model to account for these reflections. This point is certainly of importance and stresses the need for additional experimental data useful for further model validation, but is, however, beyond the scopes of this thesis.
and

- \( I_0 \) is the amplitude of the channel-base current;
- \( \tau_1 \) is the front-time constant;
- \( \tau_2 \) is the decay-time constant;
- \( \eta \) is the amplitude correction factor;
- \( n \) is an exponent (2÷10).

Function (2.1) has been preferred to the commonly used double-exponential function since it exhibits, as opposed to the double-exponential function, a time-derivative equal to zero at \( t=0 \), consistent with measured return-stroke current wave shapes. Additionally, it allows for the adjustment of the current amplitude, the maximum current derivative and the charge transferred nearly independently by varying \( I_0, \tau_1 \) and \( \tau_2 \) respectively.

In what follows we shall describe only two return stroke current models, in particular we shall focus on the TL (Transmission line) model [Uman and McLain, 1969] and MTLE (Modified Transmission Line Exponential decay) model.

The Transmission Line (TL) model. In the TL model, it is assumed that the current wave at the ground travels undistorted and unattenuated up the lightning channel at a constant speed \( v \). Mathematically:

\[
\begin{align*}
    i(z', t) &= i\left(0, t - \frac{z'}{v}\right) & z' \leq v \ t \\
    i(z', t) &= 0 & z' > v \ t
\end{align*}
\]

(2.3)

where

- \( v \) is the lightning return stroke wave-front velocity;
- \( z \) is the vertical space variable.

The transfer of charge takes place only from the bottom of the leader channel to the top; thus no net charge is removed from the channel, this being an unrealistic situation given the present knowledge of lightning physics [Uman, 1987].

The Modified Transmission Line Exponential decay (MTLE) model. In the MTLE model [Nucci et al., 1988] the lightning current intensity is supposed to decrease exponentially while propagating up the channel as expressed by:
\[ i(z', t) = i \left( t - \frac{z'}{v} \right) e^{-\frac{z'}{\lambda}} \quad z' \leq v \ t \]
\[ i(z', t) = 0 \quad z' > v \ t \]  

where

- \( v \) is the return-stroke velocity;
- \( \lambda \) is the decay constant which allows the current to reduce its amplitude with height.

This attenuation is not to be considered as due to losses in the channel or to take into account the already mentioned decay with height of the initial peak luminosity, but has been proposed by Nucci et al. [1988] to take into account the effect of the charges stored in the corona sheath of the leader and subsequently discharged during the return stroke phase. Its value has been determined to be about 2 km by Nucci and Rachidi [1989], by means of tests with experimental results published by Lin et al. [1979,1980].

The MTLE model represents a modification of the TL model which allows net charge to be removed from the leader channel via the divergence of the return stroke current with height, and thus results in a better agreement with experimental results.

Channel-base current measurements have been performed by means of instrumented towers in some countries\(^9\), and statistical elaboration of lightning current data have been presented [e.g. Berger et al. 1975; Anderson and Eriksson, 1980]. Without loosing generality only lightning with lower negative charge to ground will be considered, here since it is generally accepted that, at least in temperate climate, positive flashes occur less frequently and have a lower peak current-derivative.

In Fig. 2.2, typical channel-base current wave shapes for negative first (Fig. 2.2a) and subsequent (Fig. 2.2b) return strokes, as reported by Berger et al. [1975], are shown. The statistics of lightning current parameters which are most significant for the evaluation of induced overvoltages (peak value and front steepness) are shown in Tab. 2.2,2.3 and in Fig. 2.3\(^{10}\).

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\(^{10}\) It is worth observing that since channel-base current is generally measured at the top of the instrumented tower, the statistical distributions of current parameters are presumably affected by current reflections at both bottom and top of the tower [Montandon and Beyeler, 1994; Beierl, 1992; Janischewskyj et al., 1992, Bermudez et al. 2001]. In principle, these reflections might alter both the front steepness and the peak value of the current depending on several factors, e.g. tower height, current wave shape and value of the reflection coefficients. To elaborate meaningful statistics on lightning current parameters, one should ‘decontaminate’
Fig. 2.2. – Typical channel-base current wave shapes: a) for first negative return strokes, b) for subsequent negative return strokes. Adapted from Berger et al. [1975].

<table>
<thead>
<tr>
<th>Stroke</th>
<th>95%</th>
<th>50%</th>
<th>5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ipeak [kA]</td>
<td>14</td>
<td>4.6</td>
<td>30</td>
</tr>
<tr>
<td>Time to crest [$\mu s$]</td>
<td>1.8</td>
<td>0.2</td>
<td>5.5</td>
</tr>
<tr>
<td>$\left(\frac{di}{dt}\right)_{max}$ [kA/ms]</td>
<td>5.5</td>
<td>12</td>
<td>12</td>
</tr>
</tbody>
</table>

Tab. 2.2. – Statistics of peak amplitude, time to crest and maximum front steepness for first ad subsequent negative return strokes. Adapted from Berger et al. [1975].

the measured currents from the mentioned ‘tower-effects’ [Guerrieri et al. 1994]. The problem of ‘decontaminating’ the lightning current is, however, beyond the scope of this thesis. In what follows we shall assume that the current distributions found in the literature [Berger et al., 1975; Anderson and Eriksson, 1980] are relevant to current measured at ‘ground level’ disregarding the presence of the elevated strike object which would require an appropriate reprocessing of the statistical data [Sabot, 1995].
2.2. – Lightning electromagnetic field appraisal

Expressions of the electromagnetic field radiated by a vertical dipole of length \( dz' \) at a height \( z' \) along the lightning channel, assumed as an antenna over a perfectly conducting plane, have been derived by Master and Uman [1983] by solving Maxwell's equations in terms of retarded scalar and vector potentials:

\[
\begin{align*}
\text{d}E_r (r, \phi, z, t) &= \frac{dz'}{4\pi\varepsilon_0} \left[ \frac{3r(z-z')}{{R}^5} \int_0^t i(z', \tau - R/c) \text{d}\tau + \frac{3r(z-z')}{{cR}^2} i(z', t - R/c) + \frac{r(z-z')}{{c^2R}^2} \frac{\partial i(z', t - R/c)}{\partial t} \right] \\
\end{align*}
\]

Tab. 2.3. – Statistics of peak amplitude, time to crest and maximum front steepness for first and subsequent negative return strokes. Adapted from Anderson and Eriksson [1980]
\[
\begin{align*}
\mathrm{d}E_z(r, \phi, z, t) &= \frac{dz'}{4\pi\varepsilon_0} \left[ \frac{2(z-z')^2 - r^2}{R^3} \right] i(z', \tau - R/c) \mathrm{d}\tau + \\
&+ \frac{2(z-z')^2 - r^2}{cR^3} i(z', t - R/c) + \\
&- \frac{r^2}{c^2R^3} \frac{\partial i(z', t - R/c)}{\partial \tau} \\
\end{align*}
\]
(2.6)

\[
\begin{align*}
\mathrm{d}B_y(r, \phi, z, t) &= \frac{\mu_0 dz'}{4\pi} \left[ \frac{r}{R} i(z', t - R/c) + \\
&- \frac{r}{cR^2} \frac{\partial i(z', t - R/c)}{\partial \tau} \right] \\
\end{align*}
\]
(2.7)

where

- \( i(z', t) \) is the current along the channel obtained from one of the return-stroke current models summarized above;
- \( c \) is the speed of light;
- \( R \) is the distance of the electric dipole from the observation point;
- \( r \) is the projection of \( R \) in the plane \( xy \) (see Fig. 2.1);

and the geometrical factors are as given in Fig. 2.1.

By integrating along the channel expressions (2.5)-(2.7), where the current distribution as a function of height and time is given by the return-stroke models discussed in previous paragraph, one obtains the electromagnetic exciting field.

For distances not exceeding a few kilometers, the perfect ground conductivity assumption is a reasonable approximation for the vertical component of the electric field and for the horizontal component of the magnetic field as shown by several authors [Djébari et al., 1981; Zeddam and Degauque, 1990; Rubinstein, 1996]. In fact, the contributions of the source dipole and of its image to these field components add constructively and, consequently, small variations in the image field due to the finite ground conductivity will have little effect on the total field.

On the other hand, the horizontal component of the electric field, is appreciably affected by a finite ground conductivity. Indeed, for such a field component, the effects of the two contributions subtract, and small changes in the image field may lead to appreciable changes in the total horizontal field. Although the intensity of the horizontal field component is generally much smaller than that of the vertical one, within the context of certain coupling models it plays an important role in the coupling mechanism [Master and Uman, 1984; Cooray and De la Rosa, 1986; Rubinstein et al., 1989; Diendorfer, 1990; Nucci et al. 1993; Ishi et al., 1994] and, hence, an accurate calculation method has to be chosen for it. Methods
for the calculation of the horizontal field using the exact Sommerfeld integrals are inefficient from the point of view of computer-time. Simplified expressions exist. One of the approach which appears now most promising is that proposed independently by Cooray and Rubinstein [Rubinstein 1991, 1996; Cooray 1992, 1994] and discussed by Wait [1997].

The Cooray-Rubinstein expression is given by

$$E_z(r, z, j\omega) = E_{zp}(r, z, j\omega) - H_{zr}(r, z = 0, j\omega) \cdot \frac{(1 + j)}{\sigma_s \delta}$$  \hspace{1cm} (2.8)

where

- $p$ is the subscript that indicates the fields calculated assuming a perfect ground;
- $\varepsilon_{rg}$ is the relative ground permittivity;
- $\sigma_s$ is the ground conductivity.
- $E_{zp}(r, z, j\omega)$ and $H_{zr}(r, 0, j\omega)$ are the Fourier-transforms of the horizontal component of the electric field at height $z$, and of the azimuthal component of the magnetic field at ground level respectively, both calculated assuming a perfect conducting ground;
- $\delta$ is the skin depth factor, $\sqrt{2 / \omega \mu_s \sigma_s}$;
- $\mu_s$ is the ground permittivity.

This approach has been shown to produce satisfactory approximation of the horizontal electric field for some significant cases: in particular, it reproduces the positive, bipolar and negative polarities of the field at close (one hundred meters), intermediate (some kilometers), and far (tens of kilometers) distances respectively, and at all these ranges it predicts results close to those predicted by more accurate expressions [Cooray, 1994; Rubinstein, 1996; Rachidi et al., 1996]. Motivated by the discussion by Wait [1997], Cooray [1998b] has proposed a modified, improved expression of (2.8).

### 2.3. – Transmission line coupling models

To solve the coupling problem, i.e. the determination of voltages and currents induced by an external field on a conducting system, use could be made of the antenna theory, the general and rigorous approach based on Maxwell’s equations [Tesche, 1992]. However, due to the length of distribution lines, the use of such theory for the calculation of lightning-induced overvoltages implies long computing times. In our case, the use of the simplest approach namely the quasi-static approximation [Johnk, 1975], according to which propagation is neglected and coupling between incident fields and the line conductors can be described by means of lumped elements (e.g. an inductance, or a capacitance), is not
appropriate. In fact, such an approach requires that the dimensions of the line conductors be smaller than about one tenth of the minimum wavelength of the electromagnetic field, an unacceptable assumption for the case of power lines illuminated by LEMP fields (above 1 MHz frequency, that is below 300 m wave length). Another possible approach is the transmission line theory \[Taylor et al., 1965\]. The basic assumptions of this approximation are that the response of the line is quasi-transverse electromagnetic (quasi-TEM) and that the transverse dimension of the line is smaller than the minimum significant wavelength. The line is represented by a series of elementary sections to which, by virtue of the above assumptions, the quasi-static approximation applies. Each section is illuminated progressively by the incident electromagnetic field so that longitudinal propagation effects are taken into account. The transmission line approximation appears to be the most promising approach for the problem of interest: indeed, in the power literature the most used coupling models are based on it, and in this thesis use will be made of it.

We shall now briefly resume the most popular coupling models based on the transmission line approximation. To do that, let us start considering a single-conductor overhead line parallel to the \(x\)-axis and contained in the \(xz\)-plane terminated on two resistances \(R_0\) and \(R_L\) (Fig. 2.5). It is important to observe that the incident external electromagnetic field \(E^e, B^e\) shown in Fig. 2.5 is the sum of the field radiated by the lightning stroke and of the ground-reflected field, both considered in absence of the wire. The total field \(E, B\) is given by the sum of the incident field \(E^e, B^e\) and the scattered field \(E_s, B_s\), which represents the reaction of the wire to the incident field. The incident electromagnetic field is related to the incident scalar potential \(\phi^i\) and to the incident vector potential \(A^e\) by the following expressions:

\[
E^e = -\left(\nabla \phi^e + \frac{\partial A^e}{\partial t}\right) \quad (2.9)
\]

\[
B^e = \nabla \times A^e \quad (2.10)
\]

![Fig. 2.5. – Geometry used for the calculation of overvoltages induced on an overhead power line by an indirect lightning return-stroke](image)
The Agrawal, Price and Gurbaxani model for a single-conductor line. The most adequate coupling model to describe the coupling between lightning return-stroke fields and overhead lines is nowadays considered the model by Agrawal et al. [1980], which is based on the transmission line approximation [Tesche et al., 1997]. It is indeed the model that allows in a straightforward way to take into account the ground resistivity in the coupling mechanism, and it is the only one that has been thoroughly tested and validated using experimental results, as will be discussed next.

The two transmission line coupling equations of the Agrawal model, expressed in the time domain, are [Agrawal et al., 1980; Tesche et al., 1997; Rachidi et al., 1999]:

\[
\frac{\partial v'(x,t)}{\partial x} + 2\pi \xi_g \ast \frac{\partial i(x,t)}{\partial t} + L' \frac{\partial i(x,t)}{\partial t} = E'_s(x,h,t)
\]

\[
\frac{\partial i(x,t)}{\partial x} + G'v'(x,t) + C' \frac{\partial v'(x,t)}{\partial t} = 0
\]

where
- \(E'_s(x,h,t)\) is the horizontal component of the incident electric field along the \(x\) axis at the conductor's height;
- \(i(x,t)\) is the current induced along the line;
- \(\xi_g'\) is the inverse Fourier transform of the ground impedance;
- \(\ast\) denotes the convolution product;
- \(L', C', G'\) are the line inductance (external), capacitance and conductance per-unit-length respectively.

In equation (2.11) \(L'\) is the external per-unit-length inductance calculated for a loss less wire above a perfectly conducting ground, \(\xi_g'\) is the inverse Fourier transform of the ground impedance. In particular \(\xi_g'\) is the inverse Fourier-transform of \(Z'_g \approx j\omega Z'_s\)

where \(Z'_s\) and \(Z'_g\) are the wire and the ground impedances respectively. Note, further, that within the frequency range of interest, the wire impedance can be neglected compared with the ground impedance [Ramo et al., 1984; Rachidi et al., 1996].

The ground impedance can be viewed as a correction factor to the line longitudinal impedance when the ground is not a perfect conductor and can be defined as Rachidi et al. [1996], Guerrieri [1997], Tesche et al. [1997], Cooray and Schuka [1998]:
Chapter 2. – Evaluation of lightning-induced voltages on transmission lines

\[ Z_g' = \frac{j \omega \int B_z^s(x, z) dx}{I} - j \omega L' \]  \hspace{1cm} (2.13)

In this thesis we shall make reference to the expression derived by Carson [1926], valid for the so-called ‘low frequency approximation’ \((\sigma_g \gg \omega \epsilon_o \epsilon_r)\), and for ground conductivities not lower than about 0.001 S/m [Rachidi et al. 1999] recently improved in [Rachidi et al., 2000]:

\[
\xi_g''(t) = \min \left\{ \frac{1}{2\pi h} \sqrt{\mu_0 \mu_r} \left[ \frac{1}{2 \sqrt{\pi}} \frac{\tau_g}{t} + \frac{1}{4} e^{-\tau_g^2/4} \text{erfc} \left( \sqrt{\frac{\tau_g}{t}} \right) - \frac{1}{4} \right] \right\} 
\]  \hspace{1cm} (2.14)

in which \(\epsilon_0\) and \(\epsilon_{rg}\) are the air and ground permittivity respectively, \(\mu_0\) is the air permeability, \(\tau_g = h^2 \mu_0 \sigma_g\) (where \(\sigma_g\) is the ground conductivity) and \(\text{erfc}\) is the complementary error function.

In (2.11) and (2.12) the wire impedance, the ground admittance and the line conductance have been assumed negligible (an approximation valid for typical overhead lines a few meters above ground, \(\sigma_g = 10^{-3}-10^{-2}\) S/m, \(\epsilon_r = 1-10\), Rachidi et al. [1996]).

Equations (2.11) and (2.12) are written in terms of scattered voltage \(v'(x, t)\) \(^{11}\). The total voltage \(v(x, t)\) is given by the sum of the scattered voltage \(v'(x, t)\) and the so-called incident voltage

\[
v' = -\int_0^h E_z^s(x, z, t) dz \approx -E_z^s(x, 0, t) \cdot h
\]  \hspace{1cm} (2.15)

namely,

\[
v(x, t) = v'(x, t) + v'(x, t)
\]  \hspace{1cm} (2.16)

The boundary conditions, written for the case of resistive terminations, are

\(^{11}\) The total field is the sum of the incident (or exciting) field and of the scattered field. The first is given by the field radiated by the lightning channel and by the ground-reflected field in absence of the line conductors; the second is given by the reaction of the overhead conductors to the incident field. To each of these fields one can associate a voltage, namely the incident voltage and the scattered one.
\[
\begin{align*}
 v'(0,t) &= -R_v \cdot i(0,t) - v'(0,t) \quad (2.17) \\
 v'(L,t) &= R_L \cdot i(L,t) - v'(L,t) \quad (2.18)
\end{align*}
\]

Eqs. (2.11)-(2.16) have the circuit representation of Fig. 2.6.

![Differential equivalent coupling circuit](image)

Fig. 2.6. – Differential equivalent coupling circuit according to the Agrawal et al. formulation for a lossless single-wire overhead line.

According to the Agrawal model, the forcing functions explicitly appearing in equations which produce the scattered voltage are: the horizontal component of the incident electric field along the line and the incident vertical electric field at the vertical line terminations. These forcing functions are represented by the voltage sources in Fig. 2.6.

An adequate coupling model for lightning-induced overvoltage calculations is a model that, given as inputs the lightning electromagnetic fields which illuminate a transmission line, predicts satisfactorily the line overvoltages induced by that field. Thus, to adequately test a coupling model, data sets of simultaneously measured fields and voltages are needed.

The Agrawal model has been first applied for the calculation of lightning induced overvoltages by Master and Uman [1984] and then employed by several other authors\(^\text{12}\). The Agrawal model is the only one that has been thoroughly tested versus experimental results in the sense above specified. Although some of the first tests did not provide satisfactory agreement between theory and measurements [Master and Uman, 1984; Master et al., 1984], in subsequent experiments the agreement was largely improved [Cooray and de la Rosa, 1986; Rubinstein et al. 1989]. Some tests of the Agrawal model have been performed making use of Nuclear Electromagnetic Pulse (NEMP) simulators [Master et al., 1984; Rubinstein et al. 1989] and results have shown a reasonably good agreement between measurements and theory. It is worth mentioning that one of the aim of this thesis is the extension of the Agrawal coupling model to the case of a line with transverse discontinuities.

\(^{12}\) [Cooray and De la Rosa, 1986; Rubinstein et al., 1989; Diendorfer, 1990; Iorio et al., 1993; Nucci et al., 1993,2000; Ishii et al., 1994].
(see paragraphs 2.4. and 3.1.2.), and that the experimental validation of such a model extension was also successfully accomplished by means of a NEMP simulator.

The Agrawal model has been shown to be completely equivalent to two other models in which the forcing functions are in terms of different components of the electromagnetic field, namely the model by Taylor et al. [1965], and the model by Rachidi [1993]. Indeed, these last three models (Agrawal et al., Taylor et al., and Rachidi) are different though equivalent formulations of the same coupling equations set in terms of different combinations of the various components of the electromagnetic field. This means that it is misleading to speak of the contribution of a given component of the electromagnetic field to the induced overvoltage without specifying the formulation of the coupling model one is using [Nucci and Rachidi, 1995]. Additionally, it has been shown that for the case of a lightning channel perpendicular to a perfectly conductive ground plane the model proposed by Rusck [1958] provides the same results as the Agrawal one. Although for different channel geometry the Rusck model is expected to predict results that might be affected by some inaccuracy, it is still very popular among power engineers, and for this reason we briefly summarize it below.

**The Rusck model.** To derive the transmission-line coupling equations of this model, Rusck [1958] started from the expression relating the total electric field on the conductor surface to the scalar and vector potentials. The coupling equations obtained by using the above procedure for a single-conductor line above perfect conducting ground are:

\[
\frac{\partial \phi(x,t)}{\partial x} + L' \frac{\partial h(x,t)}{\partial t} = 0
\]

\[
\frac{\partial h(x,t)}{\partial x} + C' \frac{\partial}{\partial t} [\phi(x,t) - \phi'(x,t)] = 0
\]

where

- $L'$ and $C'$ are the line inductance and the line capacitance per unit length respectively;
- $i(x,t)$ is the total line current;
- $\phi$ is the total induced scalar potential.

The total induced voltage $v(x,t)$ is given by the following expression

\[
v(x,t) = \phi(x,t) + \int_0^h \frac{\partial A^e_z(x,z,t)}{\partial t} \, dz
\]
where $h$ is the conductor’s height and $A_z^e$ is the vertical component of the incident vector potential. The boundary conditions for (2.19) and (2.20) are [Cooray, 1994]:

$$\phi(0,t) = -R_0 i(0,t) - \int_0^h \frac{\partial A_z^e(0,z,t)}{\partial t} dz$$

(2.22)

$$\phi(L,t) = R_L i(L,t) - \int_0^h \frac{\partial A_z^e(L,z,t)}{\partial t} dz$$

(2.23)

In the Rusck model the forcing functions that explicitly appear in the coupling equations are the incident scalar potential along the line and the vertical component of the incident vector potential at the line terminations.

Yokoyama et al. [1984, 1989] used the Rusck model to compute induced voltages starting from a measured lightning current. The field was calculated from the measured current assuming a simple return-stroke current model and a satisfactory agreement was found between measurements and calculations. Also Eriksson et al. [1982] obtained a satisfactory agreement between theory and calculations. The field they input to the Rusck model was not the one that produced the measured induced voltage but was calculated using a simple return-stroke current model starting from a typical lightning channel-base current: thus their comparison has more a qualitative than a quantitative nature.

It is worth mentioning that Rusck proposed a simplified formula that provides a first estimation of the peak value $V_{\text{max}}$ of the overvoltages induced on an infinite long line starting from the peak value $I_p$ of the lightning current, the velocity of the return-stroke wavefront and the height of the line:

$$V_{\text{max}} = Z_0 \frac{I_p \cdot h}{d} \left(1 + \frac{1}{\sqrt{2}} \frac{v}{c} \frac{1}{\sqrt{1 - 0.5 \cdot (v/c)^2}} \right)$$

(2.24)

where
- $I_p$ is the lightning peak current;
- $d$ is the distance of the stroke location from the line (in m);
- $Z_0 = \frac{1}{4\pi} \sqrt{\frac{\mu_0}{\varepsilon_0}} = 30 \, \Omega$;
- $h$ is the line height (in m);
- $v$ is the return stroke wave front velocity;
- $c$ is the speed of the light.
Discussion of the simplified Rusck formula. As mentioned earlier, the Rusck simplified analytical formula (2.24) gives the maximum value $V_{\text{max}}$ of the induced overvoltages on an infinitely long line at the nearest point to the stroke location. Figures 2.7a, b and c show the variation of the maximum amplitude of the voltage induced at the point closest to the stroke location for three different lightning current wave shapes (see Tab. 2.4) and for the different values of ground conductivity, namely infinite, 0.01 and 0.001 S/m calculated adopting the MTLE return stroke model and the Agrawal coupling model. Each case is compared with the prediction of the Rusck formula.

<table>
<thead>
<tr>
<th>Current types</th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_p$ [kA]</td>
<td>12</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>$(di/dt)_{\text{max}}$ [kA/µs]</td>
<td>12</td>
<td>40</td>
<td>120</td>
</tr>
</tbody>
</table>

Tab. 2.4a. – Subsequent return-stroke current peak values and maximum time derivatives of the adopted currents.

<table>
<thead>
<tr>
<th>Current types</th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{01}$ [kA]</td>
<td>10,7</td>
<td>10,7</td>
<td>7,4</td>
</tr>
<tr>
<td>$\tau_{11}$ [µs]</td>
<td>0,95</td>
<td>0,25</td>
<td>0,063</td>
</tr>
<tr>
<td>$\tau_{12}$ [µs]</td>
<td>4,7</td>
<td>2,5</td>
<td>0,5</td>
</tr>
<tr>
<td>$I_{02}$ [kA]</td>
<td>6,5</td>
<td>6,5</td>
<td>9</td>
</tr>
<tr>
<td>$\tau_{21}$ [µs]</td>
<td>4,6</td>
<td>2,1</td>
<td>0,27</td>
</tr>
<tr>
<td>$\tau_{22}$ [µs]</td>
<td>900</td>
<td>230</td>
<td>66</td>
</tr>
<tr>
<td>$n$</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

Tab. 2.4b. – Parameters of the Heidler functions reproducing the adopted currents. The channel base current is approximated by the sum of two Heidler functions.

By observing Fig. 2.7 we can conclude that, in general, when the channel base current exhibits a steep front (current A3) and when the ground is approximated as a perfectly conducting plane, the Rusck simplified analytical expression provides an estimation for the induced voltages close to the one predicted by the more general Agrawal model in which the forcing functions are calculated by means of the MTLE model. The disagreement between the voltage predicted by the two approaches (Rusck formula and Agrawal-MTLE models) increases with the return-stroke velocity, with the ground resistivity, and with the front duration of the lightning current. As a conclusion, in general, the Rusck simplified formula should not be applied to the case of overhead lines above a lossy ground.
Fig. 2.7. – Variation of the induced voltage magnitude at the line center as a function of distance to the stroke location. Left column: return-stroke velocity equal $1.3 \times 10^8$ m/s; right column: return-stroke velocity equal $1.9 \times 10^8$ m/s. a) lightning current $A_1$, b) lightning current $A_2$, c) lightning current $A_3$. In solid line we have reported the results obtained from the Rusck simplified analytical expression. Adapted from Borghetti et al. [2000a].
The Agrawal, Price and Gurbaxani model for a multi-conductor line. The Agrawal model for the case of a multi-conductor line above a lossy ground reads [Tesche et al., 1997, Rachidi et al., 1997]:

\[
\frac{d}{dx}[\psi'(x,t)] + \left[ L'_{ij} \right] \frac{d}{dt} [i_j(x,t)] + \left[ \xi_{gij} \right] \otimes \frac{d}{dt} [i_j(x,t)] = \left[ E'^e_x(x,h_i,t) \right] \tag{2.25}
\]

\[
\frac{d}{dx} [i_j(x,t)] + \left[ C'_{ij} \right] \frac{d}{dt} [\psi'(x,t)] = 0 \tag{2.26}
\]

where
- \( E'^e_x(x,h_i,t) \) is the horizontal component of the incident electric field along the x axis at the conductor's height;
- \( \xi_{gij} \) is the matrix of transient ground resistance;
- \( L'_{ij} \) and \( C'_{ij} \) are respectively the external inductance and the capacitance matrices per unit length of the line;
- \([i_j(x,t)]\) is the current vector;
- \( \otimes \) denotes the convolution product;
- \([\psi'(x,t)]\) is the scattered voltage vector, related to the \([v_i(x,t)]\), the total voltage vector, by the following expression:

\[
[v_i(x,t)] = [\psi'_i(x,t)] + [v'_i(x,t)] = [\psi'_i(x,t)] - \int_0^h E^e_z(x,z,t)dz \tag{2.27}
\]

The boundary conditions for the scattered voltage at both line ends are:

\[
[v'_i(0,t)] = -[R_0] [i_j(0,t)] - [v'_i(0,t)] \tag{2.28}
\]

\[
[v'_i(L,t)] = [R_L] [i_j(L,t)] - [v'_i(L,t)] \tag{2.29}
\]
Chapter 3. – LEMP-to transmission-line coupling models for distribution systems and their implementation in a transient program

The evaluation of the LEMP response of distribution networks is crucial for the assessment of outages on customers and distribution utilities caused by lightning. For this purpose the availability of a model of a LEMP illuminated line having realistic geometrical configuration is of particular interest. This chapter presents the developed complex overhead line model illuminated by a LEMP, its implementation, and the relevant distribution system model developed.

The models here proposed are included in the LIOV (lightning induced overvoltage) code. The LIOV code has been developed in the framework of an international collaboration involving the University of Bologna (Department of Electrical Engineering), the Swiss Federal Institute of Technology (Power Systems Laboratory), and the University of Rome “La Sapienza” (Department of Electrical Engineering). The LIOV code is based on the field-to-transmission line coupling formulation of Agrawal et al. [1980], suitably adapted for the case of an overhead line illuminated by an indirect lightning electromagnetic field; the return stroke electromagnetic field is calculated by assuming the MTLE engineering model and using the Cooray-Rubinstein formula for the case of lossy grounds [Nucci et al., 1993; Rachidi et al., 1995]. It allows for the calculation of lightning-induced voltages along an overhead line as a function of current wave shape (amplitude, front steepness, duration), return stroke velocity, line geometry (height, length, number and position of conductors), stroke location with respect to the line, ground resistivity and relative permittivity, and value of termination impedances.

The models here proposed are aimed at improving the LIOV code in order to permit for the treatment of more realistic overhead line and distribution systems.

3.1. – Lines with transverse discontinuities

Concerning medium voltage overhead lines the main protective measures against lightning induced overvoltages can be identified as follow: I) use of shielding wires and II) use of surge arresters.

Regarding the first protective measure, several authors have addressed theoretically the issue by assuming the shielding wire at zero potential at any point of it and at any time [Rusck, 1958; Chowdhuri, 1969,1990; Yokoyama et al., 1983,1986], an assumption that appears reasonable only when shielding wire is grounded at short intervals along the line. Further, such an approach does not allow to find the ‘optimal’ distance between two
consecutives groundings needed to accomplish the required shielding effect. In [Yokoyama 1984] an improved approach was used in which the shielding wire was considered as one of the conductors of the multi-conductor line. The coupling model adopted is the same proposed by [Rusck 1958] which has been shown to apply for the case of a lightning channel perpendicular to the ground plane.

3.1.1. Proposed model

Line transverse discontinuities are represented by the connections between the line conductors and the ground. These connections are constituted by the groundings of the shielding wire and by surge arresters connected to the phase conductors. Then the elements that represent these discontinuities could be a simple linear resistance or a non-linear component.

In this thesis, we extend the model already presented in [Rachidi et al. 1997,1999] in which the coupling between the LEMP and the multi-conductor transmission line is described by the more general coupling model by Agrawal et al. and in which the shielding wire was treated, similarly to [Yokoyama, 1984], as one of the conductors of the multi-conductor line. In particular, while in [Rachidi et al., 1997,1999] the shielding wire was grounded only at the line terminations, we here propose a model modification with allows for the treatment of multiple groundings along the line. The grounding resistance is also included in our extended model. Comparisons with experimental data obtained by using a reduced scale line model illuminated by an EMP simulator and, also, with those obtained by other authors [Yokoyama, 1984], are presented.

The above-mentioned model has also be extended in order to deal with the presence of surge arresters along the line. We shall essentially assess the effect of both shielding wires and surge arresters on voltages induced by a nearby cloud-to-ground lightning.

According to the Agrawal transmission line coupling equations (2.25) and (2.26) the scattered voltage, at node \( g \) of the conductor \( i \) at which a given impedance is connected to ground, can be expressed as follows (see Fig. 3.1):

\[
v_i^s(x,t) = \Gamma \left( i_g^s(t) \right) + \int_0^h E_z^s(x,z,t)dz
\]

(3.1)

where \( \Gamma \) is an integro-differential operator which describes the voltage drop across the impedance as function of current \( i_g \) (\( \Gamma= R_g i_g \) for the simple case of a resistance). Since the Agrawal model is expressed in terms of the scattered voltage, it is necessary to include a voltage source in series with the impedance, the so-called incident voltage, which is given
by the integral from the ground level to the line conductor of the incident vertical electric field (see Fig. 3.1).

\[
\int_{0}^{E(x,z,t)} dz
\]

Fig. 3.1. – Insertion of discontinuity point in a generic point along a multi-conductor line.

The developed extended model was tested versus experimental data obtained by means of a reduced line model illuminated by an EMP simulator. Besides, additional validation was performed by testing it with the theoretical results published by Yokoyama [1984]. The entire experimental validation results are presented in the paragraph 3.1.3.

3.1.2. – Numerical implementation

Most studies on lightning-induced voltages on overhead power lines use a direct time domain analysis because of its straightforwardness in dealing with insulation coordination problems, and its ability to handle non-linearities, which arise in presence of protective devices such as surge arresters or corona phenomenon.

One of the most popular approaches to solve the transmission line coupling equations in time domain is the finite difference time domain (FDTD) technique (e.g. [Taflove, 1995]). Such technique was used indeed by Agrawal et al. in [1980] when presenting their field-to-transmission line coupling equations. In the above publications, partial time and space derivatives were approximated using the 1st order FDTD scheme.

We have proposed, instead, the use of a second order finite difference scheme, it is based on the Lax-Wendroff algorithm [Lax and Wendroff, 1960]. In [Omick Castillo, 1993] this algorithm is applied to the classical transmission line equations excited by lumped excitation sources. We here propose an extension of such an integration scheme to take into account distributed sources due to the action of an external electromagnetic field, according to the Agrawal et al. coupling model.

We shall first deal with the case of a single conductor line above an ideal ground with no discontinuities. Then we shall propose an extension of the Agrawal model to deal with the
case of a lossy ground, and eventually we shall present the algorithm relevant to a line with transverse periodical discontinuities. In Appendix A.1 we report also the treatment of line transverse discontinuities concerning the FDTD 1st order scheme.

**Case of a single-conductor line above an ideal ground.** For convenience we write below the Agrawal coupling equations (2.11) and (2.12) which, for the case of interest, assume the following form:

\[
\frac{\partial i(x,t)}{\partial x} + L \frac{\partial e(x,t)}{\partial t} = E^e_s(x,h,t) \tag{3.2}
\]

\[
\frac{\partial e(x,t)}{\partial x} + C \frac{\partial i(x,t)}{\partial t} = 0 \tag{3.3}
\]

The meaning of the symbols that appear in (3.2) and (3.3) is the same than in equations (2.11) and (2.12).

If we differentiate with respect to the \(x\) and \(t\) variables, the system of equations (3.2) and (3.3) can be rewritten as

\[
\frac{\partial^2 i(x,t)}{\partial x^2} - L' C \frac{\partial^2 i(x,t)}{\partial t^2} = -C \frac{\partial E^e_s(x,h,t)}{\partial t} \tag{3.4}
\]

\[
\frac{\partial^2 e(x,t)}{\partial x^2} - L' C \frac{\partial^2 e(x,t)}{\partial t^2} = \frac{\partial E^e_s(x,h,t)}{\partial x} \tag{3.5}
\]

Expanding the line current and the scattered voltage using Taylor’s series applied to the time variable, and truncating after the second order term yields

\[
v^s(x,t) = v^s(x,t_0) + \frac{\partial v^s(x,t)}{\partial t} \Delta t + \frac{\partial^2 v^s(x,t)}{\partial t^2} \frac{\Delta t^2}{2} + O(\Delta t^3) \tag{3.6}
\]

\[
i(x,t) = i(x,t_0) + \frac{\partial i(x,t)}{\partial t} \Delta t + \frac{\partial^2 i(x,t)}{\partial t^2} \frac{\Delta t^2}{2} + O(\Delta t^3) \tag{3.7}
\]

where \(O(\Delta t^3)\) is the remainder term, which approaches zero as the third power of the temporal increment.

If we substitute the time derivatives in (3.6) and (3.7) with the corresponding expressions using equations (3.2), (3.3), (3.4), and (3.5), we obtain the following second order differential equation...
\[ v^s(x,t) = v^s(x,t_0) - \frac{\Delta t}{C'} \frac{\partial i(x,t)}{\partial x} - \frac{\Delta t^2}{2L'C'} \left( \frac{\partial E^s_x(x,h,t)}{\partial x} - \frac{\partial^2 v^s(x,t)}{\partial x^2} \right) + O(\Delta t^3) \quad (3.8) \]

\[ i(x,t) = i(x,t_0) - \frac{\Delta t}{L'} \left( \frac{\partial v^s(x,t)}{\partial x} - E^s_x(x,h,t) \right) + \frac{\Delta t^2}{2L'C'} \left( \frac{\partial^2 i(x,t)}{\partial x^2} + C' \frac{\partial E^s_x(x,h,t)}{\partial t} \right) + O(\Delta t^3) \quad (3.9) \]

In order to represent equations (3.8) and (3.9) using an FDTD technique, we will proceed with the discretization of time and space as follows

\[ v^s(x,t) = v^s(k\Delta x, n\Delta t) = v^n_k \quad (3.10) \]

\[ i(x,t) = i(k\Delta x, n\Delta t) = i^n_k \quad (3.11) \]

\[ E^s_x(x,h,t) = E^s_x(k\Delta x, h, n\Delta t) = E^n_h_k \quad (3.12) \]

where:
- \( \Delta x \): spatial integration step;
- \( \Delta t \): time integration step;
- \( k = 1,2,\ldots,k_{\text{max}} \);
- \( n = 1,2,\ldots,n_{\text{max}} \).

In the integration scheme, the scattered voltage and current at time step \( n \) are known for all spatial nodes. Therefore equations (3.8) and (3.9) allow us to compute the scattered voltage and current at the time step \( n+1 \).

The spatial derivatives of the scattered voltage, line current, and horizontal electric field can be written respectively as

\[ \frac{\partial v^s(x,t)}{\partial x} \bigg|_{i=n\Delta t} = \frac{v^n_{k+1} - v^n_{k-1}}{2\Delta x} + O(\Delta x) \quad (3.13) \]

\[ \frac{\partial i(x,t)}{\partial x} \bigg|_{i=n\Delta t} = \frac{i^n_{k+1} - i^n_{k-1}}{2\Delta x} + O(\Delta x) \quad (3.14) \]

\[ \frac{\partial E^s_x(x,h,t)}{\partial x} \bigg|_{i=n\Delta t} = \frac{E^n_{h,k+1} - E^n_{h,k-1}}{2\Delta x} + O(\Delta x) \quad (3.15) \]

On the other hand, the time derivative of the horizontal electric field reads
The second order spatial derivatives can be written as

$$\frac{\partial^2 v(x,t)}{\partial x^2} \bigg|_{t=n\Delta t} = \frac{v_{x+1}^n + v_{x-1}^n - 2v_x^n}{\Delta x^2} + O(\Delta x) \quad (3.17)$$

$$\frac{\partial^2 i(x,t)}{\partial x^2} \bigg|_{t=n\Delta t} = \frac{i_{x+1}^n + i_{x-1}^n - 2i_x^n}{\Delta x^2} + O(\Delta x) \quad (3.18)$$

Inserting equations (3.10)-(3.16) into (3.8) and (3.9), we obtain the 1st order FDTD scheme. If we insert equations (3.10)-(3.18) into (3.8) and (3.9), we obtain the following 2nd order FDTD scheme:

$$v_k^{n+1} = v_k^n - \frac{\Delta t}{C} \left( \frac{i_{k+1}^n - i_{k-1}^n}{2\Delta x} \right) - \frac{\Delta t^2}{2L' C} \left( \frac{Eh_x^+ - Eh_x^-}{\Delta x} \right) \left( \frac{v_{k+1}^n + v_{k}^n - 2v_{k-1}^n}{\Delta x^2} \right) \quad (3.19)$$

$$i_k^{n+1} = i_k^n - \frac{\Delta t}{L} \left( \frac{v_{k+1}^n - v_{k-1}^n - Eh_k^n}{2\Delta x} \right) + \frac{\Delta t^2}{2L' C} \left( \frac{i_{k+1}^n + i_{k-1}^n - 2i_k^n}{\Delta x^2} \right) + C' \left( \frac{Eh_x^{n+1} - Eh_x^{n-1}}{2\Delta t} \right) \quad (3.20)$$

It is worth noting that, compared to the 1st order point centered scheme adopted in [Agrawal et al., 1980; Rachidi et al., 1996], the spatial-temporal grid distribution for current and voltage nodes in the 2nd order scheme is quite different. Fig. 3.2 shows a schematic representation of the spatial-temporal grid for both integration schemes.
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Fig. 3.2. – FDTD 1st and 2nd order integration schemes applied to the case of a single-conductor lossless overhead line above a perfectly conducting ground illuminated by an external electromagnetic field. Left column: time and spatial discretization; right column: schematic representation of the spatial discretization along the line.

The boundary conditions for resistive terminations, can be expressed as follows:

\[ v_0^n = -R_0 i_0^n + \int_0^h E^e_x(0,0,t)dz \]  \hspace{1cm} (3.21)

\[ v_{k\text{ max}}^n = R_L i_{k\text{ max}}^n + \int_0^h E^e_x(L,0,t)dz \]  \hspace{1cm} (3.22)

Case of a single-conductor line above a lossy ground. We now extend the FDTD 2nd order integration scheme previously presented in order to take into account the presence of an uniform lossy ground, characterized by its conductivity \( \sigma_g \) and its relative permittivity \( \varepsilon_{rg} \). In this case, the Agrawal et al. coupling equations become [Rachidi et al., 1996]:

\[ \frac{\partial v^n(x,t)}{\partial x} + L \frac{\partial i(x,t)}{\partial t} + \int_0^t \varepsilon_g(x,t-\tau) \frac{\partial i(x,\tau)}{\partial \tau} d\tau = E^e_x(x,h,t) \]  \hspace{1cm} (3.23)
in which $\xi'_g(t)$ is the transient ground resistance [Rachidi et al., 1999], which can be evaluated using the analytical expression (2.14) [Rachidi et al., 2000].

In equations (3.23) and (3.24), the contributions from the wire impedance and the ground admittance, which have shown to be negligible for typical power lines [Rachidi et al., 1996], are deliberately disregarded. Following a procedure similar to the case of a lossless line, equations (3.23) and (3.24) become

\[
\frac{\partial^2 i(x,t)}{\partial x^2} - L'C_n \frac{\partial^2 i(x,t)}{\partial t^2} - C_n \frac{\partial v'_g(x,t)}{\partial t} = -C_n \frac{\partial E'_s(x,h,t)}{\partial x} \tag{3.25}
\]

\[
\frac{\partial^2 v'(x,t)}{\partial x^2} = L'C_n \frac{\partial^2 v'(x,t)}{\partial t^2} + \frac{\partial v'_g(x,t)}{\partial t} = \frac{\partial E'_s(x,h,t)}{\partial x} \tag{3.26}
\]

Expanding the current and the scattered voltage using Taylor’s series, and replacing the time derivative of the current and of the scattered voltage in equations (3.25) and (3.26), we obtain the following second order differential equation:

\[
v'_s(x,t) = \int_0^t \xi'_g(t - \tau) \frac{\partial i(\tau)}{\partial \tau} d\tau \tag{3.27}
\]

\[
v'(x,t) = v'(x,t_0) - \frac{\Delta t}{C'} \frac{\partial i(x,t)}{\partial x} + \frac{\Delta t^2}{2L'C'} \left( \frac{\partial E'_s(x,h,t)}{\partial x} - \frac{\partial^2 v'(x,t)}{\partial x^2} - \frac{\partial v'_g(x,t)}{\partial x} \right) + O(\Delta t^3) \tag{3.28}
\]

\[
i(x,t) = i(x,t_0) - \frac{\Delta t}{L'} \left( \frac{\partial v'(x,t)}{\partial x} - E'_s(x,h,t) + v'_g(x,t) \right) + \frac{\Delta t^2}{2L'C'} \left( \frac{\partial^2 i(x,t)}{\partial x^2} + C' E'_s(x,h,t) - C' \frac{\partial v'_g(x,t)}{\partial t} \right) + O(\Delta t^3) \tag{3.29}
\]

Following a procedure similar as that for the ideal ground case, we obtain the 2nd order FDTD discretized equations:
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\[ v_{k+1}^{n+1} = v_k^n - \frac{\Delta t}{C^l} \left( i_{k+1}^n - i_{k-1}^n \right) - \frac{\Delta t^2}{2 L^c C^l} \left( E_{k+1}^n - E_{k-1}^n \right) - \frac{v_{k+1}^n + v_{k-1}^n - 2v_k^n}{2\Delta x^2} \]  

\[ + \frac{\Delta t^2}{2 L^c C^l} \left( v_{g_{k+1}}^n - v_{g_{k-1}}^n \right) \]  

\[ i_{k+1}^{n+1} = i_k^n - \frac{\Delta t}{L^c} \left( v_{k+1}^n - v_{k-1}^n - E_{k+1}^n + E_{k-1}^n \right) + \]  

\[ + \frac{\Delta t^2}{2 L^c C^l} \left( i_{k+1}^n + i_{k-1}^n - 2i_k^n + E_{k+1}^{n+1} - E_{k-1}^{n+1} \right) - \frac{\Delta t^2}{2 L^c} \left( v_{g_{k+1}}^n - v_{g_{k-1}}^n \right) \]  

Extension to the case of a multi-conductor line. The Agrawal et al. field-to-transmission line coupling equations for a multi-conductor line above a lossy ground, reported in paragraph 2.3 (equations (2.25) and (2.26)), can be numerically integrated by means of the same FDTD 2nd order scheme used for the single-conductor case, obtaining the following second-order differential equation:

\[ [v'_j(x,t)] = [v_j(x,t_0)] - \Delta t[C_j]\frac{\partial[i_j(x,t)]}{\partial t} + \]  

\[- \frac{\Delta t^2}{2} [L_j][C_j]^{-1}\left( \frac{\partial^2 [E_j^e(x,h,t)]}{\partial x^2} - \frac{\partial^2 [v'_j(x,t)]}{\partial x^2} \right) + \]  

\[ + \frac{\Delta t^2}{2} [L_j][C_j]^{-1}\left( \frac{\partial^2 [v'_j(x,t)]}{\partial x^2} \right) \]  

\[ [i_j(x,t)] = [i_j(x,t_0)] - \Delta t[L_j]^{-1}\left( \frac{\partial[v'_j(x,t)]}{\partial x} - [E_j^e(x,h,t)] + [v'_g_j(x,t)] \right) + \]  

\[ + \frac{\Delta t^2}{2} [C_j][L_j]^{-1}\left( \frac{\partial^2 [i_j(x,t)]}{\partial x^2} + [C_j]\frac{\partial[E_j^e(x,h,t)]}{\partial t} \right) + \]  

\[- \frac{\Delta t^2}{2} [C_j][L_j]^{-1}\left( [C_j]\frac{\partial[v'_g_j(x,t)]}{\partial t} \right) \]  

in which

\[ [v'_g_j(x,t)] = [E_j^g]\otimes\frac{\partial}{\partial t}[i_j(x,t)] \]  

And finally, the 2nd order FDTD representation of (3.32) and (3.33) reads
\[
\left[ v_{i} \right]_{k}^{n+1} = \left[ v_{i} \right]_{k}^{n} - \Delta t \left[ C_{y} \right]^{-1} \left( \frac{\left[ i_{y} \right]_{k+1}^{n} - \left[ i_{y} \right]_{k-1}^{n}}{2\Delta x} \right) + \\
- \frac{\Delta t^{2}}{2} \left[ L_{y} \right] \left[ C_{y} \right]^{-1} \left( \frac{\left[ E_{h} \right]_{k+1}^{n} - \left[ E_{h} \right]_{k-1}^{n}}{2\Delta x} - \frac{\left[ v_{i} \right]_{k+1}^{n} + \left[ v_{i} \right]_{k-1}^{n} - 2\left[ v_{i} \right]_{k}^{n}}{\Delta x^{2}} \right) + \\
+ \frac{\Delta t^{2}}{2} \left[ L_{y} \right] \left[ C_{y} \right]^{-1} \left( \frac{\left[ v'_{y} \right]_{k+1}^{n} - \left[ v'_{y} \right]_{k-1}^{n}}{2\Delta x} \right)
\]

\[
\left[ i_{k} \right]_{k}^{n+1} = \left[ i_{k} \right]_{k}^{n} - \Delta t \left[ L_{y} \right]^{-1} \left( \frac{\left[ v_{i} \right]_{k+1}^{n} - \left[ v_{i} \right]_{k-1}^{n}}{2\Delta x} \right) + \\
+ \frac{\Delta t^{2}}{2} \left[ C_{y} \right] \left[ L_{y} \right]^{-1} \left( \frac{\left[ i_{y} \right]_{k+1}^{n} + \left[ i_{y} \right]_{k-1}^{n} - 2\left[ i_{y} \right]_{k}^{n}}{\Delta x^{2}} \right) + \\
+ \frac{\Delta t^{2}}{2} \left[ C_{y} \right] \left[ L_{y} \right]^{-1} \left( \frac{\left[ E_{h} \right]_{k+1}^{n} - \left[ E_{h} \right]_{k-1}^{n}}{2\Delta t} \right) \\
- \frac{\Delta t^{2}}{2} \left[ C_{y} \right] \left[ L_{y} \right]^{-1} \left( \frac{\left[ v'_{y} \right]_{k+1}^{n} - \left[ v'_{y} \right]_{k-1}^{n}}{\Delta t} \right)
\]
\]

\[(3.35)\]

\[(3.36)\]

**Treatment of line transverse discontinuities in FDTD 2nd order scheme for a single-conductor line.** In the proposed FDTD 2nd order scheme, the voltages and currents nodes are coincident, which allows simplifying the equations for the treatment of the periodical groundings. The treatment of a shunt impedance representing one of the conductor groundings for an overhead line illuminated by an external electromagnetic field is schematically illustrated in Fig. 3.3.

---

**Fig. 3.3.** – Insertion of discontinuity point, in a generic point along a single-conductor line, in the 2nd order Finite Difference integration scheme.
Applying equation (3.1) the node voltage $v_{k}^{n+1}$ (see Fig. 3.3) can be expressed as follows:

$$v_{k}^{n+1} = \Gamma (i_{g}^{n+1}) + \int_{0}^{h} E_{z}(x,z,t)dz$$  \hspace{1cm} (3.37)

where

- $i_{g}^{n+1}$ is the current flowing in the grounding impedance;
- $\Gamma$ is an integral-differential operator, which describes the voltage drop across the shunt impedance as a function of current $i_{g}$ (e.g., $\Gamma = R_{g} \cdot i_{g}$ for the simple case of a resistance).

As for the FDTD 1st order scheme (see appendix A.1) current $i_{g}^{n+1}$ can be expressed as function of the current $i_{k}^{n+1}$, $i_{k}^{n-1}$ applying Kirchhoff’s law on the currents at the grounding node

$$i_{g}^{n+1} = i_{k}^{n+1} - i_{k}^{n-1}$$  \hspace{1cm} (3.38)

Currents $i_{k}^{n+1}$, $i_{k}^{n-1}$ can be expressed as a function of the adjacent current nodes assuming the following linear spatial interpolation:

$$i_{k}^{n+1} = 2i_{k-1}^{n+1} - i_{k-2}^{n+1}$$  \hspace{1cm} (3.39)
$$i_{k}^{n-1} = 2i_{k+1}^{n+1} - i_{k+2}^{n+1}$$  \hspace{1cm} (3.40)

By introducing (3.38) (3.39) and (3.40) in (3.37) we obtain the equation for the scattered voltage at the grounding point:

$$v_{k}^{n+1} = \Gamma (2i_{k-1}^{n+1} - i_{k-2}^{n+1} - 2i_{k+1}^{n+1} + i_{k+2}^{n+1}) + \int_{0}^{h} E_{z}(x,z,t)dz$$  \hspace{1cm} (3.41)

The node current $i_{k}^{n}$, needed in equations (3.19) (3.20) to compute voltages and currents at nodes $k-1$ and $k+1$, must be substituted with (3.39) and (3.40) respectively with equations written for the node $k-1$ and $k+1$.

_Treatment of line transverse discontinuities in FDTD 2nd order scheme for a multi-conductor line._ The treatment of periodical groundings in the FDTD 2nd order integration scheme for a multi-conductor line, is a simple extension in terms of current and voltage
vector of equations (3.37)-(3.41). In particular the equation for the scattered voltage at the
grounding of the conductor point reads:

\[
[v_i]_{k+1} = [\Gamma K] \left[ 2[i_{i_{k+1}}]_{k+1} - [i_{i_{k-2}}]_{k-2} - 2[i_{i_{k-1}}]_{k-1} + [i_{i_{k+2}}]_{k+2} \right] + \int_0^h E_z(x,z,t)dz
\]  

(3.42)

Equations (3.39) and (3.40) become:

\[
[i_{i_{k+1}}]_{k+1} = 2[i_{i_{k+1}}]_{k+1} - [i_{i_{k-2}}]_{k-2} \quad (3.43)
\]

\[
[i_{i_{k+1}}]_{k+1} = 2[i_{i_{k+1}}]_{k+1} - [i_{i_{k+2}}]_{k+2} \quad (3.44)
\]

where

\[
[i_{i_k}]_n = \begin{pmatrix}
  i_{1k}^n \\
  i_{2k}^n \\
  \vdots \\
  i_{N_{iK}}^n
\end{pmatrix}, \quad [v_{i_k}]_n = \begin{pmatrix}
  v_{1k}^n \\
  v_{2k}^n \\
  \vdots \\
  v_{N_{iK}}^n
\end{pmatrix}
\]

\[
\begin{pmatrix}
  [\int_0^h E_z(x,z,t)dz] \\
  0 \\
  \vdots \\
  0
\end{pmatrix}
\]

(3.46)

and matrix \([\Gamma K]\):

\[
[\Gamma K] = \begin{pmatrix}
  \Gamma K_{11} & 0 & 0 & 0 \\
  0 & \Gamma K_{22} & 0 & 0 \\
  0 & 0 & \Gamma K_{33} & 0 \\
  0 & 0 & 0 & \Gamma K_{N_{iK}N_{iK}}
\end{pmatrix}
\]  

(3.47)
3.1.3. – Comparison with experimental results and simulations

The validation of the Agrawal coupling model modified for the treatment of periodical groundings was performed by means of the experimental results obtained on reduced scale line models illuminated by an EMP simulator. The reported experimental data was obtained from two different sessions performed at the Swiss Federal Institute of Technology in Lausanne on the EMP simulator of their laboratory described in more detail in [Arreghini et al., 1993]. The simulator is a bounded wave vertically-polarized type. A measurement record of the waveform of the electric vertical field inside the working volume, performed in absence of the line during the experimental campaign, is presented in Fig. 3.4.

![Fig. 3.4. – Vertical electric field in absence of the line measured in the working volume of the SEMIRAMIS EMP simulator. Adapted from Paolone et al. [2000].](image)

The reduced scale line model reproduces single and multi-conductor lines. The procedure used for the validation is based on the measurement of the electrical field generated in the simulator (see Fig. 3.4) and the measurement of the induced current in the reduced scale line model at line terminations for each conductor. The measured vertical electric field is used as input in the modified Agrawal coupling model, then the results obtained from the simulations are compared with the measured currents.

In what follows we report the results concerning both a single conductor with a shielding wire grounded at the line extremities (Fig. 3.5), and a three conductor configuration with a shielding wire grounded at line extremities and at the line center (Fig. 3.7).

For the configuration with a single conductor, the shielding wire was placed above and under the phase conductor at different heights (as shown in Fig. 3.5). The comparison between the measurement and the simulation, reported in Figs. 3.6, show the current at line terminations on the phase conductor with and without the presence of the shielding wire.
Fig. 3.5. – Reduced scale line model used in the SEMIRAMIS EMP Simulator composed by a single conductor with shielding wire grounded at line extremities.
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\[ \text{Current [A]} \]

\[ \text{Time [\( \mu \text{s} \)]} \]

\( c) \)

\( d) \)

\( e) \)
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Fig. 3.6. – Comparison between the experimental results and simulations relevant to line configuration of Fig. 3.5: induced current on the phase conductor; a) height of shielding 14 cm; b) height of shielding 16 cm; c) height of shielding 18 cm; d) height of shielding 22 cm; e) height of shielding 24 cm; f) height of shielding 26 cm.

For the line configuration with three conductors, the shielding wire was placed above the highest phase conductor (as indicated in Fig. 3.7). The comparison between the measurement and the simulation, reported in Figs. 3.8, show the current at line terminations on all the line conductors (phase conductors and shielding wire) with and without the presence of the shielding wire.

Fig. 3.7. – Reduced scale line model used in the SEMIRAMIS EMP Simulator composed by three conductors with shielding wire grounded at line extremities.
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(a) 

(b) 

(c)
Fig. 3.8. – Comparison between the experimental results and simulations relevant to line configuration of Fig. 3.7: induced current on the line conductor; a) conductor 1; b) conductor 2; c) conductor 3; d) shielding wire.

An additional validation of the proposed model was performed by testing it with the theoretical results of Yokoyama [1984]. The line configuration considered there is presented in Fig. 3.9. Note that there is only one grounding of the shielding wire, at the line center, in front of the stroke location. Our results of computation considering different values for the grounding resistance are presented in Fig. 3.10. The results practically coincide with those presented by Yokoyama, reported in Fig. 3.10b for convenience.

Fig. 3.9. – Line geometry adopted to evaluate the effect of a shielding wire to a multi-conductor line [Yokoyama, 1984]. Lightning current: peak value 100 kA; maximum time-derivative 50 kA/μs. Ideal ground.
The results of Fig. 3.10 show that the value of the grounding resistance largely affects the amplitude of the induced voltages. Such results can be ascribed, as we shall see better in what follows, to the fact that the stroke location is situated just in front of the grounding resistance.

**Effect of the shielding wires groundings.** To better assess the effect of the shielding wire, of the number of the periodical groundings and of the value of the grounding resistance, we have considered the line geometry shown in Fig. 3.11 in which the stroke location does not ‘face’ any of the grounding points.
The adopted lightning current has the following characteristics: peak value 30 kA maximum time derivative 40 kA/ms. The following results concern the effect on lightning induced voltage peak value of the grounding step namely: 100 m; 200 m; 500 m and 1000 m, and of the grounding resistance namely: 0 \( \Omega \); 30 \( \Omega \), 100 \( \Omega \) and 1 k\( \Omega \). In Figs 3.12, we present the peak value of the induced overvoltages along the 2 km long line for a perfectly conducting ground; the same results have been computed taking into account the finite ground conductivity (\( \sigma_g=0.001 \) S/m) and they are shown in Fig. 3.13.

In addition, the following figures which relate to the ideal ground case, the mitigation effect as predicted by the Rusck formula is also shown:

\[
\eta = \frac{U'_i}{U_i} = 1 - \frac{h_{sw}}{h_i} \cdot \frac{Z_{sw-i}}{Z_{sw} + 2R_g} \quad (3.48)
\]

where:
- \( U'_i \) is the lightning induced voltage in conductor \( i \) in presence of the shielding wire;
- \( U_i \) is the lightning induced voltage in the conductor in absence of the shielding wire;
- \( h_{sw} \) is the height of the shielding wire;
- \( h_i \) is the height of conductor \( i \);
- \( Z_{sw-i} \) is the mutual surge impedance between the shielding wire and conductor \( i \);
- \( Z_{sw} \) is the surge impedance of the shielding wire;
- \( R_g \) is the grounding resistance.
It is worth reminding that the Rusck expression does not cover the case of multiple groundings of the shielding wire (it assumes that the shielding wire is grounded at single point along the line) and that is applicable in the assumption of perfectly conductive ground.
Fig. 3.12. – Induced voltage peak value along the line for a variable number of grounding step and variable grounding resistance namely: a) 0 Ω, b) 30 Ω, c) 100 Ω, d) 1 kΩ. Line configuration of Fig. 3.11 with L=2 km. Ideal ground. Adapted from Paolone et al. [2000].
The results reported in Fig. 3.10a show that, if the stroke location is located in front of a grounding point, the attenuation of the induced overvoltage on the phase conductors is very dependent from the value of the grounding resistance. Additionally, as confirmed by Yokoyama [1984], the prediction of the maximum value of the induced overvoltages obtained by the application of Rusck formula (3.48), is the same as the results obtained by considering the shielding wire an illuminated conductor.

If, instead, the stroke location does not face any grounding point, which is the more frequent case, the results show that for both cases of ideal and lossy ground, the mitigation effect of the shielding wire depends, in general, more on the number of groundings than on the value of the grounding resistance. The presence of a large number of groundings tends,
in fact, to shift the shielding wire potential to the ground one enhancing the mitigation effect as predicted, for instance, by the Rusck approach.

For the case of lossy ground the number of groundings plays a decisive role too. This differs from the case of direct stroke for which the effectiveness of the shielding wire depends strongly on the grounding resistance.

The value of the grounding resistance which appear to influence the attenuation of the induced overvoltage, for both considered ideal and lossy ground conductivity, is 100÷200 Ohms.

The comparison with the Rusck formula is possible only for the case of ideal ground: in most cases, it tends to overestimate the effect of the shielding wire. This is due to the Rusck formula assumption that considers the shielding wire as a conductor at ground potential. Indeed only with a large number of grounding points (e.g. 11-21 groundings), the results predicted by Rusck formula and the proposed model tend to provide the same values.

**Effect of the presence of surge arresters.** The presence of surge arresters along the line can be treated in a similar way as for the discontinuities caused by the periodical groundings of shielding wire. In this case, we have modeled the surge arresters representing them with a non-linear \( V-I \) characteristic (the integro-differential operator \( \Gamma \)) placed in series with the incident voltage.

To compare the mitigation effect achieved by employing surge arresters with that of the shielding wire periodical grounding technique, we have considered the same single-conductor line of the previous simulation (Fig. 3.14).

---

**Fig. 3.14.** – Line geometry adopted to evaluate the effect of surge arrester. Lightning current: peak value 30 kA maximum time derivative 40 kA/µs. Adapted from Paolone et al. [2000].
The $V$-$I$ characteristic of the surge arrester is shown in the following table.

<table>
<thead>
<tr>
<th>Voltage [V]</th>
<th>Current [A]</th>
</tr>
</thead>
<tbody>
<tr>
<td>20820</td>
<td>0.09E-02</td>
</tr>
<tr>
<td>25980</td>
<td>0.12E-02</td>
</tr>
<tr>
<td>29100</td>
<td>0.06E-01</td>
</tr>
<tr>
<td>31140</td>
<td>0.06</td>
</tr>
<tr>
<td>33300</td>
<td>0.6</td>
</tr>
<tr>
<td>47460</td>
<td>3000</td>
</tr>
<tr>
<td>51000</td>
<td>6000</td>
</tr>
<tr>
<td>56640</td>
<td>12000</td>
</tr>
</tbody>
</table>

Tab. 3.1. – Surge arrester $V$-$I$ characteristic.

We have considered a variable number of surge arresters placed along the line: 2 (at the line terminal only) 3 (each 1000 m), 5 (each 500 m) and 11 (each 200 m). The induced overvoltage amplitudes are shown in Figs. 3.15 and 3.16.

Fig. 3.15. – Induced voltage peak value along the line for a variable number of surge arresters along the line. Line configuration of Fig. 3.14 with $L=2$ km. Ideal ground. Adapted from Paolone et al. [2000].
The computed results show that an important reduction of the induced overvoltage can be achieved only with a large number of surge arresters namely 1 surge arrester each 0.2 km. It can also be seen that for some configurations, the presence of surge arresters could result in important negative peaks which are due to the reflection coefficients, associated with the surge arresters, when the induced voltage exceed the threshold voltage of this non-liner component. Indeed, depending on the line configuration, stroke location and distance between two consecutive surge arresters, the negative voltage wave generated by the non-linear components can produce the maximum amplitude of the induced overvoltage not in the point closest to the stroke location. In addition this overvoltage can be more severe than the maximum voltage amplitude induced in the absence of surge arresters (see Fig. 3.15).

By increasing the number of surge arresters the maximum amplitude of the induced overvoltage tends to stay within the range generated by the positive and negative values of the threshold voltage of the surge arrester $V-I$ non-linear characteristic (see Tab. 3.1).

### 3.2. – Distribution systems

This paragraph reports the developed illuminated distribution system model. It is obtained by means of an interface between the developed complex overhead line model and two popular programs used for the power system transient analysis, namely: the EMTP96 and the Power System Blockset within Matlab environment.

The developed illuminated distribution system model is aimed at the correct estimation of the induced overvoltages for realistic distribution systems. This estimation is necessary in order to optimize the number and location of protective devices (shielding wire groundings
and surge arresters) and to minimize the number of outages. In particular, the presence of
distribution transformers and of the relevant protection devices at the line terminations, as
well as the presence of surge arresters along the line, should be considered in the simulation.

The use of the developed illuminated distribution system model permits to perform the
analysis of the effect of the aperiodic disturbance caused by LEMP on the various power
system components (e.g. the distribution transformers). Indeed the lightning-induced
overvoltages on distribution transformers, and their propagation from the medium voltage to
the low voltage of the distribution system, is not yet considered in the literature on the
argument.

The aim of the proposed model is to give an engineering tool enabling to analyze the
above mentioned aspects.

3.2.1. – Proposed approach

For evaluating the lightning performance of distribution networks the availability of a
tool for the calculation of lightning-induced voltages is crucial. Such a tool can be a) a
simple analytical formula (as for instance the Rusck [1958] one, adopted in the IEEE WG on
lightning performance of distribution lines [1997]), b) a computer code.

An analytical formula presents the advantage of short computational times; on the other
hand its application is limited to cases which tend to be unrealistic (as reported in the
comparison of Fig. 2.7 between the Rusck simplified formula and the Agrawal model). The
distribution system model that we are aimed at developing instead must potentially permit
the treatment of realistic cases reported schematically in Fig. 3.17.

In the LIOV code [Nucci et al., 1993; Rachidi et al. 1996,1997, Nucci, 2000] the
Agrawal et al. field-to-transmission line coupling model has been implemented for dealing
with the case of multi-conductor lines closed on resistive terminations. In principle, the
LIOV code could be suitably modified, case by case, in order to take into account the
presence of the specific type of termination, line-discontinuities (e.g. surge arresters across
the line insulators along the line) and of complex system topologies. This procedure requires
that the boundary conditions for the transmission-line coupling equations be properly re-
written case by case, as discussed in [Nucci et al., 1994].

However, as proposed by other authors too [Nucci et al., 1994; Orzan et al.,1996; Orzan,
1998; Höidalen, 1997,1999], we found more convenient to link the LIOV code with the
EMTP. With these LIOV-EMTP codes it is possible to analyze the response of realistic
distribution systems. In [Nucci et al., 1994], the distribution system model is considered, as
in the proposed approach (see next paragraph), as consisting of a number of illuminated
lines connected to each other through a shunt admittance. The LIOV code has the task of
calculating the response of the various lines connecting the two-ports (see Fig. 3.17); the
EMTP has the task of solving the boundary condition and presents the advantage of making
available a large library of power components.
In this new version of the interface between the LIOV code and the EMTP96 (henceforth called LIOV-EMTP96), each LIOV line is described by the Agrawal coupling model but the partial differential equation are now solved using the previously described line model based on the FDTD 2nd order scheme in order to improve the numerical stability when non-linear phenomena are considered.

In the literature on the argument there are two other proposed models for the evaluation of the LEMP response of distribution networks, proposed by Orzan [Orzan et al., 1996; Orzan, 1998] and Høidalen [1997, 1999] respectively.

In [Orzan et al., 1996; Orzan, 1998] for each illuminated line the coupling between the external incident field and the phase conductors is reproduced, in the distribution network model, by means of equivalent current generators whose value is pre-calculated by solving the Agrawal transmission line coupling model. The current generators are then inserted in an EMTP simulation where traditional lines models replace illuminated lines. Such an approach cannot take into account non-linear local phenomena, like a variation in the line-capacitance with space, as necessary for instance when taking into account the presence of corona phenomenon [Nucci et al., 2000].

In the model proposed by Høidalen [1997, 1999], the analysis of the response of the illuminated distribution system uses the same concepts adopted in the model developed by Orzan namely each illuminated line response is reproduced by means of equivalent voltage generators. The main difference with Nucci et al. [1994] and [Orzan et al.,1996; Orzan, 1998] is in the evaluation of the coupling between the external incident field and each illuminated line: Høidalen solves it using an analytical approach valid only for the ideal ground case.

Finally, compared to [Nucci et al., 1994], the advantages of this new LIOV-EMTP96 program are that: (i) the illuminated line model is able to take into account more complex line configuration, (ii) it does not require any modification of the EMTP source code, (iii) it allows for the treatment of corona effect which was implemented.

3.2.2. – Interface between the developed line model and transient programs

Interface between LIOV and EMTP96. As previously mentioned, in order to perform in a straightforward way the analysis of the LEMP response of real distribution systems characterized by a certain topological complexity, the developed program based on the 2nd order FDTD scheme has been interfaced with the Electromagnetic Transient Program (EMTP96). Note that in principle, the developed program could have been suitably enlarged and extended case by case to take into account the specific system configuration, as discussed in [Nucci et al., 1994]. As a matter of fact, paragraph 3.1 relevant to the transverse discontinuities, report an example of such a concept.
The concept at the basis of the new interface, a beta-version of which has been presented in [Borghetti et al., 2000a; Paolone et al., 2001b], is schematically described in Fig. 3.17.

Fig. 3.17. – Electrical distribution system illuminated by LEMP.

The distribution system network is considered as consisting of a number of illuminated lines connected to each other through a shunt admittance. This admittance represents the presence of surge arresters, of groundings of shielding wires, of distribution transformers or of other power components. Each section of the distribution system between two consecutive shunt admittances is modeled as a single line henceforth called ‘LIOV-line’, while the program which results from the link between the LIOV and the EMTP96 will be called LIOV-EMTP96.

The difference from [Nucci et al., 1994] is that the new interface does not require any modification to the source code of the EMTP: the modified LIOV code is indeed contained in a dynamic link library (DLL), called within the TACS environment. The data exchange between the LIOV code and EMTP96 is realized in the following way: the induced currents at the terminal nodes, computed by the modified LIOV code are input to the EMTP via current controlled generators, and the voltages calculated by the EMTP are input to the modified LIOV code via voltage sources.

The scheme of the data exchange in the developed interface is shown in Fig. 3.18.
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The link between the LIOV-line and the EMTP96 is realized by means of a loss less Bergeron line (see Fig. 3.19). The insertion of this line in necessary to write, in an appropriate way, the link equation between the terminal nodes of the FDTD spatial discretization of the illuminated line, and the line generic EMTP96 termination. In what follows we shall refer to the left termination of a single-conductor line; the right line termination is treated in the same way.

Fig. 3.18. – Interface between LIOV code and EMTP96. Adapted from Paolone et al. [2001a].

Fig. 3.19. – Insertion of the Bergeron loss less line for the data exchange between the LIOV code and the EMTP96 environment at the left line termination.
where:

- $G_1$ and $G_2$ are the Bergeron equivalent generators;
- $v^i(t)$ is the incident voltage defined in equation (2.13);
- $Z_c$ is the line characteristic impedance;
- $v_0(t)$, $v_0^i(t)$, $i_0(t)$, $i_1(t)$ are the voltages and currents at nodes of the FDTD spatial discretization;
- $v_0(t)$, $i_0(t)$ are the voltage and current at the EMTP node;
- $\Gamma_0(t)$ is the EMTP termination type.

The generators due to the incident voltages $v^i(t)$, can be introduced in the Bergeron $G_1$ and $G_2$ generators respectively. From the Bergeron equations in a fixed time step $n$ we can write the following equations:

$$G_1 = v_0^n - Z_c i_0^n + E_{z0}^n h$$  \hspace{1cm} (3.49)
$$G_2 = v_0^{n+1} - Z_c i_0^{n+1} + E_{z0}^{n+1} h$$  \hspace{1cm} (3.50)

where:

- $Z_c$ is the characteristic impedance of the line;
- $h$ is the conductor height;
- $E_{z0}$ is the vertical component of the electric field at the left line termination.

We can write, for node ‘0’ of the FDTD spatial discretization of the LIOV line, the following equations. In particular if we consider the node ‘0’ as connected with the Bergeron line we can write:

$$v_0^{n+1} = G_1 + E_{z0}^{n+1} h - Z_c i_0^{n+1}$$  \hspace{1cm} (3.51)

Now if we consider node ‘0’ as connected to the LIOV line we can obtain the scattered voltage $v_0^{n+1}$ from the discretization of the second equation of the Agrawal coupling model for a single-conductor line (2.12):

$$\frac{i_1^{n+1} - i_0^{n+1}}{\Delta x} + C \frac{v_0^{n+1} - v_0^n}{\Delta t} = 0$$  \hspace{1cm} (3.52)
$$v_0^{n+1} = \frac{\Delta t}{C \Delta x} (i_1^{n+1} - i_0^{n+1}) + v_0^n$$  \hspace{1cm} (3.53)
From (3.52) and (3.53) we can write the following equation, from which we can obtain variable $i_0^{n+1}$:

$$i_0^{n+1} = \frac{1}{\Delta t} \left( v_0^{n+1} + \frac{\Delta t}{C\Delta x} i_1^{n+1} - E_{z0}^{n+1} h - G_i \right)$$  

(3.54)

In summary the algorithm for the data exchange between the LIOV line and the EMTP consists of the following steps:

- calculation of the input voltage for the generators $G_j$ located at line left and right terminations with the (3.49) (see Fig. 3.19);
- calculation, at time step $n+1$, of the scattered voltages and currents at all FDTD spatial nodes;
- calculation of the currents $i_0^{n+1}$ and $i_{k_{max}}^{n+1}$ from (3.54);
- calculation of the scattered voltages $v_0^{n+1}$ and $v_{k_{max}}^{n+1}$ from (3.53);
- calculation of the values of the output variable for generators $G_2$, located at line left and right terminations, from (3.50).

**Interface between LIOV and Power System Blockset within Matlab environment.** The procedure above described for the interface between the LIOV code with EMTP96, has been also implemented in a Matlab core code (Mat-LIOV). In the Mat-LIOV code, as in the LIOV one, the model adopted to describe the lightning return stroke is the MTLE model, the calculation of the incident electromagnetic field is performed in the presence of a finite conducting ground by using the Cooray-Rubinstein approach [Rubinstein 1991, 1996; Cooray 1992, 1994], and the coupling between the external incident field and a single multi-conductor line is modeled by means of the Agrawal et al. [1980] coupling model.

The basic concept of the developed interface is the same developed for the EMTP96 one: leave the task of solving the transmission line coupling equations relevant to each line to the Mat-LIOV code, and that of solving the boundary conditions to the Power System Blockset. The link between the Mat-LIOV and the Power System Blockset is realized also in this case by using a Bergeron line, which accomplish the data exchange between the two mentioned tasks (see Fig. 3.19, 3.20).

The Mat-LIOV task is contained in an s-function developed to exchange line terminals voltages and currents with the Power System Blockset environment as shown in Fig. 3.20. In particular, the s-function calculates the necessary line parameters (inductance,

\[ k_{max} \] is the last FDTD spatial node at line right termination.
capacitance and surge impedance matrix) and solves, for each line, the discretized transmission line coupling equations by means of the 2nd order FDTD scheme.

The computational procedure is the following: at a given time step the Power System Blockset sends the nodal voltages at each line termination to the s-function block which solves the discretized transmission line coupling equations. Then, one time step later, the s-function block sends the voltage at the line terminations back to the Power System Blockset; this information exchange is accomplished step by step by means of a Bergeron line as described in the previous paragraph for the interface with the EMTP96 (see Fig. 3.19).

![Fig. 3.20. – Interface scheme between Mat-LIOV code and Power System Blockset in Matlab environment. Adapted from Gutierrez et al. [2001].](image)

### 3.2.3. – Comparison with experimental results and simulations

An experimental validation of the the developed LIOV-EMTP96 program and relevant algorithm, is realized by means of the experimental results obtained by Piantini and Janischewskyj [1992]. The measurements have been performed on reduced scale models set up at the University of Sao Paulo in Brazil, which reproduce a typical overhead distribution system (main feeder plus branches) including surge arresters, neutral grounding, T-junctions (between line branches) and shunt capacitors aimed at modeling distribution transformers. The surge arresters are simulated by means of a combination of diodes and resistances, and their $V-I$ non-linear characteristic is shown in Fig. 3.22. For cases 1.1 and 1.2 the lines of such system have 4 conductors and the line geometry is represented in Fig. 3.21a in the real scale. For cases 2.1, 2.2 and 2.3 the lines of the system have a simplified configuration with 2 conductors (see Fig. 3.21b).

The system that simulates the lightning current generate a current wave shape that can be approximated with a triangular profile with a time to peak value equal to 2 µs and a half time equal to 85 µs. For all cases the lightning return stroke velocity is, in the real scale, equal to 0.33·10^8 m/s, the lightning channel height is 600 m and the lightning return stroke model can be represented with the TL model. For each case different value for the lightning peak current are considered.
Case 1.1. The network configuration is shown in Fig. 3.23, scaled to real dimension. In Figs. 3.24 are reported the connection types of the line terminations. The stroke location is represented in Fig. 3.23 at point ‘r.s.m.’, two different value of lightning current are considered for this case, their characteristics are given in Tab. 3.2.

<table>
<thead>
<tr>
<th>Time (µs)</th>
<th>Lightning current (kA)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Case 1.1.a</td>
</tr>
<tr>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>2.</td>
<td>34.</td>
</tr>
<tr>
<td>85.</td>
<td>17.</td>
</tr>
</tbody>
</table>

Tab. 3.2. – Lightning current for the case 1.1.
Fig. 3.23. – Network configuration for the case 1.1.

Fig. 3.24. – Termination types for the network configuration of Fig. 3.23.

Fig. 3.25 reports a comparison between the voltage measured at node M1 (see Fig. 3.23), and the simulation performed with the LIOV-EMTP96 for lightning current peak values equal respectively to 34 kA and 70 kA. The measurement and simulation regard the phase conductor close to the stroke location.
Case 1.2. The network configuration is reported in Fig. 3.26. In Figs. 3.27 are shown the connection types of the line terminations. The stroke location is represented in Fig. 3.26 at point ‘r.s.m.’, three different value of lightning current are considered for this case, their characteristics are given in Tab. 3.3.

<table>
<thead>
<tr>
<th>Time (µs)</th>
<th>Lightning current (kA)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Case 1.2.a</td>
</tr>
<tr>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>2.</td>
<td>34.</td>
</tr>
<tr>
<td>85.</td>
<td>17.</td>
</tr>
</tbody>
</table>

Tab. 3.3. – Lightning current for the case 3.2.
Fig. 3.26. – Network configuration for the case 1.2.

Fig. 3.27. – Termination types for the network configuration of Fig. 3.26.
Fig. 3.28 reports a comparison between the voltage measured in the node M1 (see Fig. 3.26), and the simulation performed with the LIOV-EMTP96 for various lightning current peaks namely 34 kA (Fig. 3.28a), 50 kA (Fig. 3.28b), 60 kA (Fig. 3.28c) and 70 kA (Fig. 3.28d).
Case 2.1. The network configuration is reported in Fig. 3.29. In Figs. 3.30 are shown the connection types of the line terminations. The stroke location is represented in Fig. 3.29 at point ‘r.s.m.’, the lightning current characteristics considered for this case are given in Tab. 3.4.

<table>
<thead>
<tr>
<th>Time (µs)</th>
<th>Lightning current (kA)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Case 2.1</td>
</tr>
<tr>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>2.</td>
<td>70.</td>
</tr>
<tr>
<td>85.</td>
<td>35.</td>
</tr>
</tbody>
</table>

Tab. 3.4. – Lightning current for the cases 2.1, 2.2, 2.3.
Fig. 3.29. – Network configuration for the case 2.1.

Fig. 3.30. – Termination types for the network configuration of Fig. 3.29.

Fig. 3.31 reports a comparison between the voltage measured in the node M2 (see Fig. 3.29), and the simulation performed with the LIOV-EMTP96 for lightning current peak equal to 70 kA.
Fig. 3.31. – Comparison between measurement and simulation for the case 2.1. Lightning current peak value equal to 70 kA.

**Case 2.2.** The network configuration is reported in Fig. 3.32. In Figs. 3.33 are shown the connection types of the line terminations. The stroke location is represented in Fig. 3.32 at point ‘r.s.m.’, the lightning current characteristics considered for this case are the same for the case 2.1 and are given in the previous Tab. 3.4.

Fig. 3.32. – Network configuration for the case 2.2.
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\[ C_t = 0.5 \text{ nF} \]
\[ R = 50 \text{ Ohm} \]
\[ L = 22 \text{ \mu H} \]

\[ L = 1 \text{ \mu H} \]
\[ C = 13.5 \text{ \mu F} \]

\[ I_{pr} \ [A] \quad V_{pr} \ [V] \]
\[ 0. \quad 0. \]
\[ 36. \quad 27000. \]
\[ 54. \quad 27900. \]
\[ 360. \quad 29700. \]
\[ 1620. \quad 32220. \]

\[ R = 50 \text{ Ohm} \]
\[ L = 22 \text{ \mu H} \]
\[ M2 \]

\[ N \]
\[ C_p \]
\[ C_p = 0.65 \text{ nF} \]

Fig. 3.33. – Termination types for the network configuration of Fig. 3.32.

Fig. 3.34 reports a comparison between the voltage measured in the node M2 (see Fig. 3.32), and the simulation performed with the LIOV-EMTP96 for lightning current peak namely 70 kA.

Fig. 3.34. – Comparison between measurement and simulation for the case 2.2. Lightning current peak value equal to 34 kA.

Case 2.3. The network configuration is reported in Fig. 3.35. In Figs. 3.36 are shown the connection types of the line terminations. The stroke location is represented in Fig. 3.35 at point ‘r.s.m.’, the lightning current characteristics considered for this case are the same for the case 2.1-2.2 and are given in the previous Tab. 3.4.
Fig. 3.35. – Network configuration for the case 2.3.

Fig. 3.36. – Termination types for the network configuration of Fig. 3.35.

Fig. 3.37 reports a comparison between the voltage measured in the node M2 (see Fig. 3.35), and the simulation performed with the LIOV-EMTP96 for lightning current peak equal to 70 kA.
The results above reported show a good agreement between the simulations and the measurements. For case 1.1 the agreement between the simulations and the measurements is better than for case 1.2. This difference is probably due to the presence, for case 1.2, of a larger number of surge arresters than for case 1.1. This is supported by the results shown in Fig. 3.34 for case 2.2 where the observation point is placed on a surge arrester. Indeed, at this point the LEMP-response of the distribution system is due, mainly, to the surge arrester non-linear characteristic, which means that the difference between the simulation and the results are mainly affected by the surge arrester representation.

Which refer to the simplest line geometry and the reduced number of surge arresters, cases 2.1 and 2.3 show the better agreement between simulations and measurements.

Additional reasons for disagreement between measurements and simulations, for all cases, can be addressed also to: a) measuring errors: the overall uncertainty of the measuring system used in the tests was less than ±5%; b) high frequencies oscillations associated with the switching device of the generating system; c) variation of the current propagation velocity, its distortion and attenuation as it progresses upwards along the ‘stroke’ channel.

However, if we consider the complexity of the considered distribution systems and the relative low disagreement between measurements and model simulations, the experimental validation of the proposed model can be considered satisfactory.

Case of corona. The developed LIOV-EMTP96 interface allows also for the computation of lightning-induced overvoltages in presence of corona. The same model described in [Guerrieri, 1997; Correia de Barros et al., 1999; Nucci et al., 2000] has been indeed implemented in the illuminated line model described in paragraph 3.1. According to this model, from a macroscopic point of view, corona can be described by a charge-voltage
diagram [Wagner et al., 1954; Gary et al., 1978]. After an initial linear increase of the charge with the voltage, a threshold voltage \( v_{th}(x,t) \) is reached and a sudden change of the derivative of charge with respect to the voltage takes place. This derivative defines a voltage-dependent dynamic capacitance \( C_{dyn} \). We here consider a simplified corona model given by [Correia de Barros and Borges da Silva, 1984; Correia de Barros, 1985] for a single-conductor line:

\[
C_{dyn}(x,t) = C' \quad \text{for} \quad v(x,t) < v_{th}(x,t)
\]

\[
C_{dyn}(x,t) = C'(k_1 + k_2 (v(x,t) - v_{th}(x,t)) / v_{th}(x,t)) \quad \text{for} \quad v(x,t) > v_{th}(x,t) \quad \text{and} \quad \frac{\partial v(x,t)}{\partial t} > 0
\]

\[
C_{dyn}(x,t) = C' \quad \text{for} \quad \frac{\partial v(x,t)}{\partial t} < 0
\]

(3.55)

where

- \( k_1 (\geq 1) \) is related to the sudden change of the capacitance when the voltage exceeds the corona threshold \( v_{th} \) (typical values are in the range 1.5-3);
- \( k_2 (\geq 0) \) is related to the gradual increase of the capacitance when the voltage is rising above the threshold.

A value of \( k_2=0 \) corresponds to the simplest approach to model corona, that is considering that the dynamic capacitance switches between only two values.

The dynamic capacitance \( C_{dyn} \) is then inserted in the Agrawal equations in lieu of \( C' \).

Fig. 3.39 reports some results obtained using the LIOV-EMTP96 program for the line geometry shown in Fig. 3.38 relevant to a lightning stroke with current amplitude of 35 kA, maximum time-derivative of 42 kA/\( \mu \)s, ground conductivity of 0.01 S/m.

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**Fig. 3.38.** – Line geometry adopted to evaluate the effect of corona. Lightning current: peak value 35 kA; maximum time-derivative 42 kA/\( \mu \)s. ground conductivity 0.01 S/m.
Fig. 3.39. – Induced voltages along the line in presence of corona. Line configuration of Fig. 3.38.
Chapter 4. – Influence of lightning-induced overvoltages on power quality

The overvoltages induced by lightning on distribution and transmission networks, can cause phase-to-ground and phase-to-phase flashovers that need, for their removal, the use of recloser breakers. In the case of distribution networks the use of these breakers determines a temporal decrease of consumers voltage feeding, known as ‘voltage sags’. The duration of these voltage sags depends on the cycle type of the recloser breakers, which, in turn, depends on the fault removal time.

As known, voltage sags cause serious malfunction of a large number of apparatus as, in particular, of the electrical d.c. and a.c. motors. As a matter of fact, some types of speed controls for electrical motors interrupt their normal operation for a 15% decrease of the voltage supply that have a duration of one half cycle [Bollen, 1997].

There are different causes for voltage sags; for example, the start of electric motors with large rated power, or inrush currents due to transformer insertion. There is, however, clear evidence that in electrical systems located in regions with high value of isoceraunic level, thunderstorm days with lightning activity are associated to more than 80% of the voltage sags determining apparatus malfunction [Boonseng and Kinnares, 2001] (see Fig. 4.1 and 4.2). Nevertheless, lightning activity is not the only one responsible of the voltage sag related to the thunderstorm days because, associated with the same days, there are other natural causes [Dugan et al., 1996] such as is the wind effect, that may cause short circuits (e.g. branch of trees-ground; conductor-conductor).

Therefore it results of fundamental importance the estimation of the lightning performance of distribution lines in order to infer whether lightning is natural cause of outages or voltage sags during thunderstorm days or not.

![Fig. 4.1. – Influence of lightning flash density on voltage sags. Adapted from Gunther et al. [1995].](image-url)
The basic question for a power engineer is how many lightning faults per year a certain distribution line may experience, as a function of its insulation and of its design. The issue has been the object of several studies\(^1\).

In [Rusk, 1977; Pettersson, 1991] the frequency of overvoltages exceeding a given insulation level is evaluated by means of analytical methods for the case of an infinite long line over a perfect conducting plane. The amplitude of the lightning current at the channel base is considered as a random variable taking into account its probability distribution, while the front time of the lightning current and the return stroke velocity are considered fixed.

In [Chowdhuri, 1989b; Jankov and Grzybowski, 1997] a statistical method is employed. Both the probability distribution of amplitude and that of front time of the lightning current are considered. In [Chowdhuri, 1989b] the coefficient correlation between the two above-mentioned parameters is taken into account. The striking distance of the indirect stroke from the line (lightning strokes occurring within a certain distance from the line will directly strike the line) is evaluated as a function of the return stroke peak current (while in [Rusk, 1977] it is considered as independent of the current). The return stroke velocity is fixed and the ground is considered as a perfectly conductive plane.

All the above studies deal with single conductor lines, of infinite length, above a perfectly conducting ground.

In [Hermosillo and Cooray, 1995] the Monte Carlo method has been employed to solve the problem. The induced voltages are calculated at the termination of a 2 km overhead line

\(^1\) [Rusk, 1977; Chowdhuri, 1989b; Pettersson, 1991; Hermosillo and Cooray, 1995; Jankov and Grzybowski, 1997].
above a ground of finite conductivity. The Monte Carlo simulation involves $10^4$ events taking place over a surface covering the line and 1000 m away from it. The same striking distance equation from the line as in [Chowdhuri, 1989b; Jankov and Grzybowski, 1997] is adopted. The correlation between peak value and front time of lightning current distributions is disregarded.

In this thesis, to estimate the frequency of lightning induced overvoltages exceeding a given insulation level on an overhead distribution line, we propose a procedure based on the Monte Carlo method, whose basis are found in [Borghetti and Nucci, 1998]. The proposed procedure allows to evaluate the lightning performance of distribution lines taking into account not only more realistic configurations than those usually considered in the literature, namely lines provided with shielding wires or neutral conductors with periodical groundings and surge arresters above a lossy ground, but also the line steady-state voltage and the type of flashover induced by lightning, namely phase-to-ground or phase-to-phase. Besides, according to the types of distribution transformer connections, the developed procedure allows also to take into account the change of the coupling factor between the phase conductors and the ground when a flashover occurs, and the relevant fault occurrence.

The proposed procedure will be first compared with the one proposed for the same purpose by IEEE [IEEE WG on the lightning performance of distribution lines, 1997], and eventually applied for a sensitivity analysis and for the lightning performance assessment of a typical Italian distribution line.

4.1. – Statistical evaluation of the lightning performance of distribution lines

4.1.1. – Procedure based on the Monte Carlo method and on the developed LEMP-to-transmission-line coupling models for distribution lines

In the developed procedure lightning-induced voltages are calculated using the proposed models for the LEMP illuminated distribution lines. For our purposes, and in order to reduce the computation time, the wave shape of the return stroke current at the channel base is approximated with a ramp until the peak value $I_p$ is reached at time $t_f$, then the current value is kept constant. The Monte Carlo method is applied to generate a significant number of events (larger than $10^4 \div 10^3$), each characterized by four random variables: the peak value of the lightning current $I_p$, its front time $t_f$, and the two coordinates of the stroke location.

We assume a correlation coefficient ($\rho$) between current amplitude and front duration equal to 0.47 [Chowdhuri, 1989b] and the statistical parameters of log-normal distribution of the peak and the front time published in [Anderson and Eriksson, 1980]. The stroke locations are supposed uniformly distributed within a certain surface around the line. The
‘striking area’ relevant to the surface that we consider in our study, has to be chosen wide enough to include all the lightning events that can induce a voltage along the line with maximum amplitude greater than the considered insulation level. The maximum dimension of this area ($x_{\text{max}}$, $y_{\text{max}}$ of Fig. 4.3), is preliminarily determined by evaluating the induced overvoltages due to the maximum lightning current of the statistical distribution at various distances and position from the line.

![Image](image-url)

Fig. 4.3. – Indirect stroke area to overhead line (top view).

The ‘striking area’ does not include the points whose distance from the line is less than the values $d_l$ (lateral attractive distance, see Fig. 4.3).

In this study, the expression adopted by the IEEE Working Group on Lightning performance of transmission lines [IEEE WG on the lightning performance of distribution lines, 1997] is used for the evaluation of the lateral attractive distance $d_l$. Such expression, is based on the ‘Electro-Geometric’ model of the last step of the lightning flash, which gives the following relation between the critical distances (the striking distances to the wire ($r_s$) and to ground ($r_g$)) and the lightning current $I$ [IEEE WG on estimation lightning performance of transmission lines, 1985]):

$$r_s = 10 \cdot I^{0.65}, \quad r_g = 0.9 \cdot r_s$$

(4.1)

where $r_s$ and $r_g$ are expressed in m and the lightning current $I$ in kA. As shown in Fig. 4.4, the value $d_l$ is then determined from

$$d_l = \sqrt{r_s^2 - (r_g - h)^2}$$

(4.2)
where \( h \) is the line height (in m).

If the distance of the stroke location from the line is beyond the lateral distance, the event is considered an indirect flash and the maximum amplitude of the induced overvoltages is computed.

In order to infer the classical flashover vs BIL curves, for each event we calculate the maximum of the voltage amplitudes induced along the line. Note that we do not limit the calculation to the point closest to the stroke location as done in all procedures which assume an infinitely long line and the ground as a perfect conductor, since for that case the maximum amplitude occurs indeed at that point [Hermosillo and Cooray, 1995]. This because, when the ground is not a perfect conductor, stroke locations close to one of the line terminations can induce the larger overvoltages at the opposite one [de la Rosa et al., 1988]

![Diagram](image)

**Fig. 4.4.** – Striking distances to a conductor \((r_s)\) and to ground \((r_g)\) and lateral attractive distance \((d_l)\) of a line. Adapted from Chowdhuri [1989b].

Then we compute the annual number of events that, for 100 km of line and for a given ground flash density, induce an overvoltage with amplitude exceeding the various insulation levels of the line. For all calculations the ground flash density is \(N_g=1\) flash/km\(^2\)/year and, otherwise specified, the line length is 2 km and its height 10 m.

### 4.1.2. – Comparison with the IEEE Std 1410-1997

A brief description of the IEEE Std 1410 *IEEE WG on the lightning performance of distribution lines, 1997* will be given. Concerning the model used for the calculation of the lightning induced overvoltages, the IEEE Std 1410 adopts the simplified formula presented by Rusck [1958] (hereafter called the Rusck formula). As earlier mentioned that formula has been inferred by the same Rusck from the more general model he proposed in [Rusck, 1958]; it applies to the simple case of a step current and of an infinitely-long single-conductor line above a perfectly conducting ground. It is worth remanding that the Rusck
formula (equation (2.24) of the paragraph 2.3) gives the maximum value $V_{\text{max}}$ (in kV) of the induced overvoltages at the point of the line nearest to the stroke location.

Concerning the statistical procedure used to infer the lightning performance of a distribution line, the IEEE Std 1410 follows the procedure presented in [Chowdhuri, 1989b], which we summarize here for convenience. The amplitude of the stroke current is varied from 1 to 200 kA in intervals of 1 kA. The number of annual insulation flashovers per km of distribution line $F_p$ is obtained as the summation of the contributions from all intervals considered as expressed by

$$F_p = 2 \cdot \sum_{i=1}^{200} \left( y_{\text{max}}^i - y_{\text{min}}^i \right) \cdot N_g \cdot P_i$$  \hspace{1cm} (4.3)

where $N_g$ is the ground flash density and $P_i$ is the probability of current peak to be within interval $i$; it is determined as the difference between the probability for current to be equal or larger than the lower limit and the probability for current to reach or exceed the higher limit of the interval. For the probabilistic distribution of the lightning current peak, the following expression is adopted [Anderson, 1982]:

$$P(I_p \geq I_p^*) = \frac{1}{1 + \left( I_p^*/31 \right)^{2.6}} \quad I_p^* \leq 200 \text{ kA}$$  \hspace{1cm} (4.4)

For each current value, $y_{\text{min}}$ and $y_{\text{max}}$ of equation (4.3) are the minimum distance for which lightning will not divert to the line, and the maximum distance at which the stroke may produce an insulation flashover, respectively. The value of $y_{\text{min}}$ is obtained by expression (4.2). The value of $y_{\text{max}}$ is obtained by solving equation (2.24) for $d$, by taking $I_p$ as the lower limit of the interval and by taking $V_{\text{max}} = 1.5 \cdot \text{CFO}$. The results for an ungrounded overhead 10 m high line are shown in Fig. 4.5. The value of $v$ in (2.29) is chosen equal to $1.2 \cdot 10^8 \text{ m/s}$. The results relevant to an ungrounded and grounded neutral or shielding wire are obtained in [IEEE WG on the lightning performance of distribution lines, 1997] from the preceding ones by applying a scale factor of 0.75 to the induced voltages.
We have first verified that the proposed statistical procedure gives the same results of the IEEE Std 1410 when the induced voltages are evaluated by using the Rusk formula instead of using LIOV, and when the same probability distribution (4.4) of the lightning current, the same values of return-stroke velocity \(v=1.2\cdot10^8\) m/s, line height (10 m) and lateral distance expression (4.2) are assumed. Then, we have found that the adoption of the peak distribution of [Anderson and Eriksson, 1980] instead of (4.4) does not have a significant influence on the results.

We have then compared the IEEE Std 1410 guide results of Fig. 4.5 with those obtained by using our procedure (LIOV plus Monte Carlo), assuming the statistical distributions of peak current and rise time of Fig. 2.3 (paragraph 2.1), a 2 km long line (beyond such a line length the line illumination by the LEMP field for a perfectly conducting ground does not affect the results [Borghetti and Nucci, 1999]). The comparison is shown in Fig. 4.6.

Fig. 4.5. – Number of annual induced flashovers vs distribution-line insulation level. Taken from IEEE WG on the lightning performance of distribution lines [1997].
Fig. 4.6. – Comparison between the lightning performance evaluated by using the IEEE Std 1410 procedure and the proposed one. $t_f$ is lognormally distributed with a median value of 3.83 μs. Adapted from Borghetti et al. [2001a].

The difference between our results and those of the IEEE Std 1410 can be explained by observing that the simplified Rusck formula applies to the case of a step wave shape for the lightning current [Rusck, 1958]. We therefore repeated our computation by keeping constant the value of $t_f$ throughout Monte Carlo simulation. Fig. 4.7 shows the results of four different computations relevant to four different values of $t_f$: 0.5, 1, 3 and 5 μs respectively. From Fig. 4.7 we can observe that when $t_f$ decrease, the two procedures predict basically the same results.

Fig. 4.7. – Comparison between the lightning performances evaluated by using the IEEE Std 1410 procedure and the proposed one, with different fixed value of $t_f$. 
4.1.3. – Sensitivity analysis

This paragraph describe the results of a sensitivity analysis carried out using the proposed procedure. In particular we shall deal first with the case of a single conductor line where we analyze the effect of the ground conductivity and the presence of surge arresters along the line. Then for the multi-conductor case we analyze the effect of the grounding resistance of the shielding wire, of the grounding intervals and of the different shielding wire height.

Single conductor line. Since the maximum values of the induced voltage amplitudes could be higher than for the case of a line above an ideal ground [Guerrieri et al., 1996,1997] for these calculations a larger indirect stroke area is considered. In particular the results refer to a 1 km long overhead line and to a 20 km² indirect stroke area, with $x_{\text{max}}$ equal to 5 km, $y_{\text{max}}$ equal to 2 km. The relative permittivity of the ground is assumed equal to 10. Since, for line lengths shorter than about 2 km and for ground conductivity not lower than 0.001 S/m, the effect of the lossy ground on surge attenuation along the line is not significant [Rachidi et al., 1996], the presence of a lossy ground in our calculations is taken into account only in the propagation of the incident electromagnetic field.

Fig. 4.8 shows the results obtained with two different ground conductivities, namely $\sigma_g$=0.01 S/m and $\sigma_g$=0.001 S/m. These results are also compared with those obtained with an ideal ground. It can be observed that lower values of ground conductivity result in higher flashover rates, a result which is explained by the fact that, as already mentioned, the ground resistivity enhances the induced voltages depending on the stroke location and on the observation point [Guerrieri et al., 1996,1997].

![Fig. 4.8. Effect of the ground conductivity, $t_f$ is lognormally distributed with a median value of 3.83 µs.](image)
To illustrate the influence of the presence of surge arresters along the line, we have considered the line configuration presented in Fig. 4.9 (we consider, for this case, only common-mode voltages). The surge arresters are represented by the non-linear \( V-I \) characteristic given in Tab. 3.1.

As discussed in paragraph 3.1.3 the presence of surge arrester along the line, can produce the maximum amplitude of the induced overvoltage not in the point closest to the stroke location. In order to reduce this effect, we have connected, at both line terminations in parallel with the surge arresters, two resistive loads equal to the line characteristic impedance.

The computed results are shown in Fig. 4.10, where we can observe that the presence of surge arresters affects considerably the lightning performance of distribution lines.
Multi conductor line. In this section we first evaluate the lightning performance of a typical three-phase Italian distribution line (see Fig. 4.11). For this purpose we consider a 1.8 km long line matched at both terminations, within an indirect stroke area of 7.6 km².

The calculations have been carried out assuming two different values of ground conductivity $\sigma_g$ (infinite and 0.001 S/m). They are plot in Fig. 4.12, which shows the number of events resulting in overvoltages with amplitude exceeding the value indicated in abscissa. We observe the larger number of overvoltages at phase $b$ than at phases $a$ and $c$, due to the higher position of the phase $b$ conductor. Note, again, the increase of the number of induced voltages due to the poor ground conductivity.

![Fig. 4.11. – Geometrical configuration of an Italian MV line.](image)

![Fig. 4.12. – Statistical evaluation of lightning induced voltage in the Italian MV line of Fig. 4.11. Cases for ideal ground and lossy ground ($\sigma_g=0.001$ S/m). Adapted from Borghetti et al. [2001a].](image)
We now consider a 2 km long, two-conductor line (phase conductor and shielding wire) over a perfectly conducting plane, a case similar to the one discussed in [IEEE WG on the lightning performance of distribution lines, 1997]. As shown in Fig. 4.13, the shielding wire is grounded at the line terminations and at intervals along the line. The spacing between two adjacent groundings is 2000 m (2 grs), 1000 m (3 grs), or 400 m (6 grs). The simulations are carried out with the aim of evaluating the influence of the presence of the shielding wire, of its height, of the spacing between two adjacent groundings as well as the value of the grounding resistance.

![Diagram of line configuration with shielding wire](image)

Fig. 4.13. – Line configuration with shielding wire.

Fig. 4.14 shows the results relevant to the phase conductor obtained with three different shielding wire heights, namely 9.7 m, 10 m, 10.4 m. The results are affected by the presence of the shielding wire, but the influence of the shielding wire height for the considered cases is limited.

![Graph showing influence of shielding wire height](image)

Fig. 4.14. – Influence of shielding wire height. The spacing between to adjacent grounding is 400 m (i.e. 6 groundings overall) and the grounding resistance is equal 10 Ω.
Chapter 4. – Influence of lightning-induced overvoltages on power quality

a)  

b)
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Fig. 4.15. – Influence of grounding resistance. a) The shielding wire is grounded only at its terminations (overall 2 grs). b) The spacing between two adjacent grounding is 1 km (overall 3 grs). c) The spacing between two adjacent grounding is 400 m (overall 6 grs).

Fig. 4.15 shows the influence of the grounding resistance (the shielding wire in this case is at 10 m above ground). Three values of grounding resistance are considered, namely 0, 10 and 100 Ohm. The results do not differ considerably, at least when the number of grounding is low (2 and 3 grs). On the other hand, the spacing between two adjacent grounding appears to have a larger influence, as also illustrated in Fig. 4.16 for a value of grounding resistance of 10 Ohm and a shielding wire height of 10 m.

Fig. 4.16. – Influence of spacing between adjacent grounding.
It has been shown that the shielding wire appears to have a large influence on the performance of the distribution line; additionally, as analyzed for single stroke location in paragraph 3.1.3, the spacing between two adjacent grounding of the shielding wire seems to have the major impact in decreasing the number of induced voltages exceeding a given value, in accordance with the results presented in paragraph 3.1.3.

The results obtained by using the three values of grounding resistance do not differ considerably, at least when the number of grounding is low (2 and 3 grs). On the other hand, the spacing between two adjacent grounding appears to have a larger influence, as also illustrated in Fig. 4.16 for a value of grounding resistance of 10 Ohm and a shielding wire height of 10 m, again, in accordance with the results of paragraph 3.1.3.

4.2. – Case of a typical Italian distribution line

To improve the power quality assessment of a distribution line which experience induced overvoltages, some additional features of the proposed models have been developed and are described in this section. In particular, they are aimed at investigating the effects of two parameters that are not considered in the previous model namely: the steady-state voltage and the change of coupling factor among the line conductors and the ground at the location where flashovers occur.

In the following paragraph the proposed model extensions are described, then an application of procedure is made.

4.2.1. – Model extension

The steady-state voltage at industrial-frequency, taken into account both in the generation procedure of the events and in the overvoltages calculations, is assumed constant, due to the high frequency content and the short duration of the induced voltages.

In the generation procedure of the events, uniformly distributed random values of the phase voltage are generated for one of the three phases; the voltages of the two remaining phases are assumed to form, together with the first one, a positive system for each event.

In the overvoltages calculation, the steady-state values of each phase voltage is taken into account by simply adding it to the incident voltage of equation (2.27). The reason for this is that the current circulating in the line before any lightning event does not effect the amplitude of the induced voltages, and therefore the coupling equations (2.25) and (2.26) remain unvaried.

Another additional feature of the proposed procedure, compared to [Borghetti and Nucci, 1998], is that it takes into account also the change in the coupling factor among the line conductors and the ground at the location where a flashover occurs. We consider each phase-conductor connected to ground through a resistance $R_{gp}$ at each pole: in normal
conditions, the value of $R_{gp}$ is equal to infinite; when a phase-to-ground flashover occurs at a certain pole, the relevant value of $R_{gp}$ is set at a value corresponding to the grounding resistance of the specific pole and the fault impedance. With such a model, we are able to investigate up to which extent a flashover at of one phase of the line can affect the overvoltages on the other two conductors. The scattered voltage at the points $x_p$ of the phase conductors where the induced voltage exceeds the CFO are calculated by means of

$$
\left[ v_j'(x_p, t) \right] = \left[ R_{gp} \right] \left[ i_{gp} (t) \right] + \int_{b}^{h} E''_{z}(x, z, t) dz
$$

(4.6)

where

- $[i_{gp}]$ is the matrix of the induced currents diverted to ground in correspondence of the pole (see Fig. 3.1);
- $[R_{gp}]$ is the diagonal matrix of the phase-to-ground resistances of the poles.

We consider that an overvoltage causes a flashover when it exceeds the value of 1.5+CFO. The 1.5 factor is an approximation that accounts for the turn up in the insulation volt-time curve, which is the same criterion proposed by the IEEE Std 1410 [IEEE WG on the lightning performance of distribution lines, 1997].

Fig. 4.17 shows the voltages induced by a lightning event with $I_p=40$ kA, $t_f=3$ µs, on a typical Italian distribution line (see Fig. 4.11). We consider a 1.8 km long line matched at both terminations and stroke location equidistant to the line termination, 50 m from the line.

The line span between two consecutive poles is 150 m. The electromagnetic field radiated by lightning is calculated assuming the ground conductivity equal to 0.01 S/m. Also, we assume a phase to ground voltage equal to 16 kV on phase $b$ (where the flashover occur) and a resistance $R_{gp}$ of the poles during a phase to ground flashover equal to 10 Ω.

Figs. 4.17 a–c shows the behavior of the induced voltage amplitude on phase conductors $a$, $b$ and $c$ respectively at the various poles. The flashover of phase $b$ at pole 6 at about 2 µs causes a voltage reduction at the same pole, which propagates in both directions along the line. This flashover causes an instant voltage reduction both on phases $a$ and phase $c$ where the flashover does not occur, producing in turn a shielding effect similar to that of a shielding wire.
Fig. 4.17. – Voltage amplitude induced on phase b by a lightning event with \( I_p = 40 \text{kA} \), \( t_f = 3 \mu \text{s} \), stroke location equidistant to the line termination, 50 m from the line of Fig. 4.9. The LEMP is calculated assuming a ground conductivity \( \sigma_g = 0.01 \text{ S/m} \). The grounding resistance of the poles \( R_{gp} = 10 \Omega \). a) phase a; b) phase b; c) phase c. Adapted from Borghetti et al. [2001b].
4.2.2. – Results

In this paragraph we evaluate the lightning performance of the typical Italian distribution line previously described (see Fig. 4.9). We consider the same line length and indirect stroke area assumed in the paragraph 4.2.1.: 1.8 km long line matched at both terminations, within an indirect stroke area of 7.6 km², the number of considered events is $2 \cdot 10^5$. The poles are made out of concrete or steel and are assumed to have a grounding resistance $R_{gp}$ in the range 10-100 Ω. The line span between two consecutive poles is 150 m. The amplitude of the r.m.s. value of the steady-state phase-to-phase voltage is 20 kV and the CFO of the line is 125 kV.

We consider also the pole grounding resistance at those poles of the line where the CFO is exceeded by the induced overvoltages and consequently a flashover occurs. The LEMP is now calculated assuming two different values for the ground conductivity, namely 0.1 S/m and 0.01 S/m.

In Figs. 4.18-4.21 we show the results of the statistical analysis (80000 events overall) carried out on the Italian MV line configuration of Fig. 4.10, for two different values of ground conductivity, and of pole grounding resistance. In particular, they report the expected number of flashovers, distinguishing among one phase to ground, and two or three phases to ground, caused by a single event.

To better assess the importance of taking into account the steady-state voltage in the calculations, in Figs 4.18-4.21 we report also the results for the case in which the steady-state voltage is disregarded.

To better assess the range of influence of the grounding resistance, Fig. 4.22 report the number of flashovers for a fixed value of ground conductivity and three different values of grounding resistances namely 100, 200 and 500 Ω.

![Graph showing number of flashovers with and without steady-state voltage](image)

Fig. 4.18. – Number of flashovers along the line, $R_{gp}=10$ Ω and $\sigma_g=0.1$ S/m. Adapted from Borghetti et al. [2001b].
Fig. 4.19. – Number of flashovers along the line, $R_{gp}=10\ \Omega$ and $\sigma_g=0.01\ \text{S/m}$. Adapted from Borghetti et al. [2001b].

Fig. 4.20. – Number of flashovers along the line, $R_{gp}=100\ \Omega$ and $\sigma_g=0.1\ \text{S/m}$. Adapted from Borghetti et al. [2001b].

Fig. 4.21. – Number of flashovers along the line, $R_{gp}=100\ \Omega$ and $\sigma_g=0.01\ \text{S/m}$. Adapted from Borghetti et al. [2001b].
As regard the presence of the steady-state voltage, a parameter that is not usually considered in the literature, we can observe that the taking into account of it results in a larger number of flashovers: 1.5 instead of 1.3 in Figs. 4.18 and 4.20, 3.7 instead of 3.2 in Figs. 4.19 and 4.21. For all cases the average increase of the number of flashovers/100 km/yr obtained taking into account the steady-state voltage is about 15%.

Concerning the line geometry influence, we can observe the larger number of ground flashovers at phase \( b \) than at the two others phases, independently of the steady-state voltage. This is a consequence of the higher position of phase \( b \) (see also Fig. 4.11).

Concerning the increase of the pole grounding resistance we observe that produces, in general, an increase of the number of simultaneous faults to ground (two or three phases). The shielding effect produced by the phase where a flashover occurs tends in fact to decrease with the increase of the grounding resistance \([Paolone \ et \ al., \ 2000]\). The value of the poles grounding resistance starting from which this effect becomes important is of about 100÷200 \( \Omega \).
Chapter 5. – Conclusions

Aims of this thesis are 1. – the development of models and their implementation in a computer programs for an accurate estimation of the lightning-induced overvoltages on distribution power networks, and their test against experimental results; 2. – the improved assessment of the lightning performance of distribution systems in view of the solution of power quality problems.

In what follows are summarized the contributions of this thesis thought to be original.

**Development of a complex overhead line model illuminated by external electromagnetic field.** We have developed an overhead line model, based on a modification of the Agrawal coupling model, originally introduced for the calculation of lightning-induced voltages on uniform lines, to take into account the presence of overhead line transverse discontinuities, such as those caused by periodically grounded shielding wires and surge arresters.

For solving the proposed model equations, we have proposed a 2nd order FDTD integration scheme. The developed integration scheme applies to multi-conductor lines above a frequency-dependent lossy ground. It has been compared with a similar one, also developed within the framework of this thesis, based on the 1st order FDTD integration scheme, and it has been shown to be numerically more stable when considering frequency-dependent parameters and/or non-linearities like surge arresters.

Compared to similar models developed in the literature, which consider the shielding wire at ground potential at any point along it and at any time, the proposed model permits a more accurate evaluation of the lightning-induced voltages. The developed model allows to evaluate the induced overvoltages on overhead lines above a lossy ground taking into account the resistance of the groundings and the spacing between the groundings of shielding wires. This feature is determinant if one has to found the ‘optimal’ distance between two adjacent groundings of the shielding wire.

Its experimental validation has allowed to show its adequacy. The proposed model has in fact been tested and validated versus experimental results obtained using a reduced scale multi-conductor line model illuminated by an EMP simulator. The model was given different line geometries of the conductors and of the groundings of the shielding wire.

**Analysis of the effect of shielding wires and surge arresters on lightning-induced overvoltages.** The performed sensitivity analysis has shown that in the mitigation effect of the shielding wire the number of groundings plays a decisive role: the mitigation effect has been found to be more dependent on the number of groundings than on the value of the grounding resistance, both in the case of lossy ground and of ideal ground. This differs from
the case of direct strokes, for which the effectiveness of the shielding wire depends strongly on the grounding resistance.

For the cases of an ideal ground, the mitigation effect as predicted by the Rusck formula has been analyzed too, and the results have shown that the Rusck simplified formula tends to overestimate the effectiveness of the shielding wires. It is worth reminding that the Rusck expression does not cover the case of multiple grounded shielding wire (it assumes that the shielding wire is grounded at single point along the line) and that is applicable only in the assumption of perfectly conductive ground.

Concerning the effectiveness of surge arresters, it depends strongly on their number and position along the line. An ‘optimum’ number appears, for the considered case, one surge arrester each 200 m.

*Development of a distribution power system model.* In order to simplify the achievement of the LEMP response of distribution systems, the developed line model has been interfaced with the Electromagnetic Transient Program (EMTP96). Such an interface is based on the use of a Bergeron loss less line which makes it possible the data exchange between the two programs (LIOV and EMTP96). In addition, as opposed to [Nucci et al. 1994], the developed interface does not require any modification of the source code of the EMTP96 because it use a Dynamic Link Library (DLL) called from the EMTP96-TACS environment. Compared to other ones proposed in the literature, allows also for the treatment of the corona phenomenon as opposed to [Orzan et al.,1996; Orzan, 1998], and line losses not dealt with in the interface of Høidalen [1997,1999].

The advantages of merging the developed line model with the EMTP96 consists essentially in the fact that (i) distribution systems of any complex configurations can be modeled (the only limit being the memory of the computer and of EMTP96) and (ii) all power component models of the EMTP library become available for lightning-induced voltage simulations.

Calculations performed with the developed LIOV-EMTP96 program have been compared with experimental results obtained by using a reduced-scale distribution line model described in [Piantini and Janiszewski, 1992, Nucci et al., 1998]. The comparison regards an overhead distribution system including surge arresters, shielding wire groundings and shunt capacitors. Overall, the agreement between the numerical results and measurements is quite satisfactory. We can conclude that LIOV-EMTP96 program represent a promising tool for the analysis of the response of a complex distribution system to indirect lightning discharge.

*Estimation of the lightning performance of complex overhead distribution lines.* We have developed a procedure based on the LEMP-illuminated distribution system model, and
on the Monte Carlo method that allows an improved evaluation of the lightning performance of distribution networks.

Such a procedure has been compared with the one proposed in the IEEE Std 1410-1997 for the same purpose: it has been shown that for those cases in which a comparison is possible (overhead single-wire line above a perfectly conducting ground, step function for the channel base current wave shape) the two methods predict basically the same results. Additionally, opposed to the IEEE procedure, the proposed one allows to take into account the ground resistivity, which plays a fundamental role in the calculation of the lightning electromagnetic field that excites the line, as well as any line configuration which may include the presence of shielding wire and surge arresters.

The results obtained show that the shielding wire appears to have a large influence on the performance of the distribution line; additionally, as analyzed for single stroke location, the spacing between two adjacent grounding of the shielding wire, seems to have the major impact in decreasing the number of induced voltages exceeding a given value.

Additional features of the proposed procedure have been developed in order to take into account the steady-state voltage and the change of coupling factor among the line conductors and the ground at the location where flashovers occur, which allows to estimate type of flashovers and the effective number of failures.

The procedure has been eventually applied for assessing the lightning performance of a typical Italian distribution line. The results obtained show that, in general, taking into account the steady-state voltage results in a larger number of flashovers, and that the increase of the pole grounding resistance produces, in general, an increase of the number of simultaneous faults to ground (two or three phases): the phase where a flashover occurs produces in fact the same shielding effect as a shielding wire; such effect tends to decrease with the increase of the pole grounding resistance.

Future possible development of the research.

As mentioned at the beginning of chapter 4, although there is some evidence that electrical distribution systems located in regions at high isoceraunic level are somewhat more affected by voltage sags, there is however no clear evidence that lightning is the main cause for these voltage sags.

An attempt to correlate indirect lightning activity with circuit breaker operation which, in turn, depend on voltage sags, is described in [Bernardi et al., 1998] who carried out a study to infer the possible correlations between lightning indirect events and breaker interventions. They used on the one hand the lightning current amplitude and stroke location coordinates provided by the Italian lightning location system (SIRF – Sistema Italiano di Rilevamento Fulmini), and on the other hand the simplified Rusck equation to calculate the corresponding lightning-induced overvoltage. The correlation they found between lightning events and circuit breakers operations was poor.
An explanation of this result, which is in part in contradiction with experimental observations [Boonseng and Kinnares, 2001], may be the fact that the Rusck simplified coupling model was adopted without considering the distribution system complexity and the important parameters that modify, in a decisive way, the LEMP response of the distribution system (e.g. the ground conductivity and power system configuration).

We expect that, as compared with the above mentioned study, the use of LIOV-EMTP can improve the LEMP calculation, the evaluation of the coupling by means of realistic line models and eventually the evaluation of entire distribution system response.

We think that the models developed in this thesis represent an improved tool for performing the indirect lightning correlation with distribution system a-periodic disturbances, as well as for the distribution networks protection and insulation coordination, and power quality improvement.
Agrawal A.K., Price H.J., Gurbaxani S.H., “Transient response of a multiconductor transmission line excited


above, Electric Power research Institute, ch. 12, 1982.


EMP simulator with a good confinement inside the transmission line”, Proc of the 10th Int. Symposium on


SEV Bd 46, No. 5 and 9, 1955.

Berger K., “Methoden und Resultate der Blitzforschung auf dem Monye San Salvatore bei Lugano in dem Jahr


Bermudez J.L., Rubinstein M., Rachidi F., Paolone M., “A method to find the reflection coefficients at the top
and bottom of elevated strike objects from measured lightning currents”, Proc. of the 14th International Zurich

Bernardi M., Dellera L., Garbagnati E., Sartorio G., “Leader progression model of lightning: updating of the
model on the basis of recent test results”, Proc. of the International Conference on Lightning Protection,

Bernardi M., Giorni C., Biscaglia V., “Medium voltage line faults correlation with lightning events recorded
with the Italian LLP system ‘CESI-SIRF’ Proc. of the International Conference on Lightning Protection 24th


Bollen M.H.J., “Characterization of voltage sags experienced by three-phase adjustable-speed drives”, IEEE
Transactions on PD, Vol.12, no.4, pp.1666-1671, October 1997.

Boonseng C., Kinnares V., “Analysis of harmonic for 12-pulse converter under unbalanced voltage operation
due to lightning related voltage sags and short interruption”, IEEE Power Engineering Society Winter

overhead line above a lossy ground: a sensitivity analysis”, Proc. of the 24th International Conference on

Borghetti A., Nucci C.A., “Frequency distribution of lightning induced voltages on an overhead line above a
lossy ground”, Proc. of SIPDA, San Paulo, Brazil, 1999.

Lightning Electromagnetic Fields”, Proc of the 25th International Conference on Lightning Protection, ,


Lundholm R., “Induced overvoltages surges on transmission lines and their bearing on the lighting performance at medium voltage networks”, Transactions of Chalmers University of Technology, No. 188, Gothenburg, Sweden, 1957.


References


Orzan D., Baraton P., Ianoz M., Rachidi F., “Comparaison entre deux approches pour traiter le couplage entre un champ EM et des réseaux de lignes”, 8ème Colloque International sur la Compatibilité Electromagnétique, Lille, 2-5 Septembre 1996.


TI: New insights into lightning processes gained from triggered-lightning experiments in Florida and Alabama


Rusck S., “Induced lightning overvoltages on power transmission lines with special reference to the overvoltage protection of low voltage networks”, Transactions of the Royal Institute of Technology, Stockholm, No. 120, 1958.


References


Wagner C.F., Cross I.W., Lloyd B.L., 2High voltage impulse tests on transmission lines”, AIEE Trans., Vol. 73, Part III, pp. 196-209, April 1954.


Appendix A.1. – Numerical treatment of line transverse discontinuities in 1\textsuperscript{st} order FDTD scheme

The 1\textsuperscript{st} order spatial Point Centered Finite Difference integration scheme for a single-conductor line illuminated by an external electro magnetic field, around grounding point is shown in Fig. A.1

\begin{equation}
\int_{h_i}^{i+1} z E_z(x,z,t)dz = 0
\end{equation}

\begin{equation}
\begin{bmatrix}
+ & - & - & 1 & - & - & 0 \\
- & + & - & 0 & 0 & 0 & 0 \\
0 & 0 & + & 0 & 0 & 0 & 0 \\
- & 0 & - & + & 0 & 0 & 0 \\
0 & 0 & 0 & - & + & 0 & 0 \\
0 & 0 & 0 & 0 & - & + & 0 \\
0 & 0 & 0 & 0 & 0 & - & + \\
0 & 0 & 0 & 0 & 0 & 0 & - \\
\end{bmatrix}
\end{equation}

Fig. A.1. – Insertion of discontinuity point, at a generic point along a single-conductor line, in the 1\textsuperscript{st} order Point Centered Finite Difference integration scheme.

The treatment of a grounding point, located anywhere along the line, involves a modification of the numerical solution of the Agrawal model. It is handled as a discontinuity point in the spatial grid of the Point Centered Finite Difference integration scheme.

In the numerical discretization of the Agrawal single-conductor coupling equations [Nucci et al., 1994] the equation that allows to extract a generic current node at time step \( n \), known all variables at time step \( n-1 \), is the following:

\begin{equation}
i_k^n = A_3 \left[ \frac{Eh_k^n + Eh_{k-1}^{n-1}}{2} - \frac{v_{k+1}^n - v_k^n}{\Delta x} + A_4 i_k^{n-1} \right]
\end{equation}

where the sense of the symbol that appear in (A.1) is the same of the paragraph 3.1.2. \( A_3 \) and \( A_4 \) are constants depending on the line inductance \( L' \):

\begin{equation}
A_3 = \left( \frac{L'}{\Delta t} \right)^{-1}; \quad A_4 = \left( \frac{L'}{\Delta t} \right)
\end{equation}
If we write equation (A.1) for nodes \( k \) and \( k+1 \) of Fig. A.1 we obtain the first two equations of the solution system:

\[
\begin{align*}
    i^n_k &= A_n \left[ \frac{E^n_x + E^{n-1}_x}{2} - \frac{v^n_{k+1} - v^n_k}{\Delta x} + A_i^{n-1} \right] \\
    i^n_{k+1} &= A_n \left[ \frac{E^n_{x+1} + E^{n-1}_{x+1}}{2} - \frac{v^n_{k+2} - v^n_{k+1}}{\Delta x} + A_i^{n-1} \right]
\end{align*}
\] (A.3) (A.4)

To obtain the third equation of the solution system we apply the Kirchoff current equation, at the grounding node, for currents \( i^n_{gn} \), \( i^n_k \), \( i^n_{k'} \):

\[
i^n_g = i^n_k - i^n_{k'}
\] (A.5)

Currents \( i^n_k \), \( i^n_{k'} \) can be expressed as a function of the adjacent current node assuming the following linear spatial interpolation:

\[
\begin{align*}
    i^n_k &= \frac{3i^n_{k+1} - i^n_{k-1}}{2} \\
    i^n_{k'} &= \frac{3i^n_{k+1} - i^n_{k+2}}{2}
\end{align*}
\] (A.6) (A.7)

Besides, the node voltage \( v^n_{k+1} \) can be expressed as in (3.1):

\[
v^n_{k+1} = \Gamma (i^n_g) + \int_0^h E^n_z(x,z,t)dz
\] (A.8)

By introducing (A.5), (A.6) and (A.7) in (A.8) we obtain the third equation of the solution system:

\[
v^n_{k+1} = \Gamma \left( \frac{3i^n_k - i^n_{k-1} - 3i^n_{k+1} + i^n_{k+2}}{2} \right) + \int_0^h E^n_z(x,z,t)dz
\] (A.9)

**FDTD 1st order multi-conductor line.** The model for a multi-conductor line discontinuity treatment is similar to that of the single-conductor line. We shall consider a discontinuity at one of the conductors of the bundle as a discontinuity of all conductors, in that the conductors which are not connected to ground are considered grounded through an infinite resistance. We can rewrite (A.3), (A.4) and (A.9) for a multi-conductor line as follows (\( j \) is the grounded conductor):
\[
\begin{align*}
[i]_k^n &= [A_3]\left(\frac{[Ex]_k^n + [Ex]_{k-1}^n}{2} - \frac{[v]_{k+1}^n - [v]_k^n}{\Delta x} + [A_4][i]_{k+1}^{n-1}\right) \\
[i]_{k+1}^n &= [A_3]\left(\frac{[Ex]_{k+1}^n + [Ex]_{k}^{n-1}}{2} - \frac{[v]_{k+2}^n - [v]_{k+1}^n}{\Delta x} + [A_4][i]_{k+2}^{n-1}\right) \\
[v]_{k+1}^n &= [\Gamma K]\left(3[i]_k^n - [i]_{k-1}^n - 3[i]_{k+1}^n + [i]_{k+2}^n\right) + \left[\int_0^h E_\varepsilon(x,z,t)dz\right] \\
\end{align*}
\]

where

\[
[i]_k^n = \begin{pmatrix}
i1_k^n \\
i2_k^n \\
i3_k^n \\
\vdots \\
iNC_k^n \\
\end{pmatrix},
[v]_k^n = \begin{pmatrix}
v1_k^n \\
v2_k^n \\
v3_k^n \\
\vdots \\
vNC_k^n \\
\end{pmatrix},
[Ex]_k^n = \begin{pmatrix}
Ex1_k^n \\
Ex2_k^n \\
Ex3_k^n \\
\vdots \\
ExNC_k^n \\
\end{pmatrix}
\]

\[
\int_0^h E_\varepsilon(x,z,t)dz = \begin{pmatrix}
\int_0^{h_1} E_\varepsilon(x,z,t)dz \\
\int_0^{h_2} E_\varepsilon(x,z,t)dz \\
\int_0^{h_3} E_\varepsilon(x,z,t)dz \\
\vdots \\
\int_0^{h_w} E_\varepsilon(x,z,t)dz \\
\end{pmatrix}
\]

\[
[A_3] = \frac{[L_y]^{-1}}{\Delta t}; [A_4] = \frac{[L_y]}{\Delta t}
\]

and matrix \([\Gamma K]\):

\[
[\Gamma K] = \begin{pmatrix}
\Gamma K_{11} & 0 & 0 & | & 0 \\
0 & \Gamma K_{22} & 0 & | & 0 \\
0 & 0 & \Gamma K_{33} & | & 0 \\
0 & 0 & 0 & | & \Gamma K_{NCNC} \\
\end{pmatrix}
\]
Appendix A.2. – Comparison between 1st and 2nd order FDTD integration schemes

A first comparison of the newly proposed FDTD 2nd order integration scheme with the 1st order one has been performed making reference to a 2 km long, 10 m high single-conductor line above a lossy ground shown in Fig. A.2. The ground conductivity is 0.001 S/m and its relative permittivity is 10. The stroke location is at 50 m from the left-end line terminal. The lightning channel base current peak value is 60 kA and its maximum time derivative is 120 kA/ms. The return stroke speed is $1.2 \times 10^8$ m/s. The LEMP is computed adopting the MTL return stroke model and the value of the spatial and temporal steps adopted for the simulations are 10 m and $10^{-8}$ s respectively.

![Fig. A.2. – Line geometry for the comparison between FDTD 1st and 2nd order in presence of lossy ground.](image1)

In Fig. A.3 we show the results calculated both with the 1st order and 2nd order FDTD algorithms. For both cases, we show the results considering the effect of the ground resistivity both in the electromagnetic field and in the calculation of line parameters (‘lossy line’) and the results obtained taking into account ground losses only in the electromagnetic field calculation (‘ideal line’).

A comparison between the two methods has been performed also for the case of a line with surge arresters. To perform these simulations, we have used the LIOV- EMTP96 program (see paragraph 3.2).
Fig. A.3. – Lightning induced overvoltages calculated at three different observation points of the line of Fig. A.2, a) x=0 m, b) x=500 m, c) x=2 km, using 1st and 2nd order FDTD scheme. Field calculation: lossy ground (0.001 S/m), line impedance: ideal line and lossy line.
It can be seen that the wave-shapes computed using the 2nd order FDTD algorithm are less affected by numerical oscillations, especially for observation points approaching the line far-end.

Fig. A.4 shows the geometry of the line used for the simulations and Fig. A.5 shows the numerical results. Again, it can be seen that the proposed 2nd order scheme leads to an improvement of the computed results, in terms of numerical stability.

Fig. A.4. – Line geometry for the comparison between FDTD 1st order and 2nd order in presence of surge arresters, using the developed interface between LIOV and EMTP96.
c) Fig. A.5. – Lightning induced overvoltage at two observation points a) x=0 m, b) x=500 m, c) x=2000 m of Fig. A.4. Comparison between FDTD 1st order and 2nd order in presence of surge arresters.

With the presence of non-linear components the above reported comparison show, again, that the proposed 2nd order scheme improve the computed results in terms of numerical stability. Indeed the increase numerically stability is obtained without significant increase in the computation time\(^1\).

---

\(^1\) The calculation of the exciting lightning electromagnetic field representing, for the problem of interest, the bulk of the computation time.