Non-Linear Analysis and Design of Synchronous Bearingless Multiphase Permanent Magnet Machines and Drives

Ph.D. Thesis

of

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# Table of contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Table of contents</td>
<td>I</td>
</tr>
<tr>
<td>Introduction</td>
<td>1</td>
</tr>
<tr>
<td><strong>CHAPTER 1</strong></td>
<td></td>
</tr>
<tr>
<td><strong>TWO-DIMENSIONAL ANALYSIS OF MAGNETIC FIELD DISTRIBUTIONS</strong></td>
<td></td>
</tr>
<tr>
<td><strong>IN THE AIRGAP OF ELECTRICAL MACHINES</strong></td>
<td></td>
</tr>
<tr>
<td>1.1 Introduction</td>
<td>9</td>
</tr>
<tr>
<td>1.2 Analytical methods in literature</td>
<td>10</td>
</tr>
<tr>
<td>1.3 Main assumptions and case study</td>
<td>12</td>
</tr>
<tr>
<td>1.4 Analytical solution of the problem</td>
<td>14</td>
</tr>
<tr>
<td>1.4.1 Analysis in the Region 1</td>
<td>16</td>
</tr>
<tr>
<td>1.4.2 Analysis in the Region 2</td>
<td>18</td>
</tr>
<tr>
<td>1.4.3 Common boundary conditions</td>
<td>19</td>
</tr>
<tr>
<td>1.5 Current sheet distribution of the magnets</td>
<td>23</td>
</tr>
<tr>
<td>1.6 Conclusion</td>
<td>27</td>
</tr>
<tr>
<td>1.7 References</td>
<td>28</td>
</tr>
</tbody>
</table>
# CHAPTER 2

**AN ALGORITHM FOR NON-LINEAR ANALYSIS OF MULTIPHASE BEARINGLESS SURFACE-MOUNTED PM SYNCHRONOUS MACHINES**

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Introduction</td>
<td>29</td>
</tr>
<tr>
<td>2.2</td>
<td>The magnetic circuit model</td>
<td>34</td>
</tr>
<tr>
<td>2.2.1</td>
<td>Analytical models of the reluctances</td>
<td>35</td>
</tr>
<tr>
<td>2.3</td>
<td>The numerical solving process</td>
<td>38</td>
</tr>
<tr>
<td>2.4</td>
<td>Simulating the movement</td>
<td>44</td>
</tr>
<tr>
<td>2.5</td>
<td>Co-energy, torque and radial forces</td>
<td>47</td>
</tr>
<tr>
<td>2.6</td>
<td>Results and comparison with FEA software</td>
<td>49</td>
</tr>
<tr>
<td>2.6.1</td>
<td>Machine A</td>
<td>51</td>
</tr>
<tr>
<td>2.6.2</td>
<td>Machine B</td>
<td>53</td>
</tr>
<tr>
<td>2.7</td>
<td>Conclusion</td>
<td>57</td>
</tr>
<tr>
<td>2.8</td>
<td>References</td>
<td>58</td>
</tr>
</tbody>
</table>

## Appendix A2.1

The Programming Code - Part 1

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>A2.1.1</td>
<td>The main program</td>
<td>61</td>
</tr>
</tbody>
</table>

## Appendix A2.2

The Programming Code - Part 2

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>A2.2.1</td>
<td>The MMF array (simplified version)</td>
<td>91</td>
</tr>
<tr>
<td>A2.2.2</td>
<td>The MMF array (original version)</td>
<td>94</td>
</tr>
</tbody>
</table>

# CHAPTER 3

**PRINCIPLES OF BEARINGLESS MACHINES**

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>Introduction</td>
<td>99</td>
</tr>
</tbody>
</table>
### 3.2 General principles of magnetic force generation

3.3 Bearingless machines with a dual set of windings

3.4 Bearingless machines with a single set of windings

3.5 Rotor eccentricity

3.6 Conclusion

3.7 References

### CHAPTER 4

**AN ANALYTICAL METHOD FOR CALCULATING THE DISTRIBUTION OF FORCES IN A BEARINGLESS MULTIPHASE SURFACE-MOUNTED PM SYNCHRONOUS MACHINE**

4.1 Introduction

4.2 Definition of variables

4.3 Analysis of flux density distribution in the airgap

4.4 Calculation of the force acting on the rotor

4.4.1 Normal components of the force

4.4.2 Tangential components of the force

4.4.3 Projections of the tangential force

4.5 Simulations and results

4.6 Conclusion

4.7 References

### Appendix A4.1

**Magnetic field distribution in the airgap of multiphase electrical machines**

A4.1.1 Introduction

A4.1.2 The multiphase rotating magnetic field
CHAPTER 5
DESIGN AND DEVELOPMENT OF A CONTROL SYSTEM FOR MULTIPHASE SYNCHRONOUS PERMANENT MAGNET BEARINGLESS MACHINES

5.1 Introduction ............................................................................................................. 199
5.2 Mechanical equations ............................................................................................ 201
5.3 General structure of the control system ................................................................. 206
5.4 Detailed analysis of the control system ................................................................. 211
   5.4.1 Levitation forces block ..................................................................................... 211
         A. Position errors .................................................................................................. 211
         B. PID controllers ................................................................................................. 211
         C. Force Controller block ..................................................................................... 212
         D. Electromagnetic model block .......................................................................... 216
         E. Forces to moments matrix block........................................................................ 227
   5.4.2 Euler’s equations block .................................................................................... 228
         A. Applied moments block .................................................................................... 228
         B. Euler’s equations block .................................................................................... 229
5.5 The setting of PID controllers ................................................................................ 230
   A. Equilibrium along the y-axis ................................................................................. 230
   B. Equilibrium along the z-axis ................................................................................. 231
5.6 Simulations and results .......................................................................................... 235
5.7 Conclusion .............................................................................................................. 252

Conclusion ...................................................................................................................... 255
List of papers .................................................................................................................. 259
Introduction

The use of multiphase motors over conventional three-phase motors gives a series of benefits that can be summarized as follows: possibility of dividing the power between multiple phases, higher reliability in case of failure of a phase, use of various harmonic orders of airgap magnetic field to obtain better performances in terms of electromagnetic torque and possibility to create multi-motor drives by connecting several machines in series controlled by a single power converter [1]-[9]. These features are appreciated when high power, high reliability and low dc bus voltage are requested as it happens in ship propulsion, electrical vehicles and aerospace applications. In recent years, suitable techniques have been applied in order to reduce the power losses in multiphase IGBT inverters [10].

Bearingless motors are spreading because of their capability of producing rotor suspension force and torque avoiding the use of mechanical bearings and achieving in this way much higher maximum speed. There are two typologies of winding configurations: dual set and single set of windings. The first category comprises two separated groups of three-phase windings, with a difference in their pole pair numbers equal to one: the main one carries the ‘motor currents’ for driving the rotor, while the other carries the ‘levitation currents’, to suspend the rotor [11].

The windings belonging to the latter category produce torque and radial forces by means of injecting different current sequences to give odd and even harmonic orders of magnetic field, using the properties of multiphase current systems, which have multiple orthogonal d-q planes. One of them can be used to
control the torque. The additional degrees of freedom can be used to produce levitation forces [12].

The main advantages of bearingless motors with a single set of windings (i.e., the assets of bearingless and multiphase motors together) lead to a simpler construction process, better performances in control strategy and torque production with relatively low power losses [13]. This kind of technology is expected to have very large developments in the future, particularly in the design of high power density generators, actuators and motors of More Electric Aircraft (MEA), mainly for the ability of achieving higher speed in comparison to conventional electrical machines [14]. In addition, it can be supposed that the cooperation of bearingless control techniques and the adoption of magnetic bearing could be of large interest in the MEA field.

An important target in the design of electrical machines is the analysis and comparison of a large number of solutions, spending less time than is possible but also providing an accurate description of electromagnetic phenomena. The main problems are related to the calculation of global and local quantities like linkage fluxes, output torque, flux densities in various areas of the device. The difficulties increase especially in presence of magnetic saturation, in fact in the case of non-linear magnetic problems it would be necessary to provide in-depth analyses by using complex software based on accurate analytical methods, like Finite Element Analysis (FEA). Simultaneously, it would be useful to save time, not only in terms of reducing computing time, but mainly for the need of re-designing the model of the machine in a CAD interface when changing some electrical or geometrical parameter. In order to solve this problems, some authors present analyses based on equivalent magnetic or lumped parameter circuit models [15], [16], [17].

In this thesis, a method for non-linear analysis and design of Surface-Mounted Permanent Magnet Synchronous Motors (SPMSM) is presented. The
relevant edge consists in the possibility of defining the machine characteristics in a simple user interface. Then, by duplicating an elementary cell, it is possible to construct and analyze whatever typology of windings and ampere-turns distribution in a pole-pair. Furthermore, it is possible to modify the magnet width-to-pole pitch ratio analyzing various configurations or simulating the rotor movement in sinusoidal multiphase drives or in a user-defined current distribution. Previous papers proposed the analysis of open-slot configurations with prefixed structure of the motor, with given number of poles and slots, or for only a particular position of the rotor with respect to the stator. The performances of the proposed non-linear model of SPMSM have been compared with those obtained by FEA software in terms of linkage fluxes, co-energy, torque and radial force. The obtained results for a traditional three-phase machine and for a 5-phase machine with unconventional winding distribution showed that the values of local and global quantities are practically coinciding, for values of the stator currents up to rated values. In addition, they are very similar also in the non-linear behavior even if very large current values are injected.

When developing a new machine design the proposed method is useful not only for the reduction of computing time, but mainly for the simplicity of changing the values of the design variables, being the numerical inputs of the problem obtained by changing some critical parameters, without the need for redesigning the model. For a given rotor position and for given stator currents, the output torque as well as the radial forces acting on the moving part of a multiphase machine can be calculated. The latter feature makes the algorithm particularly suitable in order to design and analyze bearingless machines. For these reasons, it constitutes a useful tool for the design of a bearingless multiphase synchronous PM machines control system.

Another important section of this thesis concerns an analytical model for radial forces calculation in multiphase bearingless SPMSM. It allows to predict
amplitude and direction of the force, depending on the values of the torque current, of the levitation current and of the rotor position. It is based on the space vectors method, letting the analysis of the machine not only in steady-state conditions but also during transients.

When designing a control system for bearingless machines, it is usual to consider only the interaction between the main harmonic orders of the stator and rotor magnetic fields. In multiphase machines, this can produce mistakes in determining both the module and the spatial phase of the radial force, due to the interactions between the higher harmonic orders. The presented algorithm allows to calculate these errors, taking into account all the possible interactions; by representing the locus of radial force vector, it allows the appropriate corrections.

In addition, the algorithm permits to study whatever configuration of SPMSM machine, being parameterized as a function of the electrical and geometrical quantities, as the coil pitch, the width and length of the magnets, the rotor position, the amplitude and phase of current space vector, etc.

The design of a control system for bearingless machines constitutes another contribution of this thesis. It implements the above presented analytical model, taking into account all the possible interactions between harmonic orders of the magnetic fields to produce radial force and provides in this way an accurate electromagnetic model of the machine.

This latter is part of a three-dimensional mechanical model where one end of the motor shaft is constrained, to simulate the presence of a mechanical bearing, while the other is free, only supported by the radial forces developed in the interactions between magnetic fields, to simulate a bearingless system with three degrees of freedom. The complete model represents the design of the experimental system to be realized in the laboratory.
References


Introduction


Chapter 1

**TWO-DIMENSIONAL ANALYSIS OF MAGNETIC FIELD DISTRIBUTIONS IN THE AIRGAP OF ELECTRICAL MACHINES**

1.1 Introduction

The aim of this chapter is the development of a method proposed in literature [1] to study the distributions of the magnetic vector potential, magnetic field and flux density in the airgap of axial flux permanent magnet electrical machines by applying a two-dimensional model. With respect to [1], the contribution of this chapter consists in the execution of the complete calculations, not reported in the original work, to get the solution of the problem. They were conducted by using the techniques of mathematical analysis applied to physical
and engineering problems, with particular reference to [2].

In the origin, the method has been applied to the design of axial flux PM machines, but it can be generalized to the analysis of any typology of electrical machine in the case of neglecting slotting effects and with the assumption of developing the machine linearly in correspondence of the mean airgap radius.

1.2 Analytical methods in literature

The works [3]-[6] represent a series of papers for a complete 2-d analysis of the magnetic field distribution in brushless PM radial-field machines. In [3] is presented an analytical method for determining the open-circuit airgap field distribution in the internal and external rotor typologies. The solution is given by the governing field equations in polar coordinates applied to the annular magnets and airgap regions of a multi-pole slotless motor, with an uniform radial magnetization in the magnets.

In [4] the analysis is conducted to determine the armature reaction field produced by a 3-phase stator currents and to take into account the effect of winding current harmonic orders on the airgap field distribution.

In [5], the method developed in [3], [4] is integrated with a model to predict the effect of stator slotting on the magnetic field distribution, using a 2-d permeance function which realizes a much higher accuracy than the conventional 1-dimensional models.

Finally, [6] presents a model to analyze the load operating conditions of the motor, by combining the armature reaction field component with the open-circuit field component produced by the magnets, studied in [3]. All the cases [3]-[6] were compared with the results of FE analysis, showing an excellent agreement.

The paper [7] presents an analytical method to study magnetic fields in permanent-magnet brushless motors, taking into consideration the effect of stator
slotting, by studying the magnetic field distribution in the situations where the magnet passes over the slot opening. In such situations it is difficult to interpret the correct method for determining, with the properly accuracy, the flux density distribution and, consequently, the magnetic forces and cogging torque.

In [8] the effects of slotting in a brushless dc motor (BLDCM) are determined by calculating the airgap permeance distribution using the Schwarz-Christoffel transformation. The analytical calculations of no-load air-gap magnetic field distribution, armature field distribution, and phase electromotive force, are implemented. Then, a three-phase circuital model is realized for determining the phase current waveforms and the instantaneous magnetic field distribution in load conditions, during the actual operations of the drive. The computation of electromagnetic torque and the analysis of torque ripple complete the features of the algorithm.

The paper [9] presents a method for the accurate calculation of magnetic field distribution in the motors with big airgap, by means of the magnetic potential superimposed calculation, since in the examined case the computing error resulting by conventional formulas can’t be neglected as happens in the small airgap machines.

In [10] a general analytical method to predict the magnetic field distribution in surface-mounted brushless permanent magnet machines is presented, considering a two-dimensional model in polar coordinates which solves the Laplacian equations in the airgap and magnets areas, with no constraints about the recoil permeability of the magnets. The analysis is applicable to internal/external rotor typologies, to radial/parallel magnetization of the magnets, to slotless/slotted motors.
1.3 Main assumptions and case study

In the following, the main assumptions of the case study are presented:

I) The permeability of iron is infinite;

II) The considered model is a slotless machine, or a slotted one with slot-openings supposed of infinitesimal width, so that the slotting effects are negligible;

III) In correspondence of the rotor and stator boundary surfaces, the magnetic field lines have only normal component;

IV) The mean airgap radius is assumed infinite, so that the airgap path can be considered as having a linear development, ignoring curvature;

V) Extremity effects are neglected.

VI) The effects of the leakage fluxes are neglected.

The Ampere-turns distributions are analyzed by means of the current sheet technique; the innovative aspect of the analysis, presented in [1], is the process of solving the electromagnetic problem depending on a generalized current distribution, whatever be the generating source, and then applying to the general solution the current-sheet related to the particular case study (ampere-turns distribution of the stator, equivalent distribution of the magnets, etc.). Consider the 2-D model presented in Fig. 1.1:
The lower surface, placed at \( y = 0 \), represents the rotor iron; the higher one, placed at \( y = Y_2 \), represents the stator iron. A generalized current sheet distribution, given by \( K_n(x) = \hat{K}_n \sin(ux) \), is placed at \( y = Y_1 \) coordinate: this parameter can be assumed as a variable height, dividing the airgap in two areas and determining different solutions of the magnetic vector potential in everyone of them. In this way, the current sheet \( K_n(x) \) can be considered in one case the stator current distribution (in the presented example, by substituting \( Y_1 = Y_2 \)) in the other case the equivalent ampere-turns distribution produced by rotor magnets (in the presented example, by substituting \( Y_1 = Y_m \), being this latter the magnet height). So, it is possible firstly to solve the problem for a generalized distribution and then to apply it to the particular case to represent.
1.4 Analytical solution of the problem

Consider by assumption that the magnetic vector potential $\vec{A}$ has the only non-zero component $A_z$, not dependent on $z$-coordinate (i.e., the analysis is carried out by operating on $xy$-planes where all the magnetic and electrical quantities are supposed invariant with respect to the $z$-axis). With these assumptions, the Laplace operator $\nabla^2 A$ can be written as:

$$\vec{A} = A_z(x,y)\hat{k}$$  \hspace{1cm} (1.1)

$$\nabla^2 \vec{A} = \nabla^2 A_z = \nabla \cdot (\nabla A_z) = \left( \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \right) \left( \frac{\partial A_z}{\partial x} \hat{i} + \frac{\partial A_z}{\partial y} \hat{j} + \frac{\partial A_z}{\partial z} \hat{k} \right) = \frac{\partial^2 A_z}{\partial x^2} + \frac{\partial^2 A_z}{\partial y^2}$$  \hspace{1cm} (1.2)

The $x$- and $y$-components of flux density and magnetic field distributions can be determined as:

$$\vec{B} = \nabla \times \vec{A} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & A_z \end{bmatrix} = \frac{\partial A_z}{\partial y} \hat{i} + \left( -\frac{\partial A_z}{\partial x} \right) \hat{j}$$  \hspace{1cm} (1.3)

$$B_x = \frac{\partial A_z}{\partial y}, \quad B_y = -\frac{\partial A_z}{\partial x}, \quad H_x = \frac{B_x}{\mu_0} = \frac{1}{\mu_0} \frac{\partial A_z}{\partial y}, \quad H_y = \frac{B_y}{\mu_0} = -\frac{1}{\mu_0} \frac{\partial A_z}{\partial x}$$  \hspace{1cm} (1.4)

The equation to be solved in the domain of study (1.5), with its related boundary conditions (1.6), is the characteristic Laplace’s equation considered in a two-dimensional domain:

$$\nabla^2 \vec{A} = 0 \quad \Rightarrow \quad \frac{\partial^2 A_z}{\partial x^2} + \frac{\partial^2 A_z}{\partial y^2} = 0$$  \hspace{1cm} (1.5)
Two-dimensional analysis of magnetic field distributions in the airgap of electrical machines

1) \( H_{x1}(y = 0) = 0 \)
2) \( H_{x2}(y = Y_2) = 0 \)
3) \( H_{x2}(y = Y_1) - H_{x1}(y = Y_1) = K_n(x) \)  
4) \( H_{y2}(y = Y_1) = H_{y1}(y = Y_1) \)  

(1.6)

Since the boundary conditions are homogeneous, it is possible to apply the method of separation of variables. Let us assume, therefore, that \( A_z \) is of the form:

\[
A_z = X(x)Y(y) \quad \Rightarrow \quad \nabla^2 A_z = Y(y)\frac{\partial^2 X(x)}{\partial x^2} + X(x)\frac{\partial^2 Y(y)}{\partial y^2} = 0
\]

(1.7)

hence, multiplying both sides by \( \frac{1}{X(x)Y(y)} \) and rewriting the second derivatives in a different way for brevity, we obtain:

\[
\frac{1}{X(x)}\frac{\partial^2 X(x)}{\partial x^2} + \frac{1}{Y(y)}\frac{\partial^2 Y(y)}{\partial y^2} = 0 \quad \Rightarrow \quad \frac{1}{X(x)} X^{xx}(x) + \frac{1}{Y(y)} Y^{yy}(y) = 0
\]

(1.8)

By isolating in different members the terms respectively dependent on \( x \) and \( y \) we obtain:

\[
\frac{Y^{yy}(y)}{Y(y)} = -\frac{X^{xx}(x)}{X(x)}
\]

(1.9)

Note that the members of the equation are absolutely independent from each other, since the first one is a function of the variable \( x \) only, the second one of the \( y \) only: having to be equivalent for any value assumed by the two variables, it is deduced that they have to be both equal to a constant term, which we define as \( u^2 \), assumed positive. By further developing the calculations, two separate differential equations are obtained, each one as a function of a single variable:

\[
-\frac{X^{xx}(x)}{X(x)} = u^2 \quad \Rightarrow \quad X^{xx}(x) + u^2 X(x) = 0
\]

(1.10)
\[
\frac{Y_{yy}(y)}{Y(y)} = u^2 \quad \Rightarrow \quad Y_{yy}(y) - u^2Y(y) = 0
\]  \hspace{1cm} (1.11)

Are obtained from (1.10), (1.11) the respective characteristic equations and their solutions:

\[
p^2 + u^2 = 0 \quad \Rightarrow \quad p_{1,2} = \pm \sqrt{j^2 u^2} = \pm ju
\]  \hspace{1cm} (1.12)

\[
q^2 - u^2 = 0 \quad \Rightarrow \quad q_{1,2} = \pm \sqrt{u^2} = u
\]  \hspace{1cm} (1.13)

Recalling the general expression of the solutions associated with the characteristic equations (1.12), (1.13):

\[
X(x) = A_x e^{jux} + B_x e^{-jux}
\]  \hspace{1cm} (1.14)

\[
Y(y) = A_y e^{uy} + B_y e^{-uy}
\]  \hspace{1cm} (1.15)

where \(A_x, B_x, A_y, B_y\), are constant terms to be evaluated using the boundary conditions. Recalling (1.7) is possible to write:

\[
A_z(x, y) = X(x)Y(y) = \left(A_x e^{jux} + B_x e^{-jux}\right)\left(A_y e^{uy} + B_y e^{-uy}\right)
\]  \hspace{1cm} (1.16)

By introducing the Euler’s formulas, presented in the follows:

\[
e^{jux} = \cos(ux) + j \sin(ux), \quad e^{-jux} = \cos(ux) - j \sin(ux)
\]  \hspace{1cm} (1.17)

\[
e^{uy} = \cosh(uy) + \sinh(uy), \quad e^{-uy} = \cosh(uy) - \sinh(uy)
\]  \hspace{1cm} (1.18)

and using (1.17) and (1.18) in (1.16), the general solution can be expressed in a trigonometric form:

\[
A_z(x, y) = X(x)Y(y) = \left(A \sin(ux) + B \cos(ux)\right)\left[C \sinh(uy) + D \cosh(uy)\right]
\]  \hspace{1cm} (1.19)

**1.4.1 Analysis in the Region 1**

Assume for region 1 the following general expression for the magnetic vector
Two-dimensional analysis of magnetic field distributions in the airgap of electrical machines

potential:

\[ A_{z1}(x,y) = X(x)Y(y) = \left[ A_1 \sin(ux) + B_1 \cos(ux) \right] \left[ A_2 \sinh(uy) + B_2 \cosh(uy) \right] \]

(1.20)

The value of the \( x \)-component of the magnetic field in the lower boundary of the region 1, leads to the first boundary condition:

\[ H_{x1}(y = 0) = 0 \]

(1.21)

\[ H_{x1}(x,y) = \frac{1}{\mu_0} \frac{\partial A_{z1}}{\partial y} = \frac{u}{\mu_0} \left[ A_1 \sin(ux) + B_1 \cos(ux) \right] \left[ A_2 \cosh(uy) + B_2 \sinh(uy) \right] \]

(1.22)

By applying the condition (1.21) in (1.22) and considering that the equation has to be verified for any value of \( x \) and \( y \), we obtain (1.23):

\[ H_{x1}(y = 0) = \frac{u}{\mu_0} \left[ A_1 \sin(ux) + B_1 \cos(ux) \right] A_2 = 0 \quad \Rightarrow \quad A_2 = 0 \]

(1.23)

By substituting the result of (1.23) into (1.20):

\[ A_{z1}(x,y) = B_2 \cosh(uy) \left[ A_1 \sin(ux) + B_1 \cos(ux) \right] \]

(1.24)

where, defining the constant terms \( k_1 = B_2 A_1 \) and \( k_2 = B_2 B_1 \):

\[ A_{z1}(x,y) = \cosh(uy) \left[ k_1 \sin(ux) + k_2 \cos(ux) \right] \]

(1.25)

By substituting the result of (1.23) into (1.22):

\[ H_{x1}(x,y) = \frac{u}{\mu_0} \sinh(uy) \left[ k_1 \sin(ux) + k_2 \cos(ux) \right] \]

(1.26)

By executing similar calculations is possible to write the \( H_{y1} \) component of the magnetic field as:

\[ H_{y1}(x,y) = -\frac{1}{\mu_0} \frac{\partial A_{z1}}{\partial x} = \frac{u}{\mu_0} \cosh(uy) \left[ k_2 \sin(ux) - k_1 \cos(ux) \right] \]

(1.27)
1.4.2 Analysis in the Region 2

Assume for region 2 the following general expression for the magnetic vector potential:

\[ A_{z2}(x,y) = X(x)Y(y) = \left[ C_1 \sin(ux) + D_1 \cos(ux) \right] \left[ C_2 \sinh(uy) + D_2 \cosh(uy) \right] \]  

(1.28)

The value of the \( x \)-component of the magnetic field in the higher boundary of the region 2, leads to the second boundary condition:

\[ H_{x2}(y = Y_2) = 0 \]  

(1.29)

\[ H_{x2}(x,y) = \frac{1}{\mu_0} \frac{\partial A_{z2}}{\partial y} = \frac{u}{\mu_0} \left[ C_1 \sin(ux) + D_1 \cos(ux) \right] \left[ C_2 \cosh(uy) + D_2 \sinh(uy) \right] \]  

(1.30)

By applying the condition (1.29) in (1.30) and considering that the equation has to be valid for any value of \( x \) and \( u \), we obtain (1.31), (1.32):

\[ H_{x2}(y = Y_2) = \frac{u}{\mu_0} \left[ C_1 \sin(ux) + D_1 \cos(ux) \right] \left[ C_2 \cosh(uy) + D_2 \sinh(uy) \right] = 0 \]  

(1.31)

\[ C_2 \cosh(uy)Y_2 + D_2 \sinh(uy)Y_2 = 0 \quad \Rightarrow \quad D_2 = -C_2 \coth(uy) \]  

(1.32)

By substituting (1.32) in (1.28):

\[ A_{z2}(x,y) = C_2 \left[ C_1 \sin(ux) + D_1 \cos(ux) \right] \left[ \sinh(uy) - \cosh(uy) \coth(uy) \right] \]  

(1.33)

where, in a similar way to what was done for the region 1, by introducing the constant terms \( k_3 = C_1 C_2 \) and \( k_4 = D_1 C_2 \), it gives:

\[ A_{z2}(x,y) = \left[ k_3 \sin(ux) + k_4 \cos(ux) \right] \left[ \sinh(uy) - \cosh(uy) \coth(uy) \right] \]  

(1.34)
Two-dimensional analysis of magnetic field distributions in the airgap of electrical machines

which can be written, by explicating $coth(uY_2)$, as:

$$A_{z2}(x,y) = [k_3 \sin(ux) + k_4 \cos(ux)] \left[ \frac{\sinh(uy) \sinh(uY_2) - \cosh(uy) \cosh(uY_2)}{\sinh(uY_2)} \right]$$

(1.35)

By considering that:

$$\cosh u(Y_2 - y) = \cosh(uY_2 - uy) = \cosh(uY_2) \cosh(uy) - \sinh(uY_2) \sinh(uy)$$

(1.36)

The relationship (1.35) can be simplified as in (1.37):

$$A_{z2}(x,y) = -[k_3 \sin(ux) + k_4 \cos(ux)] \frac{\cosh u(Y_2 - y)}{\sinh(uY_2)}$$

(1.37)

and, by means of (1.38), the related components of the magnetic field in region 2 can be calculated as in (1.39), (1.40):

$$H_{x2}(x,y) = \frac{1}{\mu_0} \frac{\partial A_{z2}}{\partial y}, \quad H_{y2}(x,y) = -\frac{1}{\mu_0} \frac{\partial A_{z2}}{\partial x}$$

(1.38)

$$H_{x2}(x,y) = \frac{u}{\mu_0} [k_3 \sin(ux) + k_4 \cos(ux)] \frac{\sinh u(Y_2 - y)}{\sinh(uY_2)}$$

(1.39)

$$H_{y2}(x,y) = \frac{u}{\mu_0} [k_3 \cos(ux) - k_4 \sin(ux)] \frac{\cosh u(Y_2 - y)}{\sinh(uY_2)}$$

(1.40)

1.4.3 Common boundary conditions

Considering a current sheet described by means of an harmonic distribution given in the generic form:

$$K_n(x) = \hat{K}_n \sin(ux)$$

(1.41)

where $\hat{K}_n$ depends on the actual current distribution and has to be evaluated in
any particular considered case, while $u$ is defined as follows:

$$u = \frac{\pi}{\tau_p} \quad (1.42)$$

The discontinuity between the $x$-component values of the magnetic field in the current sheet region, leads to the third boundary condition:

$$H_{x2}(y = Y_1) - H_{x1}(y = Y_1) = K_n(x) \quad (1.43)$$

which can be expressed by calculating (1.26) and (1.39) in correspondence of the particular value $y = Y_1$. By substituting them in (1.43) it gives:

$$\frac{u}{\mu_0} [k_3 \sin(u x) + k_4 \cos(u x)] \left[ \frac{\sinh(u Y_2 - Y_1)}{\sinh(u Y_2)} \right] - \frac{u}{\mu_0} [k_1 \sin(u x) + k_2 \cos(u x)] \sinh(u Y_1) = \hat{K}_n \sin(u x) \quad (1.44)$$

By collecting the common terms in (1.44):

$$\sin(u x) \left[ k_3 \frac{\sinh(u Y_2 - Y_1)}{\sinh(u Y_2)} - k_1 \sinh(u Y_1) - \frac{\mu_0 \hat{K}_n}{u} \right] +$$

$$\sin(u x) \left[ k_4 \frac{\sinh(u Y_2 - Y_1)}{\sinh(u Y_2)} - k_2 \sinh(u Y_1) \right] = 0 \quad (1.45)$$

Note that (1.45) has to be verified for any value of $u$ and $x$, so the only possibility is that both the coefficients of $\sin(u x)$ and $\cos(u x)$ are equal to zero:

$$k_3 \frac{\sinh(u Y_2 - Y_1)}{\sinh(u Y_2)} - k_1 \sinh(u Y_1) - \frac{\mu_0 \hat{K}_n}{u} = 0 \quad (1.46)$$

$$k_4 \frac{\sinh(u Y_2 - Y_1)}{\sinh(u Y_2)} - k_2 \sinh(u Y_1) = 0 \quad (1.47)$$

After a few steps (1.46) and (1.47) give, respectively, the relations $k_3 = f(k_1)$ and $k_4 = f(k_2)$:
Two-dimensional analysis of magnetic field distributions in the airgap of electrical machines

\[ k_3 = \frac{\sinh(uY_2)\left[\mu_0 \hat{K}_n + uk_1 \sinh(uY_1)\right]}{u \sinh u(Y_2 - Y_1)} \quad (1.48) \]

\[ k_4 = \frac{\sinh(uY_1) \sinh(uY_2)}{\sinh u(Y_2 - Y_1)} \quad (1.49) \]

Putting (1.48) and (1.49) in (1.40):

\[ H_{y2}(x, y) = \frac{ucosh u(Y_2 - y)}{\mu_0} \left[ \frac{\mu_0 \hat{K}_n + uk_1 \sinh(uY_1)}{u \sinh u(Y_2 - Y_1)} \cos(ux) + \right. \]
\[ \left. - \frac{\sinh(uY_1)}{\sinh u(Y_2 - Y_1)} k_2 \sin(ux) \right] \quad (1.50) \]

The continuity between the \( y \)-components of the magnetic field in the current sheet region, leads to the fourth boundary condition:

\[ H_{y2}(y = Y_1) = H_{y1}(y = Y_1) \quad (1.51) \]

By calculating (1.50) and (1.27) in \( y = Y_1 \) it respectively gives (1.52) and (1.53); by substituting them in (1.51), it gives (1.54):

\[ H_{y2}(y = Y_1) = \frac{ucosh u(Y_2 - Y_1)}{\mu_0} \left[ \frac{\mu_0 \hat{K}_n + uk_1 \sinh(uY_1)}{u \sinh u(Y_2 - Y_1)} \cos(ux) + \right. \]
\[ \left. - \frac{\sinh(uY_1)}{\sinh u(Y_2 - Y_1)} k_2 \sin(ux) \right] \quad (1.52) \]

\[ H_{y1}(y = Y_1) = \frac{u}{\mu_0} \cosh(uY_1) \left[ k_2 \sin(ux) - k_1 \cos(ux) \right] \quad (1.53) \]

\[ \frac{ucosh u(Y_2 - Y_1)}{\mu_0} \left[ \frac{\mu_0 \hat{K}_n + uk_1 \sinh(uY_1)}{u \sinh u(Y_2 - Y_1)} \cos(ux) - \frac{\sinh(uY_1)}{\sinh u(Y_2 - Y_1)} k_2 \sin(ux) \right] = \]
\[ = \frac{u}{\mu_0} \cosh(uY_1) \left[ k_2 \sin(ux) - k_1 \cos(ux) \right] \quad (1.54) \]

By collecting common terms in (1.54):
\[
\begin{align*}
\left[ \mu_0 \kappa_n + u k_1 \sinh(u Y_1) \right] \coth u (Y_2 - Y_1) + k_1 \frac{u}{\mu_0} \cosh(u Y_1) \cos(\mu x) & = \\
\left[ u k_2 \sinh(u Y_1) \coth u (Y_2 - Y_1) + \frac{u}{\mu_0} \cosh(u Y_1) k_2 \right] \sin(\mu x)
\end{align*}
\]

(1.55)

As seen before, (1.55) has to be verified for any value of \( u \) and \( x \), so the only possibility is that both the coefficients of \( \sin(\mu x) \) and \( \cos(\mu x) \) are equal to zero.

From (1.55) the equation (1.56):

\[
\kappa_n \coth u (Y_2 - Y_1) + k_1 \frac{u \sinh(u Y_1)}{\mu_0} \coth u (Y_2 - Y_1) + k_1 \frac{u}{\mu_0} \cosh(u Y_1) = 0
\]

(1.56)

which results after a few steps in (1.57):

\[
k_1 = -\frac{\mu_0 \kappa_n}{u} \frac{\coth u (Y_2 - Y_1)}{\sinh(u Y_1) \coth u (Y_2 - Y_1) + \cosh(u Y_1)}
\]

(1.57)

and also the equation (1.58):

\[
k_2 \left[ u \sinh(u Y_1) \frac{\coth u (Y_2 - Y_1)}{\mu_0} + \frac{u}{\mu_0} \cosh(u Y_1) \right] = 0
\]

(1.58)

which results in (1.59):

\[
k_2 = 0
\]

(1.59)

Note that (1.57) can be simplified, by simplifying the term \( \coth u (Y_2 - Y_1) \) in the numerator and denominator. After a few steps, it gives:

\[
k_1 = -\frac{\mu_0 \kappa_n}{u} \frac{\cosh u (Y_2 - Y_1)}{\sinh(u Y_2)}
\]

(1.60)

By substituting (1.60) in (1.48) and performing some similar calculations, a simplified form for \( k_3 \) can be obtained:

\[
k_3 = \frac{\mu_0 \kappa_n}{u} \cosh (u Y_1)
\]

(1.61)
Finally, using the result of (1.59) in (1.49), it gives:

\[ k_4 = 0 \]  \hspace{1cm} (1.62)

All the coefficients are now known; thus, is possible to determine the expression of magnetic vector potential and of magnetic field in the regions of the machine. By substituting (1.59) and (1.60) in (1.25), it immediately gives:

\[ A_{z1}(x, y) = -\frac{\mu_0 K_n}{u} \frac{\cosh u(Y_2 - Y_1)}{\sinh(uY_2)} \sin(ux) \cosh(uy) \]  \hspace{1cm} (1.63)

Similarly, by substituting (1.61) and (1.62) in (1.37):

\[ A_{z2}(x, y) = -\frac{\mu_0 K_n}{u} \frac{\cosh(uY_1)}{\sinh(uY_2)} \sin(ux) \cosh(Y_2 - y) \]  \hspace{1cm} (1.64)

From (1.63) and (1.64) are derived the following relationships (1.65)-(1.68):

\[ H_{x1}(x, y) = \frac{1}{\mu_0} \frac{\partial A_{z1}}{\partial y} = \frac{\hat{K}_n}{u} \frac{\cosh u(Y_2 - Y_1)}{\sinh(uY_2)} \sin(ux) \sinh(uy) \]  \hspace{1cm} (1.65)

\[ H_{y1}(x, y) = -\frac{1}{\mu_0} \frac{\partial A_{z1}}{\partial x} = \frac{\hat{K}_n}{u} \frac{\cosh u(Y_2 - Y_1)}{\sinh(uY_2)} \cos(ux) \cosh(uy) \]  \hspace{1cm} (1.66)

\[ H_{x2}(x, y) = \frac{1}{\mu_0} \frac{\partial A_{z2}}{\partial y} = \frac{\hat{K}_n}{u} \frac{\cosh(uY_1)}{\sinh(uY_2)} \sin(ux) \sinh(Y_2 - y) \]  \hspace{1cm} (1.67)

\[ H_{y2}(x, y) = -\frac{1}{\mu_0} \frac{\partial A_{z2}}{\partial x} = \frac{\hat{K}_n}{u} \frac{\cosh(uY_1)}{\sinh(uY_2)} \cos(ux) \cosh(Y_2 - y) \]  \hspace{1cm} (1.68)

### 1.5 Current sheet distribution of the magnets

As a particular example of a current sheet distribution \( K_n(x) \), will be examined the equivalent current density distribution of the magnets. Each magnet is represented by two current pulses at its edges, assuming to flow in a tending to
zero thickness, having an angular width equal to $2\delta_m$.

$$J(x) = \sum_{n=1,3,5,\ldots}^{\infty} \hat{J}_n \sin(n\theta) = \sum_{n=1,3,5,\ldots}^{\infty} \hat{J}_n \sin\left(n\frac{\pi}{\tau_p} x\right) = \sum_{n=1,3,5,\ldots}^{\infty} \hat{J}_n \sin(ux)$$  \hspace{1cm} (1.69)

The function is represented by means of the Fourier harmonic series distribution, the coefficients of which are calculated in the following:

$$\hat{J}_n = \frac{1}{\pi} \int_{-\pi}^{\pi} j(\theta)\sin(n\theta)d\theta = \frac{2}{\pi} \int_{0}^{\pi} j(\theta)\sin(n\theta)d\theta =$$

$$= \frac{2}{\pi} \int_{\theta_{mp}+\delta_m}^{\theta_{mp}} J\sin(n\theta)d\theta + \frac{2}{\pi} \int_{\pi-\theta_{mp}+\delta_m}^{\pi-\theta_{mp}-\delta_m} J\sin(n\theta)d\theta =$$

$$= \frac{2}{\pi} J \left\{ \cos(n\theta)(\theta_{mp}+\delta_m) + \cos(n\theta)(\pi-\theta_{mp}+\delta_m) \right\} =$$

$$= \frac{2J}{n\pi} \left\{ 2\sin(n\theta_{mp})\sin(n\delta_m) + 2\sin(n\pi-n\theta_{mp})\sin(n\delta_m) \right\} =$$

$$= \frac{4J}{n\pi} \left\{ \sin(n\delta_m)\left[\sin(n\theta_{mp}) + \sin(n\pi-n\theta_{mp}) \right] \right\} =$$

$$= \frac{4J}{n\pi} \left\{ 2\sin(n\delta_m)\sin\left(n\frac{\pi}{2}\right)\cos\left(\frac{2n\theta_{mp}-n\pi}{2}\right) \right\}$$  \hspace{1cm} (1.70)

Considering that $n$ is an odd number, the value of $\cos(n\pi/2)$ in (1.71) is always zero:

$$\cos\left(\frac{2n\theta_{mp}-n\pi}{2}\right) = \cos(n\theta_{mp})\cos\left(n\frac{\pi}{2}\right) + \sin(n\theta_{mp})\sin\left(n\frac{\pi}{2}\right) = \sin(n\theta_{mp})\sin\left(n\frac{\pi}{2}\right)$$  \hspace{1cm} (1.71)

By substituting the result of (1.71) in (1.70), it gives:

$$\hat{J}_n = \frac{8J}{n\pi} \sin(n\delta_m)\sin^2\left(n\frac{\pi}{2}\right)\sin(n\theta_{mp}) = \frac{8J}{n\pi} \sin(n\delta_m)\sin(n\theta_{mp})$$  \hspace{1cm} (1.72)

The equivalent surface current density related to the magnets is expressed by (1.73):
Two-dimensional analysis of magnetic field distributions in the airgap of electrical machines

\[ J = \frac{H \pi}{2 \delta_m \tau_p} = \frac{B_{\text{rem}}}{\mu_m} \frac{\pi}{2 \delta_m \tau_p} \left[ \frac{A}{m^2} \right] \]  

(1.73)

By substituting (1.73) in (1.72), it is also necessary to calculate the limit as \( \delta_m \) tends to zero, considering every edge of the magnet as a current pulse:

\[ J_n = \lim_{\delta_m \to 0} \frac{8 B_{\text{rem}}}{\mu_m} \frac{\pi}{2 \tau_p} \frac{\sin(n\delta_m)}{n\delta_m} \sin(n\theta_{mp}) \]  

(1.74)

Being:

\[ \lim_{\delta_m \to 0} \frac{\sin(n\delta_m)}{n\delta_m} = 1 \quad \forall n \]  

(1.75)

It results:

\[ J_n = \frac{4 B_{\text{rem}}}{\mu_m \tau_p} \sin(n\theta_{mp}) = \frac{4 B_{\text{rem}}}{\mu_m \tau_p} \sin\left(n \frac{\pi}{2 \tau_p} \right) \]  

(1.76)

By substituting the relationship (1.76) in (1.69), it gives:

\[ J(x) = \sum_{n=1,3,5,..}^{\infty} \frac{4 B_{\text{rem}}}{\mu_m \tau_p} \sin\left(n \frac{\pi}{2 \tau_p} \right) \sin(n\theta) \]  

(1.77)

Considering that:

\[ u = \frac{\pi}{\tau_p} n \quad \Rightarrow \quad n\theta = \left( n \frac{\pi}{\tau_p} \right) x = ux \]  

(1.78)

By substituting (1.78) in (1.77), it results:

\[ J(x) = \sum_{n=1,3,5,..}^{\infty} \frac{4 B_{\text{rem}}}{\mu_m \tau_p} \sin\left(n \frac{\pi}{2 \tau_p} \right) \sin(ux) \]  

(1.79)

To define the function of distribution \( K_n(x) \), it is important to note that the magnets are constituted by a succession of current sheets, each one of
Chapter 1

infinitesimal width \( dy \), thus characterized by a linear current density given as:

\[
K_n(x) = \hat{J}_n dy \sin(ux) \quad [A/m]
\]  \hspace{1cm} (1.80)

The expression of magnetic vector potential in the region 2, given by the magnets distribution (1.80) can be obtained by integrating (1.64) over the magnet thickness \( Y_m \):

\[
A_{z2}(x,y) = -\int_0^{Y_m} \frac{\mu_0 \hat{J}_n \cosh(uy)}{u \sinh(uY_2)} \sin(ux) \cosh(uY - y) dy_1 =
\]

\[
= -\frac{\mu_0 \hat{J}_n \sinh(uY_m)}{u^2 \sinh(uY_2)} \sin(ux) \cosh(uY - y)
\]  \hspace{1cm} (1.81)

Note that the particular form of equation (1.80), which represents in this case the magnets distribution, replaces the general function \( \hat{K}_n \sin(ux) \) in the equation (1.64).
1.6 Conclusion

In this chapter a method proposed in literature was developed to study the distributions of the magnetic vector potential, magnetic field and flux density in the airgap of axial flux permanent magnet electrical machines by applying a two-dimensional model.

The contribution of this chapter with respect to the examined work, consists in the execution of the complete calculations, which are not presented in the original paper, to get the solution of the problem. They were conducted by using the techniques of mathematical analysis applied to physical and engineering problems.

This method can be generalized to the analysis of any typology of electrical machine in the case of neglecting slotting effects and with the assumption of developing the machine linearly in correspondence of the mean airgap radius.
1.7 References


Chapter 2

AN ALGORITHM FOR NON-LINEAR ANALYSIS OF MULTIPHASE BEARINGLESS SURFACE-MOUNTED PM SYNCHRONOUS MACHINES

2.1 Introduction

In recent years, more and more advanced technologies and an impressive rise in the use of electronics, both in civil as in the industrial sector, given a contribution to reduce the cost of the components, allowing the use of complex technologies which in the past had high costs and therefore of little industrial
interest. In the field of electrical machines this evolution led not only to the realization of power drives controlled by an inverter, capable of ensuring performance significantly better than those obtained with the previous control systems, but also the advent of a new type of machines with a different number of phases from the traditional three-phase, usually employed in generation and distribution of electric energy. This has reawakened the interest in the study of multi-phase electrical machines.

In [1], a general modulation strategy is presented to be used in multimotor drives and in multiphase motor drives for improving the torque density.

In [2] a scheme, functional to implement a space vector PWM control of a twelve-phase permanent magnet synchronous motor is analyzed, to reduce the switching losses without affecting performances.

A rotor field oriented based on the space vector PWM (SVPWM) technique for a 5-phase synchronous reluctance motor is developed in [3] and verified using a dedicated inverter.

In [4] the stator of an induction machine is rewound with two three-phase winding sets displaced from each other by 30 electrical degrees, showing that this winding configuration eliminates rotor copper losses and torque harmonics of particular orders and the sixth harmonic dominant torque ripple.

In [5] an inverter-fed 5-phase induction motor is compared with a corresponding 3-phase motor, showing that the amplitude of the torque fluctuation is reduced to approximately one third.

The space vector decomposition technique is presented in [6], where the analytical modeling and control of the machine are developed in three 2-dimensional orthogonal subspaces which permits to decouple the variables related to the control of harmonic contributions.

In [7] a novel multiphase SVPWM strategy is presented, able to synthesize the d-q subspace voltage vectors to accomplish the control requirements and
make null the resultant voltage vectors on other subspace, minimizing the switching losses.

The advantages of multiphase machines are explained and discussed in [8]: capability of improve the torque production by injecting harmonic of currents in the motor, a better torque and flux adjustment in DTC control, the fault resilient current control of multi phase drive under loss of phases and the possibility of controlling multi motors through a single inverter.

The space vector control and direct torque control (DTC) schemes are presented in [9], applied to the operation of a 5-phase induction motor and using a fully digital implementation. Experimental results show that an optimal control capability is obtained for both methods, further validating the theoretical concepts. In the last years, various techniques have been applied in order to reduce the power losses in multiphase IGBT inverters [10].

The multiphase feature results particularly suitable in bearingless machines, capable of producing rotor suspension force and torque avoiding the use of mechanical bearings and achieving in this way much higher maximum speed [11]. There are two typologies of winding configurations: dual set and single set of windings. The first category comprises two separated groups of three-phase windings, with a difference in their pole pair numbers equal to one: the main one carries the ‘motor currents’ for driving the rotor, while the other carries the ‘levitation currents’, to suspend the rotor [11].

The windings belonging to the latter category produce torque and radial forces by means of injecting different current sequences to give odd and even harmonic orders of magnetic field, using the properties of multiphase current systems, which have multiple orthogonal d-q planes. One of them can be used to control the torque, the additional degrees of freedom can be used to produce levitation forces [12], as will be explained in chapter 3. The main advantages of bearingless motors with a single set of windings (i.e. of multiphase type) consist
of a simpler construction process, better performances in control strategy and torque production with relatively low power losses [13]. This kind of technology is expected to have very large developments in the future, particularly in the design of high power density generators, actuators and motors of More Electric Aircraft (MEA), mainly for the ability of achieving higher speed in comparison to conventional electrical machines [14].

In addition, it can be supposed that the characteristics of bearingless control techniques and the use of magnetic bearing could be of large interest in the MEA field.

The possibility of making quick analyses, with the comparison of a large number of solutions, nevertheless providing an accurate calculation of electromagnetic quantities, represents a relevant goal in the design of electrical machines, by analyzing global and local quantities as output torque, magnetic energy and co-energy, linkage fluxes, magnetic fields and flux densities in many parts of the machine. The difficulties increase especially in presence of magnetic saturation; in order to solve these problems, the equivalent magnetic circuit method lets fast modifications of the geometrical and electrical parameters simply by varying numerical inputs and, at the same time, obtaining an high accuracy in calculations with respect to other software based on more in-depth analytical methods, as Finite Element Analysis (FEA).

Previous papers proposed the analysis of open-slot configurations with a prefixed structure of the motor, with a given number of poles and slots [15], or by studying only particular positions of the rotor with respect to the stator without relative movement [16], [17]. This chapter presents an algorithm for non-linear magnetic analysis of multiphase surface-mounted permanent-magnet machines with semi-closed slots. The relevant edge of the method consists in the possibility of defining the machine characteristics in a simple user interface. Then, by duplicating an elementary cell, it is possible to construct and analyze
An algorithm for non-linear analysis of multiphase bearingless surface-mounted pm synchronous machines

whatever typology of windings and ampere-turns distribution in a pole-pair. Furthermore, it is possible to modify the magnet width-to-pole pitch ratio analyzing various configurations in order to minimize the cogging torque, or simulating the rotor movement in sinusoidal multiphase drives or in a user-defined current distribution.

Finally, the capability of radial forces calculation allows to determine the optimal ampere-turns distribution in the design of a bearingless control of the motor. The choice of using a software based on the equivalent magnetic circuit method allows relevant time saving for this kind of analysis with respect to a FEA software, not only due to the reduction of computing time, but mainly for the simple change of electrical and geometrical parameters (i.e. the numerical inputs of the problem), without the need of re-designing the model in a CAD interface.
2.2 The Magnetic Circuit Model

The basic element of the magnetic network is shown in Fig. 2.1, whose the related reluctances are highlighted.

![Fig. 2.1. The basic element of the magnetic network](image)

It consists of one tooth and the adjacent two semi-slots, being composed of 18 reluctances representing sub-domains of the machine, i.e. volumes of teeth, sections of the airgap, of the magnets, branches of yokes, semi-slots, etc.

Beside of considering longitudinal components, in the model were provided transverse components of the magnetic fluxes as, for example, in the slot area and in the slot-opening to take into account the leakage paths [18], in the tips of the tooth and in the branches of stator and rotor yokes.

To construct the whole model of a motor, the $i$-th basic cell is connected to the previous one through four transverse reluctances: $N_{cv} + i - 1$, $2N_{cv} + i - 1$
An algorithm for non-linear analysis of multiphase bearingless surface-mounted pm synchronous machines

(stator and rotor yokes), $3N_{cv} + i - 1$ (slot area), $8N_{cv} + i - 1$ (slot-opening area). Furthermore, the $i$-th basic cell is connected to the following one through the elements $N_{cv} + i$, $2N_{cv} + i$, $3N_{cv} + i$, and $8N_{cv} + i$.

Considering one pole pair of the model, that comprises two gaps between the magnets (each one of them provides 4 additional terms), the whole network results in a number of reluctances, i.e. unknown terms, equal to $18N_{cv} + 8$, being $N_{cv}$ the number of slots per pole pair.

2.2.1 Analytical Models of the Reluctances

In this subsection some formulas and criteria used to determine the most relevant parameters of the magnetic circuit are described.

Fig. 2.2. Configuration of the network in the case of uniform magnet

Elements $i = 1$ to $N_{cv}$, $i = 5N_{cv} + 1$ to $6N_{cv}$

To provide a more realistic representation of the flux lines crossing the airgap,
the total magnetic flux in a slot pitch, passing through the magnet, was divided into three tubes (Fig. 2.1): the one in the middle presents the tooth surface as cross-sectional area, depending on the mean radius $R_m$ for the magnet zone (2.1) and on the mean radius $R_g$ for the airgap zone (2.2). The related reluctances are respectively calculated as:

$$\mathcal{R}_i = \frac{1}{\mu_m} \frac{L_m}{R_m \alpha_{Ldg} L} \quad (2.1)$$

$$\mathcal{R}_i = \frac{1}{\mu_0} \frac{g}{R_g \alpha_{Ldg} L} \quad (2.2)$$

where the meaning of the symbols is shown in Fig. 4 and in Tab. I.

Elements $i = 7N_{cv}+1$ to $8N_{cv}$, $i = 9N_{cv}+1$ to $10N_{cv}$, elements $i = 10N_{cv}+1$ to $11N_{cv}$, $i = 11N_{cv}+1$ to $12N_{cv}$

The two tubes in left and right sideways positions with respect to the tooth, develop their paths across the airgap in a succession of a straight line and a circumferential arc, closing in the tooth tips [14], [18]. The related reluctances are calculated as:

$$\mathcal{R}_i = \frac{1}{L\mu_0} \frac{1}{\frac{1}{g + \pi r/2}dr} = \frac{\pi}{2\mu_0 L \ln \left( \frac{2g + \pi h_{cl}}{2g} \right)} \quad (2.3)$$

In the magnet, flux paths are developed in radial direction, using the simple conventional formula:

$$\mathcal{R}_i = \frac{1}{\mu_m} \frac{L_m}{R_m \alpha_{sap} L} \quad (2.4)$$
An algorithm for non-linear analysis of multiphase bearingless surface-mounted pm synchronous machines

Elements $i = 12N_{cv} + 1$ to $14N_{cv}$

Furthermore, a description of the leakage flux in the gap between the magnets is provided by using transverse reluctances. These create a closed loop including the magnets, the airgap and the tooth tips. This situation has a not negligible effect on PM machines [15] and is described in the literature [19]. The reluctance used to describe a tooth tip is given by the series of two elements, a rectangular-shaped one and a trapezoidal-shaped one (Fig. 2.3 a):

![Fig. 2.3. Reference systems for calculating the transverse reluctance: a) tooth tip, b) slot area.](image)

\[
R_i = \frac{1}{\mu_{i(B,H)}} \frac{L_{dt}}{2h_{bd}L} + \frac{1}{\mu_{i(B,H)}} \frac{L_{dg} - L_{dt}}{2(h_{cl} - h_{bd})L} \ln \left( \frac{h_{cl}}{h_{bd}} \right) \quad (2.5)
\]

Note that the reluctances placed in the iron have a value of the permeability which depends on the magnetic characteristic of the material, thus characterized as $\mu_{i(B,H)}$.

Elements $i = 3N_{cv} + 1$ to $4N_{cv}$, $i = 8N_{cv} + 1$ to $9N_{cv}$

For evaluating the leakage flux produced by the stator currents were used two transverse reluctances: one through the air of the slot (2.6), calculated as the
parallel connection of two reluctances (Fig. 2.3 (b)), the other across the slot opening (2.7):

\[
\mathcal{R}_i = \frac{1}{\mu_0 L \left( \frac{h_{avv}}{L_{tc} - L_{fc}} \right) \ln \left( \frac{L_{tc}}{L_{fc}} \right) + \mu_0 L \left( \frac{h_{bd} - h_{cl}}{L_{cl} - L_{tc}} \right) \ln \left( \frac{L_{cl}}{L_{tc}} \right)}
\]  

(2.6)

\[
\mathcal{R}_j = \frac{1}{\mu_0} \frac{L_{cl}}{h_{cl} L}
\]  

(2.7)

**Other elements**

The reluctances related to other elements are not reported because of their simple form. Note that, in general, the non-linear sub-domains have a value of permeability that depends on the B-H curve, as in (2.5).

### 2.3 The Numerical Solving Process

The problem is described through a non-linear system of \( n \) equations \( (n = \) 18\(N_{cv} + 8\) where the unknowns are the values of the magnetic fluxes \( \varphi_1, \varphi_2, \ldots \varphi_n \) in every sub-domain of the machine.

Two principles of electromagnetism are used to write the equations: Hopkinson’s law, applied along closed paths identified in the machine, and Gauss’s law, i.e. conservation of the magnetic fluxes incoming in and outgoing from the nodes of the network (continuity equations).

Overall, the system includes \( 9N_{cv} + 5 \) equations of the first typology and \( 9N_{cv} + 3 \) of the second typology, with a matrix form defined as follows:

\[
[A]\Phi = \bar{F}_M
\]  

(2.8)
An algorithm for non-linear analysis of multiphase bearingless surface-mounted pm synchronous machines

Matrix $[A]$ in (2.8) can be seen as composed by some blocks depending on the following characterization: its coefficients $a_{ij}$ represent magnetic reluctances in the rows related to Hopkinson’s law equations, whereas they have the value $\pm 1$ in the rows related to Gauss’ law equations, being essentially algebraic sum of fluxes:

$$a_{ij} \equiv \Re(\mu, \theta_{xm}) \quad i = 1..5N_{cv} + 1, 12N_{cv} + 1..14N_{cv}, 16N_{cv} + 1..18N_{cv}, 18N_{cv} + 1, 18N_{cv} + 2, 18N_{cv} + 5, 18N_{cv} + 6$$

$$a_{ij} \equiv \pm 1 \quad i = 5N_{cv} + 2..12N_{cv}, 14N_{cv} + 1..16N_{cv}, 18N_{cv} + 3, 18N_{cv} + 4, 18N_{cv} + 7, 18N_{cv} + 8$$

(2.9)

The system, divided in groups of equations according to the different areas of the machine, is specified in detail as follows (2.10 – 2.28). Note that the index $i$ varies in every group from 1 to $N_{cv}$ depending on the basic cell related to the examined equation, except for group (2.16), where the first equation of the group is substituted by (2.15):

**Tooth to tooth across the airgap ($i = 1$ to $N_{cv}$)**

$$\Re(i)\Phi(i) - \Re(i+1)\Phi(i+1) - \Re(N_{cv}+i)\Phi(N_{cv}+i) - \Re(2N_{cv}+i)\Phi(2N_{cv}+i) +$$

$$\Re(4N_{cv}+i)\Phi(4N_{cv}+i) - \Re(4N_{cv}+i+1)\Phi(4N_{cv}+i+1) + \Re(5N_{cv}+i)\Phi(5N_{cv}+i) +$$

$$\Re(5N_{cv}+i+1)\Phi(5N_{cv}+i+1) + \Re(6N_{cv}+i)\Phi(6N_{cv}+i) +$$

$$\Re(6N_{cv}+i+1)\Phi(6N_{cv}+i+1) = F_M(i)$$

(2.10)

**Higher slot area between two teeth ($i = N_{cv} + 1$ to $2N_{cv}$)**

$$-\Re(N_{cv}+i)\Phi(N_{cv}+i) - \Re(3N_{cv}+i)\Phi(3N_{cv}+i) + \Re(6N_{cv}+i)\Phi(6N_{cv}+i) +$$

$$-\Re(6N_{cv}+i+1)\Phi(6N_{cv}+i+1) = F_M(N_{cv}+i)$$

(2.11)
Tooth to tooth around the slot area \((i = 2N_{cv} + 1 \text{ to } 3N_{cv})\)

\[-R_{(N_{cv}+i)}\varphi(N_{cv}+i) + R_{(4N_{cv}+i)}\varphi(4N_{cv}+i) - R_{(4N_{cv}+i+1)}\varphi(4N_{cv}+i+1) +
+ R_{(6N_{cv}+i)}\varphi(6N_{cv}+i) - R_{(6N_{cv}+i+1)}\varphi(6N_{cv}+i+1) - R_{(8N_{cv}+i)}\varphi(8N_{cv}+i) +
- R_{(12N_{cv}+i)}\varphi(12N_{cv}+i+1) - R_{(13N_{cv}+i)}\varphi(13N_{cv}+i) = F_M(2N_{cv}+i)\] (2.12)

Right tooth tip across the airgap \((i = 3N_{cv} + 1 \text{ to } 4N_{cv})\)

\[R_{(i)}\varphi(i) + R_{(5N_{cv}+i)}\varphi(5N_{cv}+i) - R_{(9N_{cv}+i)}\varphi(9N_{cv}+i) +
- R_{(11N_{cv}+i)}\varphi(11N_{cv}+i) + R_{(13N_{cv}+i)}\varphi(13N_{cv}+i) = 0\] (2.13)

Left tooth tip across the airgap \((i = 4N_{cv} + 1 \text{ to } 5N_{cv})\)

\[-R_{(i)}\varphi(i) - R_{(5N_{cv}+i)}\varphi(5N_{cv}+i) + R_{(7N_{cv}+i)}\varphi(7N_{cv}+i) +
+ R_{(10N_{cv}+i)}\varphi(10N_{cv}+i) + R_{(12N_{cv}+i)}\varphi(12N_{cv}+i) = 0\] (2.14)

Closing equation \((i = 5N_{cv} + 1)\)

\[
\sum_{i=1}^{N_{cv}} R_{(N_{cv}+i)}\varphi(N_{cv}+i) = 0 .
\] (2.15)

Nodes \((1-2-4-7-9-3-5)\)
\((i = 5N_{cv} + 2 \text{ to } 6N_{cv})\)

\[\varphi(N_{cv}+i-1) - \varphi(N_{cv}+i) - \varphi(6N_{cv}+i) - \varphi(16N_{cv}+i) - \varphi(17N_{cv}+i) = 0\] (2.16)
An algorithm for non-linear analysis of multiphase bearingless surface-mounted pm synchronous machines

\[ (i = 6N_{cv} + 1 \text{ to } 7N_{cv}) \]

\[ \Phi_{(3N_{cv}+i-1)} - \Phi_{(3N_{cv}+i)} + \Phi_{(4N_{cv}+i)} - \Phi_{(6N_{cv}+i)} + \Phi_{(14N_{cv}+i)} + \Phi_{(15N_{cv}+i)} - \Phi_{(16N_{cv}+i)} - \Phi_{(17N_{cv}+i)} = 0 \]  \hspace{1cm} (2.17)

\[ (i = 7N_{cv} + 1 \text{ to } 8N_{cv}) \]

\[ \Phi_{(4N_{cv}+i)} - \Phi_{(5N_{cv}+i)} - \Phi_{(12N_{cv}+i)} + \Phi_{(13N_{cv}+i)} = 0 \]  \hspace{1cm} (2.18)

\[ (i = 8N_{cv} + 1 \text{ to } 9N_{cv}) \]

\[ \Phi_{(i)} - \Phi_{(5N_{cv}+i)} = 0 \]  \hspace{1cm} (2.19)

\[ (i = 9N_{cv} + 1 \text{ to } 10N_{cv}) \]

\[ \Phi_{(i)} - \Phi_{(2N_{cv}+i-1)} + \Phi_{(2N_{cv}+i)} + \Phi_{(10N_{cv}+i)} + \Phi_{(11N_{cv}+i)} = 0 \]  \hspace{1cm} (2.20)

\[ (i = 10N_{cv} + 1 \text{ to } 11N_{cv}) \]

\[ \Phi_{(7N_{cv}+i)} + \Phi_{(8N_{cv}+i-1)} - \Phi_{(12N_{cv}+i)} - \Phi_{(14N_{cv}+i)} = 0 \]  \hspace{1cm} (2.21)

\[ (i = 11N_{cv} + 1 \text{ to } 12N_{cv}) \]

\[ \Phi_{(8N_{cv}+i)} - \Phi_{(9N_{cv}+i)} - \Phi_{(13N_{cv}+i)} + \Phi_{(15N_{cv}+i)} = 0 \]  \hspace{1cm} (2.22)

Right semi-slot area \( (i = 12N_{cv} + 1 \text{ to } 13N_{cv}) \)

\[ \Re_{(4N_{cv}+i)}\Phi_{(4N_{cv}+i)} + \Re_{(6N_{cv}+i)}\Phi_{(6N_{cv}+i)} - \Re_{(13N_{cv}+i)}\Phi_{(13N_{cv}+i)} + \Re_{(15N_{cv}+i)}\Phi_{(15N_{cv}+i)} - \Re_{(17N_{cv}+i)}\Phi_{(17N_{cv}+i)} = F_M(12N_{cv}+i) \]  \hspace{1cm} (2.23)
Lower tooth area between two slots \( (i = 13N_{cv} + 1 \text{ to } 14N_{cv}) \)

\[
- \mathcal{R}(12N_{cv}+i)\varphi(12N_{cv}+i) - \mathcal{R}(13N_{cv}+i)\varphi(13N_{cv}+i) + \mathcal{R}(14N_{cv}+i)\varphi(14N_{cv}+i) - \mathcal{R}(15N_{cv}+i)\varphi(15N_{cv}+i) = F_M(13N_{cv}+i)
\]  \( (2.24) \)

Nodes (6-8)

\( (i = 14N_{cv} + 1 \text{ to } 15N_{cv}) \)

\[
\varphi(7N_{cv}+i) - \varphi(10N_{cv}+i) = 0
\]  \( (2.25) \)

\( (i = 15N_{cv} + 1 \text{ to } 16N_{cv}) \)

\[
\varphi(9N_{cv}+i) - \varphi(11N_{cv}+i) = 0
\]  \( (2.26) \)

Higher left semi-slot area \( (i = 16N_{cv} + 1 \text{ to } 17N_{cv}) \)

\[
- \mathcal{R}(6N_{cv}+i)\varphi(6N_{cv}+i) + \mathcal{R}(16N_{cv}+i)\varphi(16N_{cv}+i) = F_M(16N_{cv}+i)
\]  \( (2.27) \)

Higher right semi-slot area \( (i = 17N_{cv} + 1 \text{ to } 18N_{cv}) \)

\[
\mathcal{R}(6N_{cv}+i)\varphi(6N_{cv}+i) - \mathcal{R}(17N_{cv}+i)\varphi(17N_{cv}+i) = F_M(17N_{cv}+i)
\]  \( (2.28) \)

The form of the remaining eight equations is described in the next sections: they are used for describing additional branches which are formed during the movement of the rotor. The known terms \( F_M(i), \ i = 1 \text{ to } n \), represent the ampere-turns linked by the paths related to the Hopkinson’s law equations.

Note that the row \( 5N_{cv}+1 \) (14) represents the equation which makes the system
An algorithm for non-linear analysis of multiphase bearingless surface-mounted pm synchronous machines

solvable: it can be seen as a boundary condition equation, closing the path along the branches of the stator yoke.

The solving process is based on the method of Gaussian elimination, for reducing the matrix of coefficients to a triangular one. It is applied iteratively $k$-times, being some coefficients dependent on the rotor position ($a_{ij(k)} = f(\theta_{xm})$), some other also dependent on the value of magnetic permeability of the $i$-th sub-domain ($a_{ij(k)} = f(\mu_{i(k)}, \theta_{xm})$). For a given rotor position $\theta_{xm}$ the length and thickness for the flux tubes that change dimensions or position are recalculated.

Starting with initial random values of $\Re_i(1)$, $i = 1$ to $n$, the solution of the system in the $k$-th order of iteration is obtained in terms of $\varphi_i(k)$, $i = 1$ to $n$, by solving the system (2.8): it is then possible to determine the values of flux densities $B_i(k)$ being known the flux tubes cross-sectional areas $S_i$. An interpolation of the magnetic characteristic $H_i = f(B_i)$ is implemented for domains occupied by non-linear magnetic material, while a constant value of $\mu_i$ is used for linear domains.

It is then possible to calculate the $k$-th value $\mu_{i(k)}$ by combining the actual value $\hat{\mu}_{i(k)}$ and the previous value $\mu_{i(k-1)}$ and, for every order of iteration, re-calcultating the magnetic permeabilities $\mu_{i(k)}$ related to non-linear domains, following the general criterion, to facilitate the convergence process [15]:

$$\hat{\mu}_{i(k)} = \frac{B_{i(k)}}{H_{i(k)}}$$

(2.29)

$$\mu_{i(k)} = \mu_{i(k)}^d \mu_{i(k-1)}^{1-d}$$

(2.30)

where the value of $d$, the damping constant, is chosen equal to 0.1. Consequently, the reluctances $\Re_{i(k)} = f(\mu_{i(k)}, \theta_{xm})$ are updated, leading to a further step for the Gauss method, until the following condition is satisfied [15]:

$$\frac{|\mu_{i(k)} - \mu_{i(k-1)}|}{\mu_{i(k-1)}} \leq \delta$$

(2.31)
being $\delta$ the requested accuracy. At this step, all the magnetic quantities related to each sub-domain of the machine are known: permeabilities $\mu_i$, magnetic field $H_i$, fluxes $\varphi_i$, flux densities $B_i$.

### 2.4 Simulating the Movement

The analysis in the presence of movement proceeds with an external loop that sets the rotor angular position $\theta_{xm}$, representing the N-magnet position in a generic time instant, with respect to a fixed reference system. The origin of the reference system lies on the axis of symmetry of a chosen slot.

By varying the value of $\theta_{xm}$, the geometrical parameters of the flux tubes are modified. Consequently, the related coefficients $a_{ij}(k) = f(\mu_i(k), \theta_{xm})$ and the continuity equations in the nodes involved in changes are modified also. The solution process is then repeated by solving every step in the same way described in Section 2.3. When the gap between the magnets is comprised under a tooth, as
shown in Fig. 2.4, the software modifies the configuration of the magnetic network by adding two new branches and four new reluctances of variable cross section, depending on the value of $\theta_{xm}$ in a generic time instant. Consequently, there are four new unknowns per gap. Instead of only one flux tube, as in the case of uniform magnet (Fig. 2.2), in this situation the area under the tooth can be divided into three flux tubes related, respectively, to N-magnet portion, magnet gap and S-magnet portion, as shown in Fig. 2.4.

The index $i_{dt}$ assumes an integer value to identify the tooth that comprises the gap, so that the reluctances $\mathcal{R}_{(idt)}$, $\mathcal{R}_{(5Ncv+idt)}$, $\mathcal{R}_{(18Ncv+1idt)}$, $\mathcal{R}_{(18Ncv+3idt)}$ involved in the movement, change their reference angles $\theta_{A1} = f(\theta_{xm})$ and $\theta_{B1} = f(\theta_{xm})$, that subtend the related cross-sectional area, according to:

$$\theta_{A1} = \left[\theta_{xm} - \alpha_{sap} - (i_{dt} - 1)\alpha_{cv}\right]$$  (2.32)

$$\alpha_{spm} = \frac{s_{pm}}{R_{m}}$$  (2.33)

$$\theta_{B1} = \left[\alpha_{cv} - \alpha_{sap} - \theta_{xm} - \alpha_{spm} + (i_{dt} - 1)\alpha_{cv}\right]$$  (2.34)

The three flux tubes under the tooth are characterized by six reluctances: two of them also existing in the case of uniform magnet, $\mathcal{R}_{(idt)}$ and $\mathcal{R}_{(5Ncv+idt)}$, but in the present case modified in the cross-sections and four additional reluctances, from the $\mathcal{R}_{(18Ncv+1)}$ to the $\mathcal{R}_{(18Ncv+4)}$.

$$\mathcal{R}_{(idt)} = \frac{1}{\mu_{m}} \frac{L_{m}}{R_{m} \theta_{A1}(\theta_{xm}, i_{dt})L}$$  (2.35)

$$\mathcal{R}_{(5Ncv+idt)} = \frac{1}{\mu_{0}} \frac{L_{m}}{R_{g} \theta_{A1}(\theta_{xm}, i_{dt})L}$$  (2.36)

$$\mathcal{R}_{(18Ncv+1)} = \frac{1}{\mu_{m}} \frac{L_{m}}{R_{m} \theta_{B1}(\theta_{xm}, i_{dt})L}$$  (2.37)
\[ R_{(18N_{cv}+3)} = \frac{1}{\mu_0} R \frac{g}{\Theta_B(\Theta_{xm}, i_{dt})} L \]  
(2.38)

\[ R_{(18N_{cv}+2)} = \frac{1}{\mu_0} \frac{L_m}{R_m \alpha_{spm} L} \]  
(2.39)

\[ R_{(18N_{cv}+4)} = \frac{1}{\mu_0} \frac{g}{R \alpha_{spm} L} \]  
(2.40)

The cross-sectional areas in the formulas are calculated at the reference radius of every sub-domain. Four additional equations (2.41)-(2.44) are added to the system for taking into account the new unknown fluxes with respect to the case of uniform magnet: the Hopkinson’s law applied to \( N \) - and \( S \) -magnet, the continuity equations applied to nodes 7(2) and 7(3):

\[ R_{(idt)} \Phi_{(idt)} + R_{(5N_{cv}+idt)} \Phi_{(5N_{cv}+idt)} - R_{(18N_{cv}+2)} \Phi_{(18N_{cv}+2)} + \]
\[ - R_{(18N_{cv}+4)} \Phi_{(18N_{cv}+4)} = F_M(18N_{cv}+1) \]  
(2.41)

\[ - R_{(18N_{cv}+1)} \Phi_{(18N_{cv}+1)} + R_{(18N_{cv}+2)} \Phi_{(18N_{cv}+2)} - R_{(18N_{cv}+3)} \Phi_{(18N_{cv}+3)} + \]
\[ + R_{(18N_{cv}+4)} \Phi_{(18N_{cv}+4)} = F_M(18N_{cv}+2) \]  
(2.42)

\[- \Phi_{(18N_{cv}+2)} + \Phi_{(18N_{cv}+4)} = 0 \]  
(2.43)

\[- \Phi_{(18N_{cv}+1)} + \Phi_{(18N_{cv}+3)} = 0 \]  
(2.44)

Obviously, other equations already comprised in the system have to be modified according to the variations in the magnetic circuit, as for example in the node 4, where the continuity equation written in the presence of gap (2.45) is compared with the situation of uniform magnet (2.46):

\[ \Phi_{(4N_{cv}+idt)} - \Phi_{(5N_{cv}+idt)} - \Phi_{(12N_{cv}+idt)} + \Phi_{(13N_{cv}+idt)} + \]
\[ - \Phi_{(18N_{cv}+3)} - \Phi_{(18N_{cv}+4)} = 0 \]  
(2.45)

\[ \Phi_{(4N_{cv}+i)} - \Phi_{(5N_{cv}+i)} - \Phi_{(12N_{cv}+i)} + \Phi_{(13N_{cv}+i)} = 0 \]  
(2.46)
The same logic can be applied to the other additional equations (with indexes from $18N_{cv} + 5$ to $18N_{cv} + 8$) considering the second gap between the magnets, depending on the related reference angles and on a new index $i_{dt2}$ which identifies the tooth comprising the gap, with reluctances $\mathbb{R}(i_{dt2})$, $\mathbb{R}(5N_{cv} + i_{dt2})$, $\mathbb{R}(18N_{cv} + 5)$, $\mathbb{R}(18N_{cv} + 6)$, $\mathbb{R}(18N_{cv} + 7)$, $\mathbb{R}(18N_{cv} + 8)$.

### 2.5 Co-energy, Torque and Radial Forces

One of the features of the proposed algorithm is the capability of determining the total magnetic co-energy of the machine in a series of different rotor positions: this allows calculating the electromagnetic torque acting between stator and rotor. In order to do this, the volumes $\tau_i$ and flux paths of every sub-domain are developed in circular shape, based on the average radius of the related part of the machine [15], [18]. The equation used to determine the magnetic co-energy $W_i'$ of the $i$-th sub-domain is [20]:

$$W_i' = \int_{\tau_i}^{\pi} B_i(\bar{H}) dH_i d\tau$$  \hfill (2.47)

To evaluate (2.47), in case of non-linear sub-domains, the algorithm uses the numerical integration method of trapezoids by interpolating the magnetic characteristic of the material in order to find the couples of consequent values $B_{j+1}$ and $B_j$ in $m$-1 steps, where $m$ depends on the desired accuracy:

$$W_{i NL} = \sum_{j=1}^{m-1} \frac{B_{j+1}(H_{j+1}) + B_j(H_j)}{2} \Delta H_j \tau_i$$  \hfill (2.48)

In the case of linear sub-domains, the following relationship is used:
The electromagnetic torque $T$ is calculated through the finite difference approximation of the first derivative of the total co-energy $W'$ obtained as summation of all the $W'_i$ [21]:

$$T = \lim_{\Delta \theta_{rm} \to 0} \frac{\Delta W'}{\Delta \theta} \bigg|_{\mu_f = \text{const.}} \approx \frac{\partial W'}{\partial \theta} \bigg|_{\mu_f = \text{const.}}$$

(2.50)

where $\Delta \theta_{rm}$ is the angular rotor displacement between two different steps and the calculation is done by maintaining constant values of the phase currents.

The radial component $F_r$ of the forces was determined by applying the Maxwell Stress Tensor method (42) on a closed surface enveloping the frontal cross sections of the teeth in the stator, taking into account the normal and tangential components of flux density and of magnetic field acting in every tooth.

$$\vec{F} = \vec{H}(\vec{B} \cdot \hat{n}) - \frac{1}{2} (\vec{B} \cdot \vec{H}) \hat{n} = H_n B_n \hat{i} + \frac{1}{2} (H_n B_n - H_t B_t) \hat{n}$$

(2.51)

Integrating (2.51) on the above described surface and considering the discretization of the model, the contribution related to the $i$-th tooth is given, for radial components acting along the axis of the tooth (normal to the surface), as

$$F_{rn(i)} = \frac{1}{2\mu_0} \left( S_{dg} B_{2N_{cv}+i}^2 + S_{sap} B_{7N_{cv}+i}^2 + S_{sap} B_{9N_{cv}+i}^2 \right) +$$

$$- \frac{1}{2\mu_0} \left( S_{sap} B_{8N_{cv}+i-1}^2 + S_{sap} B_{8N_{cv}+i}^2 \right)$$

(2.52)

and, for radial components acting perpendicularly to the axis of the tooth (tangential to the surface), is given as

$$F_{rt(i)} = \frac{1}{\mu_0} \left( S_{sap} B_{7N_{cv}+i} B_{8N_{cv}+i-1} + S_{sap} B_{9N_{cv}+i} B_{8N_{cv}+i} \right)$$

(2.53)
Note that, in (2.52) and (2.53), $S_{dg}$ represents the frontal cross-sectional area of the tooth, $S_{sap}$ the area related to a semi-slot opening, while the considered flux densities values are described in Figs. 2.2-2.4. The projections of the normal and tangential components on a reference system centered on the motor axis are finally given as

$$F_{xr(i)} = F_{m(i)} \cos(\alpha_{dt(i)}) + F_{r(i)} \sin(\alpha_{dt(i)})$$  

(2.54)

$$F_{yr(i)} = F_{m(i)} \sin(\alpha_{dt(i)}) - F_{r(i)} \cos(\alpha_{dt(i)})$$  

(2.55)

where $\alpha_{dt(i)}$ represents the angular position of the axis of the $i$-th tooth with respect to the $x$ axis; by summing the terms in (2.54), (2.55) for all the teeth, it gives the components $F_{xr}$ and $F_{yr}$ of the resultant radial force applied on the rotor.

### 2.6 Results and Comparison with FEA Software

The 2D FEA software “FEMM 4.2” [22] was used in order to verify the accuracy of the proposed analysis. Two different typologies of synchronous PM machine were considered, as shown in Fig. 2.5, supplied by sinusoidal drives.

![Machine A and Machine B](image)

Fig. 2.5. The PMSM machines considered in the analysis
Machine A: 2-pole pairs, 3-phase, 24 slots, traditional winding (only odd mmf harmonic components), 1 - 6 double layer shortened pitch.

Machine B: 1-pole pair, 5-phase, 30 slots, special winding (odd and even mmf harmonic components), 6 slots per phase per pole in each layer, one phase occupying 72° in angular space of the stator.

In order to obtain an accurate electromagnetic analysis, about 108'000 nodes were used for meshing the models in FEMM. In Tab. I the main geometrical dimensions of the machines are given.

Fig. 2.6. The B-H characteristic of the material

The non-linear B-H curve of the magnetic material is shown in Fig. 2.6, the same for Machine A and for Machine B. The initial relative permeability is 4380. In Tab. I the main data of the two machines are presented.

In the following the results of the comparisons are presented in graphical form.
TABLE I. MAIN DATA OF MACHINES A E B

<table>
<thead>
<tr>
<th>Param.</th>
<th>Description</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_s )</td>
<td>number of slots</td>
<td>24</td>
<td>30</td>
</tr>
<tr>
<td>( p )</td>
<td>pole pairs of the machine</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>( m )</td>
<td>number of phases</td>
<td>3</td>
<td>5</td>
</tr>
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<td>( I_{ph} )</td>
<td>rated phase current (A_{rms})</td>
<td>74.78</td>
<td>59.82</td>
</tr>
<tr>
<td>( T_n )</td>
<td>rated torque (Nm)</td>
<td>42.41</td>
<td>30.29</td>
</tr>
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<td>( g )</td>
<td>airgap width (mm)</td>
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<td>1</td>
</tr>
<tr>
<td>( D_e )</td>
<td>stator outer diameter (mm)</td>
<td>210</td>
<td>230</td>
</tr>
<tr>
<td>( D_i )</td>
<td>stator inner diameter (mm)</td>
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<td>120</td>
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<td>( D_m )</td>
<td>mean diameter of the magnet (mm)</td>
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<td>116</td>
</tr>
<tr>
<td>( D_{cv,ext} )</td>
<td>diameter at the bottom of the slot (mm)</td>
<td>170</td>
<td>170</td>
</tr>
<tr>
<td>( D_{cv,int} )</td>
<td>diameter at the top of the slot (mm)</td>
<td>130.2</td>
<td>126.3</td>
</tr>
<tr>
<td>( D_r )</td>
<td>rotor outer diameter (mm)</td>
<td>114</td>
<td>114</td>
</tr>
<tr>
<td>( D_{alb} )</td>
<td>rotor inner diameter (mm)</td>
<td>74</td>
<td>60</td>
</tr>
<tr>
<td>( \alpha_{ldg} )</td>
<td>angle underlying the tooth surface</td>
<td>13.1°</td>
<td>10.1°</td>
</tr>
<tr>
<td>( \alpha_{sap} )</td>
<td>semi-angle underlying the slot opening</td>
<td>0.95°</td>
<td>0.95°</td>
</tr>
<tr>
<td>( \alpha_{spm} )</td>
<td>angle underlying the magnet gap</td>
<td>4°</td>
<td>8°</td>
</tr>
<tr>
<td>( \alpha_{cv} )</td>
<td>slot pitch angle</td>
<td>15°</td>
<td>12°</td>
</tr>
<tr>
<td>( h_{cl} )</td>
<td>slot opening height (mm)</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>( L )</td>
<td>axial length of the machine (mm)</td>
<td>180</td>
<td>180</td>
</tr>
<tr>
<td>( L_m )</td>
<td>magnet width (mm)</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>( L_{dt} )</td>
<td>tooth-body width (mm)</td>
<td>7.5</td>
<td>8</td>
</tr>
<tr>
<td>( L_{cl} )</td>
<td>slot opening width (mm)</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>( L_{tc} )</td>
<td>slot width at the top slot radius (mm)</td>
<td>9.6</td>
<td>5.23</td>
</tr>
<tr>
<td>( L_{fc} )</td>
<td>slot width at the bottom slot radius (mm)</td>
<td>14.8</td>
<td>9.7</td>
</tr>
<tr>
<td>( \tau_{cv} )</td>
<td>slot pitch at the inner stator radius (mm)</td>
<td>15.71</td>
<td>12.57</td>
</tr>
</tbody>
</table>

2.6.1 Machine A

With reference to Machine A, Figs. 2.7, 2.8 show the results of the comparison for the same rotor position. In particular, Fig. 2.7 shows the linkage fluxes. As can be seen, the linkage fluxes of the three phases calculated using the proposed method are in very good agreement with those obtained by FEA analysis.
Small discrepancies appear only for very high values of the current (highly saturated machine). In Fig. 2.8 the torque values (p.u.) and the magnetic co-energy values, calculated using the proposed method, are compared with those obtained by FEA.

For low values of the currents the two approaches give the same results. For
higher current values (non-linear behavior) some small differences appear. However, the torque reduction with respect to the supposed linear behavior is very well represented.

2.6.2 Machine B

With reference to the 5-phase Machine B, Figs. 2.9 to 2.11 show the results of the comparison for the same rotor position.

In particular, Fig. 2.9 shows the linkage fluxes of the five phases. Also in this case, the results obtained using the proposed method are in very good agreement with those obtained by FE analysis. Fig. 2.10 shows the torque (p.u.) and the magnetic co-energy values of the system calculated using the proposed and the FEA approach. Referring to the magnetic co-energy, a very good agreement appears, with only small differences when the machine is highly saturated; about the torque, is possible to observe a very good agreement for low and medium (rated) current values. Some small differences appear in the highy saturated behavior.
A further comparison has been made by supplying the five phases of Machine B with a balanced system of sinusoidal currents with a time-phase displacement of $4\pi/5$ (sequence 2). The considered winding arrangement produces an airgap distribution of the mmf having odd and even harmonic components.
Taking the winding arrangement into account and the particular system of currents, it is possible to align the 4 pole harmonic component of the stator mmf with the magnet axis. In these conditions a resultant radial force exists acting on the rotor along the direction of the magnet axis [12]. Fig. 2.11 shows the calculated magnitude of the radial force compared with that obtained by FEA. As can be seen, in this case also there is a good agreement between the proposed method and the results of FEA. In Fig. 2.12 the $x$- and $y$-components of the radial force are shown. The force has been calculated for 9 values, equally spaced of $\pi/16$, of the phase angle of the second harmonic component of the stator mmf with respect to the magnet axis ($y$-axis in Fig. 2.12), and for 3 values of the stator current amplitude: the rated value, two-times and four-times the rated value.

Also in this kind of analysis, the obtained results are very similar to those obtained by FEA. It is interesting to note that, for the same currents amplitude,
the magnitude of the radial force is not constant but changes with the phase angle of the mmf. This behavior is due to the presence of higher harmonic orders in the magnet and stator mmf distribution.
2.7 Conclusion

In this chapter an algorithm for the non-linear magnetic analysis of multiphase surface-mounted PM machines with semi-closed slots has been presented.

The basic element of the geometry is duplicated allowing to build and analyze whatever typology of windings and ampere-turns distribution in a pair of poles.

The performances of the proposed method have been compared with those of a well known FEA software in terms of linkage fluxes, co-energy, torque and radial force. The obtained results for a traditional three-phase machine and for a 5-phase machine with unconventional winding distribution showed that the values of local and global quantities are practically coinciding for values of the stator currents up to rated values. Furthermore, they are very similar also in the non-linear behavior even if very large current values are injected.

When developing a new machine design the proposed method is useful not only for the reduction of computing time, but mainly for the simplicity of changing the values of the design variables, being the numerical inputs of the problem obtained by changing some critical parameters, without the need for re-designing the model in a CAD interface. It can be concluded that the proposed method provides an accurate description of electromagnetic phenomena taking magnetic saturation into account. For a given rotor position and for given stator currents, the output torque as well as the radial forces acting on the moving part of a multiphase machine can be easily and quickly calculated.

The latter feature makes the algorithm particularly suitable in order to design and analyze bearingless machines.
2.8 References


An algorithm for non-linear analysis of multiphase bearingless surface-mounted pm synchronous machines


Appendix A2.1

THE PROGRAMMING CODE

Part 1

In the following, the programming code of the algorithm is presented. This is the VBA version, since there is an advanced variant, implemented in Visual Basic 6.0 and not reported here. Some input files are necessary to run the program: ordpr.txt, ordsc.txt, which describe respectively the disposition in the stator slots of the first and second winding layers; aM(pp).txt, which describes the position of the equivalent ampere-turns distribution of the magnets. The magnetic characteristic of the material is provided through the .xls main file and the stator ampere-turns distribution is created by the program with the subroutine presented in Appendix A2.2.

A2.1.1 The main program

Const pi = 3.1415927
Const muzero As Double = 1.25664 * 10 ^ -6
Public a, aI, aI_p, aIp, aIs, aM, ASP, ainiz, noti, x, x_p, y, B, Hdt, Flux_dt, deltaBH, normx As Variant
Public Fluxcs, mu, mu_p, mur_dt, Rildt, Rils, Bcs, Hcs, changer_R, changer_C As Variant
Public Fluxcvs, Hg_dt, Bg_dt, Aspcv, Bdt_r, Hdt_r, mu_dt_r As Variant
Public Blam, Hlam, Flux_tdt, Flux_dsp, dFlux_f, Flux_f, Ifs As Variant
Public piv_R, piv_C, mem, cv As Variant
Public n, m, z, fault, undteeth_1, zero_ctrl As Variant
Public pivot, a_logic, b_logic As Variant
Public xprec, Bprec, diffx, Flux_tcv, Flux_tcv_ As Variant
Appendix A2.1

Public dEnM, EnM, EnM_p, deltaEnM, Vol, Ril, Sez As Variant
Public H As Variant
Public Ncv, nc, Ia0, Ib0, Ic0, Hc, mu_magr, lm, Ldt, Lcvx, Lcl, hcl, adt, acs, acr, L,
g, havv, tollx, ta, xm, spm, dx, dx_cost, ixm_max, wm As Variant
Public Tau_cv, Tau_cv Rs, Tau_cv_int, Ldg, Ldg Rs, hbd, alfa_tcv, Ltd, mu_mag, Taup,
Brm, zm2 As Double
Public ind_dt, ind_dt2 As Variant
Public ordpr, ordsc, Ifse, Ifse_p As Variant
Public npr, ns, nsfs, w, fi_in, Irms As Variant

Public filepath As String
Public pp, kw, st As Integer
Public LdA 1, LdA 2, LdB 1, LdB 2, Ldx 1, Ldx 2, Lddx 1, Lddx 2, LdA 1 Rs, LdA 2 Rs,
LdB 1 Rs, LdB 2 Rs As Double
Public Fx, Fy, Fr, r, Fr tsx, Fr txdx, Fr mod, alfa_dt, alfa_dt_deg As Variant
Public Fx, Fy, Fr, r, Fr As Double

Public pr01, pr02, pr03 As Double
Public Rcv_ext, Lfc, Ltc, Rcv_int, thcv, Vol setcv, VolH setcv, VolL setcv, Lcvx med,
Re, Rr, Rs, Ralb, alfa cv, Lcs med, Lcr med, Vol cs, Vol cr As Double
Public Rcv efz, Rcv ifz, Rcv mfz, LcvxH, LcvxL As Double
Public Vol0cv, Volcv As Variant

Private Sub OptionButton2_Click()
If OptionButton2.Value = True Then
    MsgBox "Preparare il file ASP_TOT.txt premendo il pulsante nel foglio successivo", vbOKOnly, "SATSOLVER"
End If
End Sub

Public Sub Triang_Click()

Range("AE5:AG65492").ClearContents
Range("AH5").ClearContents
Range("AI37:IV65492").ClearContents
Range("E5:AB65492").ClearContents

ReDim Blam(1 To 452), Hlam(1 To 452), Flux_tdt(13 To 15), Flux_dsp(13 To 15), dFlux_f(1 To 3), Ifs(1 To 3) As Double

'INPUT

'numero di cave per coppia polare
Ncv = Int(Cells(4, 4))
'conduttori in cava
'I strato
npr = Cells(5, 4)
'I strato
ns = Cells(6, 4)
'passo polare
Taup = Cells(7, 4)
'Taup = Taup * 0.001
corrente efficace
Irms = Cells(8, 4)
'campo coercitivo intrinseco del magnete (A/m)
Hc = Cells(9, 4)
'permeabilità relativa magnetite
mu magr = Cells(10, 4)
'spessore magnetite
Im = Cells(11, 4)
Im = Im * 0.001
'spessore del corpo del dente

62
Ldt = Cells(12, 4)
Ldt = Ldt * 0.001
'apertura di cava (reale)
Lcl = Cells(14, 4)
Lcl = Lcl * 0.001
'altezza collarino
hcl = Cells(15, 4)
hcl = hcl * 0.001
'altezza dente statore
adt = Cells(16, 4)
adt = adt * 0.001
'spessore corona statore
acs = Cells(17, 4)
acs = acs * 0.001
'spessore corona rotore
acr = Cells(18, 4)
acr = acr * 0.001
'profondità di macchina
L = Cells(19, 4)
L = L * 0.001
'spessore traferro
g = Cells(20, 4)
g = g * 0.001
'altezza dell'avvolgimento
havv = Cells(21, 4)
havv = havv * 0.001
'scarico massimo ammesso fra due soluzioni successive del sistema
tollx = Cells(22, 4)
tollx = tollx * 0.01
'parametro adimensionale che definisce il rapporto fra
'la misura della parte bassa e quella dell'intero dente
xm = Cells(23, 4)
'posizione iniziale del magnete N
xm = Cells(24, 4)
xm = xm * 0.001
'step di sospensione del movimento (<= dx)
dx_cost = Cells(25, 4)
dx_cost = dx_cost * 0.001
'step di spostamento rotore
dx = Cells(26, 4)
dx = dx * 0.001
'numero di step nel movimento
mxm_max = Cells(27, 4)
'fase iniziale
fi_in = Cells(28, 4)
'amministrazione del magnet
wm = Cells(29, 4)
w = wm * 0.001
'numero di fasi
nfasi = Cells(30, 4)

******************************************************************************

'INPUT CARATTERISTICA DI MAGNETIZZAZIONE

For i = 1 To 452
Blam(i) = Cells(4 + i, 29)
Blam(i) = Blam(i) * 0.001
Next i

******************************************************************************

'frequenza elettrica
f = 50
kw = 1
st = 2
'raggio al fondo cava
Rcv_ext = 0.085
'coppie polari
pp = 1
'larghezza cava al fondo
Appendix A2.1

Lfc = 0.0097
'larghezza cava in testa
Ltc = 0.00523
filepath = CStr(TextBox1.Text)
If ta <= 1 - havv / adt Then
    MsgBox ("Valore di ta troppo basso!")
    Exit Sub
End If

'angolo di cava (rad. mecc.)
alfa_cv = 2 * pi / pp / Ncv
'raggio esterno statore
Re = Rcv_ext + acs
'raggio rotoare alla base del magnete
Rr = Rcv_ext - adt - g - Lm
'raggio rotoare interno (raggio albero)
Ralb = Rr - acr
'raggio interno statore
Rs = Rcv_ext - adt
'raggio di cava in testa
Rcv_int = Rcv_ext - havv
'angolo corrispondente alla cava (rad.)
thcv = 2 * Atn((Lfc - Ltc) / 2 / havv)
'passo di cava statore
Taucvs = 2 * Taup / Ncv
'passo di cava al raggio interno statore
Taucvs_Rs = 2 * pi * Rs / pp / Ncv
'passo di cava al raggio di cava in testa
Taucvs_int = 2 * pi * Rcv_int / pp / Ncv
'larghezza dente al diametro di riferimento
Ldg = Taucvs - Lcl
'larghezza dente al raggio interno statore
Ldg_Rs = 2 * pi * Rs / pp / Ncv
'larghezza cava statore in testa
Lcvs = Taucvs_int - Ldt
'spazio intermagnetico
spm = Taup - wm

*NUOVI PARAMETRI ************************************************************

'raggio medio magnete
Rm = Rs - g - Lm / 2
'raggio esterno magnetne
Rme = Rs - g
'raggio medio traferro
Rg = Rs - g / 2
'angolo corresp. alla semiapertura di cava
alfa_sap = Lcl / 2 / Rs
'angolo corresp. alla testa del dente
alfa_Ldg = alfa_cv - 2 * alfa_sap

*conduttori in cava
nc = npr + nsc
'pulsazione elettrica
w = 2 * pi * f
'altezza della base del dente
hbd = adt - havv - hcl
'angolo testa di cava
alfa_tcv = Atn((Lcvs - Lcl) / 2 / hbd)
'linea di sezione testa del dente
Ltd = hbd / Cos(alfa_tcv) + hcl
'permeabilità assoluta del magnete
mu_mag = mu_magr * muzero
'permeabilità relativa del magnete
Brm = mu_mag * Hc
'posizione iniziale del magnete S
The programming code - Part 1

\[ \text{x} \text{m2} = \text{x} \text{m} + \text{spm} + \text{wm} \]

'CONTROLLO DELLA POSIZIONE INIZIALE DEL MAGNETE
'Alla variabile "undteeth_1" viene assegnato valore True
'se la discontinuità N-S si trova sotto un qualsiasi dente

undteeth_1 = 0
For i = 1 To Ncv
  a_logic = (xm >= (Lcl / 2 + (i - 1) * Taucvs) And xm <= (Lcl / 2 + Ldg + (i - 1) * Taucvs))
  b_logic = ((xm + spm) >= (Lcl / 2 + (i - 1) * Taucvs) And (xm + spm) <= (Lcl / 2 + Ldg + (i - 1) * Taucvs))
  If a_logic And b_logic = True Then
    undteeth_1 = i
    GoTo 20
  End If
Next i
MsgBox "ATTENZIONE: uno dei due o entrambi i magneti" & Chr(13) & "si trovano sotto l'apertura di cava", vbCritical, "SATSOLVER"
Exit Sub

20 'Alla variabile "undteeth_2" viene assegnato valore True
'se la discontinuità S-N si trova sotto un qualsiasi dente
undteeth_2 = 0
For i = 1 To Ncv
  c_logic = (xm2 >= (Lcl / 2 + (i - 1) * Taucvs) And xm2 <= (Lcl / 2 + Ldg + (i - 1) * Taucvs))
  d_logic = ((xm2 + spm) >= (Lcl / 2 + (i - 1) * Taucvs) And (xm2 + spm) <= (Lcl / 2 + Ldg + (i - 1) * Taucvs))
  If c_logic And d_logic = True Then
    undteeth_2 = i
    GoTo 30
  End If
Next i
MsgBox "ATTENZIONE: uno dei due o entrambi i magneti" & Chr(13) & "si trovano sotto l'apertura di cava", vbCritical, "SATSOLVER"
Exit Sub

30 '*******************************************************************************
If undteeth_1 <> 0 And undteeth_2 <> 0 Then
  n = 18 * Ncv + 8
Else
  n = 18 * Ncv
End If
m = Int(n + 1)
ReDim a(n, m), aI(n), aI_p(n), aIp(n), aIs(n), aM(n), ASP(ixm_max * n), ainiz(n, m),
noti(n), x(1 To n), x_p(1 To n), y(n, m), B(n), Hdt(n), Flux_dt(1 To Ncv), deltaBH(n),
normx(199) As Double
ReDim Fluxcs(n), mu(n), mu_p(n), mu_dt_r(n), Rildt(n), Rilcs(n), Bcs(n), Hcs(n),
changer_R(m), changer_C(n) As Double
ReDim Fluxcv(n), Hg_dt(n), Bg_dt(n), Aspcv(n), Bdt_r(n), Hdt_r(n), mu_dt_r(n) As Double
ReDim piv_R(n), piv_C(n), mem(n), cv(1 To Ncv) As Integer
ReDim xprec(1 To n), bprec(1 To n), diffx(1 To n), Flux_tcv(1 To Ncv), Flux_tcv_(1 To Ncv) As Double
ReDim dEnM(1 To n), EnM(1 To n), EnM_p(1 To n), deltaEnM(1 To n), Vol(1 To n),
Vollcv(1 To 2 * Ncv), VolLcv(1 To 2 * Ncv), Volr(1 To n), Sez(1 To n) As Double
ReDim H(1 To n) As Double
ReDim ordpr(1 To Ncv), ordsc(1 To Ncv) As Integer
ReDim Ifse(1 To nfasi), Ifse_p(1 To nfasi), Flux_f(1 To nfasi) As Double
ReDim Fxr(1 To Ncv), Fyr(1 To Ncv), Fr_n(1 To Ncv), Fr_tsz(1 To Ncv), Fr_tdx(1 To Ncv), Fr_mod(1 To Ncv), alfa_dt(1 To Ncv), alfa_dt_deg(1 To Ncv) As Double

'PARAMETRI CIRCOLARI DI MACCHINA *********************************************

'CAVA
'NB: i raggi "fittizi" seguono i fianchi della cava,
' NON convergendo al centro del motore (0,0)
'raggio esterno fittizio
Rcv_efz = Lfc / 2 / Tan(thcv / 2)
'raggio interno fittizio
Rcv_ifz = Ltc / 2 / Tan(thcv / 2)
'raggio medio fittizio
Rcv_mfz = Rcv_efz - (1 - ta) * adt
'volumi nella zona alta di cava (H) e bassa (L)
VolH_setcv = thcv * L / 2 * (Rcv_efz ^ 2 - Rcv_mfz ^ 2)
VolL_setcv = thcv * L / 2 * (Rcv_mfz ^ 2 - Rcv_ifz ^ 2)
'volume dell'area di cava tra il raggio TCV e FCV
Vol_setcv = thcv * L / 2 * (Rcv_efz ^ 2 - Rcv_ifz ^ 2)

Lcvx_med = thcv * Rcv_mfz
If Lcvx_med < 0 Then
MsgBox ("Aumentare ta!")
Stop
End If

LcvxH = thcv / 2 * (Rcv_efz + Rcv_mfz)
LcvxL = thcv / 2 * (Rcv_mfz + Rcv_ifz)

'larghezza di cava al raggio medio
Lcvx_med = thcv * Rcv_mfz
If Lcvx_med < 0 Then
MsgBox ("Aumentare ta!")
Stop
End If

'CORONE

Vol_cs = alfa_cv * L / 2 * (Re ^ 2 - Rcv_ext ^ 2)
Vol_cr = alfa_cv * L / 2 * (Rr ^ 2 - Ralb ^ 2)

'LARGHEZZA DEI RAGGI

pr01 = (adt - havv) * L
pr02 = 2 * hbd * L / (Ldg - Ldt)
pr03 = 2 * hbd * L / (Lcvx_med - Lcl)

'PARAMETRI COSTANTI

'Riluttanze MAGNETE SOTTO IL DENTE md(i)
For i = 1 To Ncv
    Sez(i) = Rm * alfa_Ldg * L
    Vol(i) = Vol_mLdg
    Ril(i) = 1 / mu_mag * Lm / Sez(i)
Next i

'Riluttanze CORONA STATORICA
For i = Ncv + 1 To 2 * Ncv
The programming code - Part 1

```
Sez(i) = acs * L
Vol(i) = Vol_cs
Ril(i) = Rnd() * 3000
Next i

'Riluttanze CORONA ROTORICA
For i = 2 * Ncv + 1 To 3 * Ncv
   Sez(i) = acr * L
   Vol(i) = Vol_cr
   Ril(i) = Rnd() * 3000
Next i

'Riluttanze trasversali CAVE cvx(i)
For i = 3 * Ncv + 1 To 4 * Ncv
   Sez(i) = (Rcv_ext - Rcv_int) * L
   Vol(i) = Vol_cr
   Ril(i) = 2 / muzero * 1 / pr03 * Log((adt - hcl) / havv) + 1 / muzero * Lcl / (adt - hcl) / L
Next i

'Riluttanze DENTE LOW
For i = 4 * Ncv + 1 To 5 * Ncv
   Sez(i) = Ldt * L
   Vol(i) = adt * (1 - ta) * Ldt * L
   Ril(i) = Rnd() * 3000
Next i

'Riluttanze TRAFERRO SOTTO IL DENTE gd(i)
For i = 5 * Ncv + 1 To 6 * Ncv
   Sez(i) = Rg * alfa_Ldg * L
   Vol(i) = Vol_gLdg
   Ril(i) = 1 / muzero * g / Sez(i)
Next i

'Riluttanze DENTE HIGH
For i = 6 * Ncv + 1 To 7 * Ncv
   Sez(i) = Ldt * L
   Vol(i) = adt * (1 - ta) * Ldt * L
   Ril(i) = Rnd() * 3000
Next i

'Riluttanze TRAFERRO SOTTO SEMICAVA SX gcs(i)
For i = 7 * Ncv + 1 To 8 * Ncv
   Sez(i) = Rg * alfa_sap * L
   Vol(i) = Vol_gsap + pi * hcl ^ 2 / 4 * L
   Ril(i) = pi / (2 * muzero * L * Log((2 * g + pi * hcl) / 2 / g))
Next i

'Riluttanze trasversali COLLARINO clx(i)
For i = 8 * Ncv + 1 To 9 * Ncv
   Sez(i) = hcl * L
   Vol(i) = hcl * Lcl * L
   Ril(i) = 1 / muzero * Lcl / (hcl * L)
Next i

'Riluttanze TRAFERRO SOTTO SEMICAVA DX gcd(i)
For i = 9 * Ncv + 1 To 10 * Ncv
   Sez(i) = Rg * alfa_sap * L
   Vol(i) = Vol_gsap + pi * hcl ^ 2 / 4 * L
   Ril(i) = pi / (2 * muzero * L * Log((2 * g + pi * hcl) / 2 / g))
Next i

'Riluttanze MAGNETE SOTTO SEMICAVA SX mcs(i)
For i = 10 * Ncv + 1 To 11 * Ncv
   Sez(i) = Rm * alfa_sap * L
   Vol(i) = Vol_msap
   Ril(i) = 1 / mu_mag * Lm / Sez(i)
Next i

'Riluttanze MAGNETE SOTTO SEMICAVA DX mcd(i)
```
Appendix A2.1

For i = 11 * Ncv + 1 To 12 * Ncv  
Sez(i) = Rm * alfa_sap * L  
Vol(i) = Vol_msap  
Ril(i) = 1 / mu_mag * Lm / Sez(i)  
Next i

'Riluttanze nel SEMIDENTE SX e DX
For i = 12 * Ncv + 1 To 14 * Ncv  
Sez(i) = Ldg / 2 / (Ldt / 2 / (adt - havv) / L + 1 / pr02 * Log(pr01 / (pr01 - pr02 * (Ldg - Ldt) / 2)))  
Vol(i) = (Ldg / 2 + Ldt / 2) * hbd / 2 * L + Ldg / 2 * hcl * L  
Ril(i) = Rnd() * 3000  
Next i

'Riluttanze longitudinali BASSE SEMICAVE Lcvys(i), Lcvyd(i)
For i = 14 * Ncv + 1 To 16 * Ncv  
Sez(i) = LcvxL / 2 * L  
Vol(i) = VolLcv(i - 14 * Ncv)  
Ril(i) = 2 / muzero * hcl / (Lcl * L) + 2 / muzero * (hbd - hcl) / (Ltc - Lcl) / L * Log(Ltc / Lcl) + 2 / muzero * (ta * adt - hbd) / (Lcvx_med - Ltc) / L * Log(Lcvx_med / Ltc)  
Next i

'Riluttanze longitudinali ALTE SEMICAVE Hcvys(i), Hcvyd(i)
For i = 16 * Ncv + 1 To 18 * Ncv  
Sez(i) = LcvxH / 2 * L  
Vol(i) = VolHcv(i - 16 * Ncv)  
Ril(i) = 2 / muzero * adt * (1 - ta) / (Lfc - Lcvx_med) / L * Log(Lfc / Lcvx_med)  
Next i

***************************************************************************************
*ARRAY DEI TERMINI NOTI
***************************************************************************************
For i = 1 To n  
aI(i) = 0  
aM(i) = 0  
Next i

'Lettura del file relativo al I strato dell'avvolgimento statorico
Open filepath & \"ordpr.txt\" For Input As #5  
For i = 1 To Ncv  
Input #5, ordpr(i)  
Next i  
Close #5

'Lettura del file relativo al II strato dell'avvolgimento statorico
Open filepath & \"ordsc.txt\" For Input As #6  
For i = 1 To Ncv  
Input #6, ordsc(i)  
Next i  
Close #6

'Lettura del file contenente le amperprie equivalenti del magnete
Open filepath & \"aM(pp).txt\" For Input As #2  
For i = 1 To n  
Input #2, aM(i)  
Next i  
Close #2

If OptionButton2.Value = True Then
'Lettura del file contenente le amperprie con andamento SINUSOIDALE
Open filepath & \"\ASP_TOT.txt\" For Input As #1  
For i = 1 To (n * ixm_max)  
Input #1, ASP(i)  
Next i  
Close #1
Else
'Lettura del file contenente le amperprie COSTANTI
Open filepath & \"\ASP_COST.txt\" For Input As #8  
For i = 1 To n  
Input #8, aI(i)
The programming code - Part 1

Next i
Close #8
End If

'Creazione file di output
Open filepath & "\PP_18Ncv_FX(1.7st).txt" For Output As #3
Call SUBR_1
End Sub

Public Sub SUBR_1()

'Azzeramento coenergia magnetica
EnM_tp = 0
For i = 1 To n
EnM_p(i) = 0
Next i

'Inizializzazione variabile di controllo delle soluzioni nulle
zero_ctrl = 0

'CICLO DI POSIZIONAMENTO DEL MAGNETE
' NB: se le amperspire cambiano di posto nel vettore dei termini noti al variare
della posizione del magnete, è necessario inserire l'input nel ciclo "ixm"

For ixm = 1 To ixm_max
Range("AES:AO65492").ClearContents
For i = 1 To Ncv
a_logic = (xm >= (Lcl / 2 + (i - 1) * Taucvs) And xm <= (Lcl / 2 + Ldg + (i - 1) * Taucvs))
b_logic = ((xm + spm) >= (Lcl / 2 + (i - 1) * Taucvs) And (xm + spm) <= (Lcl / 2 + Ldg + (i - 1) * Taucvs))
If a_logic And b_logic = False Then
MsgBox "ATTENZIONE: uno dei due o entrambe i magneti" & Chr(13) & "si trovano sotto l'apertura di cava", vbCritical, "SATSOLVER"
GoTo 50
End If
c_logic = (xm2 >= (Lcl / 2 + (i - 1) * Taucvs) And xm2 <= (Lcl / 2 + Ldg + (i - 1) * Taucvs))
d_logic = ((xm2 + spm) >= (Lcl / 2 + (i - 1) * Taucvs) And (xm2 + spm) <= (Lcl / 2 + Ldg + (i - 1) * Taucvs))
If c_logic And d_logic = False Then
MsgBox "ATTENZIONE: uno dei due o entrambe i magneti" & Chr(13) & "si trovano sotto l'apertura di cava", vbCritical, "SATSOLVER"
GoTo 50
End If
Next i

'INDIVIDUAZIONE POSIZIONE MAGNETE
'Alle variabili ind_dt, ind_dt2 viene assegnato l'indice
"numerico dei denti sotto i quali si trovano i "buchi"
ind_dt = 0
ind_dt2 = 0

For i = 1 To Ncv
If xm >= (Lcl / 2 + (i - 1) * Taucvs) And xm <= (Lcl / 2 + Ldg + (i - 1) * Taucvs) Then
ind_dt = i
GoTo 120
End If
Next i
120
For i = 1 To Ncv
If xm2 >= (Lcl / 2 + (i - 1) * Taucvs) And xm2 <= (Lcl / 2 + Ldg + (i - 1) * Taucvs) Then
ind_dt2 = i
GoTo 130
End If
Next i
Appendix A2.1

If ind_dt = ind_dt2 Then
  MsgBox "ATTENZIONE: ind_dt = ind_dt2!" & Chr(13) & "CONDIZIONE IMPOSSIBILE!", vbCritical, "SATSOLVER"
  GoTo 50
End If

'***********************************************************************
'PARAMETRI DIPENDENTI DALLA POSIZIONE
'***********************************************************************

'PARAMETRI GEOMETRICI f(xm,ind_dt) *************************************

LdA_1 = xm - Lcl / 2 - (ind_dt - 1) * Taucvs
LdA_2 = xm2 - Lcl / 2 - (ind_dt2 - 1) * Taucvs
LdB_1 = Taucvs - Lcl / 2 - xm - spm + (ind_dt - 1) * Taucvs
LdB_2 = Taucvs - Lcl / 2 - xm2 - spm + (ind_dt2 - 1) * Taucvs
Ldx_1 = xm - Lcl / 2 + spm / 2 - (ind_dt - 1) * Taucvs
Ldx_2 = xm2 - Lcl / 2 + spm / 2 - (ind_dt2 - 1) * Taucvs
Lddx_1 = Taucvs - Lcl / 2 - xm - spm / 2 + (ind_dt - 1) * Taucvs
Lddx_2 = Taucvs - Lcl / 2 - xm2 - spm / 2 + (ind_dt2 - 1) * Taucvs

'parametri per il calcolo della forza radiale riferiti
'al diametro interno di statore

xm_Rs = Rs / Rm * xm
xm2_Rs = Rs / Rm * xm2
spm_Rs = Rs / Rm * spm

LdA_1_Rs = xm_Rs - Lcl / 2 - (ind_dt - 1) * Taucvs_Rs
LdA_2_Rs = xm2_Rs - Lcl / 2 - (ind_dt2 - 1) * Taucvs_Rs
LdB_1_Rs = Taucvs_Rs - Lcl / 2 - xm_Rs - spm_Rs + (ind_dt - 1) * Taucvs_Rs
LdB_2_Rs = Taucvs_Rs - Lcl / 2 - xm2_Rs - spm_Rs + (ind_dt2 - 1) * Taucvs_Rs

'NUOVI PARAMETRI ************************************************************

'angolo corrispondente allo spazio intermagnetico
alfa_spm = spm / Rm
'angoli corrisp. alle posizioni dei magneti
th_xm = xm / Rm
th_xm2 = xm2 / Rm
'angoli corrisp. ai parametri variabili
tha_1 = th_xm - alfa_sap - (ind_dt - 1) * alfa_cv
tha_2 = th_xm2 - alfa_sap - (ind_dt2 - 1) * alfa_cv
thB_1 = alfa_cv - alfa_sap - th_xm - alfa_spm + (ind_dt - 1) * alfa_cv
thB_2 = alfa_cv - alfa_sap - th_xm2 - alfa_spm + (ind_dt2 - 1) * alfa_cv

' ***********************************************************************************

'RICALCOLO DI RILUTTANZE, SEZIONI E VOLUMI NELLE ZONE SOGGETTE A MODIFICHE DELLA GEOMETRIA

If ind_dt <> 0 And ind_dt2 <> 0 Then

'DISCONTINUITA' N-S

Vol(ind_dt) = tha_1 / 2 * L * (Rme ^ 2 - Rr ^ 2)
Sez(ind_dt) = tha_1 * (Rme + Rr) / 2 * L
Ril(ind_dt) = 1 / mu_mag * Lm / Sez(ind_dt)

Vol(5 * Ncv + ind_dt) = thB_1 / 2 * L * (Rs ^ 2 - Rme ^ 2)
Sez(5 * Ncv + ind_dt) = thB_1 * (Rs + Rme) / 2 * L
Ril(5 * Ncv + ind_dt) = 1 / muzero * g / Sez(5 * Ncv + ind_dt)

Vol(18 * Ncv + 1) = thB_1 / 2 * L * (Rme ^ 2 - Rr ^ 2)
Sez(18 * Ncv + 1) = thB_1 * (Rme + Rr) / 2 * L
Ril(18 * Ncv + 1) = 1 / mu_mag * Lm / Sez(18 * Ncv + 1)

******************************************************************************
The programming code - Part 1

\[
\begin{align*}
\text{Vol}(18 \ast Ncv + 2) &= \text{alfa}_spm / 2 \ast L \ast (Rme ^ 2 - Rr ^ 2) \\
\text{Sez}(18 \ast Ncv + 2) &= \text{alfa}_spm \ast (Rme + Rr) / 2 \ast L \\
\text{Ril}(18 \ast Ncv + 2) &= 1 / \muzero \ast Lm / \text{Sez}(18 \ast Ncv + 2) \\
\text{Vol}(18 \ast Ncv + 3) &= \text{thB}_1 / 2 \ast L \ast (Rs ^ 2 - Rme ^ 2) \\
\text{Sez}(18 \ast Ncv + 3) &= \text{thB}_1 \ast (Rs + Rme) / 2 \ast L \\
\text{Ril}(18 \ast Ncv + 3) &= 1 / \muzero \ast g / \text{Sez}(18 \ast Ncv + 3) \\
\text{Vol}(18 \ast Ncv + 4) &= \text{alfa}_spm / 2 \ast L \ast (Rs ^ 2 - Rme ^ 2) \\
\text{Sez}(18 \ast Ncv + 4) &= \text{alfa}_spm \ast (Rs + Rme) / 2 \ast L \\
\text{Ril}(18 \ast Ncv + 4) &= 1 / \muzero \ast g / \text{Sez}(18 \ast Ncv + 4)
\end{align*}
\]

'DISCONTINUITA' S-N

\[
\begin{align*}
\text{Vol}(\text{ind_dt2}) &= \text{thA}_2 / 2 \ast L \ast (Rme ^ 2 - Rr ^ 2) \\
\text{Sez}(\text{ind_dt2}) &= \text{thA}_2 \ast (Rme + Rr) / 2 \ast L \\
\text{Ril}(\text{ind_dt2}) &= 1 / \muMag \ast Lm / \text{Sez}(\text{ind_dt2}) \\
\text{Vol}(5 \ast Ncv + \text{ind_dt2}) &= \text{thA}_2 / 2 \ast L \ast (Rs ^ 2 - Rme ^ 2) \\
\text{Sez}(5 \ast Ncv + \text{ind_dt2}) &= \text{thA}_2 \ast (Rs + Rme) / 2 \ast L \\
\text{Ril}(5 \ast Ncv + \text{ind_dt2}) &= 1 / \muMag \ast g / \text{Sez}(5 \ast Ncv + \text{ind_dt2}) \\
\text{Vol}(18 \ast Ncv + 5) &= \text{thB}_2 / 2 \ast L \ast (Rme ^ 2 - Rr ^ 2) \\
\text{Sez}(18 \ast Ncv + 5) &= \text{thB}_2 \ast (Rme + Rr) / 2 \ast L \\
\text{Ril}(18 \ast Ncv + 5) &= 1 / \muMag \ast g / \text{Sez}(18 \ast Ncv + 5) \\
\text{Vol}(18 \ast Ncv + 6) &= \text{alfa}_spm / 2 \ast L \ast (Rme ^ 2 - Rr ^ 2) \\
\text{Sez}(18 \ast Ncv + 6) &= \text{alfa}_spm \ast (Rme + Rr) / 2 \ast L \\
\text{Ril}(18 \ast Ncv + 6) &= 1 / \muzero \ast Lm / \text{Sez}(18 \ast Ncv + 6) \\
\text{Vol}(18 \ast Ncv + 7) &= \text{thB}_2 / 2 \ast L \ast (Rs ^ 2 - Rme ^ 2) \\
\text{Sez}(18 \ast Ncv + 7) &= \text{thB}_2 \ast (Rs + Rme) / 2 \ast L \\
\text{Ril}(18 \ast Ncv + 7) &= 1 / \muzero \ast g / \text{Sez}(18 \ast Ncv + 7) \\
\text{Vol}(18 \ast Ncv + 8) &= \text{alfa}_spm / 2 \ast L \ast (Rs ^ 2 - Rme ^ 2) \\
\text{Sez}(18 \ast Ncv + 8) &= \text{alfa}_spm \ast (Rs + Rme) / 2 \ast L \\
\text{Ril}(18 \ast Ncv + 8) &= 1 / \muzero \ast g / \text{Sez}(18 \ast Ncv + 8)
\end{align*}
\]

End If

If OptionButton2.Value = True Then
  If Int(ixm / 2) <> ixm / 2 Then
    'step di spostamento DISPARI: le correnti vengono calcolate
    'secondo l'esatto valore istantaneo correlato alla posizione
    't = pi * xm / w / Taup
    'Memorizzazione del valore delle correnti allo step precedente
    For i = 1 To nfasi
      Ifse_p(i) = Ifse(i)
    Next i
    For j = 1 To nfasi
      'Andamento cosinusoidale delle correnti
      Ifse(j) = -Sqr(2) * Irms * Cos(kw * w * t - (j - 1) * kw * st * 2 * pi / nfasi + fi_in)
    Next j
  Else
    For i = 1 To nfasi
      Ifse(i) = Ifse_p(i)
    Next i
  End If
End If

'***************************************************************************************
'COSTRUZIONE DELLA MATRICE A+B
'***************************************************************************************

For i = 1 To n
  For j = 1 To m
    a(i, j) = 0
  Next j
Next i

If OptionButton2.Value = True Then
For i = 1 To n
    a(i, m) = ASP((ixm - 1) * n + i)
Next i
Else
For i = 1 To n
    a(i, m) = aI(i) + aM(i)
Next i
End If

'PARTE INVARIANTE DELLA MATRICE (A)

'SNcv+2 to 6Ncv ******************************
'NODO 1: CONTINUITÀ DENTE H - COR.STAT.
For i = 2 To Ncv
    a(5 * Ncv + i, Ncv + i - 1) = 1
    a(5 * Ncv + i, Ncv + i) = -1
    a(5 * Ncv + i, 6 * Ncv + i) = -1
    a(5 * Ncv + i, 16 * Ncv + i) = -1
    a(5 * Ncv + i, 17 * Ncv + i) = -1
Next i

'SNcv+1 to 7Ncv ******************************
'NODO 2: CONTINUITÀ DENTE L - DENTE H
a(6 * Ncv + 1, 4 * Ncv) = 1
a(6 * Ncv + 1, 3 * Ncv + 1) = -1
a(6 * Ncv + 1, 4 * Ncv + 1) = 1
a(6 * Ncv + 1, 6 * Ncv + 1) = -1
a(6 * Ncv + 1, 14 * Ncv + 1) = 1
a(6 * Ncv + 1, 15 * Ncv + 1) = 1
a(6 * Ncv + 1, 16 * Ncv + 1) = -1
a(6 * Ncv + 1, 17 * Ncv + 1) = -1
For i = 2 To Ncv
    a(6 * Ncv + i, 3 * Ncv + i - 1) = 1
    a(6 * Ncv + i, 3 * Ncv + i) = -1
    a(6 * Ncv + i, 4 * Ncv + i) = 1
    a(6 * Ncv + i, 6 * Ncv + i) = -1
    a(6 * Ncv + i, 14 * Ncv + i) = 1
    a(6 * Ncv + i, 15 * Ncv + i) = 1
    a(6 * Ncv + i, 16 * Ncv + i) = -1
    a(6 * Ncv + i, 17 * Ncv + i) = -1
Next i

'SNcv+1 to 8Ncv ******************************
'NODO 4

'SEquazioni standard
For i = 1 To Ncv
    a(7 * Ncv + i, 4 * Ncv + i) = 1
    a(7 * Ncv + i, 5 * Ncv + i) = -1
    a(7 * Ncv + i, 12 * Ncv + i) = -1
    a(7 * Ncv + i, 13 * Ncv + i) = 1
Next i

'SVariazioni
If ind_dt <> 0 Then
    a(7 * Ncv + ind_dt, 18 * Ncv + 3) = -1
End If
If ind_dt2 <> 0 Then
    a(7 * Ncv + ind_dt2, 18 * Ncv + 4) = -1
End If
The programming code - Part 1

*8Ncv+1 to 9Ncv **************************************************************
*NODO 7(1)

For i = 1 To Ncv
  a(8 * Ncv + i, i) = 1
  a(8 * Ncv + i, 5 * Ncv + i) = -1
Next i

*9Ncv+1 to 10Ncv **************************************************************
*NODO 9: CONTINUITA' MAGNETE - COR.ROT.

a(9 * Ncv + 1, 1) = 1
a(9 * Ncv + 1, 2 * Ncv + 1) = 1
a(9 * Ncv + 1, 3 * Ncv) = -1
a(9 * Ncv + 1, 10 * Ncv + 1) = 1
a(9 * Ncv + 1, 11 * Ncv + 1) = 1

For i = 2 To Ncv
  a(9 * Ncv + i, i) = 1
  a(9 * Ncv + i, 2 * Ncv + i) = 1
  a(9 * Ncv + i, 2 * Ncv + i - 1) = -1
  a(9 * Ncv + i, 10 * Ncv + i) = 1
  a(9 * Ncv + i, 11 * Ncv + i) = 1
Next i

If ind_dt <> 0 Then
  a(9 * Ncv + ind_dt, 18 * Ncv + 1) = 1
  a(9 * Ncv + ind_dt, 18 * Ncv + 2) = 1
End If

If ind_dt2 <> 0 Then
  a(9 * Ncv + ind_dt2, 18 * Ncv + 5) = 1
  a(9 * Ncv + ind_dt2, 18 * Ncv + 6) = 1
End If

*10Ncv+1 to 11Ncv **************************************************************
*NODO 3

a(10 * Ncv + 1, 9 * Ncv) = 1
a(10 * Ncv + 1, 7 * Ncv + 1) = 1
a(10 * Ncv + 1, 12 * Ncv + 1) = -1
a(10 * Ncv + 1, 14 * Ncv + 1) = -1

For i = 2 To Ncv
  a(10 * Ncv + i, 8 * Ncv + i - 1) = 1
  a(10 * Ncv + i, 7 * Ncv + i) = 1
  a(10 * Ncv + i, 12 * Ncv + i) = -1
  a(10 * Ncv + i, 14 * Ncv + i) = -1
Next i

*11Ncv+1 to 12Ncv **************************************************************
*NODO 5

For i = 1 To Ncv
  a(11 * Ncv + i, 8 * Ncv + i) = 1
  a(11 * Ncv + i, 9 * Ncv + i) = -1
  a(11 * Ncv + i, 13 * Ncv + i) = -1
  a(11 * Ncv + i, 15 * Ncv + i) = 1
Next i

*14Ncv+1 to 15Ncv **************************************************************
*NODO 6

For i = 1 To Ncv
  a(14 * Ncv + i, 7 * Ncv + i) = 1
  a(14 * Ncv + i, 10 * Ncv + i) = -1
Next i

*15Ncv+1 to 16Ncv **************************************************************
*NODO 8
Appendix A2.1

For i = 1 To Ncv
    a(15 * Ncv + i, 9 * Ncv + i) = 1
    a(15 * Ncv + i, 11 * Ncv + i) = -1
Next i

If OptionButton2.Value = False Then
    For i = 1 To n
        a(i, m) = aI(i) + aM(i)
    Next i
Else
    For i = 1 To n
        a(i, m) = ASP((ixm - 1) * n + i)
    Next i
End If

For i = 1 To (5 * Ncv + 1)
    For j = 1 To n
        a(i, j) = Rnd() * 3000
    Next j
Next i

For i = (12 * Ncv + 1) To (14 * Ncv)
    For j = 1 To n
        a(i, j) = Rnd() * 3000
    Next j
Next i

For i = (16 * Ncv + 1) To n
    For j = 1 To n
        a(i, j) = Rnd() * 3000
    Next j
Next i

For i = 1 To (5 * Ncv + 1)
    For j = 1 To n
        a(i, j) = Rnd() * 3000
    Next j
Next i

For i = (12 * Ncv + 1) To (14 * Ncv)
    For j = 1 To n
        a(i, j) = Rnd() * 3000
    Next j
Next i

For i = (16 * Ncv + 1) To n
    For j = 1 To n
        a(i, j) = Rnd() * 3000
    Next j
Next i

Call SOLVESYS

CALCOLO DI FLUSSI NELLE VARIE ZONE DI MACCHINA

'flusso nel passo di cava
For i = 1 To Ncv
    Flux_tcv_(i) = x(i + 5 * Ncv) + x(i + 7 * Ncv) + x(i + 9 * Ncv)
Next i

'flusso per polo al traferro
Flux_pg = 0
For i = 1 To Ncv
    Flux_pg = Flux_pg + Flux_tcv(i)
Next i

'flusso nel dente basso - alto
Flux_pDL = 0
Flux_pDH = 0
For i = 1 To Ncv
    Flux_pDL = Flux_pDL + x(4 * Ncv + i)
    Flux_pDH = Flux_pDH + x(6 * Ncv + i)
Next i
*Flusso all'altezza dei denti
Flux_pd = ta \cdot Flux_{pdL} + (1 - ta) \cdot Flux_{pdH}

*Flussi nei denti
For i = 1 To Ncv
Flux_dt(i) = ta \cdot x(4 \cdot Ncv + i) + (1 - ta) \cdot x(6 \cdot Ncv + i)
Next i

*Flussi nel passo di cava (inteso come semicava sx - dente - semicava dx)
For i = 1 To Ncv
Flux_tcv_(i) = Flux_dt(i) + x(14 \cdot Ncv + i) + x(15 \cdot Ncv + i)
Next i

*FLUSSI CONCATENATI CON LE FASI
For i = 1 To nfasi
Flux_f(i) = 0
Next i

*Flussi concatenati I strato
For j = 1 To nfasi
For i = 1 To Ncv
If ordpr(i) = j Then
Flux_f(j) = Flux_f(j) + npr \cdot x(Ncv + i)
ElseIf ordpr(i) = -j Then
Flux_f(j) = Flux_f(j) - npr \cdot x(Ncv + i)
End If
Next i
Next j

*Flussi concatenati II strato
For j = 1 To nfasi
For i = 1 To Ncv
If ordsc(i) = j Then
Flux_f(j) = Flux_f(j) + nsc \cdot x(Ncv + i)
ElseIf ordsc(i) = -j Then
Flux_f(j) = Flux_f(j) - nsc \cdot x(Ncv + i)
End If
Next i
Next j

*FORZA RADIALE
For i = 1 To Ncv
Fyr(i) = 0
Fxr(i) = 0
Fr_n(i) = 0
Fr_tsx(i) = 0
Fr_tdx(i) = 0
Next i
Fyr_ = 0
Fxr_ = 0

*NB: il sistema di riferimento considerato nel calcolo delle forze ha gli assi coincidenti con il sistema di rif.
  ' x-y standard di FEMM. L'asse della fase 1, secondo l'interpretazione del segno
delle correnti in FEMM (- se
  ' ENTRANTI nello schermo, + se USCENTI), è ruotato invece di 180 gr.m. in senso
orario rispetto al sistema di
  ' riferimento in questione. Gli angoli sono valutati rispetto all'asse (x-standard).

*angolo dell'asse del dente (gr.m.)
For i = 1 To Ncv
alfa_dt(i) = pi / 2 - (i - 1) \cdot alfa_cv
alfa_dt_deg(i) = 180 / pi \cdot alfa_dt(i)
Next i

*Superfici frontali dei DENTI
For i = 1 To Ncv
Select Case i
Case ind_dt
'dente ind_dt

Appendix A2.1

If ind_dt = 1 Then
'Componente NORMALE della forza radiale
Fr_n(i) = 1 / 2 / muzero * L * (LdA_1_Rs * B(5 * Ncv + i) ^ 2 + spm_Rs * B(18 * Ncv + 4) ^ 2 + LdB_1_Rs * B(18 * Ncv + 3) ^ 2 + Lcl / 2 * B(7 * Ncv + i) ^ 2 + Lcl / 2 * L * B(9 * Ncv + i) ^ 2)
Fr_n(i) = Fr_n(i) - 1 / 2 / muzero * Lcl / 2 * L * B(7 * Ncv + i) * B(9 * Ncv)
Fr_tdx(i) = 1 / muzero * Lcl / 2 * L * B(9 * Ncv + i) * B(8 * Ncv + i)
Else 'Componente TANGENZIALI della forza radiale
Fr_tsx(i) = 1 / muzero * Lcl / 2 * L * B(7 * Ncv + i) * B(9 * Ncv)
Fr_tdx(i) = 1 / muzero * Lcl / 2 * L * B(9 * Ncv + i) * B(8 * Ncv + i)
End If

Case ind_dt2
'dente ind_dt2
If ind_dt2 = 1 Then
'Componente NORMALE della forza radiale
Fr_n(i) = 1 / 2 / muzero * L * (LdA_1_Rs * B(5 * Ncv + i) ^ 2 + spm_Rs * B(18 * Ncv + 8) ^ 2 + LdB_1_Rs * B(18 * Ncv + 7) ^ 2 + Lcl / 2 * B(7 * Ncv + i) ^ 2 + Lcl / 2 * L * B(9 * Ncv + i) ^ 2)
Fr_n(i) = Fr_n(i) - 1 / 2 / muzero * Lcl / 2 * L * B(8 * Ncv + i - 1) ^ 2 - 1 / 2 / muzero * Lcl / 2 * L * B(8 * Ncv + i) ^ 2
'Componente TANGENZIALI della forza radiale
Fr_tsx(i) = 1 / muzero * Lcl / 2 * L * B(7 * Ncv + i) * B(8 * Ncv + i - 1)
Fr_tdx(i) = 1 / muzero * Lcl / 2 * L * B(9 * Ncv + i) * B(8 * Ncv + i)
End If

Case Else
If i = 1 Then
'Componente NORMALE della forza radiale
Fr_n(i) = 1 / 2 / muzero * L * (Ldg_Rs * B(5 * Ncv + i) ^ 2 + Lcl / 2 * B(7 * Ncv + i) ^ 2 + Lcl / 2 * L * B(8 * Ncv + i) ^ 2)
Fr_n(i) = Fr_n(i) - 1 / 2 / muzero * Lcl / 2 * L * B(7 * Ncv + i) * B(9 * Ncv)
Fr_tdx(i) = 1 / muzero * Lcl / 2 * L * B(9 * Ncv + i) * B(8 * Ncv + i)
Else 'Componente TANGENZIALI della forza radiale
Fr_tsx(i) = 1 / muzero * Lcl / 2 * L * B(7 * Ncv + i) * B(9 * Ncv)
Fr_tdx(i) = 1 / muzero * Lcl / 2 * L * B(9 * Ncv + i) * B(8 * Ncv + i)
End If

End Select
The programming code - Part 1

*Componenti secondo (y) della forza radiale in ogni dente
Fyr(i) = Fr_n(i) * Sin(alfa_dt(i)) - Fr_tsx(i) * Cos(alfa_dt(i)) * 
Cos(alfa_dt(i))
*Componenti secondo (x) della forza radiale
Fxr(i) = Fr_n(i) * Cos(alfa_dt(i)) + Fr_tsx(i) * Sin(alfa_dt(i)) + Fr_tdx(i) * 
Sin(alfa_dt(i))
*Modulo della forza radiale in ogni dente
Fr_mod(i) = Sqr(Fxr(i) ^ 2 + Fyr(i) ^ 2)

Next i

*Calcolo della risultante delle forze RADIALI
For i = 1 To Ncv
  Fyr_ = Fyr_ + Fyr(i)
  Fxr_ = Fxr_ + Fxr(i)
Next i
*RISULTANTE
Fr = Sqr(Fxr_ ^ 2 + Fyr_ ^ 2)

**************************************************************************************
*CICLO DI CALCOLO COENERGIA MAGNETICA DEL SISTEMA

If OptionButton1.Value = True Then

*Memorizzazione dei valori step precedente
*Coenergia magnetica complessiva
EnM_tp = EnM_t

*Memorizzazione dei valori step precedente
*Coenergia magnetica nel singolo volume
For i = 1 To n
  EnM_p(i) = EnM(i)
Next i
EnM_t = 0
nH = 640

*MAGNETE (SOTTO IL DENTE)
For i = 1 To Ncv
  dEnM(i) = 1 / 2 * B(i) ^ 2 / mu(i)
  EnM(i) = dEnM(i) * Vol(i)
  EnM_t = EnM_t + EnM(i)
Next i

*CORONA STAT., CORONA ROT.
For i = Ncv + 1 To 3 * Ncv
  dEnM(i) = 1 / 2 * B(i) ^ 2 / mu(i)
  EnM(i) = dEnM(i) * Vol(i)
  EnM_t = EnM_t + EnM(i)
Next i

*CAVE (Componenti orizzontali CVX)
For i = 3 * Ncv + 1 To 4 * Ncv
  dEnM(i) = 1 / 2 * B(i) ^ 2 / mu(i)
  EnM(i) = dEnM(i) * Vol(i)
  EnM_t = EnM_t + EnM(i)
Next i

*DENTE BASSO
For i = 4 * Ncv + 1 To 5 * Ncv
\[ d\text{EnM}(i) = 0 \]
\[ dH = \text{Abs}(H(i)) / nH \]
For \( j = 1 \) To \( nH \)
\[ H1 = (j - 1) * dH \]
\[ H2 = j * dH \]
\[ B1 = \text{Induction}(n, H1, Hlam, Blam) \]
\[ B2 = \text{Induction}(n, H2, Hlam, Blam) \]
\[ dA = (B2 + B1) * dH / 2 \]
\[ d\text{EnM}(i) = d\text{EnM}(i) + dA \]
Next \( j \)
\[ \text{EnM}(i) = d\text{EnM}(i) * \text{Vol}(i) \]
\[ \text{EnM}_t = \text{EnM}_t + \text{EnM}(i) \]
Next \( i \)

*TRAFERRO sotto il DENTE*
For \( i = 5 * Ncv + 1 \) To \( 6 * Ncv \)
\[ d\text{EnM}(i) = 1 / 2 * B(i) ^ 2 / \mu(i) \]
\[ \text{EnM}(i) = d\text{EnM}(i) * \text{Vol}(i) \]
\[ \text{EnM}_t = \text{EnM}_t + \text{EnM}(i) \]
Next \( i \)

*DENTE ALTO*
For \( i = 6 * Ncv + 1 \) To \( 7 * Ncv \)
\[ d\text{EnM}(i) = 0 \]
\[ dH = \text{Abs}(H(i)) / nH \]
For \( j = 1 \) To \( nH \)
\[ H1 = (j - 1) * dH \]
\[ H2 = j * dH \]
\[ B1 = \text{Induction}(n, H1, Hlam, Blam) \]
\[ B2 = \text{Induction}(n, H2, Hlam, Blam) \]
\[ dA = (B2 + B1) * dH / 2 \]
\[ d\text{EnM}(i) = d\text{EnM}(i) + dA \]
Next \( j \)
\[ \text{EnM}(i) = d\text{EnM}(i) * \text{Vol}(i) \]
\[ \text{EnM}_t = \text{EnM}_t + \text{EnM}(i) \]
Next \( i \)

*TRAFERRO sotto la SEMICAVA SX*
*COLLARINO (TRASV.)*
*TRAFERRO sotto la SEMICAVA DX*
*MAGNETE SOTTO APERTURA CAVA SX e DX*
For \( i = 7 * Ncv + 1 \) To \( 12 * Ncv \)
\[ d\text{EnM}(i) = 1 / 2 * B(i) ^ 2 / \mu(i) \]
\[ \text{EnM}(i) = d\text{EnM}(i) * \text{Vol}(i) \]
\[ \text{EnM}_t = \text{EnM}_t + \text{EnM}(i) \]
Next \( i \)

*TESTA DENTE SX e DX*
For \( i = 12 * Ncv + 1 \) To \( 14 * Ncv \)
\[ d\text{EnM}(i) = 0 \]
\[ dH = \text{Abs}(H(i)) / nH \]
For \( j = 1 \) To \( nH \)
\[ H1 = (j - 1) * dH \]
\[ H2 = j * dH \]
\[ B1 = \text{Induction}(n, H1, Hlam, Blam) \]
\[ B2 = \text{Induction}(n, H2, Hlam, Blam) \]
\[ dA = (B2 + B1) * dH / 2 \]
\[ d\text{EnM}(i) = d\text{EnM}(i) + dA \]
Next \( j \)
\[ \text{EnM}(i) = d\text{EnM}(i) * \text{Vol}(i) \]
\[ \text{EnM}_t = \text{EnM}_t + \text{EnM}(i) \]
Next \( i \)

*SEMICAVE SX e DX*
For \( i = 14 * Ncv + 1 \) To \( 16 * Ncv \)
\[ d\text{EnM}(i) = 1 / 2 * B(i) ^ 2 / \mu(i) \]
\[ \text{EnM}(i) = d\text{EnM}(i) * \text{Vol}(i) \]
\[ \text{EnM}_t = \text{EnM}_t + \text{EnM}(i) \]
Next \( i \)
'CONTRIBUTI AGGIUNTIVI
If ind_dt <> 0 And ind_dt2 <> 0 Then
For i = 1 To 8
    dEnM(18 * Ncv + i) = 1 / 2 * B(18 * Ncv + i) ^ 2 / mu(18 * Ncv + i)
    EnM(18 * Ncv + i) = dEnM(18 * Ncv + i) * Vol(18 * Ncv + i)
    EnM_t = EnM_t + EnM(18 * Ncv + i)
Next i
End If

'VARIAZIONE COENERGIA MAGNETICA TOTALE
dEnM_t = EnM_t - EnM_tp

'VARIAZIONE COENERGIA MAGNETICA SINGOLI VOLUMI
If ixm = 1 Then
    For i = 1 To n
        deltaEnM(i) = 0
    Next i
Else
    For i = 1 To n
        deltaEnM(i) = EnM(i) - EnM_p(i)
    Next i
End If

Cells(5 + ixm, 13 + 2 * nfasi + 1) = EnM_t * pp
End If

'**************************************************************************************
If OptionButton1.Value = True And Int(ixm / 2) = ixm / 2 Then
    Fx_EM = dEnM_t / dx_cost
    'Coppia elettromagnetica
dthr_m = pi / pp / Taup * dx_cost
    C_EM = dEnM_t / dthr_m
    Cells(5 + ixm, 13 + 2 * nfasi + 2) = C_EM * pp
End If

'**************************************************************************************
'OUTPUT ARRAY
For i = 1 To Ncv
    Cells(4 + i, 5) = CStr("md")
    Cells(4 + Ncv + i, 5) = CStr("cs")
    Cells(4 + 2 * Ncv + i, 5) = CStr("cr")
    Cells(4 + 3 * Ncv + i, 5) = CStr("cvx")
    Cells(4 + 4 * Ncv + i, 5) = CStr("dl")
    Cells(4 + 5 * Ncv + i, 5) = CStr("gd")
    Cells(4 + 6 * Ncv + i, 5) = CStr("dH")
    Cells(4 + 7 * Ncv + i, 5) = CStr("gcs")
    Cells(4 + 8 * Ncv + i, 5) = CStr("clx")
    Cells(4 + 9 * Ncv + i, 5) = CStr("gcd")
    Cells(4 + 10 * Ncv + i, 5) = CStr("mcs")
    Cells(4 + 11 * Ncv + i, 5) = CStr("mcd")
    Cells(4 + 12 * Ncv + i, 5) = CStr("dsx")
    Cells(4 + 13 * Ncv + i, 5) = CStr("dxx")
    Cells(4 + 14 * Ncv + i, 5) = CStr("Lcvys")
    Cells(4 + 15 * Ncv + i, 5) = CStr("Lcvyd")
    Cells(4 + 16 * Ncv + i, 5) = CStr("Hcvys")
    Cells(4 + 17 * Ncv + i, 5) = CStr("Hcvyd")
Next i

Cells(5 + 18 * Ncv, 5) = CStr("mdB1")
Cells(6 + 18 * Ncv, 5) = CStr("spm1")
Cells(7 + 18 * Ncv, 5) = CStr("gcdB1")
Cells(8 + 18 * Ncv, 5) = CStr("gcd01")
Cells(9 + 18 * Ncv, 5) = CStr("mdB2")

79
<table>
<thead>
<tr>
<th>Cells(10 + 18 * Ncv, 5)</th>
<th>= CStr(&quot;spm2&quot;)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cells(11 + 18 * Ncv, 5)</td>
<td>= CStr(&quot;gdB2&quot;)</td>
</tr>
<tr>
<td>Cells(12 + 18 * Ncv, 5)</td>
<td>= CStr(&quot;gd02&quot;)</td>
</tr>
</tbody>
</table>

For i = 1 To n
 Cells(4 + i, 6) = H(i)
 Cells(4 + i, 7) = B(i)
 Cells(4 + i, 8) = mu(i) / muzero
 Cells(4 + i, 9) = x(i)
Next i

Cells(5 + ixm, 12) = xm
Cells(5 + ixm, 13) = Hc

For i = 1 To nfasi
 Cells(5 + ixm, 13 + i) = Ifse(i)
Next i

For i = 1 To nfasi
 Cells(5 + ixm, 13 + nfasi + i) = Flux_f(i) * pp
Next i

Cells(5 + ixm, 13 + 2 * nfasi + 3) = Fxr_
Cells(5 + ixm, 13 + 2 * nfasi + 4) = Fyr_
Cells(5 + ixm, 13 + 2 * nfasi + 5) = Fr_

If OptionButton1.Value = True Then
 Print #3, xm
 Print #3,
 Print #3, "i      VOL(m^3)      dEnM(J/m^3)   EnM(J)        H(A/m)        B(T)
 deltaEnM(J)     Ril(H-1)      x(Wb)"
 Print #3,
 For i = 1 To 18 * Ncv
 Val_00 = Format(i, "000" & "    ")
 Val_01 = Format(Vol(i), "0.00000E+" & "    ")
 Val_02 = Format(dEnM(i), "0.00000E+" & "    ")
 Val_03 = Format(EnM(i), "0.00000E+" & "    ")
 Val_04 = Format(H(i), "0000000.00" & "    ")
 Val_05 = Format(B(i), "0.000" & "    ")
 Val_06 = Format(deltaEnM(i), "0.00000E+" & "    ")
 Val_07 = Format(Ril(i), "0.00000E+" & "    ")
 Val_08 = Format(x(i), "0.000E+" & "    ")
 Print #3, Val_00 & Val_01 & Val_02 & Val_03 & Val_04 & Val_05 & Val_06 & Val_07 & Val_08
Next i

*ELEMENTI AGGIUNTIVI

For i = 18 * Ncv + 1 To n
 Val_00 = Format(i, "000" & "    ")
 Val_01 = Format(Vol(i), "0.00000E+" & "    ")
 Val_02 = Format(dEnM(i), "0.00000E+" & "    ")
 Val_03 = Format(EnM(i), "0.00000E+" & "    ")
 Val_04 = Format(H(i), "0000000.00" & "    ")
 Val_05 = Format(B(i), "0.000" & "    ")
 Val_06 = Format(deltaEnM(i), "0.00000E+" & "    ")
 Val_07 = Format(Ril(i), "0.00000E+" & "    ")
 Val_08 = Format(x(i), "0.000E+" & "    ")
 Print #3, Val_00 & Val_01 & Val_02 & Val_03 & Val_04 & Val_05 & Val_06 & Val_07 & Val_08
Next i

Print #3, Fyr(N)      Fxr(N)      alfa_dt(gr.m.)      Fr_mod(N)"
Print #3,

For i = 1 To Ncv
 Val_09 = Format(Fyr(i), "00000000.00" & "    ")
 Val_10 = Format(Fxr(i), "00000000.00" & "    ")
 Val_11 = Format(alfa_dt_deg(i), "00000000.00" & "    ")
 Val_12 = Format(Fr_mod(i), "00000000.00" & "    ")
 Print #3, Val_09 & Val_10 & Val_11 & Val_12

80
Next i
Print #3,
Print #3,
Print #3,
End If

'aggiornamento della posizione dei magneti
If Int(ixm / 2) <> ixm / 2 Then
dx1 = dx_cost
Else
dx1 = dx
End If

xm = xm + dx1
xm2 = xm + spm + wm

Cells(5, 34) = zero_ctrl

Next ixm

50
Close #3
OptionButton1.Value = False
OptionButton2.Value = False

End Sub

Public Sub SOLVESYS()

For i = 1 To n
For j = 1 To m
'matrice iniziale del sistema
ainiz(i, j) = a(i, j)
Next j
Next i

maxdiffx = 1000000
z = 1

For i = 1 To n
mu(i) = muzero
mu_p(i) = 0
Next i

Do While maxdiffx > tollx

maxdiffx = 1000000
z = 1

For i = 1 To n
mu(i) = muzero
mu_p(i) = 0
Next i

Do While maxdiffx > tollx

pivot = False
fault = 0
iperm = 0
For k = 1 To n - 1
'lo scambio avviene con le righe successive a quella considerata (k-esima)
r = k + 1
Do Until a(k, k) <> 0
'ciclo che scambia la riga k-esima con la r-esima
'nel caso in cui un elemento diagonale sia nullo
pivot = True
For icol = 1 To m
changer_R(icol) = a(k, icol)
a(k, icol) = a(r, icol)
a(r, icol) = changer_R(icol)
Next icol
Appendix A2.1

Next icol
For i = 1 To n
    piv_R(i) = 0
Next i
piv_R(k) = r
piv_R(r) = k
r = r + 1
If r > n Then
    MsgBox ("Un elemento A(k,k) rimane nullo dopo aver scorso tutte le righe (r > n)")
    Stop
End If
iperm = iperm + 1
Loop
If pivot = True Then
    mem(k) = r
Else
    mem(k) = k
End If
For i = k + 1 To n
    y(i, k) = -a(i, k) / a(k, k)
a(i, k) = y(i, k)
For j = k + 1 To m
    a(i, j) = a(i, j) + a(i, k) * a(k, j)
Next j
Next i
pivot = False
Next k

'SOLUZIONE DEL SISTEMA DI EQUAZIONI CON IL METODO DIRETTO
For i = 1 To n
    noti(i) = a(i, m)
Next i
x(n) = noti(n) / a(n, n)
For i = n - 1 To 1 Step -1
    sm = 0
    For k = n To i + 1 Step -1
        sm = sm + a(i, k) * x(k)
    Next k
    x(i) = (noti(i) - sm) / a(i, i)
Next i

'memorizzazione permeabilità allo step precedente
For j = 1 To n
    mu_p(j) = mu(j)
Next j

determinazione delle induzioni dai flussi
For i = 1 To n
    B(i) = x(i) / Sez(i)
Next i

'richiama la function per il calcolo delle permeabilità magnetiche
For j = 1 To Ncv
    Case j
End Select
Case 1 To Ncv
The programming code - Part 1

mu(j) = mu_mag
H(j) = 1 / mu_mag * B(j)

Case Ncv + 1 To 3 * Ncv
H(j) = Magfield(n, B(j), Hlam, Blam)
If B(j) < 0 Then H(j) = -H(j)
If B(j) = 0 Then
   mu(j) = (Blam(2) - Blam(1)) / (Hlam(2) - Hlam(1))
   zero_ctrl = zero_ctrl + 1
Else
   mu(j) = B(j) / H(j)
End If

Case 3 * Ncv + 1 To 4 * Ncv
mu(j) = muzero
H(j) = 1 / muzero * B(j)

Case 4 * Ncv + 1 To 5 * Ncv
H(j) = Magfield(n, B(j), Hlam, Blam)
If B(j) < 0 Then H(j) = -H(j)
If B(j) = 0 Then
   mu(j) = (Blam(2) - Blam(1)) / (Hlam(2) - Hlam(1))
   zero_ctrl = zero_ctrl + 1
Else
   mu(j) = B(j) / H(j)
End If

Case 5 * Ncv + 1 To 6 * Ncv
mu(j) = muzero
H(j) = 1 / muzero * B(j)

Case 6 * Ncv + 1 To 7 * Ncv
H(j) = Magfield(n, B(j), Hlam, Blam)
If B(j) < 0 Then H(j) = -H(j)
If B(j) = 0 Then
   mu(j) = (Blam(2) - Blam(1)) / (Hlam(2) - Hlam(1))
   zero_ctrl = zero_ctrl + 1
Else
   mu(j) = B(j) / H(j)
End If

Case 7 * Ncv + 1 To 10 * Ncv
mu(j) = muzero
H(j) = 1 / muzero * B(j)

Case 10 * Ncv + 1 To 12 * Ncv
mu(j) = mu_mag
H(j) = 1 / mu_mag * B(j)

Case 12 * Ncv + 1 To 14 * Ncv
H(j) = Magfield(n, B(j), Hlam, Blam)
If B(j) < 0 Then H(j) = -H(j)
If B(j) = 0 Then
   mu(j) = (Blam(2) - Blam(1)) / (Hlam(2) - Hlam(1))
   zero_ctrl = zero_ctrl + 1
Else
   mu(j) = B(j) / H(j)
End If

Case 14 * Ncv + 1 To 18 * Ncv
mu(j) = muzero
H(j) = 1 / muzero * B(j)
End Select
Next j

If ind_dt <> 0 Then
mu(18 * Ncv + 1) = mu_mag
H(18 * Ncv + 1) = 1 / mu_mag * B(18 * Ncv + 1)
mu(18 * Ncv + 2) = muzero
H(18 * Ncv + 2) = 1 / muzero * B(18 * Ncv + 2)
mu(18 * Ncv + 3) = muzero
H(18 * Ncv + 3) = 1 / muzero * B(18 * Ncv + 3)
mu(18 * Ncv + 4) = muzero
H(18 * Ncv + 4) = 1 / muzero * B(18 * Ncv + 4)
End If

If ind_dt2 <> 0 Then
mu(18 * Ncv + 5) = mu_mag
H(18 * Ncv + 5) = 1 / mu_mag * B(18 * Ncv + 5)
mu(18 * Ncv + 6) = muzero
H(18 * Ncv + 6) = 1 / muzero * B(18 * Ncv + 6)
mu(18 * Ncv + 7) = muzero
H(18 * Ncv + 7) = 1 / muzero * B(18 * Ncv + 7)
mu(18 * Ncv + 8) = muzero
H(18 * Ncv + 8) = 1 / muzero * B(18 * Ncv + 8)
End If

*Redefinition of magnetic permeability
For j = 1 To n
  mu(j) = mu(j) ^ 0.1 * mu_p(j) ^ 0.9
Next j

************************************************************************************
   ******************** CRITERIO DI CONVERGENZA ********************

maxdiffx = 0

*Solo elementi NON LINEARI
For j = Ncv + 1 To 3 * Ncv
  diffx(j) = Abs((mu(j) / muzero - mu_p(j) / muzero) / (mu_p(j) / muzero))
  If diffx(j) > maxdiffx Then
    maxdiffx = diffx(j)
  End If
Next j

For j = 4 * Ncv + 1 To 5 * Ncv
  diffx(j) = Abs((mu(j) / muzero - mu_p(j) / muzero) / (mu_p(j) / muzero))
  If diffx(j) > maxdiffx Then
    maxdiffx = diffx(j)
  End If
Next j

For j = 6 * Ncv + 1 To 7 * Ncv
  diffx(j) = Abs((mu(j) / muzero - mu_p(j) / muzero) / (mu_p(j) / muzero))
  If diffx(j) > maxdiffx Then
    maxdiffx = diffx(j)
  End If
Next j

For j = 12 * Ncv + 1 To 14 * Ncv
  diffx(j) = Abs((mu(j) / muzero - mu_p(j) / muzero) / (mu_p(j) / muzero))
  If diffx(j) > maxdiffx Then
    maxdiffx = diffx(j)
  End If
Next j

***************************************************************************************

*RIDEFINIZIONE MATRICE ”A” DEL SISTEMA
*vengono aggiornati i coefficienti variabili in cui compaiono i valori di permeabilità
*magnetica delle varie parti di macchina

************************************************************************************

*azzzeramento di tutti i coefficienti
For i = 1 To n
  For j = 1 To m
    a(i, j) = 0
  Next j
Next i

*RIDEFINIZIONE DELLE RILUTTANZE VARIABILI f(mu)

*corona statore
For i = Ncv + 1 To 2 * Ncv
  Ril(i) = 1 / mu(i) * Lcs_med / Sez(i)
Next i

*corona rotore
For i = 2 * Ncv + 1 To 3 * Ncv
  Ril(i) = 1 / mu(i) * Lcr_med / Sez(i)
The programming code - Part 1

Next i

'dente LOW
For i = 4 * Ncv + 1 To 5 * Ncv
    Ril(i) = 1 / mu(i) * (adt * (ta - 1) + havv) / Sez(i)
Next i

'dente HIGH
For i = 6 * Ncv + 1 To 7 * Ncv
    Ril(i) = 1 / mu(i) * (adt * (1 - ta)) / Sez(i)
Next i

'semidente SX e DX
For i = 12 * Ncv + 1 To 14 * Ncv
    Ril(i) = 1 / mu(i) / pr02 * Log(2 * pr01 / (2 * pr01 - pr02 * (Ldg - Ldt))) + 1 / mu(i) * Ldt / 2 / (adt - havv) / L
Next i

'le righe con coefficienti costanti della matrice del sistema vengono ricomposte come
'quelle iniziali per ripartire con la soluzione del sistema, aggiornato aggiungendo
'le permeabilità calcolate
For i = 5 * Ncv + 2 To 12 * Ncv
    For j = 1 To n
        a(i, j) = ainiz(i, j)
    Next j
Next i

For i = 14 * Ncv + 1 To 16 * Ncv
    For j = 1 To n
        a(i, j) = ainiz(i, j)
    Next j
Next i

'1'ultima colonna della matrice (termini noti)
'riassume i valori iniziali
For i = 1 To n
    a(i, m) = ainiz(i, m)
Next i

*PARTE VARIABILE DELLA MATRICE (A)
*1 to Ncv *****************************************

*CIRCUITAZIONI DENTE - DENTE
For i = 1 To Ncv - 1
    a(i, i) = Ril(i)
    a(i, i + 1) = -Ril(i + 1)
    a(i, i + Ncv) = -Ril(i + Ncv)
    a(i, i + 2 * Ncv) = -Ril(i + 2 * Ncv)
    a(i, i + 4 * Ncv) = Ril(i + 4 * Ncv)
    a(i, i + 6 * Ncv) = Ril(i + 6 * Ncv)
Next i

a(Ncv, Ncv) = Ril(Ncv)
a(Ncv, 1) = -Ril(1)
a(Ncv, 2 * Ncv) = -Ril(2 * Ncv)
a(Ncv, 3 * Ncv) = -Ril(3 * Ncv)
a(Ncv, 4 * Ncv) = Ril(4 * Ncv)
a(Ncv, 5 * Ncv) = Ril(5 * Ncv)
a(Ncv, 6 * Ncv) = Ril(6 * Ncv)
a(Ncv, 7 * Ncv) = Ril(7 * Ncv)
a(Ncv, 8 * Ncv) = Ril(8 * Ncv)

Select Case ind_dt
Appendix A2.1

Case 1
*Circuitaz. precedente
a(Ncv, 1) = 0
a(Ncv, 5 * Ncv + 1) = 0
a(Ncv, 18 * Ncv + 2) = -Ril(18 * Ncv + 2)
a(Ncv, 18 * Ncv + 4) = -Ril(18 * Ncv + 4)
*Circuitaz. attuale
a(1, 1) = 0
a(1, 5 * Ncv + 1) = 0
a(1, 18 * Ncv + 2) = Ril(18 * Ncv + 2)
a(1, 18 * Ncv + 4) = Ril(18 * Ncv + 4)

Case 2 To (Ncv - 1)
*Circuitaz. precedente
a(ind_dt - 1, ind_dt) = 0
a(ind_dt - 1, ind_dt + 5 * Ncv) = 0
a(ind_dt - 1, 18 * Ncv + 2) = -Ril(18 * Ncv + 2)
a(ind_dt - 1, 18 * Ncv + 4) = -Ril(18 * Ncv + 4)
*Circuitaz. attuale
a(ind_dt, ind_dt) = 0
a(ind_dt, ind_dt + 5 * Ncv) = 0
a(ind_dt, 18 * Ncv + 2) = Ril(18 * Ncv + 2)
a(ind_dt, 18 * Ncv + 4) = Ril(18 * Ncv + 4)

Case Ncv
*Circuitaz. precedente
a(Ncv - 1, Ncv) = 0
a(Ncv - 1, 6 * Ncv) = 0
a(Ncv - 1, 18 * Ncv + 2) = -Ril(18 * Ncv + 2)
a(Ncv - 1, 18 * Ncv + 4) = -Ril(18 * Ncv + 4)
*Circuitaz. attuale
a(Ncv, Ncv) = 0
a(Ncv, 6 * Ncv) = 0

Case Else
Stop
End Select

Select Case ind_dt2
Case 1
*Circuitaz. precedente
a(Ncv, 1) = 0
a(Ncv, 5 * Ncv + 1) = 0
a(Ncv, 18 * Ncv + 6) = -Ril(18 * Ncv + 6)
a(Ncv, 18 * Ncv + 8) = -Ril(18 * Ncv + 8)
*Circuitaz. attuale
a(1, 1) = 0
a(1, 5 * Ncv + 1) = 0
a(1, 18 * Ncv + 6) = Ril(18 * Ncv + 6)
a(1, 18 * Ncv + 8) = Ril(18 * Ncv + 8)

Case 2 To (Ncv - 1)
*Circuitaz. precedente
a(ind_dt2 - 1, ind_dt2) = 0
a(ind_dt2 - 1, ind_dt2 + 5 * Ncv) = 0
a(ind_dt2 - 1, 18 * Ncv + 6) = -Ril(18 * Ncv + 6)
a(ind_dt2 - 1, 18 * Ncv + 8) = -Ril(18 * Ncv + 8)
*Circuitaz. attuale
a(ind_dt2, ind_dt2) = 0
a(ind_dt2, ind_dt2 + 5 * Ncv) = 0
a(ind_dt2, 18 * Ncv + 6) = Ril(18 * Ncv + 6)
a(ind_dt2, 18 * Ncv + 8) = Ril(18 * Ncv + 8)

Case Ncv
*Circuitaz. precedente
a(Ncv - 1, Ncv) = 0
a(Ncv - 1, 6 * Ncv) = 0
a(Ncv - 1, 18 * Ncv + 6) = -Ril(18 * Ncv + 6)
a(Ncv - 1, 18 * Ncv + 8) = -Ril(18 * Ncv + 8)
*Circuitaz. attuale
a(Ncv, Ncv) = 0
a(Ncv, 6 * Ncv) = 0
The programming code - Part 1

\[
\begin{align*}
a(Ncv, 18 \times Ncv + 6) &= Ril(18 \times Ncv + 6) \\
a(Ncv, 18 \times Ncv + 8) &= Ril(18 \times Ncv + 8)
\end{align*}
\]

Case Else
Stop
End Select

'Ncv+1 to 2Ncv

\[
\begin{align*}
a(Ncv + 1, Ncv + 1) &= -Ril(Ncv + 1) \\
a(Ncv + 1, 3 \times Ncv + 1) &= -Ril(3 \times Ncv + 1) \\
a(Ncv + 1, 6 \times Ncv + 1) &= Ril(6 \times Ncv + 1) \\
a(Ncv + 1, 6 \times Ncv + 1 + 1) &= -Ril(6 \times Ncv + 1 + 1)
\end{align*}
\]

Next i

\[
\begin{align*}
a(2 \times Ncv, 2 \times Ncv) &= -Ril(2 \times Ncv) \\
a(2 \times Ncv, 4 \times Ncv) &= -Ril(4 \times Ncv) \\
a(2 \times Ncv, 7 \times Ncv) &= Ril(7 \times Ncv) \\
a(2 \times Ncv, 6 \times Ncv + 1) &= -Ril(6 \times Ncv + 1)
\end{align*}
\]

'2Ncv+1 to 3Ncv

\[
\begin{align*}
a(2 \times Ncv + i, Ncv + i) &= -Ril(Ncv + i) \\
a(2 \times Ncv + i, 4 \times Ncv + i) &= Ril(4 \times Ncv + i) \\
a(2 \times Ncv + i, 4 \times Ncv + 1 + i) &= -Ril(4 \times Ncv + 1 + i) \\
a(2 \times Ncv + i, 6 \times Ncv + i) &= Ril(6 \times Ncv + i) \\
a(2 \times Ncv + i, 6 \times Ncv + 1 + i) &= -Ril(6 \times Ncv + 1 + i) \\
a(2 \times Ncv + i, 8 \times Ncv + i) &= -Ril(8 \times Ncv + i) \\
a(2 \times Ncv + i, 12 \times Ncv + 1 + i) &= -Ril(12 \times Ncv + 1 + i) \\
a(2 \times Ncv + i, 13 \times Ncv + i) &= -Ril(13 \times Ncv + i)
\end{align*}
\]

Next i

\[
\begin{align*}
a(3 \times Ncv, 2 \times Ncv) &= -Ril(2 \times Ncv) \\
a(3 \times Ncv, 5 \times Ncv) &= Ril(5 \times Ncv) \\
a(3 \times Ncv, 9 \times Ncv) &= -Ril(9 \times Ncv) \\
a(3 \times Ncv, 12 \times Ncv + 1) &= -Ril(12 \times Ncv + 1)
\end{align*}
\]

'3Ncv+1 to 4Ncv

\[
\begin{align*}
a(3 \times Ncv + i, i) &= Ril(i) \\
a(3 \times Ncv + i, 11 \times Ncv + i) &= -Ril(11 \times Ncv + i) \\
a(3 \times Ncv + i, 5 \times Ncv + i) &= Ril(5 \times Ncv + i) \\
a(3 \times Ncv + i, 9 \times Ncv + i) &= -Ril(9 \times Ncv + i) \\
a(3 \times Ncv + i, 13 \times Ncv + i) &= Ril(13 \times Ncv + i)
\end{align*}
\]

Next i

\[
\begin{align*}
a(3 \times Ncv + ind_dt, ind_dt) &= 0 \\
a(3 \times Ncv + ind_dt, 5 \times Ncv + ind_dt) &= 0 \\
a(3 \times Ncv + ind_dt, 18 \times Ncv + 1) &= Ril(18 \times Ncv + 1) \\
a(3 \times Ncv + ind_dt, 18 \times Ncv + 3) &= Ril(18 \times Ncv + 3)
\end{align*}
\]

End If

\[
\begin{align*}
a(3 \times Ncv + ind_dt2, ind_dt2) &= 0 \\
a(3 \times Ncv + ind_dt2, 5 \times Ncv + ind_dt2) &= 0 \\
a(3 \times Ncv + ind_dt2, 18 \times Ncv + 5) &= Ril(18 \times Ncv + 5) \\
a(3 \times Ncv + ind_dt2, 18 \times Ncv + 7) &= Ril(18 \times Ncv + 7)
\end{align*}
\]

End If

'4Ncv+1 to 5Ncv

\[
\begin{align*}
a(3 \times Ncv + 18 \times Ncv + 6) &= Ril(18 \times Ncv + 6) \\
a(3 \times Ncv + 18 \times Ncv + 8) &= Ril(18 \times Ncv + 8)
\end{align*}
\]
Appendix A2.1

*CIRCUITAZIONI BASE DENTE - TRAFERRO - MAGNETE SX

For i = 1 To Ncv
a(4 * Ncv + i, i) = -Ril(i)
a(4 * Ncv + i, 10 * Ncv + i) = Ril(10 * Ncv + i)
a(4 * Ncv + i, 5 * Ncv + i) = -Ril(5 * Ncv + i)
a(4 * Ncv + i, 7 * Ncv + i) = Ril(7 * Ncv + i)
a(4 * Ncv + i, 12 * Ncv + i) = Ril(12 * Ncv + i)
Next i

*5Ncv+1 *********************************************
*CIRCUITAZIONE HcS
For i = 1 To Ncv
a(5 * Ncv + 1, Ncv + i) = Ril(Ncv + i)
Next i

*12Ncv+1 to 13Ncv *********************************************
*CIRCUITAZIONI DENTE - SEMIDENTE - SEMICAVA DX

For i = 1 To Ncv
a(12 * Ncv + i, 4 * Ncv + i) = Ril(4 * Ncv + i)
a(12 * Ncv + i, 6 * Ncv + i) = Ril(6 * Ncv + i)
a(12 * Ncv + i, 13 * Ncv + i) = -Ril(13 * Ncv + i)
a(12 * Ncv + i, 15 * Ncv + i) = -Ril(15 * Ncv + i)
a(12 * Ncv + i, 17 * Ncv + i) = -Ril(17 * Ncv + i)
Next i

*13Ncv+1 to 14Ncv *********************************************
*CIRCUITAZIONE SEMICAVA SX LOW

For i = 1 To Ncv
*a(13 * Ncv + i, 4 * Ncv + i) = -Ril(4 * Ncv + i)
a(13 * Ncv + i, 12 * Ncv + i) = -Ril(12 * Ncv + i)
a(13 * Ncv + i, 13 * Ncv + i) = -Ril(13 * Ncv + i)
a(13 * Ncv + i, 14 * Ncv + i) = Ril(14 * Ncv + i)
a(13 * Ncv + i, 15 * Ncv + i) = -Ril(15 * Ncv + i)
Next i

*16Ncv+1 to 17Ncv *********************************************
*CIRCUITAZIONI ALTA SEMICAVA - ALTO SEMIDENTE SX

For i = 1 To Ncv
*a(16 * Ncv + i, 16 * Ncv + i) = Ril(16 * Ncv + i)
a(16 * Ncv + i, 6 * Ncv + i) = -Ril(6 * Ncv + i)
Next i

*17Ncv+1 to 18Ncv *********************************************
*CIRCUITAZIONI ALTA SEMICAVA - ALTO SEMIDENTE DX

For i = 1 To Ncv
a(17 * Ncv + i, 6 * Ncv + i) = -Ril(6 * Ncv + i)
a(17 * Ncv + i, 17 * Ncv + i) = -Ril(17 * Ncv + i)
Next i

*18Ncv+1 to 18Ncv+8 *********************************************
*EQUAZIONI AGGIUNTIVE

If ind_dt <> 0 Then
*Circuitazione PRIMA discontinuità del magnete
a(18 * Ncv + 1, ind_dt) = Ril(ind_dt)
a(18 * Ncv + 1, 5 * Ncv + ind_dt) = Ril(5 * Ncv + ind_dt)
a(18 * Ncv + 1, 18 * Ncv + 4) = -Ril(18 * Ncv + 4)
a(18 * Ncv + 1, 18 * Ncv + 2) = -Ril(18 * Ncv + 2)

*Circuitazione SECONDA discontinuità del magnete
a(18 * Ncv + 2, 18 * Ncv + 2) = Ril(18 * Ncv + 2)
a(18 * Ncv + 2, 18 * Ncv + 1) = -Ril(18 * Ncv + 1)
a(18 * Ncv + 2, 18 * Ncv + 4) = Ril(18 * Ncv + 4)
a(18 * Ncv + 2, 18 * Ncv + 3) = -Ril(18 * Ncv + 3)
The programming code - Part 1

* NODI 7(2)
  a(18 * Ncv + 3, 18 * Ncv + 2) = -1
  a(18 * Ncv + 3, 18 * Ncv + 4) = 1

* NODI 7(3)
  a(18 * Ncv + 4, 18 * Ncv + 1) = -1
  a(18 * Ncv + 4, 18 * Ncv + 3) = 1

End If

If ind_dt2 <> 0 Then
  *Circuitazione PRIMA discontinuità del magnete
  a(18 * Ncv + 5, ind_dt2) = Ril(ind_dt2)
  a(18 * Ncv + 5, 5 * Ncv + ind_dt2) = Ril(5 * Ncv + ind_dt2)
  a(18 * Ncv + 5, 18 * Ncv + 8) = -Ril(18 * Ncv + 8)
  a(18 * Ncv + 5, 18 * Ncv + 6) = -Ril(18 * Ncv + 6)

  *Circuitazione SECONDA discontinuità del magnete
  a(18 * Ncv + 6, 18 * Ncv + 6) = Ril(18 * Ncv + 6)
  a(18 * Ncv + 6, 18 * Ncv + 5) = -Ril(18 * Ncv + 5)
  a(18 * Ncv + 6, 18 * Ncv + 8) = Ril(18 * Ncv + 8)
  a(18 * Ncv + 6, 18 * Ncv + 7) = -Ril(18 * Ncv + 7)

* NODI 7(2)
  a(18 * Ncv + 7, 18 * Ncv + 6) = -1
  a(18 * Ncv + 7, 18 * Ncv + 8) = 1

* NODI 7(3)
  a(18 * Ncv + 8, 18 * Ncv + 5) = -1
  a(18 * Ncv + 8, 18 * Ncv + 7) = 1

End If

Cells(4 + z, 31) = z
Cells(4 + z, 32) = maxdiffx * 100
z = z + 1
Loop
End Sub

Public Function Induction(n, x_, Hlam, Blam)
  ' NON LINEARE
  For i = 1 To 451
    If Abs(x_) >= Hlam(i) And Abs(x_) <= Hlam(i + 1) Then
      deltaBH = (Blam(i + 1) - Blam(i)) / (Hlam(i + 1) - Hlam(i))
      Induction = deltaBH * (Abs(x_) - Hlam(i)) + Blam(i)
    End If
    Next i
  If Abs(x_) >= Hlam(452) Then
    deltaBH = (Blam(452) - Blam(451)) / (Hlam(452) - Hlam(451))
    Induction = deltaBH * (Abs(x_) - Hlam(452)) + Blam(452)
  End If
End Function

Public Function Magfield(n, x_, Hlam, Blam)
  ' NON LINEARE
  For i = 1 To 451
    If Abs(x_) >= Blam(i) And Abs(x_) <= Blam(i + 1) Then
      deltaHB = (Hlam(i + 1) - Hlam(i)) / (Blam(i + 1) - Blam(i))
      Magfield = deltaHB * (Abs(x_) - Blam(i)) + Hlam(i)
    End If
    Next i
End Function
If Abs(x_) >= Blam(452) Then
    deltaHB = (Hlam(452) - Hlam(451)) / (Blam(452) - Blam(451))
    Magfield = deltaHB * (Abs(x_) - Blam(452)) + Hlam(452)
End If

End Function
Appendix A2.2

THE PROGRAMMING CODE

Part 2

In the following, the programming code of the subroutine to determine the array of the stator ampere-turns distribution, to save in the file ASP_TOT.txt combined with the magnets distribution. Two versions are presented, a simpler one and a more elaborate one.

A2.2.1 The MMF array (simplified version)

Const pi = 3.1415927

Private Sub CommandButton1_Click()

'INPUT

Ncv = Foglio2.Cells(4, 4)
npr = Foglio2.Cells(5, 4)
nscc = Foglio2.Cells(6, 4)
Taup = Foglio2.Cells(7, 4)
Taup = Taup * 0.001
Irms = Foglio2.Cells(8, 4)
adt = Foglio2.Cells(16, 4)
adt = adt * 0.001
havv = Foglio2.Cells(21, 4)
havv = havv * 0.001
ta = Foglio2.Cells(23, 4)
xm = Foglio2.Cells(24, 4)
xm = xm * 0.001
dx_cost = Foglio2.Cells(25, 4)
dx_cost = dx_cost * 0.001
dx = Foglio2.Cells(26, 4)
dx = dx * 0.001

ixm_max = Foglio2.Cells(27, 4)
fi_in = Foglio2.Cells(28, 4)
nfasi = Foglio2.Cells(30, 4)
filepath = CStr(Foglio2.TextBox1.Text)
If ta <= 1 - havv / adt Then
    MsgBox ("Valore di ta troppo basso!")
    Stop
End If
f = 50
kw = 1
st = 2
'Grandezze derivate
w = 2 * pi * f
n = 18 * Ncv + 8
ktir = adt * (1 - ta) / havv
ReDim ordpr(1 To Ncv), ordsc(1 To Ncv) As Integer
ReDim aI(1 To n), aIp(1 To n), aIs(1 To n), aI_p(1 To n), aM(1 To n), ASP(1 To n) As Double
ReDim Ifse(1 To nfasi), Ifse_p(1 To nfasi) As Double

'Lettura del file relativo al I strato dell'avvolgimento statorico
Open filepath & "\ordpr.txt" For Input As #1
For i = 1 To Ncv
    Input #1, ordpr(i)
Next i
Close #1

'Lettura del file relativo al II strato dell'avvolgimento statorico
Open filepath & "\ordsc.txt" For Input As #2
For i = 1 To Ncv
    Input #2, ordsc(i)
Next i
Close #2

'Lettura del file contenente le amperspire equivalenti del magnete
Open filepath & "\aM(pp).txt" For Input As #3
For i = 1 To n
    Input #3, aM(i)
Next i
Close #3

Open filepath & "\ASP_TOT.txt" For Output As #4
For ixm = 1 To ixm_max
    If Int(ixm / 2) <> ixm / 2 Then
        'step di spostamento DISPARI: le correnti vengono calcolate
        'secondo l'esatto valore istantaneo correlato alla posizione
        'inizializzazione dell'array
        For i = 1 To n
            aI(i) = 0
        Next i
        'Andamento sinusoidale delle correnti
        t = pi * xm / w / Taup
        'Memorizzazione del valore delle correnti allo step precedente
        For i = 1 To nfasi
            Ifse_p(i) = Ifse(i)
        Next i
        For j = 1 To nfasi
            Ifse(j) = -Sqr(2) * Irms * Cos(kw * w * t - (j - 1) * kw * st * 2 * pi / nfasi + fi_in)
        Next j
    End If
End For

'ARRAY AMPERSPIRE STATORICHE
For i = 1 To Ncv
    aI(i) = npr * Sgn(ordpr(i)) * Ifse(Abs(ordpr(i)))
The programming code - Part 2

\[
a\text{Is}(i) = nsc \times \text{Sgn}(\text{ordsc}(i)) \times \text{Ifse}(\text{Abs}(\text{ordsc}(i))) \\
\text{Next i}
\]

For \( i = 1 \) To \( Ncv \)
\[
a\text{Ip}(Ncv + i) = 0 \\
a\text{Is}(Ncv + i) = nsc \times \text{Sgn}(\text{ordsc}(i)) \times \text{Ifse}(\text{Abs}(\text{ordsc}(i))) \\
\text{Next i}
\]

For \( i = 1 \) To \( Ncv \)
\[
a\text{Ip}(2 \times Ncv + i) = npr \times \text{Sgn}(\text{ordpr}(i)) \times \text{Ifse}(\text{Abs}(\text{ordpr}(i))) \\
a\text{Is}(2 \times Ncv + i) = nsc \times \text{Sgn}(\text{ordsc}(i)) \times \text{Ifse}(\text{Abs}(\text{ordsc}(i))) \\
\text{Next i}
\]

For \( i = 1 \) To \( Ncv \)
\[
a\text{Ip}(12 \times Ncv + i) = npr \times \text{Sgn}(\text{ordpr}(i)) \times \text{Ifse}(\text{Abs}(\text{ordpr}(i))) \\
a\text{Is}(12 \times Ncv + i) = nsc \times \text{Sgn}(\text{ordsc}(i)) \times \text{Ifse}(\text{Abs}(\text{ordsc}(i))) \\
\text{Next i}
\]

\[
a\text{Ip}(13 \times Ncv + 1) = npr / 2 \times \text{Sgn}(\text{ordpr}(Ncv)) \times \text{Ifse}(\text{Abs}(\text{ordpr}(Ncv))) + npr / 2 \times \text{Sgn}(\text{ordpr}(1)) \times \text{Ifse}(\text{Abs}(\text{ordpr}(1))) \\
a\text{Is}(13 \times Ncv + 1) = 0 \\
\text{For } i = 2 \text{ To } Ncv \\
a\text{Ip}(13 \times Ncv + i) = npr / 2 \times \text{Sgn}(\text{ordpr}(i - 1)) \times \text{Ifse}(\text{Abs}(\text{ordpr}(i - 1))) + npr / 2 \times \text{Sgn}(\text{ordpr}(i)) \times \text{Ifse}(\text{Abs}(\text{ordpr}(i))) \\
a\text{Is}(13 \times Ncv + i) = 0 \\
\text{Next i}
\]

\[
a\text{Ip}(16 \times Ncv + i) = 0 \\
a\text{Is}(16 \times Ncv + i) = nsc / 2 \times \text{Sgn}(\text{ordsc}(Ncv)) \times \text{Ifse}(\text{Abs}(\text{ordsc}(Ncv))) \\
\text{For } i = 2 \text{ To } Ncv \\
a\text{Ip}(16 \times Ncv + i) = 0 \\
a\text{Is}(16 \times Ncv + i) = nsc / 2 \times \text{Sgn}(\text{ordsc}(i - 1)) \times \text{Ifse}(\text{Abs}(\text{ordsc}(i - 1))) \\
\text{Next i}
\]

For \( i = 1 \) To \( Ncv \)
\[
a\text{Ip}(17 \times Ncv + i) = 0 \\
a\text{Is}(17 \times Ncv + i) = nsc / 2 \times \text{Sgn}(\text{ordsc}(i)) \times \text{Ifse}(\text{Abs}(\text{ordsc}(i))) \\
\text{Next i}
\]

'composizione dei vettori dei due strati in un unico (ASP TOTALI STATORICHE)

For \( i = 1 \) To \( 3 \times Ncv \)
\[
aI(i) = a\text{Ip}(i) + a\text{Is}(i) \\
\text{Next i}
\]

For \( i = 12 \times Ncv + 1 \) To \( 14 \times Ncv \)
\[
aI(i) = a\text{Ip}(i) + a\text{Is}(i) \\
\text{Next i}
\]

For \( i = 16 \times Ncv + 1 \) To \( 18 \times Ncv \)
\[
aI(i) = a\text{Ip}(i) + a\text{Is}(i) \\
\text{Next i}
\]

'memorizzazione per lo step successivo

'(nel quale le correnti restano invariate)

For \( i = 1 \) To \( n \)
\[
a\text{Ip}(i) = aI(i) \\
\text{Next i}
\]

\[
dx1 = dx_{cost}
\]

Else

'step di spostamento PARI: le correnti vengono
'mantengono costanti per il calcolo della Fx

For \( i = 1 \) To \( n \)
\[
aI(i) = a\text{Ip}(i) \\
\text{Next i}
\]

\[
dx1 = dx
\]

End If
Appendix A2.2

*A composizione vettore amperspire totali*
For i = 1 To n
    ASP(i) = aI(i) + aM(i)
Next i

*scrittura su file*
For i = 1 To n
    Print #4, ASP(i)
Next i

    xm = xm + dx1
Next ixm

Close #4

End Sub

---

**A2.2.2 The MMF array (original version)**

Const pi = 3.1415927

Private Sub CommandButton1_Click()

    'INPUT
    Ncv = Foglio2.Cells(4, 4)
    npr = Foglio2.Cells(5, 4)
    nsc = Foglio2.Cells(6, 4)
    Taup = Foglio2.Cells(7, 4)
    Taup = Taup * 0.001
    Irms = Foglio2.Cells(8, 4)
    adt = Foglio2.Cells(16, 4)
    adt = adt * 0.001
    havv = Foglio2.Cells(21, 4)
    havv = havv * 0.001
    ta = Foglio2.Cells(23, 4)
    xm = Foglio2.Cells(24, 4)
    xm = xm * 0.001
    dx_cost = Foglio2.Cells(25, 4)
    dx_cost = dx_cost * 0.001
    dx = Foglio2.Cells(26, 4)
    dx = dx * 0.001
    ixm_max = Foglio2.Cells(27, 4)
    fi_in = Foglio2.Cells(28, 4)
    nfasi = Foglio2.Cells(30, 4)

    filepath = CStr(Foglio2.TextBox1.Text)

    If ta <= 1 - havv / adt Then
        MsgBox ("Valore di ta troppo basso!")
        Stop
    End If

    f = 50
    kw = 1
    st = 2
Grandezze derivate
\[ w = 2 \pi f \]
\[ n = 18 \times N_cv + 8 \]
\[ k_cir = \frac{ad_t \times (1 - t_a)}{h_{avv}} \]

ReDim ordpr(1 To N_cv) As Integer
ReDim aI(1 To n), aIp(1 To n), aI_p(1 To n), aM(1 To n), ASP(1 To n) As Double
ReDim Ifse(1 To nfasi), Ifse_p(1 To nfasi) As Double

*Lettura del file relativo al I strato dell'avvolgimento statorico
Open filepath & "\ordpr.txt" For Input As #1
For i = 1 To N_cv
  Input #1, ordpr(i)
Next i
Close #1

*Lettura del file relativo al II strato dell'avvolgimento statorico
Open filepath & "\ordsc.txt" For Input As #2
For i = 1 To N_cv
  Input #2, ordsc(i)
Next i
Close #2

*Lettura del file contenente le amper spies equivalenti del magnete
Open filepath & "\aM(pp).txt" For Input As #3
For i = 1 To n
  Input #3, aM(i)
Next i
Close #3

Open filepath & "\ASP_TOT.txt" For Output As #4
For ixm = 1 To ixm_max

  If Int(ixm / 2) <> ixm / 2 Then
    'step di spostamento DISPARI: le correnti vengono calcolate
    'secondo l'esatto valore istantaneo correlato alla posizione

    'inizializzazione dell'array
    For i = 1 To n
      aI(i) = 0
    Next i

    'Andamento sinusoidale delle correnti
    t = \pi \times \frac{xm}{w} / Taup
    'Memorizzazione del valore delle correnti allo step precedente
    For i = 1 To nfasi
      Ifse_p(i) = Ifse(i)
    Next i
    For j = 1 To nfasi
      Ifse(j) = -Sqr(2) * Irms * Cos(kw * w * t - (j - 1) * kw * st * 2 * \pi / nfasi + fi_in)
    Next j

  End If

  'ARRAY AMPER SPIRE STATORICHE
  For i = 1 To N_cv
    aIp(i) = npr * Sgn(ordpr(i)) * Ifse(Abs(ordpr(i)))
    aIs(i) = nsc * Sgn(ordsc(i)) * Ifse(Abs(ordsc(i)))
  Next i
  For i = 1 To N_cv
    If (0 <= kcir) And (kcir <= 0.5) Then
      aIp(N_cv + i) = 0
      aIs(N_cv + i) = 2 * nsc * kcir * Sgn(ordsc(i)) * Ifse(Abs(ordsc(i)))
    ElseIf (0.5 < kcir) And (kcir < 1) Then
      aIp(N_cv + i) = (2 * npr * kcir - npr) * Sgn(ordpr(i)) * Ifse(Abs(ordpr(i)))
      aIs(N_cv + i) = nsc * Sgn(ordsc(i)) * Ifse(Abs(ordsc(i)))
    Else
      aIp(N_cv + i) = npr * Sgn(ordpr(i)) * Ifse(Abs(ordpr(i)))
    End If
Appendix A2.2

\[
as(Ncv + i) = nsc \cdot \text{sgn}(\text{ordsc}(i)) \cdot \text{ifse}(\text{abs}(\text{ordsc}(i)))
\]
End If
Next i

For i = 1 To Ncv
aIp(2 * Ncv + i) = npr \cdot \text{sgn}(\text{ordpr}(i)) \cdot \text{ifse}(\text{abs}(\text{ordpr}(i)))
aIs(2 * Ncv + i) = nsc \cdot \text{sgn}(\text{ordsc}(i)) \cdot \text{ifse}(\text{abs}(\text{ordsc}(i)))
Next i

For i = 1 To Ncv
aIp(12 * Ncv + i) = npr / 2 \cdot \text{sgn}(\text{ordpr}(i)) \cdot \text{ifse}(\text{abs}(\text{ordpr}(i)))
aIs(12 * Ncv + i) = nsc / 2 \cdot \text{sgn}(\text{ordsc}(i)) \cdot \text{ifse}(\text{abs}(\text{ordsc}(i)))
Next i

If (0 <= kcir) And (kcir <= 0.5) Then
'concatena in ogni caso sempre tutto il (I)
aIp(13 * Ncv + 1) = npr / 2 \cdot \text{sgn}(\text{ordpr}(Ncv)) \cdot \text{ifse}(\text{abs}(\text{ordpr}(Ncv))) + npr / 2 \cdot \text{sgn}(\text{ordpr}(1)) \cdot \text{ifse}(\text{abs}(\text{ordpr}(1)))
'a concatena una parte del (II), a seconda del valore di \( ta \)
aIs(13 * Ncv + 1) = (nsc - 2 \cdot nsc \cdot kcir) / 2 \cdot \text{sgn}(\text{ordsc}(Ncv)) \cdot \text{ifse}(\text{abs}(\text{ordsc}(Ncv))) + (nsc - 2 \cdot nsc \cdot kcir) / 2 \cdot \text{sgn}(\text{ordsc}(1)) \cdot \text{ifse}(\text{abs}(\text{ordsc}(1)))
ElseIf (0.5 < kcir) And (kcir < 1) Then
aIp(13 * Ncv + 1) = 2 \cdot npr \cdot (1 - kcir) / 2 \cdot \text{sgn}(\text{ordpr}(Ncv)) \cdot \text{ifse}(\text{abs}(\text{ordpr}(Ncv))) + 2 \cdot npr \cdot (1 - kcir) / 2 \cdot \text{sgn}(\text{ordpr}(1)) \cdot \text{ifse}(\text{abs}(\text{ordpr}(1)))
aIs(13 * Ncv + 1) = 0
Else
aIp(13 * Ncv + 1) = 0
aIs(13 * Ncv + 1) = 0
End If

For i = 2 To Ncv
If (0 <= kcir) And (kcir <= 0.5) Then
'concatena in ogni caso sempre tutto il (I)
aIp(13 * Ncv + i) = npr / 2 \cdot \text{sgn}(\text{ordpr}(i - 1)) \cdot \text{ifse}(\text{abs}(\text{ordpr}(i - 1))) + npr / 2 \cdot \text{sgn}(\text{ordpr}(i)) \cdot \text{ifse}(\text{abs}(\text{ordpr}(i)))
'a concatena una parte del (II), a seconda del valore di \( ta \)
aIs(13 * Ncv + i) = (nsc - 2 \cdot nsc \cdot kcir) / 2 \cdot \text{sgn}(\text{ordsc}(i - 1)) \cdot \text{ifse}(\text{abs}(\text{ordsc}(i - 1))) + (nsc - 2 \cdot nsc \cdot kcir) / 2 \cdot \text{sgn}(\text{ordsc}(i)) \cdot \text{ifse}(\text{abs}(\text{ordsc}(i)))
ElseIf (0.5 < kcir) And (kcir < 1) Then
aIp(13 * Ncv + i) = 2 \cdot npr \cdot (1 - kcir) / 2 \cdot \text{sgn}(\text{ordpr}(i - 1)) \cdot \text{ifse}(\text{abs}(\text{ordpr}(i - 1))) + 2 \cdot npr \cdot (1 - kcir) / 2 \cdot \text{sgn}(\text{ordpr}(i)) \cdot \text{ifse}(\text{abs}(\text{ordpr}(i)))
aIs(13 * Ncv + i) = 0
Else
aIp(13 * Ncv + i) = 0
aIs(13 * Ncv + i) = 0
End If

If (0 <= kcir) And (kcir <= 0.5) Then
aIp(16 * Ncv + i) = 0
aIs(16 * Ncv + i) = 2 \cdot nsc \cdot kcir / 2 \cdot \text{sgn}(\text{ordsc}(Ncv)) \cdot \text{ifse}(\text{abs}(\text{ordsc}(Ncv)))
ElseIf (0.5 < kcir) And (kcir < 1) Then
aIp(16 * Ncv + i) = (2 \cdot npr \cdot kcir - npr) / 2 \cdot \text{sgn}(\text{ordpr}(Ncv)) \cdot \text{ifse}(\text{abs}(\text{ordpr}(Ncv)))
aIs(16 * Ncv + i) = nsc / 2 \cdot \text{sgn}(\text{ordsc}(Ncv)) \cdot \text{ifse}(\text{abs}(\text{ordsc}(Ncv)))
Else
aIp(16 * Ncv + i) = npr / 2 \cdot \text{sgn}(\text{ordpr}(Ncv)) \cdot \text{ifse}(\text{abs}(\text{ordpr}(Ncv)))
aIs(16 * Ncv + i) = nsc / 2 \cdot \text{sgn}(\text{ordsc}(Ncv)) \cdot \text{ifse}(\text{abs}(\text{ordsc}(Ncv)))
End If

For i = 2 To Ncv
If (0 <= kcir) And (kcir <= 0.5) Then
aIp(16 * Ncv + i) = 0
aIs(16 * Ncv + i) = 2 \cdot nsc \cdot kcir / 2 \cdot \text{sgn}(\text{ordsc}(i)) \cdot \text{ifse}(\text{abs}(\text{ordsc}(i)))
ElseIf (0.5 < kcir) And (kcir < 1) Then
aIp(16 * Ncv + i) = (2 \cdot npr \cdot kcir - npr) / 2 \cdot \text{sgn}(\text{ordpr}(i)) \cdot \text{ifse}(\text{abs}(\text{ordpr}(i)))
aIs(16 * Ncv + i) = nsc / 2 \cdot \text{sgn}(\text{ordsc}(i)) \cdot \text{ifse}(\text{abs}(\text{ordsc}(i)))
Else
    aIp(16 * Ncv + i) = npr / 2 * Sgn(ordpr(i)) * Ifse(Abs(ordpr(i)))
    aIs(16 * Ncv + i) = nsc / 2 * Sgn(ordsc(i)) * Ifse(Abs(ordsc(i)))
End If
Next i

For i = 1 To Ncv
    If (0 <= kcir) And (kcir <= 0.5) Then
        aIp(17 * Ncv + i) = 0
        aIs(17 * Ncv + i) = 2 * nsc * kcir / 2 * Sgn(ordsc(i)) * Ifse(Abs(ordsc(i)))
    ElseIf (0.5 < kcir) And (kcir < 1) Then
        aIp(17 * Ncv + i) = (2 * npr * kcir - npr) / 2 * Sgn(ordpr(i)) * Ifse(Abs(ordpr(i)))
        aIs(17 * Ncv + i) = nsc / 2 * Sgn(ordsc(i)) * Ifse(Abs(ordsc(i)))
    Else
        aIp(17 * Ncv + i) = npr / 2 * Sgn(ordpr(i)) * Ifse(Abs(ordpr(i)))
        aIs(17 * Ncv + i) = nsc / 2 * Sgn(ordsc(i)) * Ifse(Abs(ordsc(i)))
    End If
Next i

'composizione dei vettori dei due strati in un unico (ASP TOTALI STATORICHE)
For i = 1 To 3 * Ncv
    aI(i) = aIp(i) + aIs(i)
Next i
For i = 12 * Ncv + 1 To 14 * Ncv
    aI(i) = aIp(i) + aIs(i)
Next i
For i = 16 * Ncv + 1 To 18 * Ncv
    aI(i) = aIp(i) + aIs(i)
Next i

'memorizzazione per lo step successivo
'(nel quale le correnti restano invariate)
For i = 1 To n
    aI_p(i) = aI(i)
Next i
dx1 = dx_cost
Else
    'step di spostamento PARI: le correnti vengono
    'mantenute costanti per il calcolo della Fx
    For i = 1 To n
        aI(i) = aI_p(i)
    Next i
dx1 = dx
End If

'composizione vettore amperspire totali
For i = 1 To n
    ASP(i) = aI(i) + aM(i)
Next i
'scrittura su file
For i = 1 To n
    Print #4, ASP(i)
Next i
xm = xm + dx1
Next ixm
Close #4
End Sub
Appendix A2.2

Fig. A2.2-1. The flow-chart of the algorithm
3.1 Introduction

A bearingless motor is an electrical machine where the suspension and the centering of the rotor is provided by radial forces generated by the interactions between the magnetic fields acting in the airgap, avoiding the use of mechanical bearings and achieving in this way much higher maximum speed [1], as in [2] where is proposed a 60000 rpm motor for compressors and special pumps. In this way, the rotor is suspended and centered by a radial force distribution, suitably created by the interactions between different harmonic orders of the magnetic fields produced by the stator and rotor sources, whatever they are: in fact, the
principle can be applied to the permanent magnet surface-mounted synchronous machine [3], [4], [5], internal PM motors, induction motors. An interesting general method for a comparison of bearingless machines is presented in literature [6]: PM synchronous motors have the advantage that the control of rotation and levitation are independent, while the levitation force is weak. Conversely, induction motors produce a strong levitation force, but their efficiency is poor and the control of rotation and levitation are coupled. The internal permanent magnet (IPM) type bearingless motor represents a compromise, being characterized by strong levitation force and relatively easy control properties.

The radial force are generated by creating an unbalanced flux density distribution in the airgap, which results in a magnetic force acting on the rotor. In fact, in this situation by summing the force vectors related to every pole, they give a not null resultant. On the contrary, in the electrical machines of conventional typology, the magnetic poles have equal flux density and hence equal magnitudes of the attractive forces, with a null vector sum of the radial forces. An unbalanced magnetic field distribution in a bearingless machine can be obtained by two different winding distributions:

1) ‘Dual set of windings’, characterized by two systems of three-phase windings physically separated, one dedicated to the generation of tangential forces which produce torque, the other to the generation of the radial forces of levitation; a bearingless motor previously proposed in literature, presents a 2-pole radial force windings wounded in the stator of a 4-pole motor [1], could be applied to super high speed motors as well as induction and synchronous reluctance machines, as theorized in [7], [8]. An analysis and classification of 3-phase separated and concentrated windings bearingless machines is proposed by [9], which constitutes a relief in the design of this typology of motors.
2) ‘Single set of multiphase windings’, in which the two typologies of force are obtained from a single winding, exploiting the potential of multiphase motors to produce spatial harmonics of odd and even orders, by injecting different current space vectors: in this way two stator magnetic fields of different harmonic order, by acting with the rotor magnetic field, ensure one the suspension of the rotor, the other the generation of torque. In [10] is proposed a 6-phase induction machine with one set of windings instead of two sets of 3-phase windings, taking advantage of the multiple control degrees of freedom given by multiphase machines; [11] presents a 5-phase bearingless motor, explicating the principle of generating the torque and suspension forces by feeding two groups of currents projected into 2 orthogonal d-q planes. In the same paper is also presented a control system which estimates the parameters of the current levitation system by means of PID controllers, acquiring the position error of the rotor and estimates the parameters of the current motor system by means of a PI controller, acquiring the angular speed error.

### 3.2 General Principles of Magnetic Force Generation

Fig. 3.1 shows the cross section of a general bearingless motor under different operative conditions [1], [12]. In Fig. 3.1(a), the flux distribution is a symmetrical 4-pole, the flux paths around the conductors 4a being shown: the four pole flux wave ψ₄₄ produces alternating magnetic poles in the airgap. Since the flux distribution is symmetrical, the flux density distribution is identical, apart from the sign, in the airgap sections 1, 2, 3 and 4. There are attractive magnetic forces between the rotor poles and stator iron, which have identical amplitudes, but with equally distributed directions, so that the sum of radial force
acting on the rotor is zero. Fig. 3.1(b) shows the principle of radial force generation: a 2-pole winding, represented by conductors 2a, produces a magnetic flux $\psi_{2a}$ having the same direction than the one generated from the 4-pole winding in the section 1, but having the opposite direction in section 3. In this way, the flux density will increase in section 1, while will be reduced in section 3, generating a radial force $F$ according to the $x$-axis direction. It follows that the amplitude of the radial force increases as the current value in conductors 2a increases. Fig. 3.1(c) shows how a negative radial force in the $x$-axis direction is generated. The current in conductors 2a is reversed so that the flux density in airgap section 1 now decreases while that in airgap section 3 increases. Hence the magnetic force in airgap section 3 is larger than that in airgap section 1, producing a radial force in the negative $x$-axis direction.

Fig. 3.1

Fig. 3.2 shows the radial force generation in the $y$-axis direction. The conductors 2b, which have an MMF direction along the $y$-axis, produce a flux through the airgap sections 2 and 4, thus resulting in a force along the $y$-axis. The polarity of the current will dictate the direction of the force. These are the principles of the radial force generation, being its value almost proportional to the current in the
windings 2a and 2b (assuming a constant 4-pole current). The vector sum of these two perpendicular radial forces can produce a radial force in whatever direction, with any amplitude. As an essential condition to generate radial forces, the difference of pole pair numbers of the motoring and levitation fields has to be ±1.

Fig. 3.2

3.3 Bearingless Machines with a Dual Set of Windings

In [13] is presented an interesting analysis of a bearingless permanent magnet motor with a dual set of windings, reported in the following. The 4-pole magnetic field (produced by the torque current system) and the 2-pole magnetic field (produced by the levitation current system) are generated by separated, physically distinct, windings. This method allows to design the two windings independently, but has the disadvantage of reserving part of the copper surface in the slot to the levitation winding (Fig. 3.3), used to produce radial forces, causing in this way higher Joule losses to give the same torque output with respect to the
conventional electrical machines.

With reference to Fig. 3.4, the additional 2-pole windings \( N_\alpha \) and \( N_\beta \) are wound in the stator slots with the conventional 4-pole windings. The radial force is caused by the unbalanced distribution of the flux density in the airgap, caused by the existing interaction between the excitation flux \( \psi_P \), which flows in the 4 poles and in the permanent magnets, and the flux generated by the 2-pole windings currents \( i_\alpha \) and \( i_\beta \).

The current \( i_\alpha \), with a direction and orientation as in Fig. 3.4, generates the flux \( \psi_\alpha \). The flux density increases in the airgap section 2 and decreases in the airgap section 1, the radial force \( F \) is generated in the negative direction of the \( \alpha \) axis. For simplicity, it will analyzed a model of two-phase machine. The current able to generate a magneto-motive force equal to that of the permanent magnets is represented as an equivalent current in the motor windings. The currents \( i_{ap} \) and \( i_{bp} \), in the motor windings \( N_a \) and \( N_b \), are the sum of the actual components of the motor current with amplitude \( I_q \) and of the equivalent current.
The currents can be written as:

\[
i_{ap} = -I_q \sin 2\omega t + I_p \cos 2\omega t
\]

\[
i_{bp} = I_q \cos 2\omega t + I_p \sin 2\omega t
\]

Where \( \omega \) represents the angular frequency. If the bearingless motor is in open-circuit operating condition, \( I_q \) is about zero and can be neglected:

\[
i_{ap} \approx I_p \cos 2\omega t
\]

\[
i_{bp} \approx I_p \sin 2\omega t
\]

By defining \( \psi_{ap} \), \( \psi_{bp} \), \( \psi_{\alpha} \), \( \psi_{\beta} \) respectively as the 4-pole fluxes related to...
windings $N_a$, $N_b$ and the 2-pole fluxes related to windings $N_\alpha$, $N_\beta$, the relationships between them and the currents that flow in the motor and levitation windings can be written as:

$$
\begin{bmatrix}
\psi_{ap} \\
\psi_{bp} \\
\psi_\alpha \\
\psi_\beta
\end{bmatrix} =
\begin{bmatrix}
L_4 & 0 & -M'_\alpha & M'_\beta \\
0 & L_4 & M'_\beta & M'_\alpha \\
-M'_\alpha & M'_\beta & L_2 & 0 \\
M'_\beta & M'_\alpha & 0 & L_2
\end{bmatrix}
\begin{bmatrix}
i_{ap} \\
i_{bp} \\
i_\alpha \\
i_\beta
\end{bmatrix}
$$

(3.5)

Being $L_4$ and $L_2$ respectively the self-induction coefficients of motor and levitation windings, $\alpha$ e $\beta$ the rotor displacements along the $x$ and $y$ axes, $M'$ the derivative of the mutual-induction coefficient, related to the coupling between motor and levitation windings with respect to rotor displacements. $L_4$, $L_2$ and $M'$ are functions of the airgap length, number of turns and rotor dimension. By assuming a magnetic linear system, $M'$ can be written as:

$$
M' = \frac{\mu_0 \pi n_2 n_4 l r - (l_m + l_g)}{8 (l_m + l_g)^2}
$$

(3.6)

where $n_2$ and $n_4$ are the real number of turns of the windings, $l$ is the axial length of the machine, $r$ is the internal stator radius, $l_m$ the magnet thickness, $l_g$ the airgap length. Consequently, $(l_m + l_g)$ represents the distance between the internal stator surface and external rotor surface. The stored magnetic energy $W_m$ can be expressed as:

$$
W_m = \frac{1}{2} \begin{bmatrix}
i_{ap} \\
i_{bp} \\
i_\alpha \\
i_\beta
\end{bmatrix}
\begin{bmatrix}
L_4 & 0 & -M'_\alpha & M'_\beta \\
0 & L_4 & M'_\beta & M'_\alpha \\
-M'_\alpha & M'_\beta & L_2 & 0 \\
M'_\beta & M'_\alpha & 0 & L_2
\end{bmatrix}
\begin{bmatrix}
i_{ap} \\
i_{bp} \\
i_\alpha \\
i_\beta
\end{bmatrix}
$$

(3.7)

The radial force produced by the interactions between the two windings and the
Principles of bearingless machines

Rotor magnetic field can be calculated in terms of its components $F_\alpha$ and $F_\beta$ along the axes direction $\alpha$ and $\beta$:

$$F_\alpha = \frac{\partial W_m}{\partial \alpha}$$  \hspace{1cm} (3.8)

$$F_\beta = \frac{\partial W_m}{\partial \beta}$$  \hspace{1cm} (3.9)

By executing the calculations is obtained:

$$
\begin{bmatrix}
F_\alpha \\
F_\beta
\end{bmatrix} = M' I_p
\begin{bmatrix}
-\cos 2\omega t & \sin 2\omega t \\
\sin 2\omega t & \cos 2\omega t
\end{bmatrix}
\begin{bmatrix}
i_\alpha \\
i_\beta
\end{bmatrix}
$$  \hspace{1cm} (3.10)

It can be seen that the radial force is proportional to $M'$ and to the equivalent current of the permanent magnets $I_p$; thus, if the factor $M' I_p$ has a high value, the levitation windings current can be reduced. In order to do this, $I_p$ and the magnet thickness are increased but, consequently, the total gap between rotor and stator grows causing a decreasing of $M'$. So, it results very important to choose the right compromise between the optimum thickness of the permanent magnets and the motor performances.

Fig. 3.5(a).
In order to do this, was defined a performance index which correlates the radial force-versus-current ratio to the flux density in the airgap. In Fig. 3.5 is shown the cross-section of the examined permanent magnet bearingless motor, being \( R \) the rotor radius, \( \phi_s \) and \( \phi_r \) the angular positions along the stator and rotor periphery with respect to the \( \alpha \) axis, \( \omega t \) the rotor angular displacement.

The rotor iron core is made from the laminated silicon steels with the small projections. Permanent magnets are mounted on the surface of the rotor core as shown in Fig. 3.5(b). \( w \) is the width of permanent magnets. \( w' \) is the width between the small projections. Then, an area \( S' \) between small projections and an area \( S \) of a permanent magnet can be represented as \( w'l \) and \( w'l \), respectively. Fig. 3.5(c) shows a magnetic equivalent circuit of the permanent magnet having width \( w \). The magneto-motive force \( F_m \) of permanent magnet is given as

\[
F_m = \frac{l_m}{\mu_0} B_r \tag{3.11}
\]

where \( B_r \) is the remanent flux density of the permanent magnet, \( \mu_0 \) is the air
Principles of bearingless machines

permeability, \( R_g \) and \( R_m \), in Fig. 3.5(c) are the reluctances of mechanical airgap clearance and permanent magnet, respectively:

\[
R_g = \frac{l_g}{\mu_0 S} \tag{3.12}
\]

\[
R_m = \frac{l_m}{\mu_0 S} \tag{3.13}
\]

Neglecting magnetic saturation, slot ripples and permeance in the small projections, the flux linkage \( \psi_m \), caused by permanent magnets can be written as

\[
\psi_m = \frac{F_m}{R_g + R_m} = \frac{l_m B_r}{\mu_0} = \frac{S l_m}{\mu_0 S} B_r \tag{3.14}
\]

Therefore, peak values of the flux density \( B_{gp} \), in the airgap can be written as

\[
B_{gp} = \frac{\psi_m}{S'} = \frac{l_m}{l_m + l_g} \frac{S}{S'} B_r \tag{3.15}
\]

Let us define the terms \( l_{mn} \), and \( l_{gn} \), as \( l_m / r \) and \( l_g / r \), respectively. These normalized lengths with respect to the stator inner radius can be used to derive the general expression of the peak air-gap flux density. Thus, \( B_{gp} \) can be rewritten as

\[
B_{gp} = \frac{l_m}{l_m + l_g} \frac{S}{S'} B_r = \frac{l_{mn}}{l_{mn} + l_{gn}} \frac{S}{S'} B_r \tag{3.16}
\]

Fig. 3.6 shows the relationships between \( l_{mn} \), and \( B_{gp} \), with parameters of \( l_{gn} \). The term \( SB_r / S' \) in the vertical axis is a constant determined by the remanent
flux density of permanent magnets and the area density of mounted permanent magnet. In the case of Sm-Co magnets, $B_r = 1$ T. The ratio $S/S'$ is equal to 1 if a cylindrical magnet is used, but it is slightly smaller than 1 if small permanent magnets are mounted on the surface of rotor iron core as shown in Fig. 3.5(b).

The decrease in $l_{gn}$, i.e. a reduction of the ratio of mechanical air-gap width $l_g$ to radius $r$ of stator inner surface, and the increase in $l_{mn}$, i.e. the ratio of permanent magnet thickness $l_m$ to $r$ results in an increase in airgap flux density. The desired value of the peak airgap flux density is generally determined at a rated rotational speed and heat dissipation.

An important result achieved in [13], is given by analyzing the quantity radial force for unit current $F_\alpha / I_\alpha$ as a function of the machine parameters:

$$F_\alpha = \frac{n_z l B_r S}{S'} \frac{\sqrt{2}l_{mn} (1 - l_{mn} - l_{gn})}{(l_{mn} + l_{gn})^2}$$

The first term of (3.17) depends on the number of turns of levitation windings,
the rotor stack length, the remanent flux density of permanent magnets, and the area density of mounted permanent magnet. Thus, it is not possible to increase it without an increase in dimension or an improvement in materials. The second term is determined by ratios of permanent magnet thickness and mechanical airgap width to the radius of the stator inner surface. Radial force can be produced most efficiently when the second term has its maximum value. The question is if the airgap flux density value is to determine at rated motor speed and rated heat dissipation or for the optimal condition to produce radial forces. However, by differentiating (3.17) with respect to $l_{mn}$, it can be found that $F_{a}/I_{a}$ is maximum when $l_{mn}=l_{gm}$, i.e. radial forces can be produced most efficiently when permanent magnet thickness is equal to mechanical airgap width (Fig. 3.7).

Fig. 3.7.
3.4 Bearingless Machines with a Single Set of Windings

It is known that a multiphase system of currents can be represented by using temporal sequences (if in sinusoidal alternating regime) or, more generally, by using space vectors of different orders (if in non-periodic regime).

By controlling separately and in an appropriate manner various orders of the current space vectors it is possible, for example, to generate and control the torque produced by the motor with a certain order and to produce and control the suspension radial forces with other orders.

The advantage of the single set of winding consists in the possibility of implementing both the above functions using a single winding, with a simpler construction process, without designing another one which subtracts a useful section of copper, with a reduction of power losses. In this type of machines is however necessary to provide asymmetric shortened pitch windings, in order to generate even harmonic orders in the magnetic fields that permit to create a radial forces distribution.

This typology of windings has the disadvantage of reducing the available torque, so the designer must then find a compromise between the intensity of the radial force and the motor performances.

In [11] is presented an interesting analysis of a bearingless permanent magnet motor with a single set of 5-phase windings, reported in the following. The two needed magnetic fields are produced by feeding two groups of currents which are projected into 2 orthogonal d-q planes respectively.

The radial force acting on the rotor can be obtained from Maxwell stress tensor:

$$\sigma = \frac{1}{2\mu_0} \left( b_n^2 - b_i^2 \right)$$  \hspace{1cm} (3.18)
where the contribution of $b_t^2$ can be neglected with respect to $b_n^2$. By supposing that there are two revolving magnetic fields in the airgap $B_1$ and $B_2$:

$$b_n = B_1 + B_2$$  \hspace{1cm} (3.19)
$$B_1 = B_{1m} \cos(\omega_1 t - p_1 \theta + \varphi_1)$$  \hspace{1cm} (3.20)
$$B_2 = B_{2m} \cos(\omega_2 t - p_2 \theta + \varphi_2)$$  \hspace{1cm} (3.21)

where $p_1$ and $p_2$ are their corresponding numbers of pole pair, $\omega_1$ and $\omega_2$ are their corresponding angular frequencies, and $\theta$ is an arbitrary angle in stator surface. The horizontal and vertical force components are given by:

$$F_\alpha = \int \sigma \cos(\theta) l r d \theta$$  \hspace{1cm} (3.22)
$$F_\beta = \int \sigma \sin(\theta) l r d \theta$$  \hspace{1cm} (3.23)

where $l$ is the length of the stack, and $r$ is the radius of airgap. By combining the equations (3.19)-(3.21), then substituting in (3.18) and integrating by means of (3.22), (3.23), the projections of the force on the horizontal $\alpha$ axis and vertical $\beta$ axis are obtained.

$$F_\alpha = \begin{cases} \frac{l r \pi}{2 \mu_0} B_{1m} B_{2m} \cos(\omega_1 t - \omega_2 t + \varphi_1 - \varphi_2), & p_1 - p_2 = \pm 1 \\ 0, & p_1 - p_2 \neq \pm 1 \end{cases}$$  \hspace{1cm} (3.24)

$$F_\beta = \begin{cases} \frac{l r \pi}{2 \mu_0} B_{1m} B_{2m} \sin(\omega_2 t - \omega_1 t + \varphi_2 - \varphi_1), & p_1 - p_2 = -1 \\ 0, & p_1 - p_2 \neq \pm 1 \end{cases}$$  \hspace{1cm} (3.25)

So, it result that a radial force can be obtained when $p_1 = p_2 \pm 1$, and the force is
stable when $\omega_1 = \omega_2$.

The Modified Winding Function Method is applied in order to analyze the magneto-motive force. In an $n$-phase symmetric system, the winding functions of each phase can be written as:

\[
N_a = N_1 \cos \theta + N_2 \cos 2\theta + N_3 \cos 3\theta + ..
\]
\[
N_b = N_1 \cos(\theta - \xi) + N_2 \cos(2(\theta - \xi)) + N_3 \cos(3(\theta - \xi)) + ..
\]
\[
N_c = N_1 \cos(\theta - 2\xi) + N_2 \cos(2(\theta - 2\xi)) + N_3 \cos(3(\theta - 2\xi)) + ..
\]
\[...
\]
\[
N_n = N_1 \cos(\theta - (n-1)\xi) + N_2 \cos(2(\theta - (n-1)\xi)) +
\]
\[
+ N_3 \cos(3(\theta - (n-1)\xi)) + ..
\]

where $\xi = 2\pi/n$ and $N_1$, $N_2$, $N_3$ are coefficients of the winding function. The group of currents is defined as $I_k$:

\[
I_k = \begin{bmatrix}
i_{ak} \\
i_{bk} \\
i_{ck} \\
\vdots \\
i_{nk}
\end{bmatrix} = I_{km} \begin{bmatrix}
cos(\omega t + \phi_k) \\
cos(\omega t + \phi_k - k\xi) \\
cos(\omega t + \phi_k - 2k\xi) \\
\vdots \\
cos(\omega t + \phi_k - (n-1)k\xi)
\end{bmatrix}
\]

(3.27)

where $I_{km}$ is the amplitude of $I_k$ and $\phi_k$ is the initial phase of $i_{ak}$. The MMF wave of an $n$-phase symmetric system fed by $I_k$ can be expressed as

\[
F = \sum_{i=1}^{n} I_{km} \cos[\omega t + \phi_k - k(i-1)\xi] \sum_{\nu} N_{\nu} \cos[\theta - (i-1)\xi]
\]

\[
= \frac{I_{km}}{2} \sum_{i=1}^{n} \sum_{\nu} N_{\nu} \left\{ \cos[\omega t + \phi_k + \nu\theta - (k + \nu)(i-1)\xi] + 
\right.
\]
\[
\left. + \cos[\omega t + \phi_k - \nu\theta - (k - \nu)(i-1)\xi] \right\}
\]

(3.28)

Thus, the $\nu$-th harmonic mmf wave results:
From (3.29), the resultant MMF of 5-phase symmetrical windings can be shown in Tab. I. “F” and “B” indicate the harmonic orders of MMF rotating forward and backward respectively, “±” denotes a pulsating MMF.

<table>
<thead>
<tr>
<th>k=</th>
<th>Space harmonics of resultant MMF</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>I₁</td>
<td></td>
</tr>
<tr>
<td>I₂</td>
<td></td>
</tr>
<tr>
<td>I₃</td>
<td></td>
</tr>
<tr>
<td>I₄</td>
<td></td>
</tr>
<tr>
<td>I₅</td>
<td></td>
</tr>
</tbody>
</table>

In a 5-phase winding motor, \(I₁\) generates 1 pole-pair revolving magnetic field in the airgap, \(I₂\) generates 2 pole-pair revolving magnetic field when \(N₂ ≠ 0\). So the suspension force can be produced by the interaction of \(I₁\) and \(I₂\).

In the analyzed machine, the 5-phase windings are identical and each of them is 72° displaced in angular space around the stator. Fig. 3.8 shows the windings configuration and the slots where a phase goes in, in capital letter, where goes out, in lowercase letter. The phases are star-connected, as can be seen in the same Fig. 3.8.
Voltage equations in stationary reference frame are given by

\[
U_s = R_s I_s + p (L_s I_s + \Psi_r) \tag{3.30}
\]

where \( U, I, R, L \) and \( \Psi \) are matrices of voltage, current, resistance, inductance and flux linkage respectively, subscript \( s \) and \( r \) represent stator and rotor respectively, and \( p \) is the differential operator.

A transformation has to be applied to equation (3.30) in order to express the voltages and currents of stator to synchronous reference frame, given by a matrix \( C \) not reported here. The form of the voltage equations in the new reference frame results:

\[
\begin{bmatrix}
\psi_{d1r} \\
-\psi_{q1r} \\
0 \\
0
\end{bmatrix}
\]

\[
U_{st} = R_{st} I_{st} + L_{st} p I_{st} + W_s L_{st} I_{st} + \Omega I \tag{3.31}
\]

where \( \psi_{q1r}, \psi_{d1r} \) are the q- and d-axis flux linkages of rotor respectively. The
radial force acting on rotor can be calculated with Virtual Displacement Method:

\[
F_\alpha = \frac{1}{2} \begin{bmatrix}
  i_{q1s} + i_{q1r} \\
  i_{d1s} + i_{d1r} \\
  i_{q2s} \\
  i_{d2s}
\end{bmatrix}^T \frac{dL_{st}}{d\alpha} \begin{bmatrix}
  i_{q1s} + i_{q1r} \\
  i_{d1s} + i_{d1r} \\
  i_{q2s} \\
  i_{d2s}
\end{bmatrix}
\] (3.32)

\[
F_\beta = \frac{1}{2} \begin{bmatrix}
  i_{q1s} + i_{q1r} \\
  i_{d1s} + i_{d1r} \\
  i_{q2s} \\
  i_{d2s}
\end{bmatrix}^T \frac{dL_{st}}{d\beta} \begin{bmatrix}
  i_{q1s} + i_{q1r} \\
  i_{d1s} + i_{d1r} \\
  i_{q2s} \\
  i_{d2s}
\end{bmatrix}
\] (3.33)

being \( i_{d1r} = \frac{\psi_{d1r}}{L_{1m}} \), \( i_{q1r} = \frac{\psi_{q1r}}{L_{1m}} \), equivalent rotor currents and \( L_{1m}, L_{2m} \) leakage winding inductances related to 1 pole pair and 2 pole pair fields. By developing (3.32), (3.33), the radial force can be seen as composed of two parts. One part is related to eccentricity:

\[
F_\alpha = \frac{\alpha L_{1m}}{4g_0^2} \left[ (i_{q1s} + i_{q1r})^2 + (i_{d1s} + i_{d1r})^2 \right] + \frac{\alpha L_{2m}}{2g_0^2} \left( i_{q2s} + i_{d2s} \right)^2
\] (3.34)

\[
F_\beta = \frac{\beta L_{1m}}{4g_0^2} \left[ (i_{q1s} + i_{q1r})^2 + (i_{d1s} + i_{d1r})^2 \right] + \frac{\beta L_{2m}}{2g_0^2} \left( i_{q2s} + i_{d2s} \right)^2
\] (3.35)

while the other results independent of eccentricity, used to control the rotor suspension:

\[
F_\alpha = \frac{1}{2g_0 \sqrt{L_{1m}L_{2m}}} \left( \psi_{d2m} \psi_{d1m} + \psi_{q2m} \psi_{q1m} \right)
\] (3.36)

\[
F_\beta = \frac{1}{2g_0 \sqrt{L_{1m}L_{2m}}} \left( \psi_{d2m} \psi_{d1m} - \psi_{q2m} \psi_{q1m} \right)
\] (3.37)

where \( g_0 \) is the equivalent radial length of a uniform air gap, \( \alpha \) and \( \beta \) are the displacement of rotor center. The fluxes which appear in (3.36), (3.37) are
defined as:

\[ \psi_{d1m} = L_{1m}i_{d1s} + \psi_{d1r} \]
\[ \psi_{q1m} = L_{1m}i_{q1s} + \psi_{q1r} \]
\[ \psi_{d2m} = L_{2m}i_{d2s} \]
\[ \psi_{q2m} = L_{2m}i_{q2s} \]  

The rotor displacements in stationary reference frame are given by:

\[
\begin{bmatrix}
\dot{\alpha} \\
\dot{\beta}
\end{bmatrix} = \frac{1}{m} \begin{bmatrix} F_\alpha \\ F_\beta - G_r \end{bmatrix}
\]  

where \( m \) and \( G_r \) are the mass and weight of rotor respectively. The “motor currents” command \( i_{q1s}^* \) is generated from speed controller. Suspension force command \( F_\alpha^*, F_\beta^* \) are generated by rotor displacement controller, after that, “levitation currents” commands \( i_{q2s}^*, i_{d2s}^* \) are given by (3.41), (3.42), which are derived by (3.36), (3.37). Phase currents are given by the inverse transformation of \( i_{q1s}^*, i_{d1s}^* \) and \( i_{q2s}^*, i_{d2s}^* \).

\[ i_{q2s} = \frac{2g_0\sqrt{L_{1m}/L_{2m}}}{\psi_{q1m}^2 + \psi_{d1m}^2}\left(\psi_{q1m} F_\alpha + \psi_{d1m} F_\beta\right) \]  

\[ i_{d2s} = \frac{2g_0\sqrt{L_{1m}/L_{2m}}}{\psi_{q1m}^2 + \psi_{d1m}^2}\left(\psi_{d1m} F_\alpha - \psi_{q1m} F_\beta\right) \]  

In the following, the main data of the proposed motor: \( L_{1m} = 36 \) mH, \( L_{2m} = 6.3 \) mH, \( L_{0s1} = 1.3 \) mH, \( L_{0s2} = 1 \) mH, \( r_s = 1.1 \) \( \Omega \), \( \psi_f = 1.05 \) Wb, \( J = 0.012 \) kgm\(^2\), airgap thickness: 2 mm, magnet thickness 3mm, \( g_0 = 5 \) mm. The airgap length between the rotor shaft and the touchdown bearing is 0.6 mm. In Fig. 3.9 is shown the control diagram of the 5-phase PM bearingless motor.
Fig. 3.10 shows angular speed, torque and rotor displacement during the start up operation. The shaft results successfully suspended and the dynamic of rotor suspension is stable with radial displacement variations less than 50 μm. It is also obvious that the system has good speed-regulation performance.
In bearingless machines, due to the fact that the rotor is not sustained by mechanical bearings, a rotor eccentricity is determined while the motor is operating and has to be compensated by the control system. This phenomenon leads to a variation in the values of the airgap flux density, as these resulting decrease in the areas where the airgap widens, and increase in the areas where the airgap is reduced, causing a similar behavior in the radial force distribution.

\[ g(\phi_s) = g_0 - x \cos \phi_s - y \sin \phi_s \]  \hspace{1cm} (3.43)

being \( x \) and \( y \) the actual coordinates of the rotor centre. With the assumption of small displacements compared to the nominal airgap length \( g_0 \), is possible to
write:

$$\frac{1}{g_0(\phi_s)} = \frac{1}{g_0} \left( 1 + \frac{x}{g_0} \cos \phi_s + \frac{y}{g_0} \sin \phi_s \right)$$  \hspace{1cm} (3.44)$$

and, consequently, to calculate the permeance $P_0$ in the generic angular position $\phi_s$:

$$P_0(\phi_s) = \frac{\mu_0 R l}{g_0} \left( 1 + \frac{x}{g_0} \cos \phi_s + \frac{y}{g_0} \sin \phi_s \right)$$  \hspace{1cm} (3.45)$$

being $R$ the rotor radius and $l$ the axial length. By using properly (3.45) in the equations of the machine, is possible to take into account the force variation with respect to rotor position. Two typologies of eccentricity can be defined in bearingless machines:

- **Static eccentricity**: it occurs when the rotor is not centred in the stator bore;

- **Dynamic eccentricity**: it occurs when the rotor is not rotating on the rotor axis but is rotating on the centre axis.

Various authors developed models able to interpret and calculate the effects of eccentricity on the radial forces and on the rotor position. In [14] is presented, by using the nonlinear FEM and a theoretical analysis, an analytical model for calculating the levitation force under airgap eccentricity by means of the interaction between harmonic field components and a simplified modeling for levitation force control in bearingless induction machines; [15] describes a method for modeling a bearingless IPM motor, for calculating the forces on the rotor by using complex winding analysis and rotating field theory and comparing the results to FEA analysis; the model allows the introduction of levitation and main windings and rotor eccentricity. In [16] an analytical expression of the levitation force for an induction-type bearingless motor is proposed, taking into
account the rotor eccentricity, being its computation accuracy verified by ANSOFT. A real-time observation of magnetic levitation force can be realized, is then implemented a closed-loop control of levitation force on the basis of airgap-flux-oriented decoupling control system of the bearingless motor.

A very interesting work about the analysis of static and dynamic eccentricity is presented in [17], which refers to an analytical modeling technique for calculating the radial force on the rotor of a bearingless consequent-pole permanent magnet motor. The flexibility of the method permits to identify vibration components and calculate the force in various situations when the rotor is not centred in the bore, in case of either dynamic eccentricity, static eccentricity or when the rotor is vibrating. It also allows a calculations of the force in presence of a varying load or with load imbalance. The paper also studies the effects of winding design and gives a validation of the results using 2D finite element analysis.

Another particular work which could be applied also to the force calculation in bearingless machines is [18], where a general analytical model, formulated in 2-D polar coordinates, is developed to predict the unbalanced magnetic force, which results in permanent-magnet brushless ac and dc machines having a diametrically asymmetric disposition of slots and phase windings. The unbalanced magnetic force can be significant in machines having a fractional ratio of slot number to pole number, particularly when the electric loading is high. The developed model is validated by FE calculations on 9-slot/8-pole and 3-slot/2-pole machines. Finally, [19] proposes a novel approach to control the rotor radial displacement in bearingless permanent-magnet-type synchronous motors, based on the relationship between radial displacement and radial suspension force. The rotor flux orientation is adopted to decouple the electromagnetic torque and the radial suspension force. This approach, which directly controls the rotor radial displacement, was applied by designing a
suitable control system.

### 3.6 Conclusion

The main features and issues of bearingless machines were presented in this chapter, highlighting the potential of the multiphase single set of windings type, which surely represents the most effective design methodology to adopt in the future. In fact, it permits valid control strategies for suspending the rotor, for controlling the motor and, at the same time, for generating torque by using the properties of multiphase current systems, without the need of altering the physical structure of the machine by designing other groups of windings in addition to the main one.

Thus, the focus of the activity in the next chapters will be on the analysis of this typology of bearingless machines.
3.7 References


Chapter 4

AN ANALYTICAL METHOD FOR CALCULATING THE DISTRIBUTION OF FORCES IN A BEARINGLESS MULTIPHASE SURFACE-MOUNTED PM SYNCHRONOUS MACHINE

4.1 Introduction

Bearingless motors have the capability to achieve much higher maximum speed in comparison to conventional electrical machines [1],[2]. The typology “dual set windings”, has a main one which carries the ‘torque currents’ for driving the rotor and producing torque, while the other carries the ‘levitation
currents’, to suspend the rotor [3],[4]. Its advantages consist of a simpler construction process, higher flexibility in control strategy and relatively low power losses [5].

The typology “single set windings” produces torque and radial forces by means of injecting different current space vectors within the same winding to give odd and even harmonic orders of magnetic field, using the properties of multiphase current systems with multiple orthogonal d-q planes. One of them can be used to control the torque, as in [6]. The additional degrees of freedom can be used to produce levitation forces [7],[8].

An important development of this technology is expected to be in the design of electromechanical devices for More Electric Aircraft (MEA), mainly for the possibility of achieving higher speed in comparison to conventional electrical machines [9]. Also, it would be applicable in the aerospace field, where the lubricants of mechanical bearings evaporate in the presence of vacuum [10].

The multiphase motors with respect to conventional three-phase motors gives a series of advantages [11], in particular in the cases of high power, high reliability, low dc bus voltage and reduction of power losses in IGBT inverters [12] as it happens in ship propulsion, electrical vehicles and MEA applications.

In the control system of multiphase bearingless motors is necessary to calculate the levitation current able to compensate the actual error in the rotor shaft position, thus the analytical function which correlates the applied currents to the resulting suspension force on the rotor. In order to do this and to simplify the problem, some authors consider only the interactions between the main harmonic orders of the stator and rotor magnetic fields, in the particular case of steady-state AC conditions, with sinusoidal systems of currents, as done in [7]; nevertheless, by proceeding in this way, a relevant error in the prediction of the force vector can be committed, because the interactions between higher harmonic orders of the magnetic fields acting on the rotor and the effect of the torque
current on the radial force are neglected.

The aim of this chapter is to present a model allowing to predict amplitude and direction of the force for given values of the torque current, of the levitation current and of the rotor position [13], based on the space vectors method, being able to include in this way also the analysis of transients.

For this purpose, a generalized analytical model for the calculation of radial forces in multiphase bearingless Surface-Mounted Permanent Magnet Synchronous Motors (SPMSM) is presented. The stator magnetic field is represented as the sum of separated contributions given by the different current space vectors. In this way it is possible to analyze the interactions between the torque current system, the levitation current system and the rotor magnetic field. In fact, in multiphase machines the combined effect of two stator current space vectors leads to a resulting levitation force which is sensibly different from that, foreseeable, produced by the currents in the separated windings of traditional bearingless machines. In Tab. I are shown all the possible interactions between the harmonic orders of the magnetic fields. The results are compared with those of FE analysis to demonstrate their accuracy.
4.2 Definition of variables

In the following, is presented a list of the variables used in the equations and relationships of this chapter:

\[ \begin{align*}
\theta_s & \quad \text{Angular abscissa in the stator reference system} \\
p_s & \quad \text{Number of stator pole pairs} \\
p_r & \quad \text{Number of rotor pole pairs} \\
k_{1(2)} & \quad \text{Spatial harmonic order of the space vector 1 (2) stator field distribution} \\
h & \quad \text{Spatial harmonic order of the rotor field distribution} \\
\varphi_{1(2)} & \quad \text{Phase of the current space vector 1 (2)} \\
n & \quad \text{Half of the number of conductors in a slot} \\
\delta & \quad \text{Total airgap height} \\
g & \quad \text{Airgap height} \\
L_m & \quad \text{Magnet height} \\
q & \quad \text{Number of slots per pole per phase} \\
k_{dk1(2)} & \quad k\text{-th harmonic component of the winding distribution factor related to the space vector 1 (2)} \\
\gamma & \quad \text{Half of the coil pitch angle} \\
s_{v1(2)} & \quad \text{Order of the space vector 1 (2)} \\
\Delta \theta & \quad \text{Angular displacement between phase-1 axis and magnet axis} \\
B_{rem} & \quad \text{Remanent flux density of the magnets} \\
K_c & \quad \text{Karter factor} \\
\mu_0 & \quad \text{Magnetic permeability of the air} \\
\mu_{mr} & \quad \text{Relative magnetic permeability of the magnets} \\
\alpha_m & \quad \text{Angle underlying the magnet pitch} \\
i_{sv1(2)} & \quad \text{Amplitude of the current space vector 1 (2)} \\
r & \quad \text{Mean airgap radius} \\
L & \quad \text{Axial length of the machine} \\
B_{Sk1(2)} & \quad \text{peak value of the } k\text{-th harmonic order of flux density distribution produced by current space vector 1 (2)} \\
B_{Rh} & \quad \text{peak value of the } h\text{-th harmonic order of flux density distribution produced by rotor magnets} \\
n_{ac} & \quad \text{Number of slots forming the coil pitch} \\
N_0 & \quad \text{Set of natural numbers including zero} \\
N_{odd} & \quad \text{Set of odd natural numbers}
\end{align*} \]
4.3 Analysis of flux density distribution in the airgap

Considering 5 real variables \( x_1, \ldots, x_5 \), an associated set of complex variables \( \bar{x}_0, \bar{x}_1, \bar{x}_2 \) can be obtained by means of the following symmetrical linear transformations:

\[
\bar{x}_\rho = \frac{2}{5} \sum_{l=1}^{5} x_l \alpha^{\rho(l-1)}, \quad (\rho = 0, 1, 2)
\]  

(4.1)

where \( \alpha = \exp(j2\pi/5) \). The inverse transformations of (4.1) are as follows:

\[
x_l = \frac{1}{2} x_0 + \sum_{\rho=1,2} \bar{x}_\rho \cdot \bar{\alpha}^{\rho(l-1)}, \quad (l = 1, 2, \ldots, 5)
\]  

(4.2)

where the symbol “ \( \cdot \)” represents the scalar product, defined as the real part of the product between the first operand and the complex conjugate of the second.

Fig. 4.1
The considered reference system is a cartesian one, whose axes $x$- (horizontal) and $y$- (vertical) have the origin in the motor shaft center, the $y$-axis coinciding with phase “1” axis (Fig. 4.1). The assumptions taken into account are: infinite iron permeability, current distribution concentrated at the slot opening, almost unitary relative permeability of the magnets.

The method will be applied to SPMSM machine with 5 star-connected stator phase windings, symmetrically distributed within the stator slots: the windings are supposed to be shifted by $2\pi/5$ electrical radians, with a single neutral point. According to (4.1) and (4.2), it is possible in this case to represent the currents system using 2 space vectors in different $\alpha$ - $\beta$ planes, being the zero-sequence component null because of the star-connection of the phases:

$$
\mathbf{i}_{sv1} = i_{sv1}(t)e^{j\varphi_1(t)}, \quad \mathbf{i}_{sv2} = i_{sv2}(t)e^{j\varphi_2(t)} \quad (4.3)
$$

By applying (4.3) to (4.2) and by explicating some variables, the $l$-th phase current can be written as:

$$
i_p = \sum_{p=1,2} \mathbf{i}_{svp} \cdot \alpha^{p(l-1)} = \mathbf{i}_{sv1} \cdot e^{j\frac{2\pi}{5}(l-1)} + \mathbf{i}_{sv2} \cdot e^{j\frac{4\pi}{5}(l-1)}, \quad (l = 1,2,\ldots,5) \quad (4.4)
$$

By developing (4.4), it gives:

$$
i_p(t) = i_{sv1} \cos\left[\varphi_1(t) - (l-1)\frac{2\pi}{5}\right] + i_{sv2} \cos\left[\varphi_2(t) - (l-1)\frac{4\pi}{5}\right], \quad (l = 1,2,\ldots,5) \quad (4.5)
$$

When the $sv$-th space vector current is flowing in the coils, the $l$-th phase of the motor produces a magnetic field distribution whose the radial component is given by (4.6):

$$
H_{l}(\theta_s,t) = \sum_{k=1,2,3,\ldots} H_{MK} \cos\left[kp_s \theta_s - k(l-1)\frac{2\pi}{5}\right] \cos\left[\varphi(t) - s_v (l-1)\frac{2\pi}{5}\right] \quad (4.6)
$$
An analytical method for calculating the distribution of forces in a bearingless multiphase surface-mounted pm synchronous machine

with $H_{Mk}$ defined as follows:

$$H_{Mk} = \frac{4}{k\pi} \frac{n_{sv} q k_{dk}}{2\delta} \sin(k\gamma) \quad (4.7)$$

Note that the expanded Fourier harmonic series (4.6) contains odd and even orders, differently from the representations of the magnetic field distributions in the usual electrical machines, because of the particular location of the coils in the stator slots; (4.6) is also a function of the angular abscissa $\theta_s$ whose origin coincides with the $y$-axis, as shown in Fig. 4.1.

By combining the relationship (4.6) considering all the phase currents, the $k$-th harmonic order $H_{sk}$ of the 5-phase stator magnetic field, related to the current space vector $s_v$, can be expressed as:

$$H_{sk}(\theta_s, t) = \begin{cases} \frac{5}{2} H_{Mk} \cos[kp_s \theta_s \mp \varphi(t)] & \text{if } \frac{k + s_v}{5} \in N_0 \\ 0 & \text{if } \frac{k + s_v}{5} \notin N_0 \end{cases} \quad (4.8)$$

The total stator magnetic field is the summation of all the existing terms given by (4.8), depending on the value of $k$. The rotor magnetic field generated by the magnets can be written as:

$$H_r(\theta_s, t) = \sum_{h=1,3,5..}^{\infty} H_{Rh} \cos[h_p \theta_s - h_p \Delta \theta] \quad (4.9)$$

where $H_{Rh}$, $B_{RM}$ are defined as follows:

$$H_{Rh} = \frac{4}{h\pi} \frac{B_{RM}}{\mu_0} \sin(h \frac{\alpha_m}{2}) \quad B_{RM} = \frac{L_m}{L_m + \mu_m K_c g} B_{rem} \quad (4.10)$$

Two different current space vectors, one for torque production and the other for levitation (4.5), are injected: respectively $s_{v1}$ and $s_{v2}$, so that the resulting radial component of flux density in the airgap can be written by means of the principle
of superposition:

\[ B_n(\theta_s, t) = \sum_{k_1,k_2,h} [B_{sk1}(\theta_s, t) + B_{sk2}(\theta_s, t) + B_{rh}(\theta_s, t)] = \sum_{k_1} B_{sk1} \cos[k_1 p_s \theta_s + \varphi_1(t)] + \sum_{k_2} B_{sk2} \cos[k_2 p_s \theta_s + \varphi_2(t)] + \sum_h B_{rh} \cos[h p_s \theta_s - h p_s \Delta\theta] \]

(4.11)

where the values of \( k_1 \) and \( k_2 \) follow the conditions explained in (4.8), and the amplitudes of the three distributions are defined as follows:

\[ B_{sk1} = \mu_0 \frac{5}{2} H_{Mk1} = \mu_0 \frac{5}{2} \frac{4}{k_1 \pi} \frac{n_{sv1}}{2\delta} k q_{dk1} \sin(k_1 \gamma) \]  
(4.12)

\[ B_{sk2} = \mu_0 \frac{5}{2} H_{Mk2} = \mu_0 \frac{5}{2} \frac{4}{k_2 \pi} \frac{n_{sv2}}{2\delta} k q_{dk2} \sin(k_2 \gamma) \]  
(4.13)

\[ B_{rh} = \frac{4}{h\pi} B_{RM} \sin\left(\frac{h \alpha_m}{2}\right) \]  
(4.14)
4.4 Calculation of the force acting on the rotor

The resulting force acting on the rotor can be determined by applying the Maxwell Stress Tensor method on a closed surface enveloping the frontal cross sections of the teeth in the stator:

\[
\bar{T} = \frac{1}{2} (H_n B_n - H_t B_t) \hat{n} + H_t B_n \hat{i} \approx \frac{B_n^2}{2\mu_0} \hat{n} + \frac{B_t B_n}{\mu_0} \hat{i} \tag{4.15}
\]

Taking the normal and tangential components of flux density and of magnetic field into account and neglecting the term related to the product of tangential components, the expression of the elementary force \( d\bar{F} \), acting at the angular abscissa \( \theta_s \) on the elementary surface \( dS = Lr d\theta_s \), results:

\[
d\bar{F}(\theta_s,t) = T(\theta_s,t) dS \frac{B_n^2(\theta_s,t)}{2\mu_0} dSn + \frac{B_t(\theta_s,t)B_n(\theta_s,t)}{\mu_0} dSt \tag{4.16}
\]

By substituting the expression of \( dS \) in (4.16), it gives:

\[
d\bar{F}(\theta_s,t) = \frac{Lr B_n^2}{2\mu_0} \frac{\partial}{\partial \theta_s} \hat{n} \frac{d\theta_s}{dF_n} + \frac{Lr B_t B_n}{\mu_0} \frac{\partial}{\partial \theta_s} \hat{i} \frac{d\theta_s}{dF_t} \tag{4.17}
\]

where can be recognized the elementary normal component \( dF_n \) and the tangential component \( dF_t \).

4.4.1 Normal Components of the Force

With the previous assumptions the resulting force acting on the rotor can be expressed as:

\[
F_{xn} = \int_0^{2\pi} dF_n \sin \theta_s = \frac{Lr}{2\mu_0} \int_0^{2\pi} B_n^2(\theta_s,t) \sin \theta_s d\theta_s \tag{4.18}
\]
\[ F_{yn} = \int_0^{2\pi} dF_n \cos \theta_s = \frac{Lr}{2\mu_0} \int_0^{2\pi} B_n^2(\theta_s, t) \cos \theta_s d\theta_s \]  \hspace{1cm} (4.19)

Remember that the reference axis for the angle \( \theta_s \) is the \( y \)-axis. Considering the integer values of \( k_1, k_2 \) and \( h \) that makes non-zero the related harmonic components of the flux densities in (4.11), we obtain six terms representing the square of \( B_n \):

\[ B_n^2(\theta_s, t) = B_{sk1}^2 + B_{sk2}^2 + B_{rr}^2 + 2B_{sk1}B_{sk2} + 2B_{sk1}B_{rr} + 2B_{sk2}B_{rr} \]  \hspace{1cm} (4.20)

When integrating all the terms in (4.20), taking (4.18)-(4.19) into account, it is important to consider that the components of forces have to be added in summation only for the existing harmonic orders of \( k_1, k_2 (k'_2) \), which are related to the stator magnetic fields respectively given by the current space vectors \( s_{v1} \) and \( s_{v2} \), and the existing harmonic orders of \( h \), which are related to the rotor magnetic field. This fact is expressed by means of the pre-conditions (4.21)-(4.23):

\[ \frac{k_1 + s_{v1}}{5} \in N_0 \]  \hspace{1cm} (4.21)

\[ \frac{k_2 + s_{v2}}{5} \in N_0, \quad \frac{k'_2 + s_{v2}}{5} \in N_0 \]  \hspace{1cm} (4.22)

\[ h \in N_{\text{odd}} \]  \hspace{1cm} (4.23)

In (4.22) it is highlighted the interaction between different harmonic orders of the same current space vector \( s_{v2} \): in fact, as will be clear later, some components of the force depend on this phenomenon.

In Tab. I are presented all the possible interactions, in this case until the 18-th order for reasons of brevity, determined by using the equations (4.21), (4.22), (4.23). By performing the calculations (4.19) and considering term by term of (4.20):
An analytical method for calculating the distribution of forces in a bearingless multiphase surface-mounted PM synchronous machine

| TABLE I. INTERACTIONS BETWEEN HARMONIC ORDERS |
|-----------------|-----------------|-----------------|
| $k_1$ | $n$ | $k_2$ |
| 1 | 1 | 2 |
| 4 | 3 | 5 |
| 6 | 5 | 7 |
| 9 | 8 | 10 |
| 11 | 9 | 13 |
| 14 | 13 | 15 |
| 16 | 15 | 17 |
| 18 | | |

$$F_y^{(a)} = \frac{Lr}{2\mu_0} \int_0^{2\pi} B_{sk}^2(\theta, t) \cos \theta_s d\theta_s = \frac{Lr}{2\mu_0} \int_0^{2\pi} B_{sk}^2 \cos^2[kp_s \theta_s + \varphi(t)] \cos \theta_s d\theta_s$$  \hspace{1cm} (4.24)

By considering that:

$$\cos^2(\alpha) = \frac{1 + \cos(2\alpha)}{2}$$ \hspace{1cm} (4.25)

Applying (4.25) to (4.24), it gives:

$$F_y^{(a)} = \frac{LrB_{sk}^2}{4\mu_0} \int_0^{2\pi} \cos \theta_s d\theta_s + \frac{LrB_{sk}^2}{4\mu_0} \int_0^{2\pi} \cos[2kp_s \theta_s + 2\varphi(t)] \cos \theta_s d\theta_s$$ \hspace{1cm} (4.26)

By considering that:

$$\cos(\alpha)\cos(\beta) = \frac{\cos(\alpha + \beta) + \cos(\alpha - \beta)}{2}$$ \hspace{1cm} (4.27)

Applying (4.27) to (4.26), it gives:
\[ F_{yn}^{(a)} = \frac{LrB_{Sk}^2}{8\mu_0} \int_0^{2\pi} \cos\left[ (2kp_s + 1)\theta_s \mp 2\varphi(t) \right] d\theta_s + \frac{LrB_{Sk}^2}{8\mu_0} \int_0^{2\pi} \cos\left[ (2kp_s - 1)\theta_s \mp 2\varphi(t) \right] d\theta_s \]  
(4.28)

\[ F_{yn}^{(a)} = \frac{LrB_{Sk}^2}{8\mu_0} \left\{ \frac{1}{2kp_s + 1} \left[ \sin\left[ (2kp_s + 1)\theta_s \mp 2\varphi(t) \right] \right]^{2\pi}_0 + \right. 
\left. + \frac{1}{2kp_s - 1} \left[ \sin\left[ (2kp_s - 1)\theta_s \mp 2\varphi(t) \right] \right]^{2\pi}_0 \right\} \]  
(4.29)

\[ F_{yn}^{(a)} = \frac{LrB_{Sk}^2}{8\mu_0} \left\{ \frac{1}{2kp_s + 1} \left[ \sin\left[ 2\pi(2kp_s + 1) + 2\varphi \right] - \sin\left[ 2\varphi \right] \right] + \right. 
\left. + \frac{1}{2kp_s - 1} \left[ \sin\left[ 2\pi(2kp_s - 1) + 2\varphi \right] - \sin\left[ 2\varphi \right] \right] \right\} = 0 \]  
(4.30)

Note that, being both \(2kp_s + 1\) and \(2kp_s - 1\) integer numbers, the angular arguments in (4.30) differ by integer multiples of \(2\pi\), thus the sinusoidal functions assume the same value and their difference is equal to zero.

Hence, the contribution of the terms in the form (4.24) to the \(y\)-component of the force is zero:

\[ F_{yn}^{(a)} = \frac{Lr}{2\mu_0} \int_0^{2\pi} B_{sk1}^2(\theta_s, t) \cos \theta_s d\theta_s = \frac{Lr}{2\mu_0} \int_0^{2\pi} B_{sk1}^2 \cos^2 \left[ k_1 p_s \theta_s \mp \varphi_1 \right] \cos \theta_s d\theta_s = 0 \]  
(4.31)

For the same reason, the similar quadratic term related to the space vector \(s_{v2}\) and the one related to the rotor magnetic field give a null result (4.32), (4.33):

\[ F_{yn}^{(b)} = \frac{Lr}{2\mu_0} \int_0^{2\pi} B_{sk2}^2(\theta_s, t) \cos \theta_s d\theta_s = \frac{Lr}{2\mu_0} \int_0^{2\pi} B_{sk2}^2 \cos^2 \left[ k_2 p_s \theta_s \mp \varphi_2 \right] \cos \theta_s d\theta_s = 0 \]  
(4.32)

\[ F_{yn}^{(c)} = \frac{Lr}{2\mu_0} \int_0^{2\pi} B_{rh}^2(\theta_s, t) \cos \theta_s d\theta_s = \frac{Lr}{2\mu_0} \int_0^{2\pi} B_{rh}^2 \cos^2 \left[ h p_r (\theta_s - \Delta \theta) \right] \cos \theta_s d\theta_s = 0 \]  
(4.33)
Now, examine the fourth term of (4.20):

\[
F_{yn}^{(d)} = \frac{Lr}{2\mu_0} \int_0^{2\pi} 2B_{sk1}(\theta_s, t)B_{sk2}(\theta_s, t) \cos \theta_s d\theta_s = \\
= \frac{Lr}{\mu_0} B_{sk1}B_{sk2} \int_0^{2\pi} \cos [k_1 p_s \theta_s \mp \phi_1] \cos [k_2 p_s \theta_s \mp \phi_2] \cos \theta_s d\theta_s
\]

(4.34)

By applying (4.27):

\[
F_{yn}^{(d)} = \frac{Lr}{\mu_0} B_{sk1}B_{sk2} \left\{ \int_0^{2\pi} \frac{1}{2} \cos [(k_1 p_s + k_2 p_s)\theta_s \mp \phi_1 \mp \phi_2] \cos \theta_s d\theta_s + \\
+ \int_0^{2\pi} \frac{1}{2} \cos [(k_1 p_s - k_2 p_s)\theta_s \mp \phi_1 \pm \phi_2] \cos \theta_s d\theta_s \right\}
\]

(4.35)

Repeating the same process (4.27) in both the integrals of (4.35):

\[
F_{yn}^{(d)} = \frac{Lr}{2\mu_0} B_{sk1}B_{sk2} \left\{ \int_0^{2\pi} \frac{1}{2} \cos [(k_1 p_s + k_2 p_s + 1)\theta_s \mp \phi_1 \mp \phi_2] \cos \theta_s d\theta_s + \\
+ \int_0^{2\pi} \frac{1}{2} \cos [(k_1 p_s + k_2 p_s - 1)\theta_s \mp \phi_1 \mp \phi_2] \cos \theta_s d\theta_s + \\
+ \int_0^{2\pi} \frac{1}{2} \cos [(k_1 p_s - k_2 p_s + 1)\theta_s \mp \phi_1 \mp \phi_2] \cos \theta_s d\theta_s + \\
+ \int_0^{2\pi} \frac{1}{2} \cos [(k_1 p_s - k_2 p_s - 1)\theta_s \mp \phi_1 \mp \phi_2] \cos \theta_s d\theta_s \right\}
\]

(4.36)

We make the assumption (4.37), that according to (4.36) gives the relationship (4.38) and consequently (4.39):

\[
k_1 p_s - k_2 p_s + 1 = 0 \quad \Rightarrow \quad k_2 p_s = 1 + k_1 p_s
\]

(4.37)

\[
F_{yn}^{(d)} = \frac{Lr}{2\mu_0} B_{sk1}B_{sk2} \left\{ \int_0^{2\pi} \frac{1}{2} \cos [2(k_1 p_s + 1)\theta_s \mp \phi_1 \mp \phi_2] \cos \theta_s d\theta_s + \\
+ \int_0^{2\pi} \frac{1}{2} \cos [2k_1 p_s \theta_s \mp \phi_1 \mp \phi_2] \cos \theta_s d\theta_s + \\
+ \int_0^{2\pi} \frac{1}{2} \cos [\mp \phi_1 \mp \phi_2] \cos \theta_s + \int_0^{2\pi} \frac{1}{2} \cos [-2\theta_s \mp \phi_1 \mp \phi_2] \cos \theta_s d\theta_s \right\}
\]

(4.38)
As observed above, the first, second and fourth terms in (4.39) have a null result, being the difference of sine functions with multiples arguments of 2π. The only term different from zero is the third, not depending on the integration variable θ<sub>s</sub>. Finally, the result of the integral (4.34), under the hypothesis (4.37) is:

\[ F_{yn}^{(d)} = \frac{\pi Lr}{2 \mu_0} B_{Sk1} B_{Sk2} \cos(\mp \varphi_1 \mp \varphi_2) \quad \text{if} \quad k_1 p_s - k_2 p_s = -1 \]  

(4.40)

We make the assumption (4.41), that according to (4.36) gives the relationship (4.42):

\[ k_1 p_s - k_2 p_s = 0 \quad \Rightarrow \quad k_2 p_s = -1 + k_1 p_s \]  

(4.41)

\[ F_{yn}^{(d)} = \frac{Lr}{4 \mu_0} B_{Sk1} B_{Sk2} \left\{ \int_0^{2\pi} \frac{1}{2} \cos[2k_1 p_s \theta_s \mp \varphi_1 \mp \varphi_2] d\theta_s + \int_0^{2\pi} \frac{1}{2} \cos[2(\theta_s - \varphi_1 + \varphi_2)] d\theta_s + \int_0^{2\pi} \frac{1}{2} \cos[\varphi_1 \pm \varphi_2] d\theta_s \right\} \]  

(4.42)

With very similar conclusions of the other assumption, the result of the integral (4.34), under the hypothesis (4.41) is:
An analytical method for calculating the distribution of forces in a bearingless multiphase surface-mounted pm synchronous machine

\[ F_{yn}^{(d)} = \frac{\pi Lr}{2\mu_0} B_{Sk1} B_{Sk2} \cos(\mp \varphi_1 \pm \varphi_2) \quad \text{if} \quad k_1 p_s - k_2 p_s = 1 \quad (4.44) \]

Note that the assumptions regarding the sum of the numbers of pole pairs, shown in (4.45), (4.46) are not examined because they don’t correspond to real cases, being impossible to occur.

\[ k_1 p_s + k_2 p_s - 1 = 0 \quad \Rightarrow \quad k_1 p_s + k_2 p_s = 1 \quad (4.45) \]
\[ k_1 p_s + k_2 p_s + 1 = 0 \quad \Rightarrow \quad k_1 p_s + k_2 p_s = -1 \quad (4.46) \]

So, the final form of the \( y \)-projection of the normal component of the force, related to the \( d \) factor of (4.20), is given by:

\[
F_{yn}^{(d)} = \frac{\pi Lr}{2\mu_0} B_{Sk1} B_{Sk2} \cos(\mp \varphi_1 \pm \varphi_2) \begin{cases} 
\text{if} & k_1 p_s - k_2 p_s = -1 \\
\text{if} & k_1 p_s - k_2 p_s = 1 
\end{cases}
\]

with: \[ \frac{k_1 \mp s_{vl}}{m} \in N_0 \land \frac{k_2 \mp s_{v2}}{m} \in N_0 \]

It is very important to observe that the signs of the conditions of existence have to be accorded to signs of the angular phases in the same order that appears in (4.47). Performing similar calculations, by integrating the \( e \) and \( f \) factors in (4.20) which refer to the interactions between every single stator current space vector and the rotor field produced by the magnets, the following relationships can be found:

\[
F_{yn}^{(e)} = \frac{\pi Lr}{2\mu_0} B_{Sk1} B_{R_{h}} \cos(h p_r, \Delta \Theta \mp \varphi_1) \begin{cases} 
\text{if} & k_1 p_s - h p_r = -1 \\
\text{if} & k_1 p_s - h p_r = 1 
\end{cases}
\]

with: \[ \frac{k_1 \mp s_{vl}}{m} \in N_0 \land h \in N_{odd} \]

141
By adding up all the non-zero terms of the $y$-projection of the normal component of the force, is obtained:

$$F_y^{(k_1, k_2)} = \sum_{k_1} \frac{\pi L_r}{2\mu_0} B_{Sk_1} B_{Sk_2} \cos(h p_r, \Delta \theta \mp \varphi_1)$$

with:

$$k_1 \mp s_{v1} \in N_0 \quad \land \quad k_2 \mp s_{v2} \in N_0 \quad(4.50)$$

$$h = \frac{1 + k_1 p_s}{p_r} \in N_{odd}$$

$$h = \frac{-1 + k_1 p_s}{p_r} \in N_{odd}$$

$$F_y^{(k_2, k_2')} = \sum_{k_2} \frac{\pi L_r}{2\mu_0} B_{Sk_2} B_{Sk_2'} \cos(h p_r, \Delta \theta \mp \varphi_2')$$

with:

$$k_2 \mp s_{v2} \in N_0 \quad \land \quad k_2' \mp s_{v2} \in N_0$$

$$h = \frac{1 + k_2 p_s}{p_r} \in N_{odd}$$

$$h = \frac{-1 + k_2 p_s}{p_r} \in N_{odd}$$

$$F_y^{(k_1, h)} = \sum_{k_1} \frac{\pi L_r}{2\mu_0} B_{Sk1} B_{Rh} \cos(h p_r, \Delta \theta \mp \varphi_1)$$

with:

$$k_1 \mp s_{v1} \in N_0$$

$$h = \frac{1 + k_1 p_s}{p_r} \in N_{odd}$$

$$h = \frac{-1 + k_1 p_s}{p_r} \in N_{odd}$$

$$F_y^{(k_2, h)} = \sum_{k_2} \frac{\pi L_r}{2\mu_0} B_{Sk2} B_{Rh} \cos(h p_r, \Delta \theta \mp \varphi_2)$$

with:

$$k_2 \mp s_{v2} \in N_0$$

$$h = \frac{1 + k_2 p_s}{p_r} \in N_{odd}$$

$$h = \frac{-1 + k_2 p_s}{p_r} \in N_{odd}$$

$$F_y^{(k_1, k_2)} = \sum_{k_1} \frac{\pi L_r}{2\mu_0} B_{Sk1} B_{Sk2} \cos(h p_r, \Delta \theta \mp \varphi_1, \mp \varphi_2)$$

with:

$$k_2 \mp s_{v2} \in N_0$$

$$k_1 \mp s_{v1} \in N_0$$

$$h = \frac{1 + k_1 p_s}{p_r} \in N_{odd}$$

$$h = \frac{-1 + k_1 p_s}{p_r} \in N_{odd}$$

$$F_y^{(k_2, k_2')} = \sum_{k_2} \frac{\pi L_r}{2\mu_0} B_{Sk2} B_{Sk2'} \cos(h p_r, \Delta \theta \mp \varphi_2, \mp \varphi_2')$$

with:

$$k_2 \mp s_{v2} \in N_0$$

$$k_2' \mp s_{v2} \in N_0$$

$$h = \frac{1 + k_2 p_s}{p_r} \in N_{odd}$$

$$h = \frac{-1 + k_2 p_s}{p_r} \in N_{odd}$$
The equations (4.53) represent a typology of interaction not explicated in (4.20): it is the influence between different harmonic orders belonging to the same current space vector \( s_{v2} \), denoted with \( k_2 \) and \( k'_2 \). In fact, it happens in the analyzed bearingless machine that some harmonic orders of \( s_{v2} \) exist, whose number of pole pairs differ of \( \pm 1 \): in this case, they also give a contribution to the resultant force. It is important to note that the phase angle \( \varphi_{2'} \) in (4.53) represents exactly the same phase angle \( \varphi_2 \), being only a formal distinction which indicates that the sign of \( \varphi_{2'} \) depends on the existing harmonic order \( k'_2 \), while the sign of \( \varphi_2 \) is obviously related to \( k_2 \): in this way the argument of the function cosine in (4.53) has to be understood as the algebraic sum of the same variable \( \varphi_2 \), resulting in a 0, 2\( \varphi_2 \) or \(-2\varphi_2 \). Finally, the resultant \( y \)-projection of the normal component of the force transmitted to the rotor is the sum of the equations (4.50), (4.51), (4.52), (4.53):

\[
F_{yn} = F_{yn}^{(k_1,k_2)} + F_{yn}^{(k_1,h)} + F_{yn}^{(k_2,h)} + F_{yn}^{(k_2,k_2')} \tag{4.54}
\]

A similar calculation can be performed to determine the resultant \( x \)-projection, by considering term by term of (4.20) integrating with (4.18):

\[
F_{xn}^{(a)} = \frac{Lr}{2\mu_0} \int_0^{2\pi} B_{sk}^2 (\theta_s,t) \sin \theta_s d\theta_s = \frac{Lr}{2\mu_0} \int_0^{2\pi} B_{sk}^2 \cos^2 [kp_s \theta_s \mp \varphi(t)] \sin \theta_s d\theta_s \tag{4.55}
\]

By applying (4.25) to (4.55), it gives:

\[
F_{xn}^{(a)} = \frac{LrB_{sk}^2}{4\mu_0} \int_0^{2\pi} \sin \theta_s d\theta_s + \frac{LrB_{sk}^2}{4\mu_0} \int_0^{2\pi} \cos [2kp_s \theta_s \mp 2\varphi(t)] \sin \theta_s d\theta_s \tag{4.56}
\]

By considering that:

\[
\cos(\alpha) \sin(\beta) = \frac{\sin(\alpha + \beta) - \sin(\alpha - \beta)}{2} \tag{4.57}
\]
Chapter 4

By applying (4.57) to (4.56), it gives:

\[
F_{xn}^{(a)} = \frac{LrB_{skk}^2}{8\mu_0} \int_0^{2\pi} \sin[(2kp_s + 1)\theta_s + 2\varphi(t)]d\theta_s - \frac{LrB_{skk}^2}{8\mu_0} \int_0^{2\pi} \sin[(2kp_s - 1)\theta_s + 2\varphi(t)]d\theta_s
\]

(4.58)

\[
F_{xn}^{(a)} = \frac{LrB_{skk}^2}{8\mu_0} \left[ \frac{1}{2kp_s + 1} \left[ -\cos[(2kp_s + 1)\theta_s + 2\varphi] \right]_{0}^{2\pi} + \frac{1}{2kp_s - 1} \left[ -\cos[(2kp_s - 1)\theta_s + 2\varphi] \right]_{0}^{2\pi} \right]
\]

(4.59)

\[
F_{xn}^{(a)} = \frac{LrB_{skk}^2}{8\mu_0} \left[ \frac{1}{2kp_s + 1} \left[ -\cos[2\pi(2kp_s + 1) + 2\varphi] + \cos[2\pi + 2\varphi] \right] + \frac{1}{2kp_s - 1} \left[ -\cos[2\pi(2kp_s - 1) + 2\varphi] + \cos[2\pi + 2\varphi] \right] \right] = 0
\]

(4.60)

In a very similar way to the equation (4.30), being both \(2kp_s + 1\) and \(2kp_s - 1\) integer numbers, the arguments of the cosines in (4.60) differ by integer multiples of \(2\pi\), thus they assume the same value and their difference is equal to zero. Hence, the contribution of the terms in the form (4.55) to the \(x\)-component of the force is zero:

\[
F_{xn}^{(a)} = \frac{Lr}{2\mu_0} \int_0^{2\pi} B_{sk1}(\theta_s, t) \sin \theta_s d\theta_s = \frac{Lr}{2\mu_0} \int_0^{2\pi} B_{sk1}^2 \cos^2[k_1 p_s \theta_s + \varphi_1] \sin \theta_s d\theta_s = 0
\]

(4.61)

For the same reason, the similar quadratic term related to the space vector \(s_{v2}\) and the one related to the rotor magnetic field give a null result (4.62), (4.63):

\[
F_{xn}^{(b)} = \frac{Lr}{2\mu_0} \int_0^{2\pi} B_{sk2}^2(\theta_s, t) \sin \theta_s d\theta_s = \frac{Lr}{2\mu_0} \int_0^{2\pi} B_{sk2}^2 \cos^2[k_2 p_s \theta_s + \varphi_2] \sin \theta_s d\theta_s = 0
\]

(4.62)

\[
F_{xn}^{(c)} = \frac{Lr}{2\mu_0} \int_0^{2\pi} B_{rh}^2(\theta_s, t) \sin \theta_s d\theta_s = \frac{Lr}{2\mu_0} \int_0^{2\pi} B_{rh}^2 \cos^2[k p_r (\theta_s - \Delta \theta)] \sin \theta_s d\theta_s = 0
\]

(4.63)
An analytical method for calculating the distribution of forces in a bearingless multiphase surface-mounted PM synchronous machine

By integrating the fourth term of (4.20):

\[
F_{xn}^{(d)} = \frac{Lr}{2\mu_0} \int_0^{2\pi} \frac{1}{2} B_{sk1}(\theta_s, t) B_{sk2}(\theta_s, t) \sin \theta_s d\theta_s = 
\]

\[
= \frac{Lr}{2\mu_0} B_{sk1} B_{sk2} \int_0^{2\pi} \cos[k_1 p_s \theta_s \mp \varphi_1] \cos[k_2 p_s \theta_s \mp \varphi_2] \sin \theta_s d\theta_s
\]

(4.64)

By applying (4.27) to (4.64):

\[
F_{xn}^{(d)} = \frac{Lr}{\mu_0} B_{sk1} B_{sk2} \left\{ \int_0^{2\pi} \frac{1}{2} \cos[(k_1 p_s + k_2 p_s) \theta_s \mp \varphi_1 \mp \varphi_2] \sin \theta_s d\theta_s + 
\right.
\]

\[
+ \int_0^{2\pi} \frac{1}{2} \cos[(k_1 p_s - k_2 p_s) \theta_s \mp \varphi_1 \pm \varphi_2] \sin \theta_s d\theta_s \right\}
\]

(4.65)

By applying (4.57) to both the integrals of (4.65):

\[
F_{xn}^{(d)} = \frac{Lr}{2\mu_0} B_{sk1} B_{sk2} \left\{ \int_0^{2\pi} \frac{1}{2} \sin[(k_1 p_s + k_2 p_s + 1) \theta_s \mp \varphi_1 \pm \varphi_2] d\theta_s + 
\right.
\]

\[- \int_0^{2\pi} \frac{1}{2} \sin[(k_1 p_s + k_2 p_s - 1) \theta_s \mp \varphi_1 \mp \varphi_2] d\theta_s + 
\]

\[+ \int_0^{2\pi} \frac{1}{2} \sin[(k_1 p_s - k_2 p_s + 1) \theta_s \mp \varphi_1 \pm \varphi_2] d\theta_s + 
\]

\[- \int_0^{2\pi} \frac{1}{2} \sin[(k_1 p_s - k_2 p_s - 1) \theta_s \mp \varphi_1 \pm \varphi_2] d\theta_s \right\}
\]

(4.66)

We make the assumption (4.67), that according to (4.66) gives the relationship (4.68) and, consequently, (4.69):

\[k_1 p_s - k_2 p_s + 1 = 0 \quad \Rightarrow \quad k_2 p_s = 1 + k_1 p_s
\]

(4.67)

\[
F_{xn}^{(d)} = \frac{Lr}{2\mu_0} B_{sk1} B_{sk2} \left\{ \int_0^{2\pi} \frac{1}{2} \sin[2(k_1 p_s + 1) \theta_s \mp \varphi_1 \pm \varphi_2] d\theta_s + 
\right.
\]

\[- \int_0^{2\pi} \frac{1}{2} \sin[2k_1 p_s \theta_s \mp \varphi_1 \mp \varphi_2] d\theta_s + 
\]

\[+ \int_0^{2\pi} \frac{1}{2} \sin[\mp \varphi_1 \pm \varphi_2] d\theta_s - \int_0^{2\pi} \frac{1}{2} \sin[-2\theta_s \mp \varphi_1 \pm \varphi_2] d\theta_s \right\}
\]

(4.68)
The first, second and fourth terms in (4.69) have a null result, being differences between cosines with arguments multiples of $\frac{\pi}{2}$. The only term different from zero is the third, not depending on the integration variable $\theta_s$. Finally, the result of the integral (4.64), under the hypothesis (4.67) is:

$$F_{xn}^{(d)} = \frac{\pi Lr}{2\mu_0} B_{Sk1} B_{Sk2} \sin(\mp \varphi_1 \pm \varphi_2) \quad \text{if} \quad k_1 p_s - k_2 p_s = -1 \quad (4.70)$$

We make the assumption (4.71), that according to (4.66) gives the relationship (4.72) and, consequently, (4.73):

$$k_1 p_s - k_2 p_s - 1 = 0 \quad \Rightarrow \quad k_2 p_s = -1 + k_1 p_s \quad (4.71)$$

$$F_{xn}^{(d)} = \frac{Lr}{2\mu_0} B_{Sk1} B_{Sk2} \left\{ \int_{0}^{2\pi} \frac{1}{2} \sin[2k_1 p_s \theta_s \mp \varphi_1 \pm \varphi_2] d\theta_s + \int_{0}^{2\pi} \frac{1}{2} \sin[2k_1 p_s - 1] \theta_s \mp \varphi_1 \pm \varphi_2 \right\} \quad (4.72)$$

$$F_{xn}^{(d)} = \frac{Lr}{4\mu_0} B_{Sk1} B_{Sk2} \left\{ \frac{1}{2k_1 p_s} \left[ -\cos(2k_1 p_s \theta_s \mp \varphi_1 \pm \varphi_2) \right]_0^{2\pi} + \right.$$  

$$\left. + \frac{1}{2(k_1 p_s - 1)} \left[ \cos(2(k_1 p_s - 1) \theta_s \mp \varphi_1 \pm \varphi_2) \right]_0^{2\pi} - \frac{1}{2} \cos(2 \theta_s \mp \varphi_1 \pm \varphi_2) \right\} \quad (4.73)$$

With very similar conclusions of the other assumption, the result of the integral.
An analytical method for calculating the distribution of forces in a bearingless multiphase surface-mounted pm synchronous machine

(4.64), under the hypothesis (4.71) is:

$$F_\text{xn}^{(d)} = -\frac{\pi L r}{2\mu_0} B_{s k_1} B_{s k_2} \sin(\mp \varphi_1 \pm \varphi_2) \quad \text{if} \quad k_1 p_s - k_2 p_s = 1 \quad (4.74)$$

As done before, the assumptions regarding the sum of the numbers of pole pairs, replicated in (4.75), (4.76) are not examined because they don’t correspond to real cases.

$$k_1 p_s + k_2 p_s - 1 = 0 \quad \Rightarrow \quad k_1 p_s + k_2 p_s = 1 \quad (4.75)$$

$$k_1 p_s + k_2 p_s + 1 = 0 \quad \Rightarrow \quad k_1 p_s + k_2 p_s = -1 \quad (4.76)$$

So, the final form of the $x$-projection of the normal component of the force, related to the $d$ factor of (4.20), is given by:

$$F_\text{xn}^{(d)} = \begin{cases} \frac{\pi L r}{2\mu_0} B_{s k_1} B_{s k_2} \sin(\mp \varphi_1 \pm \varphi_2) \quad \text{if} \quad k_1 p_s - k_2 p_s = -1 \\ -\frac{\pi L r}{2\mu_0} B_{s k_1} B_{s k_2} \sin(\mp \varphi_1 \pm \varphi_2) \quad \text{if} \quad k_1 p_s - k_2 p_s = 1 \end{cases} \quad (4.77)$$

with: $\frac{k_1 \mp s_{v1}}{m} \in N_0 \quad \land \quad \frac{k_2 \mp s_{v2}}{m} \in N_0$

As said before, the signs of the conditions of existence have to be accorded to signs of the angular phases in the same order that appears in (4.77). Note that the sign of the $x$-component changes depending on the condition to be verified (4.67) or (4.71), differently from the $y$-component (4.47).

By integrating the $e$ and $f$ factors of (4.20) in a very similar way to what has been done for $d$, the interactions between every single stator current space vector and the rotor field produced by the magnets can be expressed as:
By adding up all the non-zero terms of the \( x \)-projections of the normal component of the force, is obtained:

\[
F_{xn}^{(k_1,k_2)} = \begin{cases} 
\sum_{k_1} \frac{\pi L_r}{2\mu_0} B_{Sk_1} B_{Rh} \sin[h p_r, \Delta \theta \mp \varphi_1] & \forall k_2 = \frac{1+k_1}{p_s} \in N \\
\sum_{k_1} -\frac{\pi L_r}{2\mu_0} B_{Sk_1} B_{Sk_2} \sin[h p_r, \varphi_1 \pm \varphi_2] & \forall k_2 = \frac{-1+k_1}{p_s} \in N 
\end{cases}
\]  
\text{with: } \frac{k_1 \mp s_{v1}}{m} \in N_0 \land \frac{k_2 \pm s_{v2}}{m} \in N_0

(4.80)

\[
F_{xn}^{(k_1,h)} = \begin{cases} 
\sum_{k_1} \frac{\pi L_r}{2\mu_0} B_{Sk_1} B_{Rh} \sin[h p_r, \Delta \theta \mp \varphi_1] & \forall h = \frac{1+k_1}{p_r} \in N_{odd} \\
\sum_{k_1} -\frac{\pi L_r}{2\mu_0} B_{Sk_1} B_{Rh} \sin[h p_r, \varphi_1 \pm \varphi_1] & \forall h = \frac{-1+k_1}{p_r} \in N_{odd} 
\end{cases}
\]  
\text{with: } \frac{k_1 \mp s_{v1}}{m} \in N_0 \land h \in N_{odd}

(4.81)
An analytical method for calculating the distribution of forces in a bearingless multiphase surface-mounted pm synchronous machine

\[
F_{xn}^{(k_2,k_2')} = \begin{cases} 
\sum_{k_2} \frac{\pi L_r}{2\mu_0} B_{Sk_2} B_{Rh} \sin[hp_r \Delta \theta \mp \varphi_2] & \forall \ h = \frac{1 + k_2 p_s}{p_r} \in N_{odd} \\
\sum_{k_2} -\frac{\pi L_r}{2\mu_0} B_{Sk_2} B_{Rh} \sin[hp_r \Delta \theta \mp \varphi_2] & \forall \ h = \frac{-1 + k_2 p_s}{p_r} \in N_{odd}
\end{cases}
\]

with: \[ \frac{k_2 \mp s_{v2}}{m} \in N_0 \land h \in N_{odd} \]

\[
F_{xn}^{(k_2,k_2')} = \begin{cases} 
\sum_{k_2} \frac{\pi L_r}{2\mu_0} B_{Sk_2} B_{Sk_2'} \sin[\varphi_2 \pm \varphi_2'] & \forall \ k_2' = \frac{1 + k_2 p_s}{p_s} \in N \\
\sum_{k_2} -\frac{\pi L_r}{2\mu_0} B_{Sk_2} B_{Sk_2'} \sin[\varphi_2 \pm \varphi_2'] & \forall \ k_2' = \frac{-1 + k_2 p_s}{p_s} \in N
\end{cases}
\]

with: \[ \frac{k_2 \mp s_{v2}}{m} \in N_0 \land \frac{k_2' \mp s_{v2}}{m} \in N_0 \]

(4.82)

The same considerations done for the equation (4.53), related to the meaning of \( k_2 \) and \( k_2' \), are valid for (4.83). So, the resultant \( x \)-projection of the normal component of the force transmitted to the rotor is the sum of the equations (4.80), (4.81), (4.82), (4.83), taking into account all the presented assumptions:

\[
F_{xn} = F_{xn}^{(k_1,k_2)} + F_{xn}^{(k_1,k_2)} + F_{xn}^{(k_2,k_2)} + F_{xn}^{(k_2,k_2')}
\]

(4.84)
4.4.2 Tangential components of the force

An analysis to determine the contribution of the tangential components of the magnetic field to the radial forces is presented, with reference to the equivalent surface current density $G(x_s,t)$, responsible for the normal component distribution of the magnetic field:

$$G(x_s,t) = \frac{\partial H_n}{\partial x_s}$$  \hspace{1cm} (4.85)

Note that $x_s$ is the linear abscissa corresponding to the angular one $\theta_s$. By considering the simplified assumption that gives the tangential component of magnetic field $H_t$ equal to the linear current density $J$, it is possible to find a correlation between $H_t$ and $H_n$:

$$H_t(x_s,t) \equiv J(x_s,t) = G\delta = \frac{\partial H_n}{\partial x_s} \delta \Rightarrow H_t \approx \frac{\partial H_n}{\partial x_s} \delta$$  \hspace{1cm} (4.86)

In order to determine the expression of (4.86), based on (4.11), it is possible to write:

$$H_n(\theta_s,t) = \sum_{k_1} H_{sk1}(k_1 p_s \theta_s,t) + \sum_{k_2} H_{sk2}(k_2 p_s \theta_s,t) + \sum_{h} H_{rh}(h p_r \theta_s,t)$$  \hspace{1cm} (4.87)

where

$$p_s \theta_s = p_r \theta_s = \frac{\pi}{\tau} x_s$$  \hspace{1cm} (4.88)

Note that as specified in (4.88), $p_s$ and $p_r$ have the same value, being the pole pair number of stator and rotor fields for the motoring torque. The derivative of $H_n$ by using (4.87), (4.88) can be expressed as:

$$\frac{\partial H_n}{\partial x_s} = \sum_{k_1} \frac{k_1 \pi}{\tau} \frac{\partial H_{sk1}(k_1 p_s \theta_s,t)}{\partial (k_1 p_s \theta_s)} + \sum_{k_2} \frac{k_2 \pi}{\tau} \frac{\partial H_{sk2}(k_2 p_s \theta_s,t)}{\partial (k_2 p_s \theta_s)} + \sum_{h} \frac{h \pi}{\tau} \frac{\partial H_{rh}(h p_r \theta_s,t)}{\partial (h p_r \theta_s)}$$  \hspace{1cm} (4.89)
By substituting (4.89) in (4.86), calculating the derivatives and explicating the values of the flux densities, \( H_r \) can be finally expressed as:

\[
H_r(\theta_s,t) = \sum_{k_1} \frac{\delta}{\mu_0} \frac{k_1 \pi}{\tau} B_{Sk1} \sin[k_1 p_s \theta_s + \varphi_1(t)] + \\
- \sum_{k_2} \frac{\delta}{\mu_0} \frac{k_2 \pi}{\tau} B_{Sk2} \sin[k_2 p_s \theta_s + \varphi_2(t)] - \sum_{h} \frac{\delta}{\mu_0} \frac{h \pi}{\tau} B_{Rh} \sin[hp_r \theta_s - hp_r \Delta \theta]
\]

(4.90)

According to the Maxwell stress tensor expression (4.15), the tangential component of \( \mathbf{T} \) is given by \( H_r B_n \), calculated by multiplying (4.11) and (4.90), resulting in the sum of the following nine terms:

\[
\sum_{k_1} \frac{k_1 \delta \pi}{\mu_0 \tau} B_{Sk1}^2 \sin[k_1 p_s \theta_s + \varphi_1(t)] \cos[k_1 p_s \theta_s + \varphi_1(t)] 
\]

(4.91)

\[
\sum_{k_1,k_2} \frac{k_1 \delta \pi}{\mu_0 \tau} B_{Sk1} B_{Sk2} \sin[k_1 p_s \theta_s + \varphi_1(t)] \cos[k_2 p_s \theta_s + \varphi_2(t)] 
\]

(4.92)

\[
\sum_{k_1,h} \frac{k_1 \delta \pi}{\mu_0 \tau} B_{Sk1} B_{Rh} \sin[k_1 p_s \theta_s + \varphi_1(t)] \cos[hp_r \theta_s - hp_r \Delta \theta] 
\]

(4.93)

\[
\sum_{k_1,k_2} \frac{k_2 \delta \pi}{\mu_0 \tau} B_{Skk1} B_{Sk2} \cos[k_1 p_s \theta_s + \varphi_1(t)] \sin[k_2 p_s \theta_s + \varphi_2(t)] 
\]

(4.94)

\[
\sum_{k_2} \frac{k_2 \delta \pi}{\mu_0 \tau} B_{Sk2}^2 \sin[k_2 p_s \theta_s + \varphi_2(t)] \cos[k_2 p_s \theta_s + \varphi_2(t)] 
\]

(4.95)

\[
\sum_{k_2,h} \frac{k_2 \delta \pi}{\mu_0 \tau} B_{Sk2} B_{Rh} \sin[k_2 p_s \theta_s + \varphi_2(t)] \cos[hp_r \theta_s - hp_r \Delta \theta] 
\]

(4.96)

\[
\sum_{h,k_1} \frac{h \delta \pi}{\mu_0 \tau} B_{Sk1} B_{Rh} \cos[k_1 p_s \theta_s + \varphi_1(t)] \sin[hp_r \theta_s - hp_r \Delta \theta] 
\]

(4.97)
As done in section 4.4.1, every summation described in (4.91)-(4.99) has to be projected along the axes directions and integrated between 0 and $2\pi$ in order to determine the $x$- and $y$-components of the resulting tangential force in the cartesian reference system (4.100), (4.101):

$$F_{xt} = \int_{0}^{2\pi} dF_{r} \cos \theta_s = Lr \int_{0}^{2\pi} H_{s} (\theta_s, t) B_{s} (\theta_s, t) \cos \theta_s d\theta_s \quad (4.100)$$

$$F_{yt} = \int_{0}^{2\pi} dF_{r} (-\sin \theta_s) = -Lr \int_{0}^{2\pi} H_{s} (\theta_s, t) B_{s} (\theta_s, t) \sin \theta_s d\theta_s \quad (4.101)$$

As seen in the previous, the result of the integrals is non-zero when the difference between the pole pairs numbers of two fields in exam is equal to $\pm 1$. Furthermore, the terms in summation are considered for the existing harmonic orders of $k_1, k_2, h$. Let us perform the calculations (4.101), considering the terms (4.91), (4.95):

$$F_{yt}^{(a.e)} = -Lr \int_{0}^{2\pi} \frac{k\delta \pi}{\mu_0 \tau} B_{sk}^2 \sin[kp_s \theta_s \mp \varphi(t)] \cos[kp_s \theta_s \pm \varphi(t)] \sin \theta_s d\theta_s \quad (4.102)$$

By considering that:

$$\sin(\alpha) \cos(\alpha) = \frac{1}{2} \sin(2\alpha) \quad (4.103)$$

Applying (4.103) to (4.102), it gives:

$$F_{yt}^{(a.e)} = \frac{1}{2} \frac{\delta \pi}{\mu_0 \tau} \frac{k B_{sk}^2 Lr}{2} \int_{0}^{2\pi} \sin[2kp_s \theta_s \mp 2\varphi(t)] \sin \theta_s d\theta_s \quad (4.104)$$

By assuming that:
An analytical method for calculating the distribution of forces in a bearingless multiphase surface-mounted pm synchronous machine

\[
sin(\alpha)sin(\beta) = \frac{\cos(\alpha-\beta) - \cos(\alpha+\beta)}{2} \tag{4.105}\]

Applying (4.105) to (4.104), it gives:

\[
F_{yil}^{(a,e)} = \frac{1}{4} \frac{\delta \pi}{\mu_0 \tau} kB_{Sk}^2 Lr \left\{ \frac{2\pi}{2kps - 1} \left[ \cos((2kps - 1)\theta_s + 2\varphi)\right] d\theta_s - \frac{2\pi}{2kps + 1} \left[ \cos((2kps + 1)\theta_s + 2\varphi)\right] d\theta_s \right\} \tag{4.106}
\]

\[
F_{yil}^{(a,e)} = \frac{1}{4} \frac{\delta \pi}{\mu_0 \tau} kB_{Sk}^2 Lr \left\{ \frac{2\pi}{2kps - 1} \left[ \sin((2kps - 1)\theta_s + 2\varphi)\right]^{\pi} + \frac{1}{2kps + 1} \left[ \sin((2kps + 1)\theta_s + 2\varphi)\right]^{\pi} \right\} = 0 \tag{4.107}
\]

Proceeding by analogy, it can be assumed that:

\[
F_{yil}^{(i)} = \frac{1}{4} \frac{\delta \pi}{\mu_0 \tau} hB_{Rh}^2 Lr \left\{ \frac{1}{2hp_r - 1} \left[ \sin((2hp_r - 1)\theta_s - 2hp_r \Delta \theta)\right]^{\pi} + \frac{1}{2hp_r + 1} \left[ \sin((2hp_r + 1)\theta_s - 2hp_r \Delta \theta)\right]^{\pi} \right\} = 0 \tag{4.108}
\]

Thus, the integrals related to the terms of the typologies a, e, i, are zero. Let us calculate the terms of the typology c, f, corresponding to equations (4.93), (4.96):

\[
F_{yil}^{(c,f)} = -Lr \int_0^{2\pi} - \frac{k\delta \pi}{\mu_0 \tau} B_{Sk} B_{Rh} \sin[kp_s \theta_s + \varphi(t)] \cos[hp_r(\theta_s - \Delta \theta)] \sin \theta_s d\theta_s \tag{4.109}
\]

By considering (4.110) and applying it to (4.109), it gives (4.111):

\[
sin(\alpha)cos(\beta) = \frac{\sin(\alpha + \beta) + \sin(\alpha - \beta)}{2} \tag{4.110}
\]

\[
F_{yil}^{(c,f)} = \frac{1}{2} \frac{\delta \pi}{\mu_0 \tau} kB_{Sk} B_{Rh} Lr \left\{ \int_0^{2\pi} \sin[(kp_s + hp_r)\theta_s + \varphi - hp_r \Delta \theta] \sin \theta_s d\theta_s + \int_0^{2\pi} \sin[(kp_s - hp_r)\theta_s + \varphi + hp_r \Delta \theta] \sin \theta_s d\theta_s \right\} \tag{4.111}
\]
By using (4.105) in (4.111):

\[
F_{yt}^{(c,f)} = \frac{1}{4 \mu_0} k B_{Sk} B_{Rh} L r \left\{ \int_0^{2\pi} \cos \left( \frac{2\pi}{4} \left( (kp_s + hp_r - 1)\theta_s \mp \varphi - hp_r \Delta \theta \right) \right) d\theta_s + \\
- \int_0^{2\pi} \cos \left( (kp_s + hp_r + 1)\theta_s \mp \varphi - hp_r \Delta \theta \right) d\theta_s + \\
+ \int_0^{2\pi} \cos \left( (kp_s - hp_r - 1)\theta_s \mp \varphi - hp_r \Delta \theta \right) d\theta_s + \\
- \int_0^{2\pi} \cos \left( (kp_s - hp_r + 1)\theta_s \mp \varphi + hp_r \Delta \theta \right) d\theta_s \right\} \tag{4.112}
\]

Consider, as done above, that the sum of the pole pairs of the stator and rotor fields cannot be equal to \( \pm 1 \), conditions impossible to verify, thus the hypothesis (4.113), (4.114) aren’t admitted:

\[
k p_s + hp_r - 1 = 0 \quad \Rightarrow \quad kp_s + hp_r = 1 \tag{4.113}
\]

\[
k p_s + hp_r + 1 = 0 \quad \Rightarrow \quad kp_s + hp_r = -1 \tag{4.114}
\]

Let us suppose that (4.115):

\[
k p_s - hp_r - 1 = 0 \quad \Rightarrow \quad kp_s - hp_r = 1 \tag{4.115}
\]

By using (4.115) in (4.112), it gives (4.116) and consequently (4.117):

\[
F_{yt}^{(c,f)} = \frac{1}{4 \mu_0} k B_{Sk} B_{Rh} L r \left\{ \int_0^{2\pi} \cos \left( 2(kp_s - 1)\theta_s \mp \varphi - hp_r \Delta \theta \right) d\theta_s + \\
- \int_0^{2\pi} \cos \left( 2kp_s \theta_s \mp \varphi - hp_r \Delta \theta \right) d\theta_s + \int_0^{2\pi} \cos (\varphi + hp_r \Delta \theta) d\theta_s + \\
- \int_0^{2\pi} \cos \left( 2\theta_s \mp \varphi + hp_r \Delta \theta \right) d\theta_s \right\} \tag{4.116}
\]

\[
F_{yt}^{(c,f)} = \frac{1}{4 \mu_0} k B_{Sk} B_{Rh} L r \left\{ \frac{1}{2(kp_s - 1)} \left[ \sin \left( 2(kp_s - 1)\theta_s \mp \varphi - hp_r \Delta \theta \right) \right]_0^{2\pi} + \\
- \frac{1}{2kp_s} \sin \left( 2kp_s \theta_s \mp \varphi - hp_r \Delta \theta \right)_0^{2\pi} + 2\pi \cos (\varphi + hp_r \Delta \theta) + \\
- \frac{1}{2} \sin \left( 2\theta_s \mp \varphi + hp_r \Delta \theta \right)_0^{2\pi} \right\} \tag{4.117}
\]
An analytical method for calculating the distribution of forces in a bearingless multiphase surface-mounted pm synchronous machine

For the reasons already mentioned several times, all the integrals dependent on $\theta$ calculated on the interval $[0, 2\pi]$ are zero; so, the third term is the only contribution to the force:

$$F_{yf}^{(c.f)} = k \frac{\pi^2}{2} \frac{\delta L_r}{\mu_0 \tau} B_{Sk} B_{Rh} \cos[hp_r \Delta \theta \mp \varphi(t)] \quad \text{if} \quad kp_s - hp_r = 1 \quad (4.118)$$

Let us suppose that (4.119) is verified:

$$kp_s - hp_r + 1 = 0 \quad \Rightarrow \quad kp_s - hp_r = -1 \quad (4.119)$$

By using (4.119) in (4.112), it gives:

$$F_{yf}^{(c.f)} = \frac{1}{4} \frac{\delta \pi}{\mu_0 \tau} k B_{Sk} B_{Rh} L_r \left\{ \int_0^{2\pi} \cos(2kp_s \theta_s \mp \varphi - hp_r \Delta \theta) d\theta_s + \int_0^{2\pi} \cos(-2\theta_s \mp \varphi + hp_r \Delta \theta) d\theta_s + \right\}$$

(4.120)

By executing the integrals in (4.120):

$$F_{yf}^{(c.f)} = \frac{1}{4} \frac{\delta \pi}{\mu_0 \tau} k B_{Sk} B_{Rh} L_r \left\{ \frac{1}{2kp_s} \left[ \sin(2kp_s \theta_s \mp \varphi - hp_r \Delta \theta) \right]_0^{2\pi} + \right\}$$

(4.121)

The only not null contribution to the force is the fourth term of (4.121):

$$F_{yf}^{(c.f)} = \frac{1}{4} \frac{\delta \pi}{\mu_0 \tau} k B_{Sk} B_{Rh} \cos[hp_r \Delta \theta \mp \varphi(t)] \quad \text{if} \quad kp_s - hp_r = -1 \quad (4.122)$$

So, it can be concluded that the contribution of the terms of the typologies $c$ and $f$ to the $y$-component of the tangential force acting on the rotor is given as:
Obviously, this contribution has to be considered in the interaction between everyone of the current space vector $s_{v1}$, $s_{v2}$, and the rotor magnetic field produced by the magnets:

\[
F_{yi}^{(c,f)} = \begin{cases} 
  k \frac{\pi^2}{2} \frac{\delta Lr}{\mu_0 \tau} B_{Sk} B_{Rh} \cos[h p_s \Delta \theta \mp \varphi(t)] & \text{if } k p_s - h p_r = 1 \\
  -k \frac{\pi^2}{2} \frac{\delta Lr}{\mu_0 \tau} B_{Sk} B_{Rh} \cos[h p_s \Delta \theta \mp \varphi(t)] & \text{if } k p_s - h p_r = -1
\end{cases}
\]  

(4.123)

\[
F_{yi}^{(c)} = \begin{cases} 
  k_1 \frac{\pi^2}{2} \frac{\delta Lr}{\mu_0 \tau} B_{Sk1} B_{Rh} \cos[h p_s \Delta \theta \mp \varphi_1(t)] & \text{if } k_1 p_s - h p_r = 1 \\
  -k_1 \frac{\pi^2}{2} \frac{\delta Lr}{\mu_0 \tau} B_{Sk1} B_{Rh} \cos[h p_s \Delta \theta \mp \varphi_1(t)] & \text{if } k_1 p_s - h p_r = -1
\end{cases}
\]  

(4.124)

\[
F_{yi}^{(f)} = \begin{cases} 
  k_2 \frac{\pi^2}{2} \frac{\delta Lr}{\mu_0 \tau} B_{Sk2} B_{Rh} \cos[h p_s \Delta \theta \mp \varphi_2(t)] & \text{if } k_2 p_s - h p_r = 1 \\
  -k_2 \frac{\pi^2}{2} \frac{\delta Lr}{\mu_0 \tau} B_{Sk2} B_{Rh} \cos[h p_s \Delta \theta \mp \varphi_2(t)] & \text{if } k_2 p_s - h p_r = -1
\end{cases}
\]  

(4.125)

Let us calculate the terms of the typology $g$, $h$, corresponding to equations (4.97), (4.98):

\[
F_{yi}^{(g,h)} = -Lr \int_{0}^{\pi} \frac{h \delta \pi}{\mu_0 \tau} B_{Sk} B_{Rh} \cos[k p_s \theta_s \mp \varphi(t)] sin[h p_r (\theta_s - \Delta \theta)] sin \theta_s d \theta_s
\]  

(4.126)

By considering (4.127) and applying it to (4.126), it gives (4.128):

\[
cos(\alpha) sin(\beta) = \frac{sin(\alpha + \beta) - sin(\alpha - \beta)}{2}
\]  

(4.127)

\[
F_{yi}^{(g,h)} = \frac{1}{2} \frac{\delta \pi}{\mu_0 \tau} h B_{Sk} B_{Rh} Lr \left\{ \int_{0}^{\pi} \sin[(k p_s + h p_r) \theta_s \mp \varphi - h p_r \Delta \theta] \sin \theta_s d \theta_s + \right.
\]
\[
\left. - \int_{0}^{\pi} \sin[(k p_s - h p_r) \theta_s \mp \varphi + h p_r \Delta \theta] \sin \theta_s d \theta_s \right\}
\]  

(4.128)
By using (4.105) in (4.128):

\[ F_{yi} = \frac{1}{4 \mu_0} hB_{Sk}B_{Rh}Lr \left\{ \begin{array}{c}
\int_{0}^{2\pi} \cos [(kp_s + hp_r - 1)\theta_s + \varphi - hp_r \Delta \theta] d\theta_s + \\
- \int_{0}^{2\pi} \cos [(kp_s + hp_r - 1)\theta_s + \varphi + hp_r \Delta \theta] d\theta_s + \\
+ \int_{0}^{2\pi} \cos [(kp_s - hp_r + 1)\theta_s + \varphi + hp_r \Delta \theta] d\theta_s \\
\end{array} \right\} (4.129) \]

As done in the previous, the hypothesis (4.113), (4.114) are to be considered not valid, so suppose that:

\[ kp_s - hp_r - 1 = 0 \quad \Rightarrow \quad kp_s - hp_r = 1 \quad (4.130) \]

By using (4.130) in (4.129), it gives:

\[ F_{yi} = \frac{1}{4 \mu_0} hB_{Sk}B_{Rh}Lr \left\{ \begin{array}{c}
\int_{0}^{2\pi} \cos [2(kp_s - 1)\theta_s + \varphi - hp_r \Delta \theta] d\theta_s + \\
- \int_{0}^{2\pi} \cos (2kp_s \theta_s + \varphi - hp_r \Delta \theta) d\theta_s - \int_{0}^{2\pi} \cos (\varphi + hp_r \Delta \theta) d\theta_s + \\
+ \int_{0}^{2\pi} \cos (2\theta_s + \varphi + hp_r \Delta \theta) d\theta_s \\
\end{array} \right\} (4.131) \]

By executing the integrals in (4.131):

\[ F_{yi} = \frac{1}{4 \mu_0} hB_{Sk}B_{Rh}Lr \left\{ \begin{array}{c}
\frac{1}{2(kp_s - 1) \left[ \sin (2kp_s - 1)\theta_s + \varphi - hp_r \Delta \theta \right]^{2\pi}} + \\
- \frac{1}{2kp_s} \left[ \sin (2kp_s \theta_s + \varphi - hp_r \Delta \theta) \right]^{2\pi} - 2\pi \cos (\varphi + hp_r \Delta \theta) + \\
+ \frac{1}{2} \left[ \sin (2\theta_s + \varphi + hp_r \Delta \theta) \right]^{2\pi} \\
\end{array} \right\} (4.132) \]

The third term of (4.132) is the only contribution to the force.
Chapter 4

\[ F_{y}^{(g,h)} = -h \frac{\pi^2}{2} \frac{\delta Lr}{\mu_0 \tau} B_{Sk} B_{Rh} \cos [h_{p_r} \Delta \theta \mp \varphi(t)] \quad \text{if} \quad k_{p_s} - h_{p_r} = 1 \]  

(4.133)

Let us suppose that (4.134) is verified:

\[ k_{p_s} - h_{p_r} + 1 = 0 \quad \Rightarrow \quad k_{p_s} - h_{p_r} = -1 \]  

(4.134)

By using (4.134) in (4.129), it gives:

\[
F_{y}^{(g,h)} = \frac{1}{4} \frac{\delta \pi}{\mu_0 \tau} h B_{Sk} B_{Rh} Lr \left\{ \int_{0}^{2\pi} \cos (2k_{p_s} \theta_s \mp \varphi - h_{p_r} \Delta \theta) d\theta_s + 
- \int_{0}^{2\pi} \cos (2(k_{p_s} + 1) \theta_s \mp \varphi - h_{p_r} \Delta \theta) d\theta_s - \int_{0}^{2\pi} \cos (-2\theta_s \mp \varphi + h_{p_r} \Delta \theta) d\theta_s + 
\right. 
+ \left. \int_{0}^{2\pi} \cos (\mp \varphi + h_{p_r} \Delta \theta) d\theta_s \right\} 
\]  

(4.135)

By executing the integrals in (4.135):

\[
F_{y}^{(g,h)} = \frac{1}{4} \frac{\delta \pi}{\mu_0 \tau} h B_{Sk} B_{Rh} Lr \left\{ \frac{1}{2k_{p_s}} \left[ \sin (2k_{p_s} \theta_s \mp \varphi - h_{p_r} \Delta \theta) \right]_{0}^{2\pi} + 
\right.
- \frac{1}{2(k_{p_s} + 1)} \left[ \sin (2(k_{p_s} + 1) \theta_s \mp \varphi - h_{p_r} \Delta \theta) \right]_{0}^{2\pi} + 
\left. + \frac{1}{2} \left[ \sin (-2\theta_s \mp \varphi + h_{p_r} \Delta \theta) \right]_{0}^{2\pi} + 2\pi \cos (\mp \varphi + h_{p_r} \Delta \theta) \right\} 
\]  

(4.136)

The only not null contribution to the force is the fourth term of (4.136):

\[
F_{y}^{(g,h)} = h \frac{\pi^2}{2} \frac{\delta Lr}{\mu_0 \tau} B_{Sk} B_{Rh} \cos [h_{p_r} \Delta \theta \mp \varphi(t)] \quad \text{if} \quad k_{p_s} - h_{p_r} = -1 
\]  

(4.137)

So, it can be concluded that the contribution of the terms of the typologies \( g \) and \( h \) to the \( y \)-component of the tangential force acting on the rotor is given as:

\[
F_{y}^{(g,h)} = \begin{cases} 
- h \frac{\pi^2}{2} \frac{\delta Lr}{\mu_0 \tau} B_{Sk} B_{Rh} \cos [h_{p_r} \Delta \theta \mp \varphi(t)] & \text{if} \quad k_{p_s} - h_{p_r} = 1 \\
 h \frac{\pi^2}{2} \frac{\delta Lr}{\mu_0 \tau} B_{Sk} B_{Rh} \cos [h_{p_r} \Delta \theta \mp \varphi(t)] & \text{if} \quad k_{p_s} - h_{p_r} = -1 
\end{cases} 
\]  

(4.138)
As in the previous, this contribution has to be considered in the interaction between everyone of the current space vector $s_{v1}$, $s_{v2}$, and the rotor magnetic field produced by the magnets (4.139), (4.140):

$$F_{y(t)}^{(g)} = \begin{cases} -h \frac{\pi^2}{2} \frac{\delta L_r}{\mu_0 \tau} B_{Sk1} B_{Rh} \cos[h p_{r,} \Delta \theta \mp \varphi_1(t)] & \text{if } k_1 p_s - h p_r = 1 \\ h \frac{\pi^2}{2} \frac{\delta L_r}{\mu_0 \tau} B_{Sk1} B_{Rh} \cos[h p_{r,} \Delta \theta \mp \varphi_1(t)] & \text{if } k_1 p_s - h p_r = -1 \end{cases}$$ \hspace{1cm} (4.139)

$$F_{y(t)}^{(b)} = \begin{cases} -h \frac{\pi^2}{2} \frac{\delta L_r}{\mu_0 \tau} B_{Sk2} B_{Rh} \cos[h p_{r,} \Delta \theta \mp \varphi_2(t)] & \text{if } k_2 p_s - h p_r = 1 \\ h \frac{\pi^2}{2} \frac{\delta L_r}{\mu_0 \tau} B_{Sk2} B_{Rh} \cos[h p_{r,} \Delta \theta \mp \varphi_2(t)] & \text{if } k_2 p_s - h p_r = -1 \end{cases}$$ \hspace{1cm} (4.140)

The last typologies to analyze are $b$ and $d$, related to the equations (4.92), (4.94):

$$F_{y(t)}^{(b)} = -L_r \int_0^{2\pi} \frac{k_1 \delta \pi}{\mu_0 \tau} B_{Sk1} B_{Sk2} \sin[k_1 p_s \theta_s \mp \varphi_1] \cos[k_2 p_s \theta_s \mp \varphi_2] \sin \theta_s d\theta_s$$ \hspace{1cm} (4.141)

By applying (4.110) to (4.141), it gives (4.142):

$$F_{y(t)}^{(b)} = \frac{1}{2} \frac{\delta \pi}{\mu_0 \tau} k_1 B_{Sk1} B_{Sk2} L_r \left\{ \int_0^{2\pi} \sin[(k_1 p_s + k_2 p_s) \theta_s \mp \varphi_1 \mp \varphi_2] \sin \theta_s d\theta_s + \right. \left. \int_0^{2\pi} \sin[(k_1 p_s - k_2 p_s) \theta_s \mp \varphi_1 \pm \varphi_2] \sin \theta_s d\theta_s \right\}$$ \hspace{1cm} (4.142)

By using (4.105) in (4.142):

$$F_{y(t)}^{(b)} = \frac{1}{4} \frac{\delta \pi}{\mu_0 \tau} k_1 B_{Sk1} B_{Sk2} L_r \left\{ \int_0^{2\pi} \cos[(k_1 p_s + k_2 p_s - 1) \theta_s \mp \varphi_1 \mp \varphi_2] d\theta_s + \right. \left. \int_0^{2\pi} \cos[(k_1 p_s - k_2 p_s - 1) \theta_s \mp \varphi_1 \pm \varphi_2] d\theta_s + \right. \left. \int_0^{2\pi} \cos[(k_1 p_s - k_2 p_s + 1) \theta_s \mp \varphi_1 \pm \varphi_2] d\theta_s + \right. \left. \int_0^{2\pi} \cos[(k_1 p_s + k_2 p_s + 1) \theta_s \mp \varphi_1 \mp \varphi_2] d\theta_s \right\}$$ \hspace{1cm} (4.143)
As done in the previous, the assumptions (4.144), (4.145) are to be considered not valid:

\[ k_1p_s + k_2p_s - 1 = 0 \quad \Rightarrow \quad k_1p_s + k_2p_s = 1 \]  
(4.144)

\[ k_1p_s + k_2p_s + 1 = 0 \quad \Rightarrow \quad k_1p_s + k_2p_s = -1 \]  
(4.145)

So, let us to examine the hypothesis (4.146):

\[ k_1p_s - k_2p_s - 1 = 0 \quad \Rightarrow \quad k_1p_s - k_2p_s = 1 \]  
(4.146)

By using (4.146) in (4.143), it gives:

\[
F_{y_l}^{(b)} = \frac{1}{4} \frac{\delta \pi}{\mu_0 \tau} k_1 B_{Sk1} B_{Sk2} L r \left\{ \int_0^{2\pi} \cos [2(k_1 p_s - 1)\theta_s \mp \phi_1 \mp \phi_2] d\theta_s + \right.
\]

\[ - \int_0^{2\pi} \cos [2k_1 p_s \theta_s \mp \phi_1 \mp \phi_2] d\theta_s + \int_0^{2\pi} \cos [2\phi_1 \pm \phi_2] d\theta_s - \int_0^{2\pi} \cos [2\theta_s \mp \phi_1 \pm \phi_2] d\theta_s \right\}
\]

(4.147)

By executing the integrals in (4.147):

\[
F_{y_l}^{(b)} = \frac{1}{4} \frac{\delta \pi}{\mu_0 \tau} k_1 B_{Sk1} B_{Sk2} L r \left\{ \frac{1}{2(2k_1 p_s - 1)} [\sin [2(k_1 p_s - 1)\theta_s \mp \phi_1 \mp \phi_2]]^{2\pi} + \right.
\]

\[ - \frac{1}{2k_1 p_s} [\sin (2k_1 p_s \theta_s \mp \phi_1 \mp \phi_2)]^{2\pi}_0 + 2\pi \cos (\mp \phi_1 \pm \phi_2) + \]

\[ - \frac{1}{2} [\sin (2\theta_s \mp \phi_1 \pm \phi_2)]^{2\pi}_0 \right\}
\]

(4.148)

The third term of (4.148) is the only not null contribution to the force:

\[
F_{y_l}^{(b)} = k_1 \frac{\pi^2 \delta L r}{2 \mu_0 \tau} B_{Sk1} B_{Sk2} \cos [\mp \phi_1 (t) \pm \phi_2 (t)] \quad \text{if} \quad k_1 p_s - k_2 p_s = 1
\]

(4.149)

Let us suppose that (4.150) is verified:

\[ k_1p_s - k_2p_s + 1 = 0 \quad \Rightarrow \quad k_1p_s - k_2p_s = -1 \]  
(4.150)

By using (4.150) in (4.143), it gives:
An analytical method for calculating the distribution of forces in a bearingless multiphase surface-mounted PM synchronous machine

\[
F_{yl}^{(b)} = \frac{1}{4} \frac{\delta \pi}{\mu_0 \tau} k_1 B_{sk1} B_{sk2} L_r \left\{ \int_0^{2\pi} \cos \left[ 2k_1 p_s \theta_s + \varphi_1 \mp \varphi_2 \right] d\theta_s + \right.
\]
\[
- \int_0^{2\pi} \cos \left[ 2(k_1 p_s + 1) \theta_s + \varphi_1 \mp \varphi_2 \right] d\theta_s + \left. \int_0^{2\pi} \cos \left[ -2 \theta_s + \varphi_1 \pm \varphi_2 \right] d\theta_s \right\}
\]

By executing the integrals in (4.151):

\[
F_{yl}^{(b)} = \frac{1}{4} \frac{\delta \pi}{\mu_0 \tau} k_1 B_{sk1} B_{sk2} L_r \left\{ \frac{1}{2k_1 p_s} \left[ \sin \left[ 2k_1 p_s \theta_s + \varphi_1 \mp \varphi_2 \right] \right]^{2\pi} + \right.
\]
\[
- \frac{1}{2(k_1 p_s + 1)} \left[ \sin \left[ 2(k_1 p_s + 1) \theta_s + \varphi_1 \mp \varphi_2 \right] \right]^{2\pi} - \left. \frac{1}{2} \left[ \sin \left[ -2 \theta_s + \varphi_1 \pm \varphi_2 \right] \right]^{2\pi} \right\} - 2\pi \cos (\mp \varphi_1 \pm \varphi_2)
\]

The only not null contribution to the force is the fourth term of (4.152):

\[
F_{yl}^{(b)} = -k_1 \frac{\pi^2}{2} \frac{\delta Lr}{\mu_0 \tau} B_{sk1} B_{sk2} \cos \left[ \mp \varphi_1(t) \pm \varphi_2(t) \right] \quad \text{if} \quad k_1 p_s - k_2 p_s = -1 \quad (4.153)
\]

So, it can be concluded that the contribution of the term of the typology \( b \) to the \( y \)-component of the tangential force acting on the rotor is given as:

\[
F_{yl}^{(b)} = \begin{cases} 
 k_1 \frac{\pi^2}{2} \frac{\delta Lr}{\mu_0 \tau} B_{sk1} B_{sk2} \cos \left[ \mp \varphi_1(t) \pm \varphi_2(t) \right] & \text{if} \quad k_1 p_s - k_2 p_s = 1 \\
 -k_1 \frac{\pi^2}{2} \frac{\delta Lr}{\mu_0 \tau} B_{sk1} B_{sk2} \cos \left[ \mp \varphi_1(t) \pm \varphi_2(t) \right] & \text{if} \quad k_1 p_s - k_2 p_s = -1
\end{cases} \quad (4.154)
\]

Without the need to execute the whole calculation, the contribution of the term of the typology \( d \) can be determined by analogy with respect to (4.154):

\[
F_{yl}^{(d)} = \begin{cases} 
 -k_2 \frac{\pi^2}{2} \frac{\delta Lr}{\mu_0 \tau} B_{sk1} B_{sk2} \cos \left[ \pm \varphi_1(t) \mp \varphi_2(t) \right] & \text{if} \quad k_1 p_s - k_2 p_s = 1 \\
 k_2 \frac{\pi^2}{2} \frac{\delta Lr}{\mu_0 \tau} B_{sk1} B_{sk2} \cos \left[ \pm \varphi_1(t) \mp \varphi_2(t) \right] & \text{if} \quad k_1 p_s - k_2 p_s = -1
\end{cases} \quad (4.155)
\]
4.4.3 Projections of the tangential force

The components related to the $x$-projection were calculated in a very similar way to that shown in the case of $y$-projection; only the final results being reported here. In the following are listed all the contributions to the $y$-projection (4.156)-(4.161) and $x$-projection (4.162)-(4.167) of the tangential component of the force. Note that the equations (4.160), (4.161) and (4.166), (4.167) related to the interactions between different harmonic orders of the same current space vector $s_{\alpha\beta}$, were determined by analogy. The resultant tangential force is given by adding all the terms, respectively along the $x$ and $y$ axis.

\[
F_{yt}^{(k_1,k_2)} = \begin{cases} 
\sum_{k_1} k_1^2 \frac{\delta L r}{2} B_{Sk_1} B_{Sk_2} \cos[\mp \varphi_1 \pm \varphi_2] & \forall k_2 = \frac{-1 + k_1 p_s}{p_s} \in N \\
\sum_{k_1} -k_1^2 \frac{\delta L r}{2} B_{Sk_1} B_{Sk_2} \cos[\mp \varphi_1 \pm \varphi_2] & \forall k_2 = \frac{1 + k_1 p_s}{p_s} \in N 
\end{cases}
\]

with: $\frac{k_1 \mp s_{\alpha\beta}}{m} \in N_0 \land \frac{k_2 \mp s_{\alpha\beta}}{m} \in N_0$

(4.156)

\[
F_{xt}^{(k_2,k_1)} = \begin{cases} 
\sum_{k_1} -k_2^2 \frac{\delta L r}{2} B_{Sk_1} B_{Sk_2} \cos[\pm \varphi_1 \mp \varphi_2] & \forall k_2 = \frac{-1 + k_1 p_s}{p_s} \in N \\
\sum_{k_1} k_2^2 \frac{\delta L r}{2} B_{Sk_1} B_{Sk_2} \cos[\pm \varphi_1 \mp \varphi_2] & \forall k_2 = \frac{1 + k_1 p_s}{p_s} \in N 
\end{cases}
\]

with: $\frac{k_1 \mp s_{\alpha\beta}}{m} \in N_0 \land \frac{k_2 \mp s_{\alpha\beta}}{m} \in N_0$

(4.157)
An analytical method for calculating the distribution of forces in a bearingless multiphase surface-mounted pm synchronous machine

\[
F_{y1}^{(k_1,h)} = \begin{cases} 
\sum_{k_1} (k_1 - h) \frac{\pi^2}{2} \frac{\delta L r}{\mu_0 \tau} B_{S_{k1}} B_{R_{h}} \cos [h p_r, \Delta \theta \mp \varphi_1] & \forall h = \frac{-1 + k_1 p_s}{p_r} \in N_{\text{odd}} \\
\sum_{k_1} (h - k_1) \frac{\pi^2}{2} \frac{\delta L r}{\mu_0 \tau} B_{S_{k1}} B_{R_{h}} \cos [h p_r, \Delta \theta \mp \varphi_1] & \forall h = \frac{1 + k_1 p_s}{p_r} \in N_{\text{odd}} 
\end{cases}
\]

with: \( \frac{k_1 \mp s_{v1}}{m} \in N_0 \land h \in N_{\text{odd}} \)

(4.158)

\[
F_{y1}^{(k_2,h)} = \begin{cases} 
\sum_{k_2} (k_2 - h) \frac{\pi^2}{2} \frac{\delta L r}{\mu_0 \tau} B_{S_{k2}} B_{R_{h}} \cos [h p_r, \Delta \theta \mp \varphi_2] & \forall h = \frac{-1 + k_2 p_s}{p_r} \in N_{\text{odd}} \\
\sum_{k_2} (h - k_2) \frac{\pi^2}{2} \frac{\delta L r}{\mu_0 \tau} B_{S_{k2}} B_{R_{h}} \cos [h p_r, \Delta \theta \mp \varphi_2] & \forall h = \frac{1 + k_2 p_s}{p_r} \in N_{\text{odd}} 
\end{cases}
\]

with: \( \frac{k_2 \mp s_{v2}}{m} \in N_0 \land h \in N_{\text{odd}} \)

(4.159)

\[
F_{y1}^{(k_2,k_2')} = \begin{cases} 
\sum_{k_2} k_2 \frac{\pi^2}{2} \frac{\delta L r}{\mu_0 \tau} B_{S_{k2}} B_{S_{k2'}} \cos [\mp \varphi_2 \pm \varphi_2'] & \forall k_2' = \frac{-1 + k_2 p_s}{p_s} \in N \\
\sum_{k_2} -k_2 \frac{\pi^2}{2} \frac{\delta L r}{\mu_0 \tau} B_{S_{k2}} B_{S_{k2'}} \cos [\mp \varphi_2 \pm \varphi_2'] & \forall k_2' = \frac{1 + k_2 p_s}{p_s} \in N 
\end{cases}
\]

with: \( \frac{k_2 \mp s_{v2}}{m} \in N_0 \land \frac{k_2' \mp s_{v2}}{m} \in N_0 \)

(4.160)
\[
F_{y^l}(k_2', k_2) = \begin{cases} 
\sum_{k_2} -k'_2 \frac{\varpi^2}{2} \frac{\delta L_r}{\mu_0} B_{sk_2} B_{sk_2'} \cos[\pm \varphi_2 \mp \varphi_2'] & \forall k'_2 = -\frac{1+k_2 p_s}{p_s} \in N \\
\sum_{k_2} k'_2 \frac{\varpi^2}{2} \frac{\delta L_r}{\mu_0} B_{sk_2} B_{sk_2'} \cos[\pm \varphi_2 \mp \varphi_2'] & \forall k'_2 = \frac{1+k_2 p_s}{p_s} \in N
\end{cases}
\]

with: \[ \frac{k_2 \mp s_{v2}}{m} \in N_0 \land \frac{k'_2 \mp s_{v2}}{m} \in N_0 \]

\[ (4.161) \]

\[
F_{x^l}(k_2, k_2) = \sum_{k_2} -k_1 \frac{\varpi^2}{2} \frac{\delta L_r}{\mu_0} B_{sk_1} B_{sk_2} \sin[\mp \varphi_1 \pm \varphi_2] \begin{cases} 
\forall k_2 = -\frac{1+k_1 p_s}{p_s} \in N \\
\forall k_2 = \frac{1+k_1 p_s}{p_s} \in N
\end{cases}
\]

with: \[ \frac{k_1 \mp s_{v1}}{m} \in N_0 \land \frac{k_2 \mp s_{v2}}{m} \in N_0 \]

\[ (4.162) \]

\[
F_{x^l}(k_2, k_1) = \sum_{k_1} -k_2 \frac{\varpi^2}{2} \frac{\delta L_r}{\mu_0} B_{sk_1} B_{sk_2} \sin[\pm \varphi_1 \mp \varphi_2] \begin{cases} 
\forall k_2 = -\frac{1+k_1 p_s}{p_s} \in N \\
\forall k_2 = \frac{1+k_1 p_s}{p_s} \in N
\end{cases}
\]

with: \[ \frac{k_1 \mp s_{v1}}{m} \in N_0 \land \frac{k_2 \mp s_{v2}}{m} \in N_0 \]

\[ (4.163) \]
An analytical method for calculating the distribution of forces in a bearingless multiphase surface-mounted pm synchronous machine

\[ F_{x(k_1 h)} = \sum_{k_1} (h-k_1) \frac{\pi^2}{2} \frac{\delta L_r}{\mu_0 \tau} B_{sk1} B_{rh} \sin[h p_r \Delta \theta \mp \varphi_1] \]

\( \forall h = \frac{-1 + k_1 p_s}{p_r} \in N_{odd} \)

\( \forall h = \frac{1 + k_1 p_s}{p_r} \in N_{odd} \)

with: \( \frac{k_1 \mp s_{v1}}{m} \in N_0 \land h \in N_{odd} \)

(4.164)

\[ F_{x(k_2 h)} = \sum_{k_2} (h-k_2) \frac{\pi^2}{2} \frac{\delta L_r}{\mu_0 \tau} B_{sk2} B_{rh} \sin[h p_r \Delta \theta \mp \varphi_2] \]

\( \forall h = \frac{-1 + k_2 p_s}{p_r} \in N_{odd} \)

\( \forall h = \frac{1 + k_2 p_s}{p_r} \in N_{odd} \)

with: \( \frac{k_2 \mp s_{v2}}{m} \in N_0 \land h \in N_{odd} \)

(4.165)

\[ F_{x(k_2' k_2')} = \sum_{k_2} -k_2 \frac{\pi^2}{2} \frac{\delta L_r}{\mu_0 \tau} B_{sk2} B_{sk2'} \sin[\varphi_2 \pm \varphi_2'] \]

\( \forall k_2' = \frac{-1 + k_2 p_s}{p_s} \in N \)

\( \forall k_2' = \frac{1 + k_2 p_s}{p_s} \in N \)

with: \( \frac{k_2 \mp s_{v2}}{m} \in N_0 \land \frac{k_2' \mp s_{v2}}{m} \in N_0 \)

(4.166)

\[ F_{x(k_2' k_2)} = \sum_{k_2} -k_2 \frac{\pi^2}{2} \frac{\delta L_r}{\mu_0 \tau} B_{sk2} B_{sk2'} \sin[\pm \varphi_2 \mp \varphi_2'] \]

\( \forall k_2' = \frac{-1 + k_2 p_s}{p_s} \in N \)

\( \forall k_2' = \frac{1 + k_2 p_s}{p_s} \in N \)

with: \( \frac{k_2 \mp s_{v2}}{m} \in N_0 \land \frac{k_2' \mp s_{v2}}{m} \in N_0 \)

(4.167)
The same considerations done for the equation (4.53), related to the meaning of $k_2$ and $k'_2$, are valid for equations (4.160), (4.161), (4.166), (4.167).
4.5 Simulations and results

An electrical machine suitable for bearingless application (Tab. II) was analyzed in order to validate the relationships presented in the previous chapters, making a comparison with the results of the 2D FEA software “FEMM 4.2” [14].

<table>
<thead>
<tr>
<th>Param.</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{sl}$</td>
<td>number of slots</td>
<td>30</td>
</tr>
<tr>
<td>$p$</td>
<td>pole pairs of the machine</td>
<td>1</td>
</tr>
<tr>
<td>$m$</td>
<td>number of phases</td>
<td>5</td>
</tr>
<tr>
<td>$I_a$</td>
<td>rated phase current ($A_{rms}$)</td>
<td>64.68</td>
</tr>
<tr>
<td>$T_n$</td>
<td>rated torque (Nm)</td>
<td>30.29</td>
</tr>
<tr>
<td>$g$</td>
<td>airgap width (mm)</td>
<td>1</td>
</tr>
<tr>
<td>$D_e$</td>
<td>stator outer diameter (mm)</td>
<td>230</td>
</tr>
<tr>
<td>$D_s$</td>
<td>stator inner diameter (mm)</td>
<td>120</td>
</tr>
<tr>
<td>$D_m$</td>
<td>mean diameter of the magnet (mm)</td>
<td>116</td>
</tr>
<tr>
<td>$D_{cv, ext}$</td>
<td>diameter at the bottom of the slot (mm)</td>
<td>170</td>
</tr>
<tr>
<td>$D_{cv, int}$</td>
<td>diameter at the top of the slot (mm)</td>
<td>126.3</td>
</tr>
<tr>
<td>$D_r$</td>
<td>rotor outer diameter (mm)</td>
<td>114</td>
</tr>
<tr>
<td>$D_{rh}$</td>
<td>rotor inner diameter (mm)</td>
<td>60</td>
</tr>
<tr>
<td>$\alpha_{ldg}$</td>
<td>angle underlying the tooth surface</td>
<td>10.1°</td>
</tr>
<tr>
<td>$\alpha_{sap}$</td>
<td>semi-angle underlying the slot opening</td>
<td>0.95°</td>
</tr>
<tr>
<td>$\alpha_{m}$</td>
<td>angle underlying the magnet</td>
<td>172°</td>
</tr>
<tr>
<td>$\alpha_{cv}$</td>
<td>slot pitch angle</td>
<td>12°</td>
</tr>
<tr>
<td>$d_{st}$</td>
<td>stator slot height (mm)</td>
<td>25</td>
</tr>
<tr>
<td>$h_{cl}$</td>
<td>slot opening height (mm)</td>
<td>1</td>
</tr>
<tr>
<td>$L$</td>
<td>axial length of the machine (mm)</td>
<td>180</td>
</tr>
<tr>
<td>$L_m$</td>
<td>magnet width (mm)</td>
<td>2</td>
</tr>
<tr>
<td>$L_{dt}$</td>
<td>tooth-body width (mm)</td>
<td>8</td>
</tr>
<tr>
<td>$L_{cl}$</td>
<td>slot opening width (mm)</td>
<td>2</td>
</tr>
<tr>
<td>$L_{tc}$</td>
<td>slot width at the top slot radius (mm)</td>
<td>5.23</td>
</tr>
<tr>
<td>$L_{fc}$</td>
<td>slot width at the bottom slot radius (mm)</td>
<td>9.7</td>
</tr>
<tr>
<td>$\tau_{cv}$</td>
<td>slot pitch at the inner stator radius (mm)</td>
<td>12.57</td>
</tr>
</tbody>
</table>
The analysis was carried out considering only the normal components of the force, after having verified that the contributions of tangential components are negligible, by applying phase currents given by the sum of the two space vectors $s_{v1} = 1$ and $s_{v2} = 2$. As a general criterion, the torque current space vector is maintained in leading by 90 electrical degrees with respect to the rotor position in order to reach the maximum torque per ampere behavior of the motor. In the following, the force is calculated by varying the phase angle $\phi_2$ related to the levitation current space vector. From (4.52), (4.82) it is possible to determine the relationship between the direction of the force $F_{Fr}$, measured with respect to $y$-axis and positive when clockwise-oriented, the phase $\phi_2$ and the angular position of the rotor $\Delta \theta$, obtaining:

\[
\phi_{Fr}^{(k_2,h)} = \arctan\left(\frac{F_{xn}^{(k_2,h)}}{F_{yn}^{(k_2,h)}}\right) = hp_r\Delta \theta \mp \phi_2, \quad \forall \ h = \frac{1+k_2p_s}{p_r} \in N_{odd} \quad (4.168)
\]

\[
\phi_{Fr}^{(k_2,h)} = \arctan\left(\frac{F_{xn}^{(k_2,h)}}{F_{yn}^{(k_2,h)}}\right) = -(hp_r\Delta \theta \mp \phi_2), \quad \forall \ h = -\frac{1+k_2p_s}{p_r} \in N_{odd} \quad (4.169)
\]

Considering the effects of the second spatial harmonic component of the stator magneto-motive force ($k_2 = 2$) which interacts with the first spatial harmonic component of the PM ($h = 1$), (4.169) allows to calculate the direction of the force $F_{Fr}$ by means of the following relationship:

\[
\phi_{Fr}^{(2,1)} = \phi_2 - \Delta \theta \quad (4.170)
\]

In the equation is considered only the interaction between the main harmonic orders of the levitation current space vector and the rotor magnetic field: this is the usual approach when designing a control system for bearingless machines, following the relationship (4.170). In multiphase machines this can produce mistakes in determining both the module and the spatial phase of the radial force,
due to the interactions between the higher harmonic orders. The relationships (4.50)-(4.53) and (4.80)-(4.83), taking into account all the possible interactions, allows to calculate these errors in terms of differences, in module and phase of the radial forces, between the simplified prediction (4.170) and the actual function, representing the locus of radial force vector and allowing the appropriate corrections.

Figs. 4.2-4.3 show a comparison between the simplified equation “main harmonic orders” (4.170) and the actual radial force vector determined by using both the “proposed method”, both the FEA software. The analysis was conducted by varying the phase angle of levitation current $\varphi_2$ by multiple values of 22.5 electrical degrees. Particularly, in the Figs. 4.2-4.3 are shown the $x$- and $y$-components of the calculated radial force and the good agreement with the FE software results, represented by the red dots.

**Fig. 4.2.** $y$-component vs $x$-component of the radial force:

$i_{sv1} = 0 \, A_{min}, \, i_{sv2} = 45.74 \, A_{min}, \, \Delta \theta = 0 \, \text{mech. degrees}, \, n_{ac} = 10 \,(/15)$
In this case, the magnet axis is aligned to the phase “1” axis, consequently $\Delta \theta = 0$ and the phase $\phi_1$ of the torque current space vector is 90 electrical degrees. By comparing Fig. 4.2 to Fig. 4.3, the value of $\bar{i}_{sv1}$ was changed from 0 to the rated value of 45.74 A$_{rms}$. As it is possible to see, the effect of the torque current space vector $s_{sv1}$ determines a counterclockwise rotation of the locus that describes the position of the resultant radial force vector. In fact, for a null value of the torque current space vector, the actual direction of the radial force is practically coinciding with that of the phase angle $\phi_2$ related to the space vector $s_{sv2}$ given by (4.170), as shown in Fig. 4.2; the influence of $\bar{i}_{sv1}$ determines a significant change with respect to the actual value of $\phi_2$ and the predicted one. The pitch winding is another important factor which influences the locus of radial force vector, that is represented by a nearly ellipsoidal shape only in the particular case of a shortened pitch winding, able to eliminate the third harmonic order.
An analytical method for calculating the distribution of forces in a bearingless multiphase surface-mounted pm synchronous machine

(corresponding to a $n_{dc}$ value of 10).

Fig. 4.4. $y$-component vs $x$-component of the radial force: $i_{sv1} = 45.74$ Arms, $i_{sv2} = 45.74$ Arms, $\Delta \theta = 0$ mech. degrees, $n_{dc} = 9 (/15)$

In general, by changing this value, the locus seems to be warped with respect to this ideal shape, as can be seen in Fig. 4.4 for a $n_{dc}$ value of 9. In the Figs. 4.5-4.6, modulus and phase of the radial force are presented in terms of differences with respect to the values of the “main harmonic orders” locus: in this way, they give the corrections to be made in order to obtain the actual radial force vector.
Fig. 4.5. Modulus of the difference of the radial force:
\[ i_{sv1} = 45.74 \, A_{ms}, \quad i_{sv2} = 45.74 \, A_{ms}, \quad \Delta \theta = 0 \, \text{mech. Degrees} \]

Fig. 4.6. Phase difference of the radial force:
\[ i_{sv1} = 45.74 \, A_{ms}, \quad i_{sv2} = 45.74 \, A_{ms}, \quad \Delta \theta = 0 \, \text{mech. Degrees} \]
The influence of the rotor position, for a fixed space vector $\vec{i}_{sv1}$, represents another analysis tool, shown in Figs. 4.7-4.8. In Fig. 4.7 the rms torque current is null, while in Fig. 4.8 is equal to the rated value of 45.74 $A_{\text{rms}}$. In every picture the magnet axis is rotated in three different positions with respect to phase “1” axis: 0, 45 and 90 mechanical degrees. The analysis shows a clockwise rotation of the locus by the same mechanical angle than the magnet axis, but is important to note that the position of the corresponding points, characterized by a same value of the phase $\varphi_2$, changes by the same amount but in a counterclockwise rotation.

Fig. 4.7. y-component vs x-component of the radial force:

$i_{sv1} = 0 \ A_{\text{rms}}, \ i_{sv2} = 45.74 \ A_{\text{rms}}, \ \Delta \theta = 0, 45, 90 \ \text{mech. degrees}, \ n_{\text{dc}} = 10 \ (/15)$
In the next will be shown an in-depth analysis, conducted with the presented algorithm by varying the main parameters in a large number of possible combinations, to highlight the influence of the magnet pitch, the rotor position and the coil pitch ($\alpha_m, \Delta \theta, n_{ac}$); the rms values of the modulus of current space vectors $i_{sv1}, i_{sv2}$ are fixed to their maximum value. The results are presented in terms of modulus and phase differences, showing in this way the corrections which can be applied to the simplified function “main harmonic orders” to obtain the absolute values of the modulus and difference of the force. All the quantities are determined as functions of the levitation current space vector phase angle $\varphi_2$. 

Fig. 4.8. $y$-component vs $x$-component of the radial force:

\[ i_{sv1} = 45.74 \, A_{rms}, \ i_{sv2} = 45.74 \, A_{rms}, \ \Delta \theta = 0, 45, 90 \, \text{mech. degrees}, \ n_{ac} = 10 \, (/15) \]
An analytical method for calculating the distribution of forces in a bearingless multiphase surface-mounted pm synchronous machine

\[ \alpha_m = 172^\circ_{\text{mech}}, i_{sv1} = 45.74 \; A_{\text{rms}}, i_{sv2} = 45.74 \; A_{\text{rms}}, \Delta \theta = 0^\circ_{\text{mech}}, n_{ac} \text{ varies} \]

**Fig. 4.9.**

**Fig. 4.10.**
\begin{align*}
\alpha_m &= 172^\circ_{\text{mech}}, \quad i_{sv1} = 45.74 \, A_{\text{rms}}, \quad i_{sv2} = 45.74 \, A_{\text{rms}}, \quad \Delta \theta = 45^\circ_{\text{mech}}, \quad n_{\text{ac}} \text{ varies}
\end{align*}

Fig. 4.11.

Fig. 4.12.
An analytical method for calculating the distribution of forces in a bearingless multiphase surface-mounted pm synchronous machine

\[ \alpha_m = 120^\circ_{\text{mech}}, i_{sv1} = 45.74 \ A_{\text{rms}}, i_{sv2} = 45.74 \ A_{\text{rms}}, \Delta \Theta = 0^\circ_{\text{mech}}, n_{ac} \text{ varies} \]

Fig. 4.13.

\[ \phi_2 \ (\text{el. deg.}) \]

Fig. 4.14.
\[ \alpha_m = 90^{\circ mech}, i_{sv1} = 45.74 \, A_{rms}, i_{sv2} = 45.74 \, A_{rms}, \Delta \theta = 0^{\circ mech}, n_{ac} \text{ varies} \]

Fig. 4.15.

Fig. 4.16.
An analytical method for calculating the distribution of forces in a bearingless multiphase surface-mounted pm synchronous machine

\[ \alpha_m = 90^\circ_{\text{mech}}, i_{sv1} = 45.74 \ A_{\text{rms}}, i_{sv2} = 45.74 \ A_{\text{rms}}, \Delta \theta = 90^\circ_{\text{mech}}, n_{ac} \text{ varies} \]

Fig. 4.17.

Fig. 4.18.
4.6 Conclusion

In this chapter an analytical model for radial forces calculation in multiphase bearingless Surface-Mounted Permanent Magnet Synchronous Motors (SPMSM) is presented. It allows to predict amplitude and direction of the force, depending on the values of the torque current, of the levitation current and of the rotor position. It is based on the space vectors method, letting the analysis of the machine not only in steady-state conditions but also during transients.

When designing a control system for bearingless machines, it is usual to consider only the interaction between the main harmonic orders of the stator and rotor magnetic fields: in multiphase machines this can produce mistakes in determining both the module and the spatial phase of the radial force, due to the interactions between the higher harmonic orders. The presented algorithm allows to calculate these errors, taking into account all the possible interactions; by representing the locus of radial force vector, it allows the appropriate corrections.

In addition, the algorithm permits to study whatever configuration of SPMSM machine, being parameterized as a function of the electrical and geometrical quantities, as the coil pitch, the width and length of the magnets, the rotor position, the amplitude and phase of current space vector, etc.

Finally, the results of the proposed method have been compared with those of a most used FEA software, obtaining very similar values of the analyzed quantities.
4.7 References


Appendix A4.1

MAGNETIC FIELD DISTRIBUTION IN THE AIRGAP OF MULTIPHASE ELECTRICAL MACHINES

A4.1.1 Introduction

In recent years, more and more advanced technologies and an impressive rise in the use of electronics, both in civil as in the industrial sector, given a contribution to reduce the cost of the components, allowing the use of complex technologies which in the past had high costs and therefore of little industrial interest. In the field of electrical machines this evolution led not only to the realization of power drives controlled by an inverter, capable of ensuring performance significantly better than those obtained with the previous control systems, but also the advent of a new type of machines with a different number of phases from the traditional three-phase, usually employed in generation and distribution of electric energy. This has reawakened the interest in the study of multi-phase electrical machines.
The field of study of polyphase machines is relatively new and in rapid evolution, but it is already possible to say that these machines are able to provide better performances of the classical ones, and precisely for this reason, are currently a matter of great interest. Indeed, the multiphase machines have several advantages compared to the traditional three-phase machines, such as the amplitude reduction and the increase of the frequency of pulsating torque, the reduction of stator phase current, the increase of the fault tolerance. In addition, the multiphase machines offer a larger number of degrees of freedom with respect to the three-phase machines, which can be used to improve the performances.

Furthermore, the creation of software for analyzing the behavior of magnetic fields in electromechanical devices, based on numerical methods as the FEA, has greatly contributed to the improvement in the design of electrical machines by introducing, in addition to an excellent accuracy of the results, also a considerable saving of time and money.

**A4.1.2 The multiphase rotating magnetic field**

For an in-depth understanding on the topic of multi-phase machines, making a brief reference to the theory of the rotating magnetic field, in steady state conditions, to better understand the equations that will be proposed in the following sections.

By supposing to operate in linear regime, with an iron magnetic permeability of infinite value, it can be concluded that there are no appreciable drops of magneto-motive force (mmf) in the iron, considering in this region a null value for the magnetic field.

Thus the study is conducted only in the airgap, considering the hypothesis of the representation along a straight line, as in Fig. A4.1-1, the distribution of
the current density $\mathbf{J}(x, y, z)$ being only by means of $z$-component (A4.1-6); the current sources are located in the slots and considered as concentrated in a point. With reference to the Maxwell’s equations, develop the first of (A4.1-1):

\[
\begin{aligned}
\text{div}\vec{B} &= 0 \\
\text{rot}\vec{H} &= \vec{J}
\end{aligned}
\] (A4.1-1)

By executing the calculations, the first equation of (A4.1-1) gives three scalar components:

\[
\begin{aligned}
\frac{\partial H_y}{\partial y} - \frac{\partial H_z}{\partial z} &= J_x = 0 \\
\frac{\partial H_x}{\partial z} - \frac{\partial H_y}{\partial x} &= J_y = 0 \\
\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} &= J_z \\
\vec{J} &= 0\hat{i} + 0\hat{j} + J_z(x)\hat{k}
\end{aligned}
\] (A4.1-3) (A4.1-4) (A4.1-5) (A4.1-6)

Another very important hypothesis about the magnetic field distribution, gives
the only component in the $y$ direction, being $\vec{H} = H_y \hat{j}$: by applying this to
\begin{equation}
-\frac{\partial H_y}{\partial z} = 0
\end{equation}
\begin{equation}
0 = 0
\end{equation}
\begin{equation}
\frac{\partial H_y}{\partial x} = J_z
\end{equation}
From the first equation of (A4.1-7) it results that $H_y$ does not depend on $z$, varying only as a function of the $x$ coordinate.

In the Figs. A4.1-3 and A4.1-4 are shown the most known typologies of current density and the related magnetic field distributions, by following the relationship A4.1-8.

The presented analysis of magnetic field distribution is based on the following assumptions:

I) The permeability of iron is infinite;

II) The slots of the machine are semi-closed, having an infinitesimal slot opening width and height;
III) The magnetic field lines are radial and perpendicular to the rotor and stator boundary surfaces;
IV) The mean airgap radius of curvature is infinite, so that the airgap path can be considered as a straight line;
V) Extremity effects are neglected.
VI) The effects of the leakage fluxes are neglected.
The analysis will be conducted starting from the magnetic field distribution produced by a one slot-per-pole winding (Fig. A4.1-5). Considering, with no lack of generality, a 2-pole machine and by applying the second equation of (A4.1-1):

\[
\text{div} \vec{B} = 0 \quad \Rightarrow \quad \int_{S_c} \vec{B} \cdot n dS = B_1 \tau_p L + B_2 \tau_p L = 0 \\
\Rightarrow \quad \mu_0 H_1 \tau_p L + \mu_0 H_2 \tau_p L = 0 \quad \Rightarrow \quad H_1 = -H_2 (= H)
\]  

(A4.1-9)

Fig. A4.1-5. Magnetic field distribution (1 slot-per-pole winding)

By applying Ampere’s law on a path crossing the airgap which includes a group of \( n \) conductors, neglecting the \( x \)-components of \( \vec{H} \) and taking into account (A4.1-9) it gives:

\[
\int \vec{H} \cdot d\vec{l} = ni \quad \Rightarrow \quad H_1 \delta - H_2 \delta = ni \quad \Rightarrow \quad 2H\delta = ni 
\]  

(A4.1-10)

where it is understood that \( H \) stays for \( H_y \). After a simple step:

\[
H = \frac{ni}{2\delta}
\]  

(A4.1-11)

As shown in Fig. A4.1-5, the considered reference system has the origin in the middle of the coil. So, the spatial distribution of the magnetic field is a periodic function of \( \theta \) and can be developed in Fourier harmonic series:

\[
H_s(\theta) = \sum_{k=1,3,\ldots}^{\infty} H_k \cos(k\theta)
\]  

(A4.1-12)
where:

\[
H_k = \frac{1}{\pi} \int_{-\pi}^{\pi} H(\theta) \cos(k\theta) d\theta
\]  
(A4.1-13)

Having all the coils the same pitch, equal to \( \pi \) electrical radians, it can be easily prove that the Fourier series has only odd harmonic orders. Furthermore, it presents only cosine terms due to the choice of the reference system.

The calculation of (A4.1-13) gives:

\[
H_k = \frac{1}{\pi} \int_{-\pi}^{\pi} H(\theta) \cos(k\theta) d\theta = \frac{2}{\pi} \left[ \frac{ni}{2\delta} \cos(k\theta) d\theta + \int_{\pi/2}^{\pi/2} \frac{ni}{2\delta} \cos(k\theta) d\theta \right] =
\]

\[
\frac{2}{\pi} \left[ \frac{ni}{2\delta} \sin(k\theta) \right]_{0}^{\pi/2} - \frac{2}{\pi} \left[ \frac{ni}{2\delta} \sin(k\theta) \right]_{\pi/2}^{\pi} = \frac{4ni}{\pi} \frac{1}{2\delta} \sin \left( \frac{k\pi}{2} \right)
\]  
(A4.1-14)

To obtain:

\[
H_k = \frac{4ni}{\pi} \frac{1}{2\delta} \sin \left( \frac{k\pi}{2} \right)
\]  
(A4.1-15)

By substituting (A4.1-15) in (A4.1-12):

\[
H_s(\theta) = \sum_{k=1,3,\ldots}^{\infty} \frac{4ni}{\pi} \frac{1}{2\delta} \sin \left( \frac{k\pi}{2} \right) \cos(k\theta)
\]  
(A4.1-16)

By assumption the slot opening width and height are considered to be infinitesimal, without creating leakage fluxes; the coils of one phase are series-connected with the same current flowing in. The equation (A4.1-16) contains the following terms: \( \frac{4ni}{\pi} \frac{1}{2\delta} \) represents the amplitude of fundamental harmonic order, \( 1/k \sin(k\pi/2) \) is the harmonic factor whose value is \( \pm 1/k \), the dependence on the function \( \cos(k\theta) \) represents the spatial harmonic distribution.
The spatial distribution of the magnetic field is constituted by the superposition of sinusoidal distributions of decreasing amplitude with the harmonic order $1/k$. For each harmonic the maximum value of the field occurs at the center of the coil. The amplitude of each harmonic is proportional to the value of the current flowing in the coil.

Consider a magnetic field distribution generated by a $q$ slots-per-pole winding, as shown in Fig. A4.1-6:

![Fig. A4.1-6. Magnetic field distribution generated by a $q$ slots-per-pole winding.](image)

The field distribution is formed by the superposition of $q$ contributions, relatively displaced by an electrical angle $\alpha$ (the angle between two adjacent slots): the related representation, by means of the Fourier harmonic series, becomes:

$$
H^q(\theta) = \sum_{j=1}^{q} \sum_{k=1,3,\ldots}^{\infty} \frac{4\, n_i}{\pi} \frac{1}{2\delta} \frac{1}{k} \sin \left( \frac{\pi}{2} \right) \cos \left( k \left[ \theta - (j-1)\alpha \right] \right)
$$

(A4.1-17)

By varying the index $j$ from 1 to $q$ it is obtained a summation of cosine functions which can be calculated as:

$$
cos(k\theta) + cos(k[\theta - \alpha]) + cos(k[\theta - 2\alpha]) + \ldots + cos(k[\theta - (q-1)\alpha]) = \\
= \frac{\sin\left(\frac{kq\alpha}{2}\right)}{\sin\left(\frac{k\alpha}{2}\right)} \cos \left( k \left[ \theta - \frac{q-1}{2} \alpha \right] \right) = qK_{dk} \cos \left( k \left[ \theta - \frac{q-1}{2} \alpha \right] \right)
$$

(A4.1-18)
where the terms $K_{dk}$ is defined distribution factor of Blondel (A4.1-19):

$$K_{dk} = \frac{\sin \frac{kq\alpha}{2}}{q \sin \frac{k\alpha}{2}}$$  \hspace{1cm} (A4.1-19)

By adopting a suitable winding distribution in the slots, $K_{dk}$ allows to reduce, even remarkably, some harmonic orders. By substituting the relation (A4.1-18) in the expression (A4.1-17), it gives:

$$H^q(\theta) = \sum_{k=1,3,...}^{\infty} \frac{4 ni}{\pi 2\delta} qK_{dk} \frac{1}{k} \sin \left( \frac{k \pi}{2} \right) \cos \left( k \left[ \frac{\theta}{2} - \frac{q-1}{2} \frac{\alpha}{2} \right] \right)$$ \hspace{1cm} (A4.1-20)

Note that the diagram of the magnetic field distribution $H^q(\theta)$ moved in the original reference system with respect to (A4.1-16) due to the modification of the distribution, that now is related to $q$ slots-per-pole-per-phase: thus, is necessary to shift the reference system by an angle $(q-1)\alpha/2$, corresponding with the new peak value of the $H^q(\theta)$ distribution as the centre of the phase.

$$H^q(\theta) = \sum_{k=1,3,...}^{\infty} \frac{4 ni}{\pi 2\delta} qK_{dk} \frac{1}{k} \sin \left( \frac{k \pi}{2} \right) \cos(k\theta)$$  \hspace{1cm} (A4.1-21)

In the comparison between (A4.1-21) and (A4.1-16) the contribution of the $q$ slots-per-pole-per-phase can be recognized in the term $qK_{dk}$.

Fig. A4.1-7. A slot with double layer winding
Consider now the distribution of magnetic field produced by a winding of \( q \) slots-per-pole-per-phase in double-layer. It is shown in Fig. A4.1-7 the representation of a slot in double layer, while in Fig. A4.1-8 the representation of a double layer winding.

As can be seen, in a double layer winding it could happen to find, in the same slot, groups of conductors belonging to different coils. The magnetic field distributions produced by different layers of the same phase, can be seen as identical to (A4.1-21) but shifted by a \( \beta \) electrical angle. Also, is important to note that in (A4.1-22) the number of conductors-per-slot \( n \) is divided by 2 due to the presence of the layers. Thus, the resultant distribution can be written as:

\[
H^q(\theta) = \sum_{k=\frac{1}{2}, \frac{3}{2}, \ldots}^{\infty} \frac{4}{\pi} \frac{n}{2\delta} qK_{dk} \frac{1}{k} \sin\left(\frac{k\pi}{2}\right) \cos(k\theta) + \\
+ \sum_{k=\frac{1}{2}, \frac{3}{2}, \ldots}^{\infty} \frac{4}{\pi} \frac{n}{2\delta} qK_{dk} \frac{1}{k} \sin\left(\frac{k\pi}{2}\right) \cos(k[\theta - \beta])
\]  

(A4.1-22)

The even harmonic orders cancel each other for the two layers. Both the bottom layer than the top one, produces a field of \( q \) slots-per-pole: being a phase shift between them, it is necessary to add the two sinusoidal functions taking into account of it. The result, by applying again an angular shift to the reference system on the new center of the phase, is equivalent to multiplying by 2 and by the shortened pitch factor (A4.1-23):
Magnetic field distribution in the airgap of multiphase electrical machines

\[ K_{rk} = \cos \left( \frac{kB}{2} \right) \]  \hspace{1cm} (A4.1-23)

The relationship (A4.1-24) represents the magnetic field distribution produced by a double layer winding of one phase, having \( q \) slots-per-pole-per-phase:

\[ H(\theta) = \sum_{k=1,3,...}^{\infty} \frac{4n}{\pi} \frac{qK_{dk}K_{rk}}{2k} \sin \left( \frac{k\pi}{2} \right) \cos(k\theta) \cos \left( \frac{kB}{2} \right) \]  \hspace{1cm} (A4.1-24)

The product of the Blondel’s factor \( K_{dk} \) and the shortened pitch factor \( K_{rk} \) gives the winding factor \( K_{ak} \) related to \( k \)-th harmonic order:

\[ K_{ak} = K_{dk} K_{rk} = \frac{\sin \left( \frac{kq\alpha}{2} \right)}{q \sin \left( \frac{k\alpha}{2} \right)} \cos \left( \frac{kB}{2} \right) \]  \hspace{1cm} (A4.1-25)

Consider an instantaneous phase current \( i(t) \) which varies following a sinusoidal law (A4.1-26), being \( I \) the rms value and \( \omega \) the angular frequency:

\[ i(t) = \sqrt{2}I \cos(\omega t) \]  \hspace{1cm} (A4.1-26)

By substituting it in (A4.1-24) it gives:

\[ H(\theta,t) = \sum_{k=1,3,...}^{\infty} \frac{4\sqrt{2}nI}{\pi} \frac{qK_{ak}}{2k} \sin \left( \frac{k\pi}{2} \right) \cos(k\theta) \cos(\omega t) \]  \hspace{1cm} (A4.1-27)

The equation (A4.1-27) can be written as:

\[ H(\theta,t) = \sum_{k=1,3,...}^{\infty} H_{Mk} \cos(k\theta) \cos(\omega t) \]  \hspace{1cm} (A4.1-28)

where:

\[ H_{Mk} = \frac{4\sqrt{2}nI}{\pi} \frac{qK_{ak}}{2k} \sin \left( \frac{k\pi}{2} \right) \]  \hspace{1cm} (A4.1-29)

The distribution given by (A4.1-28) represents, for each value of \( k \), a stationary
wave with $kp$ pole pairs, shown in Fig. A4.1-9 in a succession of time instants.

Fig. A4.1-9. The stationary wave shown in different time instants

The equation (A4.1-28) can be seen also as the sum of two counter-rotating fields depending on a space-time variable $f(k\theta + \omega t)$, each one moving in or opposite to the $\theta$ axis direction without warping, as shown in Fig. A4.1-10 and in the relationship (A4.1-31).

Fig. A4.1-10. A translating wave

\[
H(\theta,t) = \sum_{k=1,3,\ldots}^\infty H_M \frac{1}{2} \left[ \cos(k\theta - \omega t) + \cos(k\theta + \omega t) \right] \quad (A4.1-30)
\]

\[
H(\theta,t) = \sum_{k=1,3,\ldots}^\infty \frac{1}{2} H_M \cos(k\theta - \omega t) + \sum_{k=1,3,\ldots}^\infty \frac{1}{2} H_M \cos(k\theta + \omega t) \quad (A4.1-31)
\]

Each distribution in (A4.1-31) represents, for each value of $k$, a rotating wave with $kp$ pole pairs: in particular, the first summation is a wave which rotates in
the same direction of $\theta$, while the second one is a wave which rotates in the opposite direction of $\theta$. In Fig. A4.1-11 is shown the dual interpretation of this phenomenon: the concepts of the stationary wave and the counter-rotating fields.

![Image of the stationary and counter-rotating distributions](image)

Fig. A4.1-11. The stationary and the counter-rotating distributions

The space-time distribution produced by one of the two rotating fields is repeated identical in the following situations: in a position $\theta_1$ at the instant of time $t_1$ and in the position $\theta_2$ at the instant of time $t_2$, such that:

$$\theta_1 \pm \frac{\omega}{k} t_1 = \theta_2 \pm \frac{\omega}{k} t_2 \quad \text{(A4.1-32)}$$

After some simple calculations:

$$\theta_2 - \theta_1 = \mp \frac{\omega}{k} t_2 \mp \frac{\omega}{k} t_1 = \pm \frac{\omega}{k} (-t_2 + t_1) = \mp \frac{\omega}{k} (t_2 - t_1) \quad \text{(A4.1-33)}$$

The electrical angular velocity $\omega_{E}$ of the distribution, measured in radians per sec., is given by:

$$\omega_{E} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \mp \frac{\omega}{k} \quad \text{(A4.1-34)}$$

The mechanical angular velocity $\omega_{E}$ of the rotating field, measured in radians
per sec., is given by:

\[ \omega_{ck} = \pm \frac{\omega}{kp} \]  \hspace{1cm} (A4.1-35)

From which follows the rpm speed of the \( k \) -th order of the magnetic field:

\[ n_k = \pm \frac{60 f}{kp} \]  \hspace{1cm} (A4.1-36)

Consider a system of currents characterized by a phase difference \( S_t(2\pi/m) \): \( S_t \)
is defined as “sequence of time”. Some examples are presented in the Figs. A4.1-12 - A4.1-14 (in the following, \( k_m \) will be assumed equal to 1):

\[ i_j(t) = \sqrt{2} I \cos \left[ k_m \omega \left( t - S_t(j-1)\frac{T}{m} \right) \right] \]  \hspace{1cm} (A4.1-37)

![Diagram](image1.png)

Fig. A4.1-12. Sequences of time with \( m = 3 \)

![Diagram](image2.png)

Fig. A4.1-13. Sequences of time with \( m = 5 \)
The sequence of time must comply with the following constraint:

\[ 0 \leq S_t \leq m - 1 \]  

(A4.1-38)

If the phases are disposed by following different spatial configurations, in this case one speaks of sequence of space \( S_s \). Consider a machine with \( p \) pole pairs, \( m \) phases shifted in space by an angle equal to \( S_s \left( \frac{2\pi}{m} \right) \), in which flows a balanced system of currents.

In the reference system centered in the phase 1, the magnetic field distribution produced by the \( j \)-th phase is given by:

\[
H_j(\theta, t) = \sum_{k=1,3,\ldots}^{\infty} H_{Mk} \cos \left( k \left( \frac{2\pi}{m} \right) j \right) \cos \left( S_t (j-1) \frac{2\pi}{m} \right) 
\]

(A4.1-39)

The resulting magnetic field produced by all the phases of the machine can be expressed as:

\[
H(\theta, t) = \sum_{j=1}^{m} \sum_{k=1,3,\ldots}^{\infty} H_{Mk} \cos \left( k \left( \frac{2\pi}{m} \right) j \right) \cos \left( S_t (j-1) \frac{2\pi}{m} \right) = \\
= \sum_{j=1}^{m} \sum_{k=1,3,\ldots}^{\infty} \frac{1}{2} H_{Mk} \cos \left( k\theta + \omega t - (j-1)kS_s + S_t \right) \frac{2\pi}{m} 
\]

(A4.1-40)
Finally is obtained:

$$H(\theta,t) = \sum_{j=1}^{n} \sum_{k=1,3,...}^{\infty} \frac{1}{2} H_{Mk} \left\{ \cos \left( k\theta + \omega t - (j-1)(kS_s + S_t) \frac{2\pi}{m} \right) \right\}$$  \hspace{1cm} \text{(A4.1-41)}$$

The resulting magnetic field is given by adding $m$ sinusoidal contributions with a phase difference equal to $(kS_s + S_t)2\pi/m$.

There are two possible cases: if all the contributions have the same phase, the result is $m$ times the contribution; if the contributions are shifted by the same angle, their resultant is null. This result is explained in (A4.1-42):

$$H(\theta,t) = \begin{cases} \sum_{k=1}^{\infty} \frac{m}{2} H_{Mk} \cos(k\theta + \omega t) & \text{if } \frac{kS_s + S_t}{m} \text{ is integer} \\ 0 & \text{if } \frac{kS_s + S_t}{m} \neq \text{integer} \end{cases}$$  \hspace{1cm} \text{(A4.1-42)}$$

For a fixed value of $k$, the magnetic field distribution has a direction of rotation:

- direct, if $\frac{kS_s - S_t}{m}$ is integer
- reverse, if $\frac{kS_s + S_t}{m}$ is integer

A symmetrical polyphase winding in which flows a balanced system of currents, produces a distribution of magnetic field in the airgap described by the following relation:

$$H(\theta,t) = \sum_{k=1,...}^{\infty} \frac{m}{2} \frac{4\sqrt{n}}{\pi} \frac{I}{2\delta} qK_{ak} \frac{1}{k} \sin \left( k \frac{\pi}{2} \right) \cos(k\theta + \omega t)$$  \hspace{1cm} \text{(A4.1-43)}$$

The terms in summation (A4.1-43) have to be considered not null only for the values of $k$ such that $\frac{kS_s + S_t}{m}$ is integer. This distribution can therefore be considered formed by the overlap of direct and inverse harmonic rotating magnetic fields.
Chapter 5

Design and Development of a Control System for Multiphase Synchronous Permanent Magnet Bearingless Machines

5.1 Introduction

In this chapter a control system for bearingless multiphase synchronous PM machines is presented, integrating the electromagnetic model seen in Chapter 4 with a three-dimensional mechanical model developed based on the Euler’s equations. One end of the motor shaft is constrained, to simulate the presence of a mechanical bearing, while the other is free, only supported by the radial forces developed in the interactions between magnetic fields, to simulate a bearingless
system with three degrees of freedom.

The body in Fig. 5.1 represents the rotor and the shaft of the machine, rotating around an axis with a fixed point. The interactions between the levitation current space vector $\vec{i}_{sv2}$ and the other sources of magnetic fields, i.e. rotor magnets and torque current space vector $\vec{i}_{sv1}$, provide to generate the levitation forces, as seen in the previous chapter by means of the analytical formulation. The system was implemented on a SIMULINK® model, representing the conceptual design of the experimental device and related control system that could be realized in a test bench application.

![Fig. 5.1](image)

![Fig. 5.2](image)
Consider three reference systems in the space:

- **Absolute (abs)**, characterized by the unit vectors \( \hat{i}, \hat{j}, \hat{k} \), Fig. 5.1;
- **Relative (rel)**, characterized by the unit vectors \( \hat{I}, \hat{J}, \hat{K} \), Fig. 5.2(a);
- **Fixed-to-rotor (ftr)**, characterized by the unit vectors \( \hat{I}, \hat{\epsilon}, \hat{\eta} \), Fig. 5.2(b).

The last two systems have a common unit vector because their first axes coincide.

### 5.2 Mechanical equations

The rotor angular speed in the absolute reference system is calculated by applying the “principle of composition of angular speed”:

\[
\bar{\omega} = \bar{\Omega} + \bar{\omega}_{\text{rel}} \tag{5.1}
\]

where \( \bar{\Omega} \) is the angular speed of the relative reference system with respect to the absolute and \( \bar{\omega}_{\text{rel}} \) is the angular speed of the rotor with respect to the relative reference system. They can be expressed as:

\[
\bar{\Omega} = \dot{\psi} \hat{k} - \dot{\phi} \hat{J} \tag{5.2}
\]

\[
\bar{\omega}_{\text{rel}} = \dot{\theta} \hat{I} \tag{5.3}
\]

By substituting (5.2), (5.3) in (5.1), it gives the rotor angular speed vector:

\[
\bar{\omega} = \dot{\psi} \hat{k} - \dot{\phi} \hat{J} + \dot{\theta} \hat{I} \tag{5.4}
\]

The equations of the motion are given by:

\[
\frac{d\bar{\Gamma}_o}{dt} = \bar{M}_o \tag{5.5}
\]
Being \( \Gamma_o \), \( \vec{M}_o \) respectively the moment of momentum and the moment of the external forces evaluated with respect to the point \( O \). \( \Gamma_o \) is given by:

\[
\Gamma_o = \vec{I}_o \cdot \vec{\omega} \tag{5.6}
\]

The fixed-to-rotor reference system is obtained by rotation of an angle \( \theta \) around the \( X \)-axis, and the relationships between its unit vectors and the ones of the relative system are (note that the unit vector \( \hat{I} \) is the same because the \( X \)-axis is in common):

\[
\dot{J} = \cos \theta \hat{e} - \sin \theta \hat{\eta} \tag{5.7}
\]

\[
\dot{K} = \sin \theta \hat{e} + \cos \theta \hat{\eta} \tag{5.8}
\]

In order to write the angular speed vector with respect to the fixed-to-rotor reference system, is necessary firstly to write it in explicit way, with respect to the relative system:

\[
\dot{k} = \sin \varphi \hat{I} + \cos \varphi \hat{K} \tag{5.9}
\]

By substituting (5.9) in (5.4):

\[
\vec{\omega} = (\psi \sin \varphi + \dot{\theta})\hat{I} - \dot{\phi} \hat{J} + \psi \cos \varphi \hat{K} \tag{5.10}
\]

By substituting (5.7) and (5.8) in (5.10):

\[
\vec{\omega} = (\psi \sin \varphi + \dot{\theta})\hat{J} + (\psi \cos \varphi \sin \theta - \dot{\phi} \cos \theta)\hat{e} + (\dot{\phi} \sin \theta + \psi \cos \varphi \cos \theta)\hat{\eta} \tag{5.11}
\]

which can be expressed in matrix form in the following:

\[
\begin{bmatrix}
\omega_I \\
\omega_J \\
\omega_K
\end{bmatrix} =
\begin{bmatrix}
\sin \varphi & 0 & 1 \\
\cos \varphi \sin \theta & -\cos \theta & 0 \\
\cos \varphi \cos \theta & \sin \theta & 0
\end{bmatrix}
\begin{bmatrix}
\psi \\
\phi \\
\dot{\theta}
\end{bmatrix} \tag{5.12}
\]

As the system chosen is principal of inertia, the \( \vec{I}_o \) is a diagonal matrix thus,
Design and development of a control system for multiphase synchronous PM bearingless machines

with reference to (5.6), is possible to write:

\[
\begin{bmatrix}
I_{oI} & 0 & 0 \\
0 & I_{oe} & 0 \\
0 & 0 & I_{on}\end{bmatrix} \begin{bmatrix}
\omega_I \\
\omega_e \\
\omega_\eta\end{bmatrix} = I_{oI} \dot{\omega}_I + I_{oe} \omega_e \dot{\hat{e}} + I_{on} \omega_\eta \dot{\hat{\eta}} \quad (5.13)
\]

Reminding that \(\overline{\omega}\) is defined with respect to a stationary observer, the derivative \(\frac{d \overline{\Gamma}_o}{dt}\) has to be determined with respect to the same observer. It is possible to write:

\[
\left(\frac{d \overline{\Gamma}_o}{dt}\right)_{(abs)} = \left(\frac{d \overline{\Gamma}_o}{dt}\right)_{(fri)} + \overline{\omega} \times \overline{\Gamma}_o \quad (5.14)
\]

By considering the equation (5.6) and that the matrix of inertia \(\overline{\mathbf{I}}_o\) doesn’t change, it can be calculated:

\[
\left(\frac{d \overline{\Gamma}_o}{dt}\right)_{(fri)} = \overline{\mathbf{J}}_o \cdot \overline{\omega} \quad (5.15)
\]

By substituting (5.6) and (5.15) in (5.14):

\[
\left(\frac{d \overline{\Gamma}_o}{dt}\right)_{(abs)} = \overline{\mathbf{J}}_o \cdot \overline{\omega} + \overline{\omega} \times \overline{\mathbf{J}}_o \cdot \overline{\omega} \quad (5.16)
\]

\[
\left(\frac{d \overline{\Gamma}_o}{dt}\right)_{(abs)} = I_{oI} \dot{\omega}_I \hat{I} + I_{oe} \dot{\omega}_e \hat{\hat{e}} + \omega_\eta \hat{\dot{\eta}} + I_{oI} \dot{\omega}_I \begin{bmatrix}
\hat{I} \\
\hat{\hat{e}} \\
\dot{\hat{\eta}}\end{bmatrix} + \begin{bmatrix}
I_{oI} & I_{oe} & I_{on}\end{bmatrix} \begin{bmatrix}
\omega_I \\
\omega_e \\
\omega_\eta\end{bmatrix} \quad (5.17)
\]

By executing the calculations:

\[
\left(\frac{d \overline{\Gamma}_o}{dt}\right)_{(abs)} = \left[I_{oI} \dot{\omega}_I + (I_{on} - I_{oe}) \omega_e \omega_\eta\right] \hat{I} + \left[I_{oe} \dot{\omega}_e + (I_{ol} - I_{oI}) \omega_I \omega_\eta\right] \hat{\hat{e}} + \left[I_{on} \dot{\omega}_\eta + (I_{oe} - I_{ol}) \omega_I \omega_e\right] \dot{\hat{\eta}} \quad (5.18)
\]
Writing the moment of the external forces vector $\mathbf{M}_o$ with respect to the point $O$ in the $\mathbf{fr}$ reference system:

$$\mathbf{M}_o = M_{ol} \mathbf{\hat{i}} + M_{oe} \mathbf{\hat{e}} + M_{o\eta} \mathbf{\hat{\eta}}$$

(5.19)

By substituting (5.18) and (5.19) in (5.5) is possible to determine the equations of the motion, also called “Euler equations”:

$$\begin{align*}
I_{ol} \dot{\omega}_l + (I_{o\eta} - I_{oe}) \omega_e \omega_\eta &= M_{ol} \\
I_{oe} \dot{\omega}_e + (I_{ol} - I_{o\eta}) \omega_l \omega_\eta &= M_{oe} \\
I_{o\eta} \dot{\omega}_\eta + (I_{oe} - I_{ol}) \omega_e \omega_e &= M_{o\eta}
\end{align*}$$

(5.20)

It is now useful to define the relationship between the moment of the external forces vector evaluated in the $\mathbf{fr}$ reference system and the one evaluated in the $\mathbf{abs}$ reference system. Thus, is necessary to write the unit vectors $\mathbf{\hat{i}}, \mathbf{\hat{j}}, \mathbf{\hat{k}}$ with respect the unit vectors $\mathbf{\hat{I}}, \mathbf{\hat{e}}, \mathbf{\hat{\eta}}$. Firstly, the equations (5.7), (5.8) are presented in matrix form:

$$
\begin{bmatrix}
\mathbf{\hat{i}} \\
\mathbf{\hat{j}} \\
\mathbf{\hat{k}}
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 \\
0 & \cos\theta & -\sin\theta \\
0 & \sin\theta & \cos\theta
\end{bmatrix}
\begin{bmatrix}
\mathbf{\hat{I}} \\
\mathbf{\hat{e}} \\
\mathbf{\hat{\eta}}
\end{bmatrix}
$$

(5.21)

The unit vector $\mathbf{\hat{n}}$ is defined as the projection of the unit vector $\mathbf{\hat{I}}$ on the $xy$ plane:

$$\mathbf{\hat{n}} = \cos \varphi \mathbf{\hat{i}} - \sin \varphi \mathbf{\hat{k}}$$

(5.22)

By substituting (5.22) in (5.23), (5.24), the relationships between the unit vectors $\mathbf{\hat{i}}, \mathbf{\hat{j}}, \mathbf{\hat{k}}$ and the unit vectors $\mathbf{\hat{I}}, \mathbf{\hat{J}}, \mathbf{\hat{K}}$ are found:

$$\begin{align*}
\mathbf{\hat{i}} &= \cos \psi \mathbf{\hat{n}} - \sin \psi \mathbf{\hat{J}} = \cos \psi \cos \varphi \mathbf{\hat{I}} - \sin \psi \mathbf{\hat{J}} - \cos \psi \sin \varphi \mathbf{\hat{K}} \\
\mathbf{\hat{j}} &= \sin \psi \mathbf{\hat{n}} + \cos \psi \mathbf{\hat{J}} = \sin \psi \cos \varphi \mathbf{\hat{I}} + \cos \psi \mathbf{\hat{J}} - \sin \psi \sin \varphi \mathbf{\hat{K}}
\end{align*}$$

(5.23) (5.24)
Design and development of a control system for multiphase synchronous PM bearingless machines

\[ \dot{k} = \sin \varphi \dot{i} + \cos \varphi \dot{K} \]  \hspace{1cm} (5.25)

By expressing (5.23)-(5.25) in the matrix form:

\[
\begin{bmatrix}
\dot{i} \\
\dot{j} \\
\dot{k}
\end{bmatrix}
= \begin{bmatrix}
\cos \psi \cos \varphi & -\sin \psi & -\cos \psi \sin \varphi \\
\sin \psi \cos \varphi & \cos \psi & -\sin \psi \sin \varphi \\
\sin \varphi & 0 & \cos \varphi
\end{bmatrix}
\begin{bmatrix}
\dot{I} \\
\dot{J} \\
\dot{K}
\end{bmatrix}
\]  \hspace{1cm} (5.26)

By combining (5.21) and (5.26), are presented the relationships between the unit vectors \((\dot{i}, \dot{j}, \dot{k})\) related to absolute reference system and the unit vectors \((\dot{I}, \dot{e}, \dot{\eta})\) related to fixed-to-rotor reference system:

\[
\begin{bmatrix}
\dot{i} \\
\dot{j} \\
\dot{k}
\end{bmatrix}
= \begin{bmatrix}
\cos \psi \cos \varphi & -\sin \psi & -\cos \psi \sin \varphi \\
\sin \psi \cos \varphi & \cos \psi & -\sin \psi \sin \varphi \\
\sin \varphi & 0 & \cos \varphi
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \theta & -\sin \theta \\
0 & \sin \theta & \cos \theta
\end{bmatrix}
\begin{bmatrix}
\dot{I} \\
\dot{e} \\
\dot{\eta}
\end{bmatrix}
\]  \hspace{1cm} (5.27)

By multiplying the matrices in (5.27) it gives:

\[
\begin{bmatrix}
\cos \psi \cos \varphi & -\sin \psi \cos \theta & -\cos \psi \sin \varphi \sin \theta & \sin \psi \sin \theta - \cos \psi \sin \varphi \cos \theta \\
\sin \psi \cos \varphi & \cos \psi \cos \theta - \sin \psi \sin \varphi \sin \theta & -\cos \psi \sin \theta - \sin \psi \sin \varphi \cos \theta \\
\sin \varphi & \cos \varphi \sin \theta & \cos \varphi \cos \theta
\end{bmatrix}
\]  \hspace{1cm} \overrightarrow{B}^{-1}(\theta, \varphi, \psi)

The matrix obtained in (5.28), called \(\overrightarrow{B}^{-1}(\theta, \varphi, \psi)\), permits to directly convert the abs unit vectors in the ftr unit vectors. It is mentioned as an inverse matrix because the main matrix \(\overrightarrow{B}(\theta, \varphi, \psi)\) is that one which defines the moment of external forces vector \(\overrightarrow{M_0}\) in the ftr reference system, being the most used.

In fact, the latter can be immediately substituted in the Euler’s equations (5.20).

By combining (5.29) and (5.30), is obtained (5.31):
\[
\begin{bmatrix}
M_{oX} \\
M_{oY} \\
M_{oZ}
\end{bmatrix} =
\begin{bmatrix}
\cos \psi \cos \varphi & \sin \psi \cos \varphi & \sin \varphi \\
-\sin \psi & \cos \psi & 0 \\
-\cos \psi \sin \varphi & -\sin \psi \sin \varphi & \cos \varphi
\end{bmatrix}
\begin{bmatrix}
M_{oX} \\
M_{oY} \\
M_{oZ}
\end{bmatrix}
\] (5.29)

\[
\begin{bmatrix}
M_{ol} \\
M_{ox} \\
M_{on}\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \theta & \sin \theta \\
0 & -\sin \theta & \cos \theta
\end{bmatrix}
\begin{bmatrix}
M_{oX} \\
M_{oY} \\
M_{oZ}
\end{bmatrix}
\] (5.30)

In (5.31), (5.32) is shown the \( \bar{B}(\theta, \varphi, \psi) \) matrix, which permits to calculate the moment of the external forces vector \( \vec{M}_o \) in the fixed-to-rotor reference system:

\[
\begin{bmatrix}
\cos \psi \cos \varphi & \sin \psi \cos \varphi & \sin \varphi \\
-\sin \psi \cos \theta - \cos \psi \sin \varphi \sin \theta & \cos \psi \cos \theta - \sin \varphi \sin \psi \sin \theta & \cos \varphi \sin \theta \\
\sin \psi \sin \theta - \cos \psi \sin \varphi \cos \theta & -\cos \psi \sin \theta - \sin \varphi \sin \psi \cos \theta & \cos \varphi \cos \theta
\end{bmatrix}
\bar{B}(\theta, \varphi, \psi)
\] (5.31)

\[
\begin{bmatrix}
M_{ol} \\
M_{ox} \\
M_{on}\end{bmatrix} =
\begin{bmatrix}
\bar{B}(\theta, \varphi, \psi) \\
M_{ox} \\
M_{oz}\end{bmatrix}
\] (5.32)

The relationship (5.32) is really important because the moments of external forces are given in the absolute reference system, but is necessary to express them in the fixed-to-rotor reference system in order to apply equations (5.20).

### 5.3 General structure of the control system

The external structure of the control system, realized with SIMULINK®, is composed by four main blocks, the other being “scope blocks” and “integrators”: Levitation forces block, Euler equations block, Lagrangian variables block,
Axis coordinates block.

Fig. 5.3. General structure of the control system

1) **Levitation forces block**: the action of PID controllers is based on the position error along the $y$ and $z$ axes, by calculating the force, of modulus $F_r$ and spatial phase $\text{fi}_F$, requested to maintain the rotor in the centre of the stator bore. Together with the angular position of the magnets $\text{theta}$, the **Force Controller** block determines the modulus and the phase of the $\tilde{t}_{sv2}$ current space vector, called as variables $\text{lrms}_-$ and $\text{fi}_\text{ln}_-$, to produce the requested force. This couple of values is necessarily provisional, being the analytic relationship that gives the $\tilde{t}_{sv2}$ parameters univocal only considering the main harmonic orders of stator and rotor magnetic fields.

The next block, **Electromagnetic Model**, determines the real value of the force to suspend the rotor, by taking into account all the possible interactions between harmonic orders of the magnetic fields. Also, this model consider the effects of both current space vectors $\tilde{t}_{sv1}$, $\tilde{t}_{sv2}$, respectively torque and
levitation. It gives in this way an exact prediction of the radial force and electromagnetic torque produced by the motor.

The drive maintains constantly the current space vector $\vec{i}_{sv1}$ in leading by 90 electrical degrees with respect to magnet axis, in order to give the requested torque in whatever operating condition. The output gives in particular the components of the force (variables $F_y$, $F_z$) which are used to calculate the moments with respect to the absolute reference system, by means of the “Forces to Moments matrix” block.

![Diagram of the Levitation Forces block](image)

**Fig. 5.4. Diagram of the Levitation Forces block**

2) **Euler equations block**: the sub-block “Applied Moments” calculates the moments of external forces with respect to the fixed-to-rotor reference system and applies them to the Euler’s equations (5.20) in the form (5.33), achievable after some simple calculations. The output is constituted by the components of angular speed vector, evaluated in the fixed-to-rotor reference system and determined by integrating (5.33), as can be seen in the SIMULINK® diagram of Fig. 5.5.
Design and development of a control system for multiphase synchronous PM bearingless machines

\[
\begin{align*}
\dot{\omega}_l &= \frac{M_{ol}}{I_{ol}} - \frac{I_{on} - I_{oe}}{I_{ol}} \omega_e \omega_\eta \\
\dot{\omega}_e &= \frac{M_{oe}}{I_{oe}} - \frac{I_{ol} - I_{on}}{I_{oe}} \omega_I \omega_\eta \\
\dot{\omega}_\eta &= \frac{M_{on}}{I_{on}} - \frac{I_{oe} - I_{ol}}{I_{on}} \omega_I \omega_e 
\end{align*}
\] (5.33)

Fig. 5.5. Diagram of the Euler equations block

3) **Lagrangian variables block**: applies the inverse matrix of (5.12), described in the equation (5.34), to the angular speed vector $\vec{\omega}$ in order to determine the derivatives with respect to time $[\dot{\psi}, \dot{\varphi}, \dot{\theta}]$ of the lagrangian variables $[\psi, \varphi, \theta]$, which are obtained by means of integration: they constitute the components of the angular speed vector $\vec{\omega}$ evaluated in the absolute reference system.

\[
\begin{bmatrix}
\dot{\psi} \\
\dot{\varphi} \\
\dot{\theta}
\end{bmatrix} =
\begin{bmatrix}
0 & \sin \theta / \cos \varphi & \cos \theta / \cos \varphi \\
0 & -\cos \theta & \sin \theta \\
1 & -\tan \varphi \sin \theta & -\tan \varphi \cos \theta
\end{bmatrix}
\begin{bmatrix}
\omega_I \\
\omega_e \\
\omega_\eta
\end{bmatrix}
\] (5.34)
4) **Axis coordinates block:** it converts the lagrangian variables $[\psi, \varphi, \theta]$, constituted by angular coordinates, in the linear coordinates $[x, y, z]$ which describe the position of the motor shaft end point. These are then used in a closed loop feedback to return as an input in the “Levitation forces block”, closing in this way the loop of the bearingless machine control system.
5.4 Detailed analysis of the control system

5.4.1 Levitation Forces Block

A) Position Errors

These blocks determine the errors along the $y$ and $z$ axes, calculating the difference between the actual position, on the $y$-$z$ plane, of the end shaft point and the reference values $y_0$ and $z_0$, which obviously have to be set to zero requiring a centered rotor (Fig. 5.8).

![Fig. 5.8. Position errors](image)

B) Pid Controllers

In Fig. 5.9 is represented the general structure of the PID controller, where it is possible to distinguish the three actions: proportional, integrator, derivative.

![Fig. 5.9. Diagram of the PID controllers](image)
The parameter $aT_d$ in the derivative branch is a time constant which permits to set the duration of the transitional regime. The output of the PIDs gives the $y$- and $z$-components of the force, necessary to stabilize the rotor.

C) Force Controller Block

The input stadium transforms the components of the force from cartesian representation to polar, providing the modulus $F_r$ and the spatial phase $\phi_{Fr}$. In addition, the angular position of the rotor $\theta$ is a required input variable. As said above, the Force Controller determines the modulus and the phase of the levitation current space vector $\vec{i}_{sv2}$ (respectively identified by the output variables $I_{rms_\_}$ and $\phi_{ln_\_}$), required to generate the input force. These values result approximate because the analytic relationship that allows the calculation of the current space vector parameters from the value of the force, is invertible only considering the interaction between the main harmonic of the stator levitation field and the main one of the rotor field. In this way, all the higher harmonic orders are neglected. However, this is not a problem because the consequent block takes into account all the possible interactions between the magnetic fields, and the PID controllers provide to stabilize the system with their feedback action; but, the function of the Force Controller is necessary to give some initial values of the $\vec{i}_{sv2}$ parameters, in absence of which would not be possible to implement the regulation process.

![Diagram of the Force Controller](image)

Fig. 5.10. The Force Controller of the model
In the following, the programming code of the **Force Controller** is shown, being a “Matlab function”.

```matlab
function ABF_inv = fcn(Fr,fi_Fr,theta)
    %definizione costanti
    Pi = 3.141592654;
    muzero = 0.000001256;

    %**************************************************************************************
    %INPUT DATI (begin)
    %**************************************************************************************

    %modulo della forza radiale 
    % Fr = 2620.06;
    %fase spaziale della forza radiale, misurata rispetto all'asse y (FEMM)
    % fi_Fr = -90 * Pi / 180;
    %massimo ordine armonico indagato
    k_max = 2;
    Br_=zeros(1,1);
    Bs_=zeros(1,1);
    Irms_=zeros(1,1);
    fi_In_=zeros(1,1);
    k_=zeros(1,1);
    hn_=zeros(1,1);
    hp_=zeros(1,1);
    %dichiarazione array
    ctr=zeros(k_max,1);
    rot=zeros(k_max,1);
    Kd=zeros(k_max,1);
    Bs_hp=zeros(k_max,1);
    Bs_hn=zeros(k_max,1);
    Br_hp=zeros(k_max,1);
    Br_hn=zeros(k_max,1);
    fi_In_hp=zeros(k_max,1);
    fi_In_hn=zeros(k_max,1);
    Fq=zeros(k_max,1);
    Fd=zeros(k_max,1);
    %Fr=zeros(k_max,1);
    %fi_Fr=zeros(k_max,1);
    Irms_hp=zeros(k_max,1);
    Irms_hn=zeros(k_max,1);
    hp=zeros(k_max,1);
    hn=zeros(k_max,1);

    %angolo meccanico fra asse M ed asse fase 1 (gr.mecc.)
    % deltath = 90;
    % deltath = deltath * Pi / 180;
    %numero di cave sottese dalla bobina
    nac = 10;
    %numero di fasili
    m = 5;
    %numero di cave/polo/fase
    q = 6;
    %numero di conduttori in cava
    n = 2;
    %numero cave statoriche
    %Ncv = 30;
    %coppie polari STATORE (riferite al campo PRINCIPALE)
    N_ = 1;
    %coppie polari ROTORE
    M_ = 1;
```
%angolo descritto dal magnete (rad.mecc.)
alfa_mag = 172;
alfa_mag = alfa_mag * Pi / 180;

%spessore del magnete
Lm = 0.002;

%spessore del traferro
g = 0.001;

%permeabilità relativa magnete
mu_mr = 1.045;

%fattore di Carter
kc = 1;

%induzione residua magnete
Bres = 1.05;

%profondità di macchina
L = 0.18;

%raggio medio al traferro
Rg = 0.0595;

%angolo di cava (rad.el.)
alfa_c = 12;
alfa_c = alfa_c * Pi / 180;
%alfa_c = Pi / m / q
%alfa_c = 2 * Pi / Ncv * N_
%alfa_c = 2 * Pi / Ncv * M

%sequenza temporale di corrente
st = 2;

%angolo elettrico correnti (pulsazione * tempo - gr.el.)
wt = 0;
wt = wt * Pi / 180;

%%%%%%%%%%%%%%%%%%%%%%%%% INPUT DATI (end) %%%%%%%%%%%%%%%%%%%%%%%%%%

%spessore traferro complessivo
deltag = Lm + g;

%semiampiezza di una bobina (rad.el.)
gamma = nac * alfa_c / 2;

%induzione al traferro generata dal magnete
BgM = Lm / (Lm + mu_mr * kc * g) * Bres;

%passo polare (al traferro)
Taup = Pi * Rg / N_;

%ordine massimo sequenza di corrente
st_max = m - 1;
if st > st_max
    return
end

for k = 1 : k_max
    ctr(k) = 0;
    rot(k) = ' ';
    Rd(k) = 0;
end

%componenti della forza radiale definite rispetto all'asse y (d) (FEMM)
Fq(k) = Fr * sin(fi_Fr);
Fd(k) = Fr * cos(fi_Fr);

for k = 1 : k_max
    if (k + st) / m == int32((k + st) / m)
        ctr(k) = 1;
        rot(k) = 'I';
    elseif (k - st) / m == int32((k - st) / m)
        ctr(k) = 2;
        rot(k) = 'D';
    else
        ctr(k) = 0;
        rot(k) = ' ';```
Design and development of a control system for multiphase synchronous PM bearingless machines

if ctr(k) == 1 || ctr(k) == 2
\*fattore di distribuzione
Kd(k) = sin(q * k * alpha_c / 2) / q / sin(k * alpha_c / 2);
\*valore h positivo
hp(k) = (1 + k * N_1) / M;
\*valore h negativo
hn(k) = (-1 + k * N_1) / M;
\%CASO hp INTERO e DISPARI
\%(sovrapponibile con hn INTERO)
if hp(k) == int32(hp(k)) && hp(k) / 2 =~ int32(hp(k) / 2)
Br_hp(k) = 4 * hp(k) / Pi * BgM * sin(hp(k) * M_1 * alpha_mag / 2);
Bs_hp(k) = 2 * mzero / Pi / L / Sq / Br_hp(k) * Fr;
Irms_hp(k) = 2 * k * Pi * 2 * deltag / muzero / m / 4 / n / sqrt(2) / q / Kd(k) /
sin(k * gamma) * Bs_hp(k);
if ctr(k) == 2
fi_In_hp(k) = hp(k) * M_1 * theta - wt - atan2(Fq(k),Fd(k));
elseif ctr(k) == 1
fi_In_hp(k) = -hp(k) * M_1 * theta - wt + atan2(Fq(k),Fd(k));
end
end
\%CASO hn INTERO e DISPARI
\%(sovrapponibile con hp INTERO)
if hn(k) == int32(hn(k)) && hn(k) / 2 =~ int32(hn(k) / 2)
Br_hn(k) = 4 / hn(k) / Pi * BgM * sin(hn(k) * M_1 * alpha_mag / 2);
Bs_hn(k) = 2 * muzero / Pi / L / Sq / Br_hn(k) * Fr;
Irms_hn(k) = 2 * k * Pi * 2 * deltag / muzero / m / 4 / n / sqrt(2) / q / Kd(k) /
sin(k * gamma) * Bs_hn(k);
if ctr(k) == 2
fi_In_hn(k) = hn(k) * M_1 * theta - wt + atan2(Fq(k),Fd(k));
elseif ctr(k) == 1
fi_In_hn(k) = -hn(k) * M_1 * theta - wt - atan2(Fq(k),Fd(k));
end
end
\%il ciclo si interrompe quando l'ordine armonico di rotore è INTERO, DISPARI ed
\%assume il valore INFERIORE tra i due: hn, hp (in realtà questo è sempre 'hn'
\%per definizione)
if (hn(k) == int32(hn(k)) && hn(k) / 2 =~ int32(hn(k) / 2)) && (hn(k) < hp(k))
Br =Br_hn(k);
Bs =Bs_hn(k);
Irms =Irms_hn(k);
fi_In =fi_In_hn(k);
k =k;
hn =hn(k);
hp =1 + k_1 * N_1) / M;
break
elseif (hp(k) == int32(hp(k)) && hp(k) / 2 =~ int32(hp(k) / 2)) && (hp(k) < hn(k))
Br =Br hp(k);
Bs =Bs_hp(k);
Irms =Irms_hp(k);
fi_In =fi_In_hp(k);
k =k;
hn =(-1 + k_1 * N_1) / M;
hp =hp(k);
break
else
Br =0;
Bs =0;
Irms =0;
fi_In =0;
k =0;
hn =0;
hp =0;
end
end
end %for k = 1 : k_max

215
Note that the code goes to a “break” when the first value of an existing harmonic order is found, both for the magnetic field produced by levitation current space vector \( \vec{i}_{sv2} \), both for the magnetic field produced by rotor magnets, taking in this way into account only the main orders.

D) Electromagnetic Model Block

The Electromagnetic Model block represents a complete model of the motor, by the electrical point of view. It determines the effective radial force necessary to support the rotor, in terms of the \( y \)- and \( z \)-components \( F_y \) and \( F_z \), taking into account all the possible interactions between the harmonic orders of stator and rotor magnetic fields, according to the conditions of existence already seen in chapter 4 and revived hereunder (5.35)-(5.39):

\[
\frac{k_1 \mp s_{sv1}}{m} \in N_0, \quad \frac{k_2 \mp s_{sv2}}{m} \in N_0, \quad \frac{k_{s}' \mp s_{sv2}}{m} \in N_0, \quad h \in N_{odd}
\]

\[
h p_r - k_1 p_s = \pm 1
\]

\[
h p_r - k_2 p_s = \pm 1
\]

\[
k_2 p_s - k_1 p_s = \pm 1
\]

\[
k_2 p_s - k_{s}' p_s = \pm 1
\]

This model also considers the effects of both space vectors \( \vec{i}_{sv1}, \vec{i}_{sv2} \) (respectively, torque and levitation), thus provides an accurate prediction of the radial force and torque generated by the motor. The drive maintains the current space vector \( \vec{i}_{sv1} \) in leading by 90 electrical degrees with respect to the magnet axis, so that the motor produces the requested torque in each operating condition of the \( \vec{i}_{sv2} \) space vector: particularly, this task is performed by the block...
represented in the following Fig. 5.11, which adds an angle of 90 electrical degrees to the electrical angle corresponding to $\theta$, angular position of the magnet axis. The obtained result, together with the value of $\text{Irms}_1$ block, completely defines the current space vector $\vec{i}_{sv1}$.

The input of **Electromagnetic model** block is given by the $\vec{i}_{sv1}$ modulus and phase $\text{Irms}_1$ and $\text{fi}_\text{In1}$, the $\vec{i}_{sv2}$ modulus and phase $\text{Irms}_2$ and $\text{fi}_\text{In2}$, the rotor position $\text{deltath}$ (it is the same variable $\theta$ used in the Force Controller, its name changed only because of formal reasons), Fig. 5.12.

In the following, the programming code of the **Electromagnetic Model** is shown, being a “Matlab function”.
%ALGORITMO COMPLETO
% interazioni st1-rotore, st2-rotore, st1-st2, st2-st2 descritte tramite
% sviluppo in serie di Fourier

function ABF = fcn(Irms1,Irms2,fi_In1,fi_In2,deltath)

%Irms=zeros(1,1);
%fi_In=zeros(1,1);
st=zeros(1,1);

%sequenze di corrente
st1=1;
st2=2;

%definizione costanti
Pi = 3.141592654;
muzero = 0.000001256;

%numero di cave sottese dalla bobina
nac = 10;

%numero di fasi
m = 5;

%numero di cave/polo/fase
q = 6;

%numero di conduttori in cava
n = 2;

%coppie polari STATORE (riferite al campo PRINCIPALE)
N_ = 1;

%coppie polari ROTORE
M_ = 1;

%angolo descritto dal magnete (rad.mecc.)
alfa_mag = 172;
alfa_mag = alfa_mag * Pi / 180;

%spessore del magnete
Lm = 0.002;

%spessore del traferro
g = 0.001;

%permeabilità relativa magnete
mu_mr = 1.045;

%fattore di Carter
kc = 1;

%induzione residua magnete
Bres = 1.05;

%profondità di macchina
L = 0.18;

%raggio medio al traferro
Rg = 0.0595;

%angolo di cava (rad.el.)
alfa_c = 12;
alfa_c = alfa_c * Pi / 180;

%massimo ordine armonico indagato
k_max = 30;
k1_max = k_max;
k2_max = k_max;

%angolo elettrico correnti (pulsazione - gr.el.)
w = 0;
w = w * Pi / 180;

******************************************************************************
%INPUT DATI (end)
******************************************************************************

%spessore traferro complessivo
deltag = Lm + g;

%semiapertura di una bobina (rad.el.)
gamma = nac * alfa_c / 2;

%induzione al traferro generata dal magnete
BgM = Lm / (Lm + mu_mr * kc * g) * Bres;
Design and development of a control system for multiphase synchronous PM bearingless machines

%passo polare (al traferro)
Taup = Pi * Rg / N;

crt=zeros(k_max,1);
rot=zeros(k_max,1);
Kd=zeros(k_max,1);
Bs=zeros(k_max,1);
Br=zeros(k_max,1);
Fq_hp1=zeros(k1_max,1);
Fd_hp1=zeros(k1_max,1);
Fq_hn1=zeros(k1_max,1);
Fd_hn1=zeros(k1_max,1);
Fq_hp1=zeros(k1_max,1);
Fd_hp1=zeros(k1_max,1);
Fq_hn1=zeros(k1_max,1);
Fd_hn1=zeros(k1_max,1);
fi_Frp1=zeros(k1_max,1);
fi_Frn1=zeros(k1_max,1);
Fq_hp2=zeros(k2_max,1);
Fd_hp2=zeros(k2_max,1);
Fq_hn2=zeros(k2_max,1);
Fd_hn2=zeros(k2_max,1);
Fq_hp2=zeros(k2_max,1);
Fd_hp2=zeros(k2_max,1);
Fq_hn2=zeros(k2_max,1);
Fd_hn2=zeros(k2_max,1);
fi_Frp2=zeros(k2_max,1);
fi_Frn2=zeros(k2_max,1);
Fq_s12=zeros(k1_max,1);
Fd_s12=zeros(k1_max,1);
Fq_s22=zeros(k2_max,1);
Fd_s22=zeros(k2_max,1);

Torque1=zeros(k_max,1);
Torque2=zeros(k_max,1);
Fq=zeros(1,1);
Fd=zeros(1,1);

% SEQUENZA DI CORRENTE ST1

for i = 1 : k1_max
    ctr(i) = 0;
    rot(i) = ' ';
    Kd(i) = 0;
    Bs(i) = 0;
    Fq_hp1(i) = 0;
    Fd_hp1(i) = 0;
    Fq_hn1(i) = 0;
    Fd_hn1(i) = 0;
end

for k1 = 1 : k1_max
    if (k1 + s1) / m == int32((k1 + s1) / m)
        ctr(k1) = 1;
        rot(k1) = 'I';
    elseif (k1 - s1) / m == int32((k1 - s1) / m)
        ctr(k1) = 2;
        rot(k1) = 'D';
    else
        ctr(k1) = 0;
        rot(k1) = ' ';
    end
end
if ctr(k1) == 1 || ctr(k1) == 2
%fattore di distribuzione
Kd(k1) = \frac{\sin(q * k1 * \alpha_c / 2)}{q / \sin(k1 * \alpha_c / 2)};
%ampiezza della k1-esima armonica di campo
B_s(k1) = muzero * m / 2 * 4 / k1 / Pi * n * sqrt(2) * Irms1 / 2 / deltag * q *
Kd(k1) * \sin(k1 * \gamma);
%valore h positivo
hp = (1 + k1 * N_) / M_;
%valore h negativo
hn = (-1 + k1 * N_) / M_;
%CASO hp INTERO
% (sovrapponibile con hn INTERO)
if hp == int32(hp) && hp / 2 ~= int32(hp / 2)
Br_hp = 4 / hp / Pi * BgM * \sin(hp * M_ * \alpha_m / 2);
if ctr(k1) == 2
F_{nq\_hp1}(k1) = \frac{1}{2} * L * R_g * B_s(k1) * Br_hp * \sin(hp * M_ * \delta - wt - \phi_{In1});
F_{q\_hp1}(k1) = F_{nq\_hp1}(k1); % F_{tq\_hp1}(k1) - F_{ttq\_hp1}(k1);
F_{nd\_hp1}(k1) = \frac{1}{2} * L * R_g * B_s(k1) * Br_hp * \cos(hp * M_ * \delta - wt - \phi_{In1});
F_{d\_hp1}(k1) = F_{nd\_hp1}(k1); % F_{td\_hp1}(k1);
elseif ctr(k1) == 1
F_{nq\_hp1}(k1) = \frac{1}{2} * L * R_g * B_s(k1) * Br_hp * \sin(hp * M_ * \delta + wt + \phi_{In1});
F_{q\_hp1}(k1) = F_{nq\_hp1}(k1); % F_{tq\_hp1}(k1) - F_{ttq\_hp1}(k1);
F_{nd\_hp1}(k1) = \frac{1}{2} * L * R_g * B_s(k1) * Br_hp * \cos(hp * M_ * \delta + wt + \phi_{In1});
F_{d\_hp1}(k1) = F_{nd\_hp1}(k1); % F_{td\_hp1}(k1);
end
%CASO hn INTERO
% (sovrapponibile con hp INTERO)
if hn == int32(hn) && hn / 2 ~= int32(hn / 2)
Br_hn = 4 / hn / Pi * BgM * \sin(hn * M_ * \alpha_m / 2);
if ctr(k1) == 2
F_{nq\_hn1}(k1) = -\frac{1}{2} * L * R_g * B_s(k1) * Br_hn * \sin(hn * M_ * \delta - wt - \phi_{In1});
F_{q\_hn1}(k1) = F_{nq\_hn1}(k1); % F_{tq\_hn1}(k1) - F_{ttq\_hn1}(k1);
F_{nd\_hn1}(k1) = \frac{1}{2} * L * R_g * B_s(k1) * Br_hn * \cos(hn * M_ * \delta - wt - \phi_{In1});
F_{d\_hn1}(k1) = F_{nd\_hn1}(k1); % F_{td\_hn1}(k1);
elseif ctr(k1) == 1
F_{nq\_hn1}(k1) = -\frac{1}{2} * L * R_g * B_s(k1) * Br_hn * \sin(hn * M_ * \delta + wt + \phi_{In1});
F_{q\_hn1}(k1) = F_{nq\_hn1}(k1); % F_{tq\_hn1}(k1) - F_{ttq\_hn1}(k1);
F_{nd\_hn1}(k1) = \frac{1}{2} * L * R_g * B_s(k1) * Br_hn * \cos(hn * M_ * \delta + wt + \phi_{In1});
F_{d\_hn1}(k1) = F_{nd\_hn1}(k1); % F_{td\_hn1}(k1);
end
end
%RISULTANTE
for i = 1 : k1_max
F_q = F_q + F_{q\_hp1(i)};
F_q = F_q + F_{q\_hn1(i)};
F_d = F_d + F_{d\_hp1(i)};
F_d = F_d + F_{d\_hn1(i)};
end
% SEQUENZA DI CORRENTE ST2
Design and development of a control system for multiphase synchronous PM bearingless machines

```matlab
for i = 1 : k2_max
    ctr(i) = 0;
    rot(i) = ' ';
    Kd(i) = 0;
    Bs(i) = 0;
    Fq_hp2(i) = 0;
    Fd_hp2(i) = 0;
    Fq_hn2(i) = 0;
    Fd_hn2(i) = 0;
end

for k2 = 1 : k2_max
    if (k2 + st2) / m == int32((k2 + st2) / m)
        ctr(k2) = 1;
        rot(k2) = 'I';
    elseif (k2 - st2) / m == int32((k2 - st2) / m)
        ctr(k2) = 2;
        rot(k2) = 'D';
    else
        ctr(k2) = 0;
        rot(k2) = ' '; 
    end
    if ctr(k2) == 1 || ctr(k2) == 2
        % fattore di distribuzione
        Kd(k2) = sin(q * k2 * alfa_c / 2) / q / sin(k2 * alfa_c / 2);
        % ampiezza della k2-esima armonica di campo
        Bs(k2) = muzero * m / 2 * 4 / k2 / Pi * n * sqrt(2) * Irms2 / 2 / deltag * q * Kd(k2) / sin(k2 * gamma);
        % valore h positivo
        hp = (1 + k2 * N_) / M_; 
        % valore h negativo
        hn = (-1 + k2 * N_) / M_; 
        % CASO hp INTERO
        % (sovrapponibile con hn INTERO)
        if hp == int32(hp) && hp / 2 == int32(hp / 2)
            Br_hp = 4 / hp / Pi * BgM * sin(hp * M_ * alfa_mag / 2); 
        else
            Fq_hp2(k2) = Pi * L * Rg / 2 / muzero * Bs(k2) * Br_hp * sin(hp * M_ * deltath - wt - fi_In2);
            Fd_hp2(k2) = Fq_hp2(k2); %+ Fvq_hp2(k2) - Ftvq_hp2(k2);
            Fd_hn2(k2) = Pi * L * Rg / 2 / muzero * Bs(k2) * Br_hp * cos(hp * M_ * deltath - wt + fi_In2);
            Fd_hp2(k2) = Fd_hn2(k2); %+ Ftd_hp2(k2);
        end
        % CASO hn INTERO
        % (sovrapponibile con hp INTERO)
        if hn == int32(hn) && hn / 2 == int32(hn / 2)
            Br_hn = 4 / hn / Pi * BgM * sin(hn * M_ * alfa_mag / 2); 
        else
            Fq_hn2(k2) = Pi * L * Rg / 2 / muzero * Bs(k2) * Br_hn * sin(hn * M_ * deltath - wt - fi_In2);
            Fq_hn2(k2) = Fq_hn2(k2); %+ Fvq_hn2(k2) - Ftvq_hn2(k2);
            Fd_hn2(k2) = Pi * L * Rg / 2 / muzero * Bs(k2) * Br_hn * cos(hn * M_ * deltath - wt - fi_In2);
            Fd_hn2(k2) = Fd_hn2(k2); %+ Ftd_hn2(k2);
        end
end
```
elseif ctr(k2) == 1
    \[ F_{nq\_hn2}(k2) = -\pi \times L \times R_g / 2 / \mu_0 \times B_s(k2) \times B_r \_hn \times \sin(hn \times M_\* \_delth + wt + \_fi\_In2); \]
    \[ F_{q\_hn2}(k2) = F_{nq\_hn2}(k2) \quad ; \]
    \[ F_{n\_hn2}(k2) = \pi \times L \times R_g / 2 / \mu_0 \times B_s(k2) \times B_r \_hn \times \cos(hn \times M_\* \_delth + wt + \_fi\_In2); \]
    \[ F_{d\_hn2}(k2) = F_{n\_hn2}(k2) \quad ; \]
end
else
    \[ F_{q\_hp2}(k2) = 0; \]
    \[ F_{d\_hp2}(k2) = 0; \]
    \[ F_{q\_hn2}(k2) = 0; \]
    \[ F_{d\_hn2}(k2) = 0; \]
end
end % Next k2

%RESULTANTE
\[
\text{for } i = 1 : k2\_max \\
F_q = F_q + F_{q\_hp2}(i); \\
F_q = F_q + F_{q\_hn2}(i); \\
F_d = F_d + F_{d\_hp2}(i); \\
F_d = F_d + F_{d\_hn2}(i); \\
\text{end}
\]

%******************************************************************************
%INTERAZIONE fra i due campi STATORICI di sequenza ST1, ST2
\[
\text{for } k1 = 1 : k1\_max \\
\%campo k1 INVERSO
\text{if (k1 + st1) / m == int32((k1 + st1) / m)}
\%
ampierza della k1-esima armonica di campo
\text{Kd\_k1 = sin(q \times k1 \times \alpha_c / 2) / q / sin(k1 \times \alpha_c / 2);} \\
B_s\_k1 = \mu_0 \times m / 2 \times 4 / k1 / Pi \times n \times \sqrt{2} \times I_{rms1} / 2 / \Delta \_t1 \times \gamma \times Kd\_k1 \\
k2p = (1 + k1 \times N_\_) / N; \\
k2n = (-1 + k1 \times N_\_) / N; \\
\text{if (k2p + st2) / m == int32((k2p + st2) / m)}
\%
Kd\_k2p = sin(q \times k2p \times \alpha_c / 2) / q / sin(k2p \times \alpha_c / 2);} \\
B_s\_k2p = \mu_0 \times m / 2 \times 4 / k2p / Pi \times n \times \sqrt{2} \times I_{rms2} / 2 / \Delta \_t1 \times \gamma \times Kd\_k2p \\
F_q\_s12(k1) = Pi \times L \times R_g / 2 / \mu_0 \times B_s\_k1 \times B_s\_k2p \times \sin((wt + \_fi\_In1) - (wt + \_fi\_In2)); \\
F_d\_s12(k1) = Pi \times L \times R_g / 2 / \mu_0 \times B_s\_k1 \times B_s\_k2p \times \cos((wt + \_fi\_In1) - (wt + \_fi\_In2)); \\
F_q = F_q + F_q\_s12(k1); \\
F_d = F_d + F_d\_s12(k1); \\
\text{end}
\text{if (k2p - st2) / m == int32((k2p - st2) / m)}
\%
Kd\_k2p = sin(q \times k2p \times \alpha_c / 2) / q / sin(k2p \times \alpha_c / 2);} \\
B_s\_k2p = \mu_0 \times m / 2 \times 4 / k2p / Pi \times n \times \sqrt{2} \times I_{rms2} / 2 / \Delta \_t1 \times \gamma \times Kd\_k2p \\
F_q\_s12(k1) = Pi \times L \times R_g / 2 / \mu_0 \times B_s\_k1 \times B_s\_k2p \times \sin((wt + \_fi\_In1) + (wt + \_fi\_In2)); \\
F_d\_s12(k1) = Pi \times L \times R_g / 2 / \mu_0 \times B_s\_k1 \times B_s\_k2p \times \cos((wt + \_fi\_In1) + (wt + \_fi\_In2)); \\
F_q = F_q + F_q\_s12(k1); \\
F_d = F_d + F_d\_s12(k1); \\
\text{end}
\text{if (k2n + st2) / m == int32((k2n + st2) / m)}
\%
Kd\_k2n = sin(q \times k2n \times \alpha_c / 2) / q / sin(k2n \times \alpha_c / 2);} \\
B_s\_k2n = \mu_0 \times m / 2 \times 4 / k2n / Pi \times n \times \sqrt{2} \times I_{rms2} / 2 / \Delta \_t1 \times \gamma \times Kd\_k2n \\
F_q\_s12(k1) = -Pi \times L \times R_g / 2 / \mu_0 \times B_s\_k1 \times B_s\_k2n \times \sin((wt + \_fi\_In1) - (wt + \_fi\_In2)); \\
F_d\_s12(k1) = Pi \times L \times R_g / 2 / \mu_0 \times B_s\_k1 \times B_s\_k2n \times \cos((wt + \_fi\_In1) - (wt + \_fi\_In2)); \\
F_q = F_q + F_q\_s12(k1); \\
\text{end}
\text{end % Next k1}
\text{end % Next k2}
Design and development of a control system for multiphase synchronous PM bearingless machines

\[
F_d = F_d + F_{d_{s12}}(k1);
\]

if \((k2n - st2) / m == \text{int32}((k2n - st2) / m)\)
\[
Kd_{k2n} = \sin(q \cdot k2n \cdot \alpha_c / 2) / q / \sin(k2n \cdot \alpha_c / 2);
\]
\[
Bs_{k2n} = \text{muzero} \cdot m / 2 \cdot 4 / k2n / \Pi \cdot n \cdot \text{sqr}(2) \cdot \text{Irm}_{s2} / 2 / \text{deltag} \cdot q \cdot Kd_{k2n} \cdot \sin(k2n \cdot \gamma);
\]
\[
F_{q_{s12}}(k1) = -\Pi \cdot L \cdot Rg / 2 / \text{muzero} \cdot Bs_{k1} \cdot Bs_{k2n} \cdot \sin((wt + fi_{In1}) + (wt + fi_{In2}));
\]
\[
F_{d_{s12}}(k1) = \Pi \cdot L \cdot Rg / 2 / \text{muzero} \cdot Bs_{k1} \cdot Bs_{k2n} \cdot \cos((wt + fi_{In1}) + (wt + fi_{In2}));
\]
\[
F_q = F_q + F_{q_{s12}}(k1);
\]
\[
F_d = F_d + F_{d_{s12}}(k1);
\]
end
end %campo k1 DIRETTO

if \((k1 - st1) / m == \text{int32}((k1 - st1) / m)\)
\[
k2n = (1 + k1 \cdot N_) / N_;
\]
\[
k2_p = (1 + k1 \cdot N_) / N_;
\]
\[
k2_n = (-1 + k1 \cdot N_) / N_;
\]
if \((k2p + st2) / m == \text{int32}((k2p + st2) / m)\)
\[
Kd_{k2p} = \sin(q \cdot k2p \cdot \alpha_c / 2) / q / \sin(k2p \cdot \alpha_c / 2);
\]
\[
Bs_{k2p} = \text{muzero} \cdot m / 2 \cdot 4 / k2p / \Pi \cdot n \cdot \text{sqr}(2) \cdot \text{Irm}_{s2} / 2 / \text{deltag} \cdot q \cdot Kd_{k2p} \cdot \sin(k2p \cdot \gamma);
\]
\[
F_{q_{s12}}(k1) = \Pi \cdot L \cdot Rg / 2 / \text{muzero} \cdot Bs_{k1} \cdot Bs_{k2p} \cdot \sin(-(wt + fi_{In1}) - (wt + fi_{In2}));
\]
\[
F_{d_{s12}}(k1) = \Pi \cdot L \cdot Rg / 2 / \text{muzero} \cdot Bs_{k1} \cdot Bs_{k2p} \cdot \cos(-(wt + fi_{In1}) - (wt + fi_{In2}));
\]
\[
F_q = F_q + F_{q_{s12}}(k1);
\]
\[
F_d = F_d + F_{d_{s12}}(k1);
\]
end
if \((k2p - st2) / m == \text{int32}((k2p - st2) / m)\)
\[
Kd_{k2p} = \sin(q \cdot k2p \cdot \alpha_c / 2) / q / \sin(k2p \cdot \alpha_c / 2);
\]
\[
Bs_{k2p} = \text{muzero} \cdot m / 2 \cdot 4 / k2p / \Pi \cdot n \cdot \text{sqr}(2) \cdot \text{Irm}_{s2} / 2 / \text{deltag} \cdot q \cdot Kd_{k2p} \cdot \sin(k2p \cdot \gamma);
\]
\[
F_{q_{s12}}(k1) = \Pi \cdot L \cdot Rg / 2 / \text{muzero} \cdot Bs_{k1} \cdot Bs_{k2p} \cdot \sin(-(wt + fi_{In1}) + (wt + fi_{In2}));
\]
\[
F_{d_{s12}}(k1) = \Pi \cdot L \cdot Rg / 2 / \text{muzero} \cdot Bs_{k1} \cdot Bs_{k2p} \cdot \cos(-(wt + fi_{In1}) + (wt + fi_{In2}));
\]
\[
F_q = F_q + F_{q_{s12}}(k1);
\]
\[
F_d = F_d + F_{d_{s12}}(k1);
\]
end
if \((k2p + st2) / m == \text{int32}((k2p + st2) / m)\)
\[
Kd_{k2p} = \sin(q \cdot k2p \cdot \alpha_c / 2) / q / \sin(k2p \cdot \alpha_c / 2);
\]
\[
Bs_{k2p} = \text{muzero} \cdot m / 2 \cdot 4 / k2p / \Pi \cdot n \cdot \text{sqr}(2) \cdot \text{Irm}_{s2} / 2 / \text{deltag} \cdot q \cdot Kd_{k2p} \cdot \sin(k2p \cdot \gamma);
\]
\[
F_{q_{s12}}(k1) = \Pi \cdot L \cdot Rg / 2 / \text{muzero} \cdot Bs_{k1} \cdot Bs_{k2p} \cdot \sin(-(wt + fi_{In1}) + (wt + fi_{In2}));
\]
\[
F_{d_{s12}}(k1) = \Pi \cdot L \cdot Rg / 2 / \text{muzero} \cdot Bs_{k1} \cdot Bs_{k2p} \cdot \cos(-(wt + fi_{In1}) + (wt + fi_{In2}));
\]
\[
F_q = F_q + F_{q_{s12}}(k1);
\]
\[
F_d = F_d + F_{d_{s12}}(k1);
\]
end
if \((k2n + st2) / m == \text{int32}((k2n + st2) / m)\)
\[
Kd_{k2n} = \sin(q \cdot k2n \cdot \alpha_c / 2) / q / \sin(k2n \cdot \alpha_c / 2);
\]
\[
Bs_{k2n} = \text{muzero} \cdot m / 2 \cdot 4 / k2n / \Pi \cdot n \cdot \text{sqr}(2) \cdot \text{Irm}_{s2} / 2 / \text{deltag} \cdot q \cdot Kd_{k2n} \cdot \sin(k2n \cdot \gamma);
\]
\[
F_{q_{s12}}(k1) = \Pi \cdot L \cdot Rg / 2 / \text{muzero} \cdot Bs_{k1} \cdot Bs_{k2n} \cdot \sin(-(wt + fi_{In1}) - (wt + fi_{In2}));
\]
\[
F_{d_{s12}}(k1) = \Pi \cdot L \cdot Rg / 2 / \text{muzero} \cdot Bs_{k1} \cdot Bs_{k2n} \cdot \cos(-(wt + fi_{In1}) - (wt + fi_{In2}));
\]
\[
F_q = F_q + F_{q_{s12}}(k1);
\]
\[
F_d = F_d + F_{d_{s12}}(k1);
\]
end
if \((k2n - st2) / m == \text{int32}((k2n - st2) / m)\)
\[
Kd_{k2n} = \sin(q \cdot k2n \cdot \alpha_c / 2) / q / \sin(k2n \cdot \alpha_c / 2);
\]
\[
Bs_{k2n} = \text{muzero} \cdot m / 2 \cdot 4 / k2n / \Pi \cdot n \cdot \text{sqr}(2) \cdot \text{Irm}_{s2} / 2 / \text{deltag} \cdot q \cdot Kd_{k2n} \cdot \sin(k2n \cdot \gamma);
\]
\[
F_{q_{s12}}(k1) = \Pi \cdot L \cdot Rg / 2 / \text{muzero} \cdot Bs_{k1} \cdot Bs_{k2n} \cdot \sin(-(wt + fi_{In1}) + (wt + fi_{In2}));
\]
\[
F_{d_{s12}}(k1) = \Pi \cdot L \cdot Rg / 2 / \text{muzero} \cdot Bs_{k1} \cdot Bs_{k2n} \cdot \cos(-(wt + fi_{In1}) + (wt + fi_{In2}));
\]
\[
F_q = F_q + F_{q_{s12}}(k1);
\]
\[
F_d = F_d + F_{d_{s12}}(k1);
\]
end
end %Next k1
%INTERAZIONE fra diversi ordini armonici della sequenza ST2

for k2 = 1 : k2_max
  %campo k2 INVERSO
  if (k2 + st2) / m == int32((k2 + st2) / m)
    Kd_k2 = sin(q * k2 * alfa_c / 2) / q / sin(k2 * alfa_c / 2);
    Bs_k2 = muzero * m / 2 * 4 / k2 / Pi * n * sqrt(2) * Irms2 / 2 / deltag * q * Kd_k2 * sin(k2 * gamma);
    k2p = (1 + k2 * N_) / N_; 
    k2n = (-1 + k2 * N_) / N_; 
    if (k2p + st2) / m == int32((k2p + st2) / m)
      Kd_k2p = sin(q * k2p * alfa_c / 2) / q / sin(k2p * alfa_c / 2);
      Bs_k2p = muzero * m / 2 * 4 / k2p / Pi * n * sqrt(2) * Irms2 / 2 / deltag * q * Kd_k2p * sin(k2p * gamma);
      Fq_s22(k2) = Pi * L * Rg / 2 / muzero * Bs_k2 * Bs_k2p * sin((wt + fi_In2) - (wt + fi_In2));
      Fd_s22(k2) = Pi * L * Rg / 2 / muzero * Bs_k2 * Bs_k2p * cos((wt + fi_In2) - (wt + fi_In2));
      Fq = Fq + Fq_s22(k2);
      Fd = Fd + Fd_s22(k2);
    end
    if (k2p - st2) / m == int32((k2p - st2) / m)
      Kd_k2p = sin(q * k2p * alfa_c / 2) / q / sin(k2p * alfa_c / 2);
      Bs_k2p = muzero * m / 2 * 4 / k2p / Pi * n * sqrt(2) * Irms2 / 2 / deltag * q * Kd_k2p * sin(k2p * gamma);
      Fq_s22(k2) = Pi * L * Rg / 2 / muzero * Bs_k2 * Bs_k2p * sin((wt + fi_In2) + (wt + fi_In2));
      Fd_s22(k2) = Pi * L * Rg / 2 / muzero * Bs_k2 * Bs_k2p * cos((wt + fi_In2) + (wt + fi_In2));
      Fq = Fq + Fq_s22(k2);
      Fd = Fd + Fd_s22(k2);
    end
  end
  if (k2n + st2) / m == int32((k2n + st2) / m)
    Kd_k2n = sin(q * k2n * alfa_c / 2) / q / sin(k2n * alfa_c / 2);
    Bs_k2n = muzero * m / 2 * 4 / k2n / Pi * n * sqrt(2) * Irms2 / 2 / deltag * q * Kd_k2n * sin(k2n * gamma);
    Fq_s22(k2) = -Pi * L * Rg / 2 / muzero * Bs_k2 * Bs_k2n * sin((wt + fi_In2) - (wt + fi_In2));
    Fd_s22(k2) = Pi * L * Rg / 2 / muzero * Bs_k2 * Bs_k2n * cos((wt + fi_In2) - (wt + fi_In2));
    Fq = Fq + Fq_s22(k2);
    Fd = Fd + Fd_s22(k2);
  end
  if (k2n - st2) / m == int32((k2n - st2) / m)
    Kd_k2n = sin(q * k2n * alfa_c / 2) / q / sin(k2n * alfa_c / 2);
    Bs_k2n = muzero * m / 2 * 4 / k2n / Pi * n * sqrt(2) * Irms2 / 2 / deltag * q * Kd_k2n * sin(k2n * gamma);
    Fq_s22(k2) = -Pi * L * Rg / 2 / muzero * Bs_k2 * Bs_k2n * sin((wt + fi_In2) + (wt + fi_In2));
    Fd_s22(k2) = Pi * L * Rg / 2 / muzero * Bs_k2 * Bs_k2n * cos((wt + fi_In2) + (wt + fi_In2));
    Fq = Fq + Fq_s22(k2);
    Fd = Fd + Fd_s22(k2);
  end
%campo k2 DIRETTO
if (k2 - st2) / m == int32((k2 - st2) / m)
  %ampiezza della k2-esima armonica di campo
  Kd_k2 = sin(q * k2 * alfa_c / 2) / q / sin(k2 * alfa_c / 2);
  Bs_k2 = muzero * m / 2 * 4 / k2 / Pi * n * sqrt(2) * Irms2 / 2 / deltag * q * Kd_k2 * sin(k2 * gamma);
  k2p = (1 + k2 * N_) / N_; 
  k2n = (-1 + k2 * N_) / N_; 
  if (k2p + st2) / m == int32((k2p + st2) / m)
    Kd_k2p = sin(q * k2p * alfa_c / 2) / q / sin(k2p * alfa_c / 2);
    Bs_k2p = muzero * m / 2 * 4 / k2p / Pi * n * sqrt(2) * Irms2 / 2 / deltag * q * Kd_k2p * sin(k2p * gamma);
    Fq_s22(k2) = Pi * L * Rg / 2 / muzero * Bs_k2 * Bs_k2p * sin((wt + fi_In2) - (wt + fi_In2));
    Fd_s22(k2) = Pi * L * Rg / 2 / muzero * Bs_k2 * Bs_k2p * cos((wt + fi_In2) - (wt + fi_In2));
    Fq = Fq + Fq_s22(k2);
    Fd = Fd + Fd_s22(k2);
  end
  if (k2p - st2) / m == int32((k2p - st2) / m)
    Kd_k2p = sin(q * k2p * alfa_c / 2) / q / sin(k2p * alfa_c / 2);
    Bs_k2p = muzero * m / 2 * 4 / k2p / Pi * n * sqrt(2) * Irms2 / 2 / deltag * q * Kd_k2p * sin(k2p * gamma);
    Fq_s22(k2) = Pi * L * Rg / 2 / muzero * Bs_k2 * Bs_k2p * sin((wt + fi_In2) + (wt + fi_In2));
    Fd_s22(k2) = Pi * L * Rg / 2 / muzero * Bs_k2 * Bs_k2p * cos((wt + fi_In2) + (wt + fi_In2));
    Fq = Fq + Fq_s22(k2);
    Fd = Fd + Fd_s22(k2);
  end
%
Design and development of a control system for multiphase synchronous PM bearingless machines

\[
\text{Bs}_k2p = \mu_{\text{zero}} \cdot m / 2 \cdot 4 \cdot k2p / \pi \cdot n \cdot \sqrt{2} \cdot \text{Irms}2 / 2 / \text{deltag} \cdot q \cdot \sin(k2p \cdot \gamma);
\]
\[
\text{Fq}_s22(k2) = \pi \cdot L \cdot \text{Rg} / 2 / \mu_{\text{zero}} \cdot \text{Bs}_k2 \cdot \text{Bs}_k2p \cdot \sin(-\omega t + \theta_{\text{In2}}) - \omega t + \theta_{\text{In2}}); \]
\[
\text{Fd}_s22(k2) = \pi \cdot L \cdot \text{Rg} / 2 / \mu_{\text{zero}} \cdot \text{Bs}_k2 \cdot \text{Bs}_k2p \cdot \cos(-\omega t + \theta_{\text{In2}}) - \omega t + \theta_{\text{In2}}); \]
\[
\text{Fs} = \text{Fq} + \text{Fq}_{s22}(k2); \]
\[
\text{Fd} = \text{Fd} + \text{Fd}_{s22}(k2); \]
\[
\text{end}
\]
\[
\text{if} (k2p - st2) / m == \text{int32}((k2p - st2) / m)
\]
\[
\text{Kd}_k2p = \sin(q \cdot k2p \cdot \alpha_c / 2) / q / \sin(k2p \cdot \alpha_c / 2);
\]
\[
\text{Bs}_k2p = \mu_{\text{zero}} \cdot m / 2 \cdot 4 / k2p / \pi \cdot n \cdot \sqrt{2} \cdot \text{Irms}2 / 2 / \text{deltag} \cdot q \cdot \sin(k2p \cdot \gamma);
\]
\[
\text{Fq}_s22(k2) = \pi \cdot L \cdot \text{Rg} / 2 / \mu_{\text{zero}} \cdot \text{Bs}_k2 \cdot \text{Bs}_k2p \cdot \sin(-\omega t + \theta_{\text{In2}}) + \omega t + \theta_{\text{In2}}); \]
\[
\text{Fd}_s22(k2) = \pi \cdot L \cdot \text{Rg} / 2 / \mu_{\text{zero}} \cdot \text{Bs}_k2 \cdot \text{Bs}_k2p \cdot \cos(-\omega t + \theta_{\text{In2}}) + \omega t + \theta_{\text{In2}}); \]
\[
\text{Fs} = \text{Fq} + \text{Fq}_{s22}(k2); \]
\[
\text{Fd} = \text{Fd} + \text{Fd}_{s22}(k2); \]
\[
\text{end}
\]
\[
\text{if} (k2n + st2) / m == \text{int32}((k2n + st2) / m)
\]
\[
\text{Kd}_k2n = \sin(q \cdot k2n \cdot \alpha_c / 2) / q / \sin(k2n \cdot \alpha_c / 2);
\]
\[
\text{Bs}_k2n = \mu_{\text{zero}} \cdot m / 2 \cdot 4 / k2n / \pi \cdot n \cdot \sqrt{2} \cdot \text{Irms}2 / 2 / \text{deltag} \cdot q \cdot \sin(k2n \cdot \gamma);
\]
\[
\text{Fq}_s22(k2) = -\pi \cdot L \cdot \text{Rg} / 2 / \mu_{\text{zero}} \cdot \text{Bs}_k2 \cdot \text{Bs}_k2n \cdot \sin(-\omega t + \theta_{\text{In2}}) - \omega t + \theta_{\text{In2}}); \]
\[
\text{Fd}_s22(k2) = \pi \cdot L \cdot \text{Rg} / 2 / \mu_{\text{zero}} \cdot \text{Bs}_k2 \cdot \text{Bs}_k2n \cdot \cos(-\omega t + \theta_{\text{In2}}) - \omega t + \theta_{\text{In2}}); \]
\[
\text{Fs} = \text{Fq} + \text{Fq}_{s22}(k2); \]
\[
\text{Fd} = \text{Fd} + \text{Fd}_{s22}(k2); \]
\[
\text{end}
\]
\[
\text{if} (k2n - st2) / m == \text{int32}((k2n - st2) / m)
\]
\[
\text{Kd}_k2n = \sin(q \cdot k2n \cdot \alpha_c / 2) / q / \sin(k2n \cdot \alpha_c / 2);
\]
\[
\text{Bs}_k2n = \mu_{\text{zero}} \cdot m / 2 \cdot 4 / k2n / \pi \cdot n \cdot \sqrt{2} \cdot \text{Irms}2 / 2 / \text{deltag} \cdot q \cdot \sin(k2n \cdot \gamma);
\]
\[
\text{Fq}_s22(k2) = -\pi \cdot L \cdot \text{Rg} / 2 / \mu_{\text{zero}} \cdot \text{Bs}_k2 \cdot \text{Bs}_k2n \cdot \sin(-\omega t + \theta_{\text{In2}}) + \omega t + \theta_{\text{In2}}); \]
\[
\text{Fd}_s22(k2) = \pi \cdot L \cdot \text{Rg} / 2 / \mu_{\text{zero}} \cdot \text{Bs}_k2 \cdot \text{Bs}_k2n \cdot \cos(-\omega t + \theta_{\text{In2}}) + \omega t + \theta_{\text{In2}}); \]
\[
\text{Fs} = \text{Fq} + \text{Fq}_{s22}(k2); \]
\[
\text{Fd} = \text{Fd} + \text{Fd}_{s22}(k2); \]
\[
\text{end}
\]
\[
\text{end}
\]
\[
\text{end}
\]
\[
\text{end}
\]

\%CALCULO RISULTANTI

\%modulo della forza radiale
\%
\%angolo misurato rispetto all'asse y (FEMM)
\%
\%FI_R = atan2(Fq, Fd);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% sequenza ST1: CALCULO COPPIA Torque1(k)

\%
\% for k = 1 : k_max
\%
\% if (k + st1) / m == int32((k + st1) / m)
\% c1 = 1;
\% rot(k) = 'I';
\% elseif (k - st1) / m == int32((k - st1) / m)
\% c2 = 2;
\% rot(k) = 'D';
\% else
\% c1 = 0;
\% rot(k) = ' ';
\% end

225
if ctr(k) == 1 || ctr(k) == 2
  % fattore di distribuzione
  Kd(k) = sin(q * k * alfa_c / 2) / q / sin(k * alfa_c / 2);
  % ampiezza della k-esima armonica di campo
  Bs(k) = muzero * m / 2 * 4 / k / Pi * n * sqrt(2) * Irms1 / 2 / deltag * q * Kd(k) *
    sin(k * gamma);
  if k / 2 ~= int32(k / 2)
    Br(k) = 4 / k / Pi * BgM * sin(k * M_ * alfa_mag / 2);
  else if ctr(k) == 1
    Torque1(k) = 1 / muzero * deltag * L * N_^2 * Taup * Bs(k) * Br(k) * sin(k * M_
      * deltath + wt + fi_In1);
  elseif ctr(k) == 2
    Torque1(k) = 1 / muzero * deltag * L * N_^2 * Taup * Bs(k) * Br(k) * sin(k * M_
      * deltath - wt - fi_In1);
  else
    Torque1(k) = 0;
    end
  else
    Br(k) = 0;
    Torque1(k) = 0;
  end
end

% CALCOLO RISULTANTE
Torque1sum = 0;
for k = 1 : k_max
  Torque1sum = Torque1sum + Torque1(k);
end

% sequenza ST2: CALCOLO COPPIA Torque2(k)
for k = 1 : k_max
  if (k + st2) / m == int32((k + st2) / m)
    ctr(k) = 1;
    rot(k) = 'I';
  elseif (k - st2) / m == int32((k - st2) / m)
    ctr(k) = 2;
    rot(k) = 'D';
  else
    ctr(k) = 0;
    rot(k) = ' ';
  end
  if ctr(k) == 1 || ctr(k) == 2
    % fattore di distribuzione
    Kd(k) = sin(q * k * alfa_c / 2) / q / sin(k * alfa_c / 2);
    % ampiezza della k-esima armonica di campo
    Bs(k) = muzero * m / 2 * 4 / k / Pi * n * sqrt(2) * Irms2 / 2 / deltag * q * Kd(k) *
      sin(k * gamma);
    if k / 2 ~= int32(k / 2)
      Br(k) = 4 / k / Pi * BgM * sin(k * M_ * alfa_mag / 2);
    else if ctr(k) == 1
      Torque2(k) = 1 / muzero * deltag * L * N_^2 * Taup * Bs(k) * Br(k) * sin(k * M_
        * deltath + wt + fi_In2);
    elseif ctr(k) == 2
      Torque2(k) = 1 / muzero * deltag * L * N_^2 * Taup * Bs(k) * Br(k) * sin(k * M_
        * deltath - wt - fi_In2);
    else
      Torque2(k) = 0;
      end
    else
      Br(k) = 0;
      Torque2(k) = 0;
    end
  end
Design and development of a control system for multiphase synchronous PM bearingless machines

```
end
else
Kd(k) = 0;
Bs(k) = 0;
end
end %(for k = 1 : k_max)

%CALCOLO RISULTANTE
Torque2sum = 0;
for k = 1 : k_max
    Torque2sum = Torque2sum + Torque2(k);
end

ABF = [Fq,Fd,Fr1,fi_Fr,Torque1sum,Torque2sum,Irms1,fi_In1,Irms2,fi_In2,deltath];
```

Note that the variables $F_q$, $F_d$, restituted as output arguments of the "Matlab function", correspond to the searched $F_y$, $F_z$ components of the radial force.

E) Forces To Moments Matrix Block

This block simply provides to calculate the moments given by the resultant radial force in the absolute reference system, by means of the contributions of its components (5.40)-(5.41):

$$
\begin{align*}
\overline{M}_{F_Y} &= -F_y \frac{L_{sh}}{2} \sin \varphi \hat{i} + F_y \frac{L_{sh}}{2} \cos \varphi \cos \psi \hat{k} \quad (5.40) \\
\overline{M}_{F_Z} &= F_z \frac{L_{sh}}{2} \cos \varphi \sin \psi \hat{i} - F_z \frac{L_{sh}}{2} \cos \varphi \cos \psi \hat{j} \quad (5.41)
\end{align*}
$$

![Fig. 5.13](image-url)
The relationships were implemented in SIMULINK® by describing every single term, as shown in Fig. 5.13.

5.4.2 Euler’s Equations Block

A) Applied Moments Block

In the Applied Moments block, the moments of the resultant radial forces are recalculated from the abs to the ftr reference system. The important feature in this block, is the “ABSOLUTE to FTR” function which provides to realize the above action; other blocks are used for auxiliary functions, as the signals scope.

The array \( MFyz \) represents the moments already calculated in the Levitation Forces block, which are added in summation to the moment produced by the weight force, given in (5.42) with respect to the abs reference system.

\[
\mathbf{M}_g = \begin{bmatrix}
\hat{i} & \hat{j} & \hat{k} \\
\frac{L_{sh}}{2} \cos \phi \cos \psi & \frac{L_{sh}}{2} \cos \phi \sin \psi & \frac{L_{sh}}{2} \sin \phi \\
0 & 0 & -mg
\end{bmatrix}
\]

\[
= mg \frac{L_{sh}}{2} (- \cos \phi \sin \psi \hat{i} + \cos \phi \cos \psi \hat{j})
\]

(5.42)
Consider the mass \( m \) of the rotor and the shaft, the center of gravity located on the axis with a fixed point at the origin \( O \) of the system at a distance \( L_{sh}/2 \) from the latter. The total moment is then expressed, by means of the \( \overline{B}(\theta, \varphi, \psi) \) matrix (5.31), with respect to the \( \text{fir} \) reference system and passed to the next block.

**B) Euler’s Equations Block**

![Euler’s Equations Block Diagram](image)

This part of the system implements the Euler’s equations, seen in (5.20) as rearranged in (5.33): the output of the block is constituted by the angular speed vector with respect to the \( \text{fir} \) reference system (Fig. 5.15). The variables marked with a capital letter which compare in Fig. 5.15 correspond to the moments of inertia seen in (5.33).
5.5 The setting of PID controllers

To set up the PID controllers, firstly the system was analyzed by disabling the Force Controller and Electromagnetic model blocks, because of the high complexity of these Matlab functions which introduce the Fourier harmonic series distribution of the various magnetic fields: in fact, considering the whole harmonic contributions, it would be very difficult to define the corresponding transfer function. On the contrary, in this way the PID controllers output gives directly the $F_y$, $F_z$ components of the resultant force. Obviously, this represents only an intermediate step to produce some provisional values of the PID coefficients $K_i$, $K_p$ and $K_d$, being not the original system, but it permitted to obtain the actual values in an easier way. In fact, after having calibrated the values of the PID coefficients by means of this simplified analysis, the two functions were reintroduced to set up and recalculate the coefficients in the actual, original configuration.

The analytical approach to the problem was formulated by expressing the force, resultant of the interactions between magnetic fields by means of its $y$- and $z$-components, as the direct result of the PID controllers regulation, thus as a combination of proportional, integral and derivative actions (5.43), (5.51). Also, the rotor is considered subject to the weight force, obviously acting along the $z$-axis:

A) Equilibrium along the $y$-axis:

$$f_y(t) = K_p \Delta y(t) + K_i \int_0^t \Delta y(\tau) d\tau + K_d \frac{d[\Delta y(t)]}{dt}$$  \hspace{1cm} (5.43)

Where $\Delta y$ represents the error with respect to $y$ coordinate, as defined in (5.45). Applying the Laplace transformation, it gives:
Design and development of a control system for multiphase synchronous PM bearingless machines

\[ F_y(s) = K_p \Delta Y(s) + \frac{K_i}{s} \Delta Y(s) + K_d \left[ s \Delta Y(s) - \Delta y(0) \right] \]  \hspace{1cm} (5.44)

\[ \Delta y(t) = y(t) - y(0) \quad \Rightarrow \quad \Delta Y(s) = Y(s) - \frac{y(0)}{s} \]  \hspace{1cm} (5.45)

By substituting (5.45) in (5.44) it gives:

\[ F_y(s) = K_p Y(s) - K_p \frac{y(0)}{s} + \frac{K_i}{s} Y(s) - \frac{K_i}{s^2} y(0) + K_d s Y(s) - K_d y(0) \]  \hspace{1cm} (5.46)

By applying the second law of motion along the y axis:

\[ m \ddot{y}(t) - f_y(t) = 0 \quad \Rightarrow \quad mL \left[ \frac{d^2 y}{dt^2} \right] = L[f_y(t)] \]  \hspace{1cm} (5.47)

By substituting the respective \( L \)-transforms in the equation (5.47) it gives:

\[ m \left[ s^2 Y(s) - sy(0) - \dot{y}(0) \right] = \]
\[ = K_p Y(s) - K_p \frac{y(0)}{s} + \frac{K_i}{s} Y(s) - \frac{K_i}{s^2} y(0) + K_d s Y(s) - K_d y(0) \]  \hspace{1cm} (5.48)

Considering that \( y(0) = \dot{y}(0) = 0 \), by substituting into (5.48) it gives:

\[ m s^2 Y(s) = K_p Y(s) + \frac{K_i}{s} Y(s) + K_d s Y(s) \]  \hspace{1cm} (5.49)

By collecting the common terms in (5.49), it gives the trivial solution (5.50):

\[ Y(s) = 0 \]  \hspace{1cm} (5.50)

Similarly, proceed to writing the equation along the z- axis:

**B) Equilibrium along the z-axis:**

\[ f_z(t) = K_p \Delta z(t) + K_i \int_0^t \Delta z(\tau) d\tau + K_d \frac{d[\Delta z(t)]}{dt} \]  \hspace{1cm} (5.51)
\[ F_z(s) = K_p \Delta Z(s) + \frac{K_i}{s} \Delta Z(s) + K_d \left[ s \Delta Z(s) - \Delta z(0) \right] \] \hspace{1cm} (5.52)

\[ \Delta z(t) = z(t) - z(0) \implies \Delta Z(s) = Z(s) - \frac{z(0)}{s} \] \hspace{1cm} (5.53)

\[ F_z(s) = K_p Z(s) - K_p \frac{z(0)}{s} + \frac{K_i}{s} Z(s) - \frac{K_i}{s^2} z(0) + K_d s Z(s) - K_d z(0) \] \hspace{1cm} (5.54)

This time the equation includes the weight force, as said above:

\[ m\ddot{z}(t) = f_z(t) - mg \implies mL \left[ \frac{d^2 z}{dt^2} \right] = L[f_z(t)] - \frac{mg}{s} \] \hspace{1cm} (5.55)

\[ m \left[ s^2 Z(s) - sz(0) - \dot{z}(0) \right] = K_p Z(s) - K_p \frac{z(0)}{s} + \frac{K_i}{s} Z(s) - \frac{K_i}{s^2} z(0) + K_d s Z(s) - K_d z(0) - \frac{mg}{s} \] \hspace{1cm} (5.56)

\[ ms^2 Z(s) = K_p Z(s) + \frac{K_i}{s} Z(s) + K_d s Z(s) - \frac{mg}{s} \] \hspace{1cm} (5.57)

By collecting the common terms in (5.57), it gives (5.58), (5.59):

\[ Z(s) \left[ ms^3 - K_d s^2 - K_p s - K_i \right] = -mg \] \hspace{1cm} (5.58)

\[ Z(s) = \frac{-mg}{ms^3 - K_d s^2 - K_p s - K_i} = \frac{-g}{s^3 - \frac{K_d}{m} s^2 - \frac{K_p}{m} s - \frac{K_i}{m}} \] \hspace{1cm} (5.59)

The \( L \)-transform of the \( z \) coordinate (5.59) describes the height of the shaft ending point with respect to the centre of the motor. By studying the stability of the equation (5.59) is possible to obtain the order of magnitude of the PID coefficients. It is not the exact solution because, as mentioned before, a simplified system configuration is examined. To find the correlation between the three poles and the coefficients of the polynomial equation in \( s \), given by putting the denominator of (5.59) equal to zero, a generic expression of a third degree
The polynomial is written:

\[(s - p_1)(s - p_2)(s - p_3) = s^3 - (p_1 + p_2 + p_3)s^2 + (p_1p_2 + p_1p_3 + p_2p_3)s - p_1p_2p_3 = 0\]  \hspace{1cm} (5.60)

By equating the coefficients of the polynomial (5.60) to those of the denominator in the equation (5.59), it gives:

\[
\begin{align*}
p_1 + p_2 + p_3 &= \frac{K_d}{m} \\
p_1p_2 + p_1p_3 + p_2p_3 &= -\frac{K_p}{m} \\
p_1p_2p_3 &= \frac{K_i}{m}
\end{align*}
\]  \hspace{1cm} (5.61)

To solve the problem, the easiest way is to choose only one pole of multiplicity equal to three and of negative value if real or, if complex, having a real part of negative value (5.62), to assure the stability of the system:

\[
p_1 = p_2 = p_3 = p \begin{cases} p < 0 & \text{if } p \in \mathbb{R} \\ |\text{Re}\{p\}| < 0 & \text{if } p \in \mathbb{C} \end{cases}
\]  \hspace{1cm} (5.62)

Finally, by substituting (5.62) in (5.61) and developing the equations, it’s possible to obtain the relationships between the PID coefficients and the pole; choosing suitably its value, are determined $K_i$, $K_p$ and $K_d$:

\[
\begin{align*}
3p &= \frac{K_d}{m} \quad \Rightarrow \quad K_d = 3mp \\
3p^2 &= -\frac{K_p}{m} \quad \Rightarrow \quad K_p = -3mp^2 \\
p^3 &= \frac{K_i}{m} \quad \Rightarrow \quad K_i = mp^3
\end{align*}
\]  \hspace{1cm} (5.63)

In the next, $p$ is chosen as a real number: the general criterion is to vary the value of $p$ until the maximum excursion of the motor shaft ending point falls within the
desired tolerance, which can be reasonably fixed in one-tenth of the amplitude of the airgap, or less if necessary.
5.6 Simulations and results

The software SIMULINK® was used in order to complete the simulations; the analyzed machine is characterized by the following parameters (Tab. I):

<table>
<thead>
<tr>
<th>Param.</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_{dl} )</td>
<td>number of slots</td>
<td>30</td>
</tr>
<tr>
<td>( p )</td>
<td>pole pairs of the machine</td>
<td>1</td>
</tr>
<tr>
<td>( m )</td>
<td>number of phases</td>
<td>5</td>
</tr>
<tr>
<td>( I_s )</td>
<td>rated phase current (A)</td>
<td>59.82</td>
</tr>
<tr>
<td>( T_n )</td>
<td>rated torque (Nm)</td>
<td>30.29</td>
</tr>
<tr>
<td>( g )</td>
<td>airgap width (mm)</td>
<td>1</td>
</tr>
<tr>
<td>( D_e )</td>
<td>stator outer diameter (mm)</td>
<td>230</td>
</tr>
<tr>
<td>( D_i )</td>
<td>stator inner diameter (mm)</td>
<td>120</td>
</tr>
<tr>
<td>( D_m )</td>
<td>mean diameter of the magnet (mm)</td>
<td>116</td>
</tr>
<tr>
<td>( D_{cv, ext} )</td>
<td>diameter at the bottom of the slot (mm)</td>
<td>170</td>
</tr>
<tr>
<td>( D_{cv, int} )</td>
<td>diameter at the top of the slot (mm)</td>
<td>126.3</td>
</tr>
<tr>
<td>( D_r )</td>
<td>rotor outer diameter (mm)</td>
<td>114</td>
</tr>
<tr>
<td>( D_{ri} )</td>
<td>rotor inner diameter (mm)</td>
<td>60</td>
</tr>
<tr>
<td>( \alpha_{cv} )</td>
<td>slot pitch angle</td>
<td>12°</td>
</tr>
<tr>
<td>( a_{st} )</td>
<td>stator slot height (mm)</td>
<td>25</td>
</tr>
<tr>
<td>( h_{cl} )</td>
<td>slot opening height (mm)</td>
<td>1</td>
</tr>
<tr>
<td>( L )</td>
<td>axial length of the machine (mm)</td>
<td>180</td>
</tr>
<tr>
<td>( L_{sh} )</td>
<td>total length of the shaft (mm)</td>
<td>320</td>
</tr>
<tr>
<td>( D_{sh} )</td>
<td>shaft diameter (mm)</td>
<td>40</td>
</tr>
<tr>
<td>( L_m )</td>
<td>magnet width (mm)</td>
<td>2</td>
</tr>
<tr>
<td>( L_d )</td>
<td>tooth-body width (mm)</td>
<td>8</td>
</tr>
<tr>
<td>( L_{cl} )</td>
<td>slot opening width (mm)</td>
<td>2</td>
</tr>
<tr>
<td>( L_w )</td>
<td>slot width at the top slot radius (mm)</td>
<td>5.23</td>
</tr>
<tr>
<td>( L_{fc} )</td>
<td>slot width at the bottom slot radius (mm)</td>
<td>9.7</td>
</tr>
<tr>
<td>( \tau_{cv} )</td>
<td>slot pitch at the inner stator radius (mm)</td>
<td>12.57</td>
</tr>
<tr>
<td>( m )</td>
<td>rotor and shaft mass (kg)</td>
<td>16.75</td>
</tr>
<tr>
<td>( I_{0f} )</td>
<td>moment of inertia, ( I ) axis (kg m²)</td>
<td>2.7·10⁻²</td>
</tr>
<tr>
<td>( I_{0c} )</td>
<td>moment of inertia, ( \varepsilon ) axis (kg m²)</td>
<td>50.6·10⁻²</td>
</tr>
<tr>
<td>( I_{0\eta} )</td>
<td>moment of inertia, ( \eta ) axis (kg m²)</td>
<td>50.6·10⁻²</td>
</tr>
</tbody>
</table>
Using the method described in the previous section, a list of values for $K_i$, $K_p$ and $K_d$ is obtained, shown in Tab. II. By applying these values in the simulation, it can be seen that by increasing the absolute value of $p$, the maximum excursion of the motor shaft ending point is progressively reduced.

<table>
<thead>
<tr>
<th>$p$</th>
<th>$K_d$</th>
<th>$K_p$</th>
<th>$K_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-20</td>
<td>-1.005E+03</td>
<td>-2.01E+04</td>
<td>-1.34E+05</td>
</tr>
<tr>
<td>-40</td>
<td>-2.01E+03</td>
<td>-8.04E+04</td>
<td>-1.072E+06</td>
</tr>
<tr>
<td>-80</td>
<td>-4.02E+03</td>
<td>-3.216E+05</td>
<td>-8.576E+06</td>
</tr>
</tbody>
</table>

In the following, will be analyzed the simulation results for $p = -80$ by representing in the Figs. 5.17, 5.18, the positions of the shaft ending point and of the axis point corresponding to the rotor stack length in the $y$, $z$ coordinates of the absolute reference system (Fig. 5.1). It is important to note that, with reference to the constrained extremity of the shaft, the rotor stack length extends up to 250 mm and its corresponding axis point position represents the parameter to be verified. The shaft ending point extends up to 320 mm, thus its excursion will be obviously greater than the latter (Fig. 5.16).
The simulations were performed at a rotor angular speed $\omega_t = 1004.8 \text{ rad/s}$, corresponding to 9595.13 rpm and the torque current has the rated value of 59.82 $A_{\text{rms}}$. 
As explained above, all the possible interactions between magnetic fields acting in the airgap are considered, by means of the “Electromagnetic model”. Thus, the analyzed situation can be considered as a complete and realistic operating condition of the bearingless machine. In addition, the locus occupied by the same points on the y-z plane, represented in Fig. 5.19, provides a clearer representation of the rotor axis position. It is also interesting to observe the behavior of the torque and levitation current space vectors, \( \vec{i}_{sv1} \) and \( \vec{i}_{sv2} \), in terms of rms value and phase, Figs. 5.20-5.21:
Design and development of a control system for multiphase synchronous PM bearingless machines

Fig. 5.20

Fig. 5.21
The current space vector 1 has an imposed constant rms value (59.82) to generate continuously the requested torque, its phase which varies remaining in leading by 90 electrical degrees with respect to magnet axis.

The modulus of current space vector 2, after having reached a maximum value of almost 4.5 Amps rms in the early instants of time, oscillates between 3 and 3.3 Amps, while the phase continuously changes its value in the whole range (0 to 360 electrical degrees), having to follow the spatial phase of the required force necessary to counterbalance the weight and the other forces generated by the interactions between harmonic orders of the magnetic fields. To give a more realistic idea, the resultant force vector acting on the rotor, in the time interval from 0 ms to 100 ms, changes its position on the $yz$ plane as shown in Fig. 5.22.

![Fig. 5.22](image)

As can be seen by comparing the Figs. 5.17-5.19, the maximum value of the excursion of the rotor stack length axis point is about 2 tenth of millimeter in the
negative $z$-axis direction and 1.5 tenth of millimeter in the negative $y$-axis direction, over the prefixed value of 1 tenth, which represents 1 tenth of the airgap width. So, it is necessary to set a different calibration of the PID controllers. The first attempt, based on the values of Tab. II corresponding to $p = -80$, is made by varying the coefficients $K_d$, $K_p$, $K_i$ and verifying the result. Proceeding in this way there is no more correlation with general criterion (5.63), but lower absolute values for parameters could be found with, consequently, an easier way to practically realize the controller. A good compromise is found by acting only on $K_p$, multiplying by three its value in Tab. II. Thus, the values of Tab. III were used and the simulation results are shown in the following:

**TABLE III. VALUES OF PID COEFFICIENTS**

<table>
<thead>
<tr>
<th>$K_d$</th>
<th>$K_p$</th>
<th>$K_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4.02E+03</td>
<td>-9.648E+5</td>
<td>-8.576E+06</td>
</tr>
</tbody>
</table>

![Graph](image)

**Fig. 5.23**
Fig. 5.26

Fig. 5.27
The modulus of current space vector $2$ has an oscillating behavior with a progressive reduction tending to the regime, standing in the range of 3 to 3.4 Amps, as can be noted in Fig. 5.27 and, clearly, in Fig. 5.28 where the same function is represented with an extended time axis.

As can be seen by comparing the Figs. 5.23-5.25, now the maximum excursion of the rotor stack length axis point is about 1 tenth of millimeter in the negative $z$-axis direction and about 0.75 tenth of millimeter in the negative $y$-axis direction. Thus, it is possible to say that the target has been achieved. As done before, in Fig. 5.29 is shown the succession of the different positions occupied by the resultant force vector on the $yz$ plane in the time interval from 0 ms to 100 ms. In the Figs. 5.30, 5.31 the time axis has been scaled up to the value of 1 second with respect to Figs. 5.23-5.24, to highlight the stable state achieved by the system.
Fig. 5.29

Fig. 5.30
The second way to proceed is by continuing to use the general criterion (5.63), which would probably permits a finest regulation of the PID coefficients. Finally, the values in Tab. IV were found with $p = -120$:

### TABLE IV. VALUES OF PID COEFFICIENTS

<table>
<thead>
<tr>
<th>$p$</th>
<th>$K_d$</th>
<th>$K_p$</th>
<th>$K_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-120</td>
<td>-6.03E+03</td>
<td>-7.236E+05</td>
<td>-2.8944E+07</td>
</tr>
</tbody>
</table>

The results of the simulation are shown in the following, from Fig. 5.32 to Fig. 5.37.
Design and development of a control system for multiphase synchronous PM bearingless machines

Fig. 5.32

Fig. 5.33
Fig. 5.36

Fig. 5.37
As can be seen by comparing the obtained results with the previous, a sensible reduction of the overshoot and a faster attainment of regime condition are achieved, even if the absolute values of the PID coefficients are in general greater than in the previous attempt. It can be noted that the system behaves as if in the instant $t = 0^-$ the rotor would be perfectly centered and the motor is off; in the instant $t = 0^+$ the weight force and the other forces, produced by the interactions between stator and rotor magnetic fields, begin their action on the rotor, with the control system trying to bring it in the requested position. In the Figs. 5.38, 5.39 is shown the analyzed 5-phase bearingless motor.

![Fig. 5.38](image)
Design and development of a control system for multiphase synchronous PM bearingless machines

Fig. 5.39
5.7 Conclusion

In this chapter a control system for bearingless multiphase synchronous PM machines is presented, integrated by a three-dimensional mechanical model based on the Euler’s equations.

The electromagnetic model of the machine, seen in Chapter 4, takes into account all the possible interactions between harmonic orders of the magnetic fields produced by the current space vector $\vec{i}_{sv1}$, which gives the torque, produced by the current space vector $\vec{i}_{sv2}$, which gives the levitation forces, and produced by rotor magnets.

Differently from other authors, which propose models that take into account only the main harmonic orders interactions between magnetic fields, the developed system is a complete one, giving in this way a more accurate modeling of the mechanical and electromagnetic phenomena.

For these reasons, it constitutes an important tool for the design of a bearingless multiphase synchronous PM machines control system and represents the design of the experimental device with related control system to realize in a test bench application.
The main scope of this Ph.D thesis is constituted by the non linear analysis and design of bearingless multiphase machines and drives.

The thesis work began with the development of a method to analyze the distributions of the magnetic vector potential, magnetic field and flux density in the airgap of a permanent magnet electrical machines by applying a two-dimensional model. The original contribution of the approach, inspired by a literature paper, consisted in the complete calculations to get the solution of the problem, conducted by using the techniques of mathematical analysis applied to physical and engineering problems. This model is characterized by a linear analysis.

The previous constraint of magnetic linearity is overcome by the second chapter of this Thesis, where an algorithm for the non-linear magnetic analysis of multiphase surface-mounted PM machines with semi-closed slots has been presented. Previous papers proposed the analysis of open-slot configurations with a prefixed structure of the motor, with a given number of poles and slots, or by studying only a particular position of the rotor with respect to the stator. In this work, the PM machine is represented by using a modular structure geometry. The basic element of the geometry is duplicated allowing to build up and analyze whatever typology of windings and ampere-turns distribution in a pair of poles.

In the third chapter the theme of the bearingless machines has been introduced, analyzing and describing the main concepts and ideas developed in the literature.

The fourth chapter presents an analytical model for radial forces
Calculation in multiphase bearingless Surface-Mounted Permanent Magnet Synchronous Motors (SPMSM). The model allows to predict amplitude and direction of the force, depending on the values of the torque current, of the levitation current and of the rotor position. It is based on the space vectors method, letting the analysis of the machine not only in steady-state conditions but also during transients. The calculations are conducted by developing the analytical functions in Fourier series, taking all the possible interactions between stator and rotor mmf harmonic components into account. The proposed method allowed to emphasize the effects of electrical and geometrical quantities like the coil pitch, the width and length of the magnets, the rotor position, the amplitude and phase of current space vector, etc.

In the last chapter a three-dimensional mechanical model model of a bearingless multiphase synchronous PM machines has been analyzed. The mechanical model is based on the Euler’s equations, while the electromagnetic model of the machine, developed in the previous chapter, takes into account all the possible interactions between harmonic orders of the magnetic fields produced by the current space vector mainly responsible for the torque, and by the current space vector injected for producing levitation forces. In the control model, implemented in MATLAB SIMULINK, the errors in the rotor position are used in order to calculate the components of the radial forces necessary to control the rotor axis position of the machine.

The performances of the proposed non-linear model of SPMSM have been compared with those obtained by FEA software in terms of linkage fluxes, co-energy, torque and radial force. The obtained results for a traditional three-phase machine and for a 5-phase machine with unconventional winding distribution showed that the values of local and global quantities are practically coinciding, for values of the stator currents up to rated values. In addition, they are very similar also in the non-linear behavior even if very large current values are
injected. The relevant edge of the method consists in the possibility of defining the machine characteristics in a simple user interface. Then, by duplicating an elementary cell, it is possible to construct and analyze whatever typology of windings and ampere-turns distribution in a pole-pair. Furthermore, it is possible to modify the magnet width-to-pole pitch ratio analyzing various configurations in order to minimize the cogging torque, or simulating the rotor movement in sinusoidal multiphase drives or in a user-defined current distribution.

When developing a new machine design the proposed method is useful not only for the reduction of computing time, but mainly for the simplicity of changing the values of the design variables, being the numerical inputs of the problem obtained by changing some critical parameters, without the need for redesigning the model in a CAD interface. For a given rotor position and for given stator currents, the output torque as well as the radial forces acting on the moving part of a multiphase machine can be calculated. The latter feature makes the algorithm particularly suitable in order to design and analyze bearingless machines. For these reasons, it constitutes a useful tool for the design of a bearingless multiphase synchronous PM machines control system.

With reference to the control system for bearingless machines the presented model allows to calculate the radial force avoiding the errors introduced by the use of only the basic mmf harmonic components. In fact, when designing a control system for bearingless machines, many authors considered only the interaction between the main harmonic orders of the stator and rotor mmfs. In multiphase machines this can produce mistakes in determining both the module and the spatial phase of the radial force, due to the interactions between the higher harmonic orders. In addition, the proposed algorithm permits to study whatever configuration of SPMSM machine, being parameterized as a function of the electrical and geometrical quantities, like the coil pitch, the width and length of the magnets, the rotor position, the amplitude and phase of current
space vector, etc. Finally, the results of the proposed method have been compared with those of a most used FEA software, obtaining very similar values of the analyzed quantities.

In conclusion, this thesis aims to be a complete reference for the design methodologies of multiphase bearingless machines and drives, in the linear and non-linear fields of application.
List of papers by Stefano Serri

