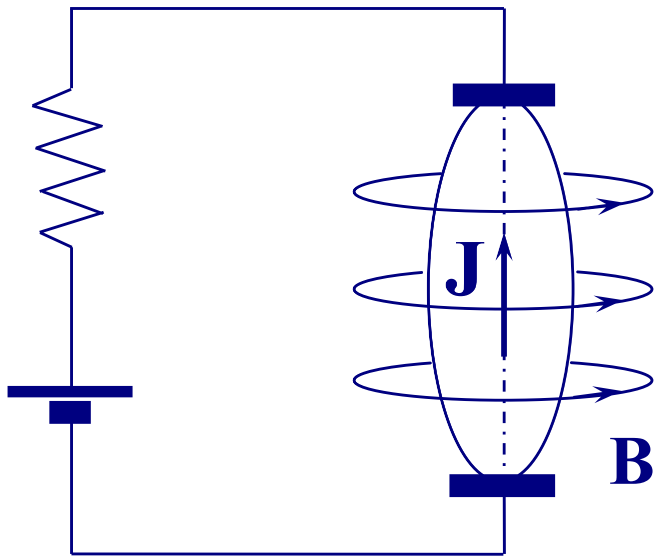
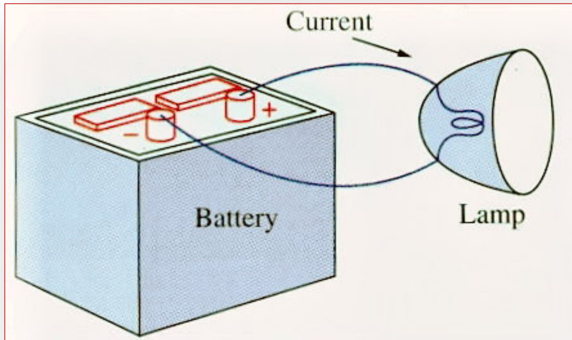
2. Electric Circuit Theory



Electric Circuit Theory

Electric circuit theory and Electromagnetic theory are the two fundamental theories upon which all branches of electrical engineering are based. Many of these branches, such as production, transmission and utilization of electric power, electric machines, control, electronics communication, and instrumentation, are based on electric circuit theory.

In electrical engineering, we want to transfer electric signals or electric power from one point to another. To do this requires an interconnection of electrical devices. A very effective interconnection is the *electric circuit,* which is constituted by electrically interconnected *circuital elements*.



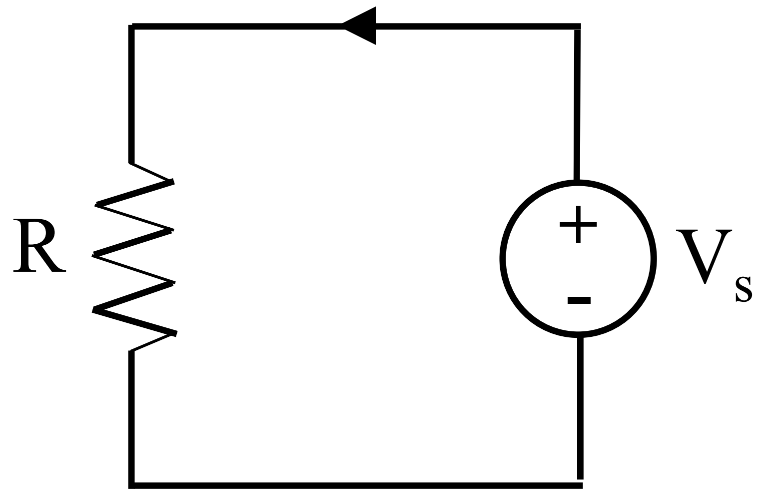
Electric Circuit Theory

- The term ***electric circuit*** indicates the physical place where electromagnetic phenomena are located. The other meaning of the term ***electric circuit*** regards the mathematical models which describes them.

- Usually the term is utilized to indicate the circuits and the relative models, that satisfy the assumption of ***lumped component models*** (also called ***lumped element models*** or ***lumped parameter models***). This assumption considers all electromagnetic phenomena concentrated and confined inside discrete bodies (named ***lumped components***, or ***circuit components,*** or also ***circuit elements***). These elements are electrically connected so that the electric charges can move between them.

Electric Circuit Theory

- In the figure a representation of a lumped model, or circuit model, made up of a ***voltage source*** and a ***resistor***, is shown. The charges are moved by the voltage of the voltage source and flows from one terminal of the voltage source. They go through the electrical resistance and flow to the other terminal of the voltage source.



- This is possible due to the ***electrical connections*** joining the two elements so that the electric charges can move between them.

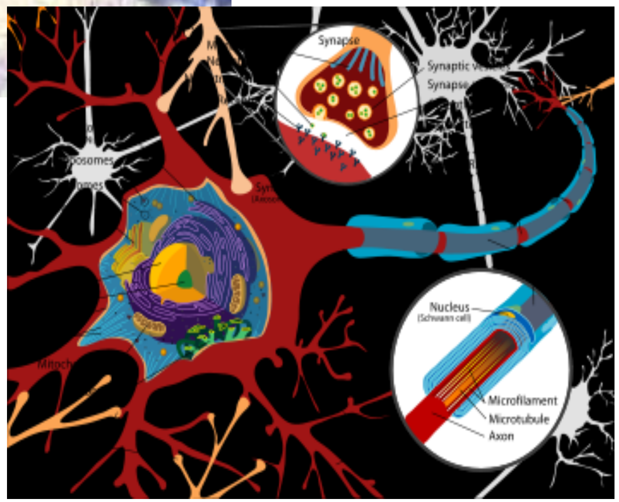
Electric Circuit Theory

- The electrical quantities which describe the electrical behavior of a circuit are integral quantities (lumped parameters, macroscopic quantities that are voltages and currents). They depend on time but not on space. These quantities can be measured in physical circuits.

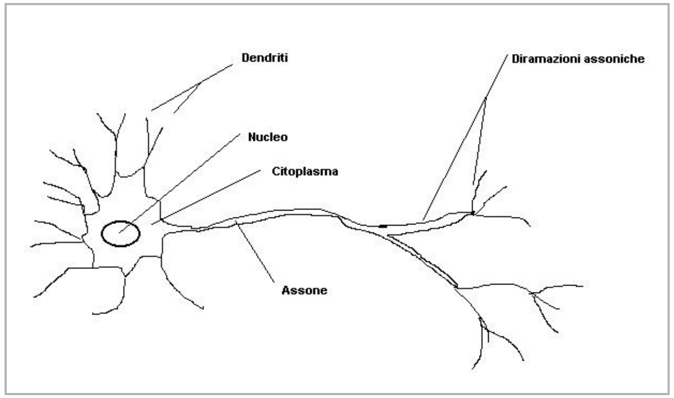
- The equations of the circuit models are ***algebraic equations*** or ***integrodifferential time dependent equations*** that in many cases can be reduced to algebraic equations.

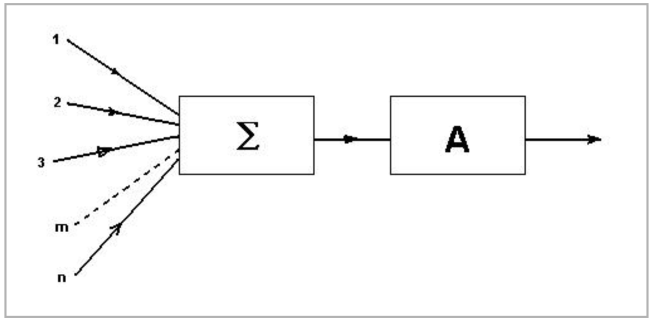
- The quantities of the ***distributed parameter*** models are usually time and space dependent quantities (differential parameters, microscopic quantities that are electric fields and current densities). The equations of these models are partial differential equations in time and space.

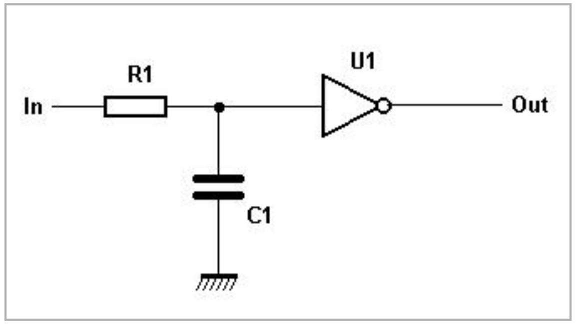
Circuits
in Nature and in Technological Applications



Neurons and their connections



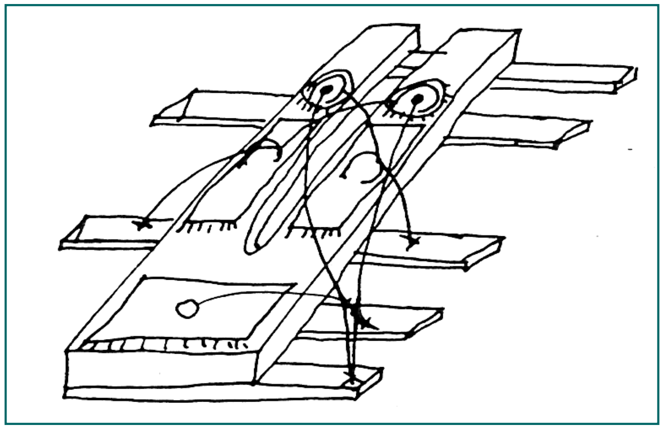




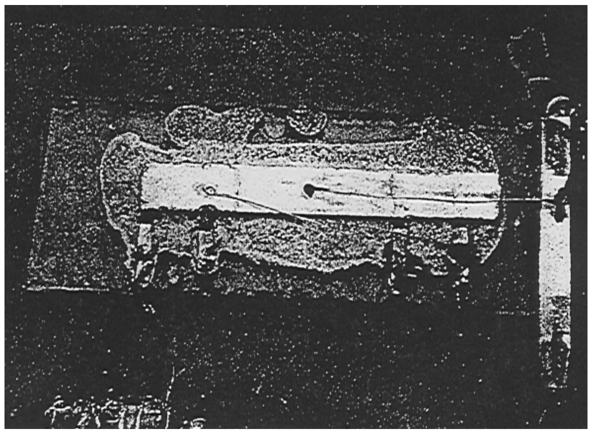
Electric Circuit Theory

- In 1958 the first ***chip*** was realized by J. St. Clair Kilby (Fig. A, B – ***chip*** is an electronic device that contains several solid state circuit elements).

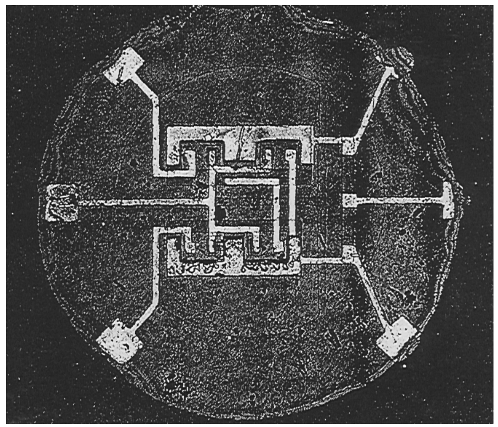
- In 1961 the first ***monolithic chip*** (Fig. C) was made . The realization of monolithic circuits with several circuit elements (***integrated circuits***) allowed to reduce the dimensions of the electric and the electronic devices followed by a rapid development of the technology in this field.



A



B



C

Electric Circuit Theory

- The characteristics of the electromagnetic fields and the properties of the materials (electric conductive, semi-conductive and insulating materials) allowed to develop the very powerful technology of the electric circuits which are studied by means of the electric circuit theory.
- The electric circuits are used in many technological applications for the treatment of information signals and of power (electrical and electronic devices, computers, control devices, telecommunication systems, electric power systems).

Electric Circuit Theory

The ***circuit theory*** aims to simulate and to predict the ***electrical behavior*** of the physical circuits for the analysis and the design of them (to enhance the performance, to decrease their cost, to analyze all working conditions, to study the fault conditions, the thermal effects, the endurance, etc.)

Applications

- ✓ ***dimensions:*** integrated circuits, hi-fi circuits, computers, electronic devices, telecommunication systems, electrical power generation, transmission and utilization systems (10^{-3} - 10^{6} m);
- ✓ ***tension:*** 10^{-6} V (noise analysis devices) - 10^{6} V(electrical power systems);
- ✓ ***current:*** 10^{-15}A (1 fA: electrometers) - 10^{6} A (power systems);
- ✓ ***frequency:*** 0 (direct current) - 10^{9} Hz (1 GHz: microwave circuits, computers);
- ✓ ***power***: 10^{-14} W (radio signals from galaxies) - 10^{9} W (electric power stations).

Integral form of the EM Equations *to be considered for the* Electric Circuit Theory

➢ An electric circuit, made of interconnected circuit elements (resistors, inductors, capacitors, diodes, transistors operational amplifiers), operates through the EM phenomenology.

$$\oint_l \mathbf{H}\cdot d\mathbf{l} = i_t$$ 1st Maxwell's law $$\left(i_t = \iint_S \mathbf{J}_t \cdot \hat{\boldsymbol{n}}\, d\boldsymbol{S} \right)$$

$$\oint_l \mathbf{E}\cdot d\mathbf{l} = -\frac{d\Phi_B}{dt}$$ 2nd Maxwell's law $$\left(\Phi_B = \iint_S \mathbf{B}\cdot \hat{\boldsymbol{n}}\, d\boldsymbol{S} \right)$$

$$\oiint_S \mathbf{J}\cdot\hat{\mathbf{n}}\, dS = -\frac{dq}{dt}$$ charge conservation law

$$\Rightarrow \quad \oiint_S \mathbf{D}\cdot\hat{\mathbf{n}}\, dS = q$$ Gauss's law

$$\Rightarrow \quad \oiint_S \mathbf{J}_t\cdot\hat{\mathbf{n}}\, dS = 0$$

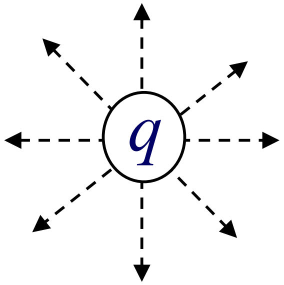
$$\Rightarrow \quad \oiint_S \mathbf{B}\cdot\hat{\mathbf{n}}\, dS = 0$$

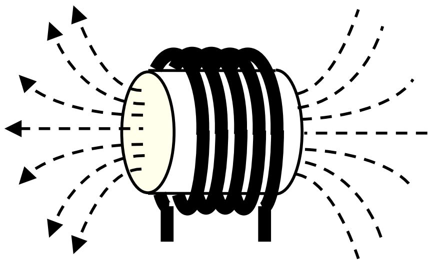
Material laws

$$\begin{cases} \mathbf{H} = \mu \mathbf{B} \\ \mathbf{D} = \varepsilon \mathbf{E} \\ \mathbf{J} = \sigma \mathbf{E} \end{cases}$$

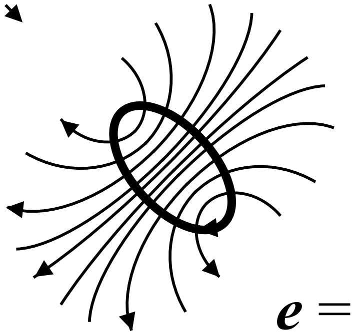
Electric Circuit Theory

$$i_{out} = -\frac{dq}{dt}$$





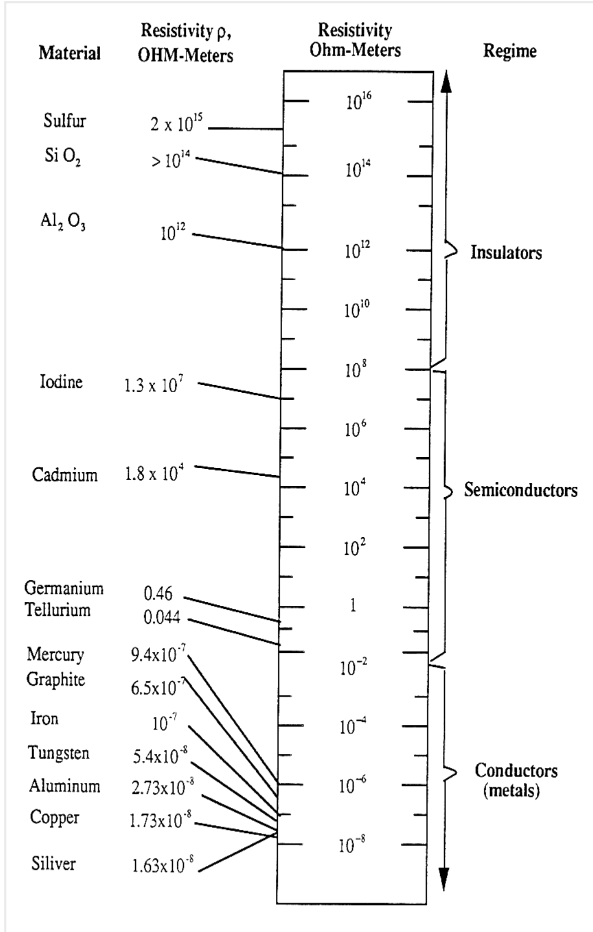
$$\Phi = L\, i$$



$$e = -\frac{d\Phi}{dt}$$

$$J = \sigma E$$

$$\rho = \frac{1}{\sigma}$$



Circuit element (circuit component): it is a region of space inside which the electromagnetic phenomena are confined.

Interconnections: they are conducing channels (usually cables) where the charges flows from a component to another..

Electric Circuit Theory Assumptions

- The ***circuit model*** following the ***lumped component model approximation*** is based on the assumption of quasi-stationary approximation of the EM equations and on the properties of materials.

- Quasi-stationary approximation states that electric displacement **D** and magnetic flux density **B** do not vary in time simultaneously in the same place

$$\frac{\partial \mathbf{B}}{\partial \mathrm{t}} \neq 0, \frac{\partial \mathbf{D}}{\partial \mathrm{t}} = 0 \quad oppure \quad \frac{\partial \mathbf{B}}{\partial \mathrm{t}} = 0, \frac{\partial \mathbf{D}}{\partial \mathrm{t}} \neq 0$$

- Moreover due to the quasi-stationary assumption the propagation of the EM quantities is assumed to be instantaneous ($\Delta t_{propag} = 0$, the propagation velocity is assumed to be infinite).

Electric Circuit Theory Assumptions

- A circuit is constituted by circuit elements connected by conductors . The circuit is immersed into an insulating material. The assumption of quasi-stationary EM, necessary to the lumped circuit model, may be done due to the materials constituting a circuit. Indeed here it is assumed that:

1. The time variation of the magnetic flux **outside circuit elements** is zero.

$$\frac{d\Phi}{dt} = 0$$

*$\mathbf{B}$ varies in time only inside elements → $\nabla\times\mathbf{E}=-\partial B/\partial t$: outside the elements $\mathbf{E}$ is **conservative**, inside the elements $\mathbf{E}$ is **not conservative**.*

2. The time variation of the charge **inside conductors,** where the conductivity is assumed to be infinite, is zero.

$$\frac{dq}{dt} = 0$$

*Inside conductors q does not varies in time as $\sigma = \infty$ → from Gauss low $\partial \mathbf{D}/\partial t = \partial \rho_C/\partial t = 0$ → $\mathbf{J} = \mathbf{J_t}$ and $\mathbf{J}$ is **solenoidal.***

- The EM phenomena are considered to be confined inside the circuit elements.

Electric Circuit Theory Assumptions

As $dq/dt = 0$ inside conductors and that they are immersed into insulating material allows to consider the conductors as flux tubes of **J**. Moreover inside connectors the displacement current density $\mathbf{J_D} = \partial \mathbf{D}/\partial t = 0$ and that $\mathbf{J_t} = \mathbf{J}+\mathbf{J_D} = \mathbf{J}$. Hence inside a conductor **J** is ***solenoidal*** ($\nabla \cdot \mathbf{J_t} = \nabla \cdot \mathbf{J} = 0$). Hence it follows:

- Due to the solenoidality of **J** ***the current is constant in any cross section of a connector***.
- Moreover, due to the solenoidality of **J** for a closed surface S which passes completely outside the circuit elements, it is:

$$\oiint_S \mathbf{J} \cdot \mathbf{n}\, dS = 0$$

The total flux of **J** through S is given by the currents $i_1, i_2, \ldots, i_n$ flowing outside of the surface. Thus it is:

$$i_1 + i_2 + \ldots + i_n = 0$$

This is the Kickoff Current Law (KCL)

Electric Circuit Theory Assumptions

As $d\Phi/dt = 0$ outside the circuit elements, outside the circuit elements the electric field is conservative and the tension in a closed line not intersecting circuit elements is zero:

$$\oint \mathbf{E} \cdot \mathbf{dl} = - d\Phi/dt = 0$$

Hence it follows that:

- The electrical tension (the voltage) between two points connected by a line in conductors connecting circuit elements running outside of them, is given by the electric potential difference of the potential in the two points:

$$\int_1^2 \mathbf{E} \cdot \mathbf{dl} = v_1 - v_2 = v_{12}$$

- For a closed line connecting the points in conductors, running outside the elements, it is :

$$v_{12} + v_{23} + \ldots + v_{n,1} = 0$$

This is the Kickoff Tension Law (KTL)

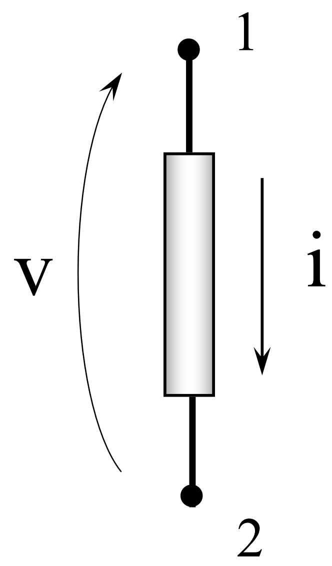
Circuit Element
The Two Terminal Circuit Element

- The ***two terminal circuit element*** (also said ***dipole***) is a circuit element consisting of a closed surface S from which two terminals come out. All EM phenomena are active inside S. Here **E** can be non-conservative and it can vary (**B** or **D** varies in time). Outside S **D** and **B** are constant in time and all EM phenomena are silent. Hence the **E** field is conservative. S has a shielding effect. In a circuit the circuit elements are connected to each other through conductors connecting their terminals.

- The status of the element is described by current and tension: i, v.

- As a consequence of the charge conservation low the current flowing through an element is the current entering from a terminal and going out from the other one.

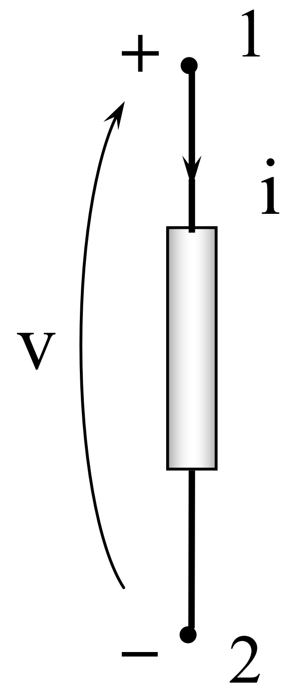
- Outside S, **E** is conservative. Therefore outside S a potential function v exists and is defined by v_1 and v_2, which are the values of v at terminal 1 and 2. $v_{12} = v_1 - v_2$ is the potential difference (or the voltage) between them.



Characteristics of the Two Terminal Element

Circuit Element

- The two terminal element (dipole) is connected through its two terminals to the other circuit elements of a circuit.
  The connections are the ***nodes*** of the circuit.
  The circuit elements are the ***branches*** of the circuit.



- $v_1 > v_2 \quad \rightarrow \quad v = v_1 - v_2 > 0$ is the ***branch tension*** (or ***branch voltage***).
- i is the ***branch current***. i is assumed to be positive when it enters into the terminal at higher potential (terminal 1 in this case) and goes out from terminal at lower potential.
- The relation between v and i, given by the ***element equation*** $v = f(i)$, is the circuit element characteristic.
- The work made by the field **E** on the charges which flows through a branch cross section per time unit is the ***two terminal element electrical power***:

$$p(t) = \lim_{\Delta t \to 0}\left(\int_1^2 \Delta q \mathbf{E} \cdot d\mathbf{l} \Big/ \Delta t \right) = \frac{dq}{dt} \int_1^2 \mathbf{E} \cdot d\mathbf{l} = i(t)\, v(t)$$

n-Terminal Circuit Element

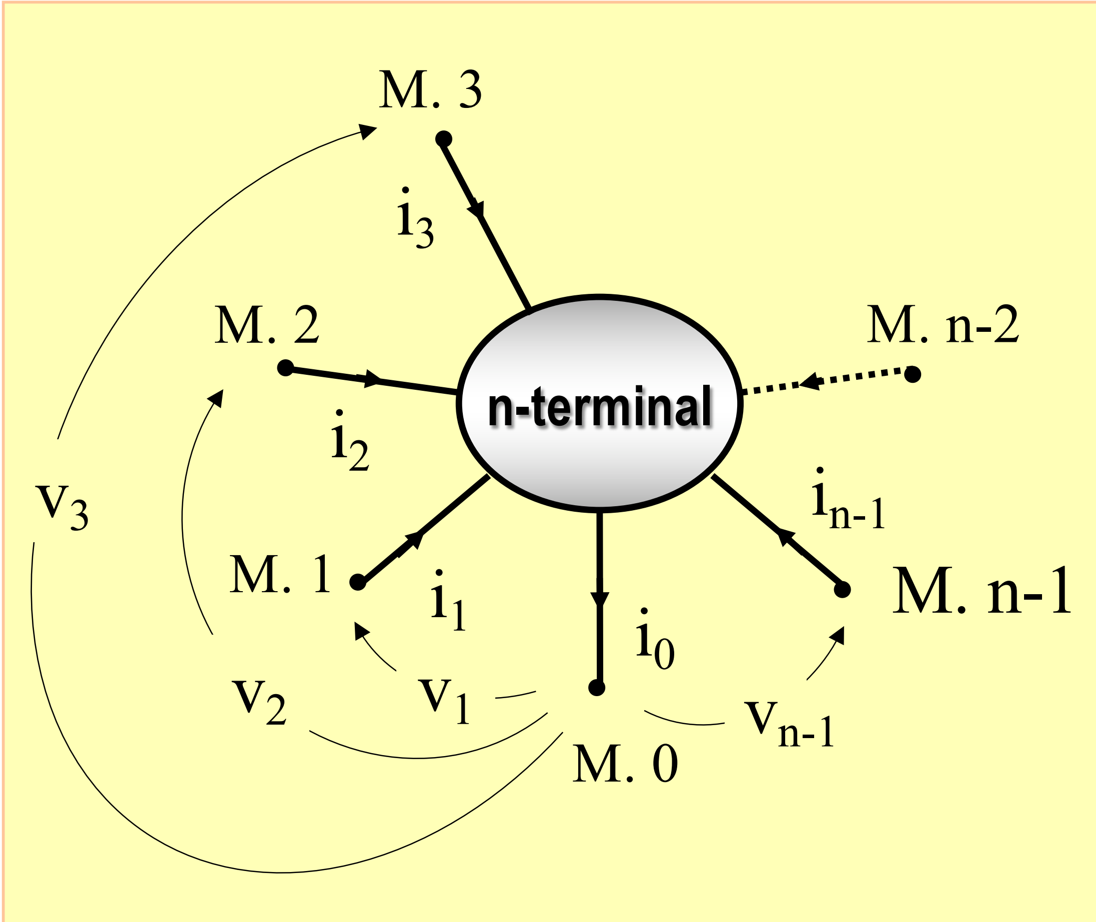
Electric circuit element having n terminals with n greater than two is said ***n-terminal circuit element***. A reference terminal is defined (M.0).

- As a consequence of the charge conservation:

$$i_0 = i_1 + i_2 + \ldots. + i_{n-1}$$

- As **E** is conservative outside of the n-terminal the following definitions are made:

$$v_1 = v_{M.1} - v_{M.0}$$
$$v_2 = v_{M.2} - v_{M.0}$$
$$v_3 = v_{M..3} - v_{M.0}$$
....
$$v_{n-1} = v_{M.n-1.} - v_{M.\,0}$$

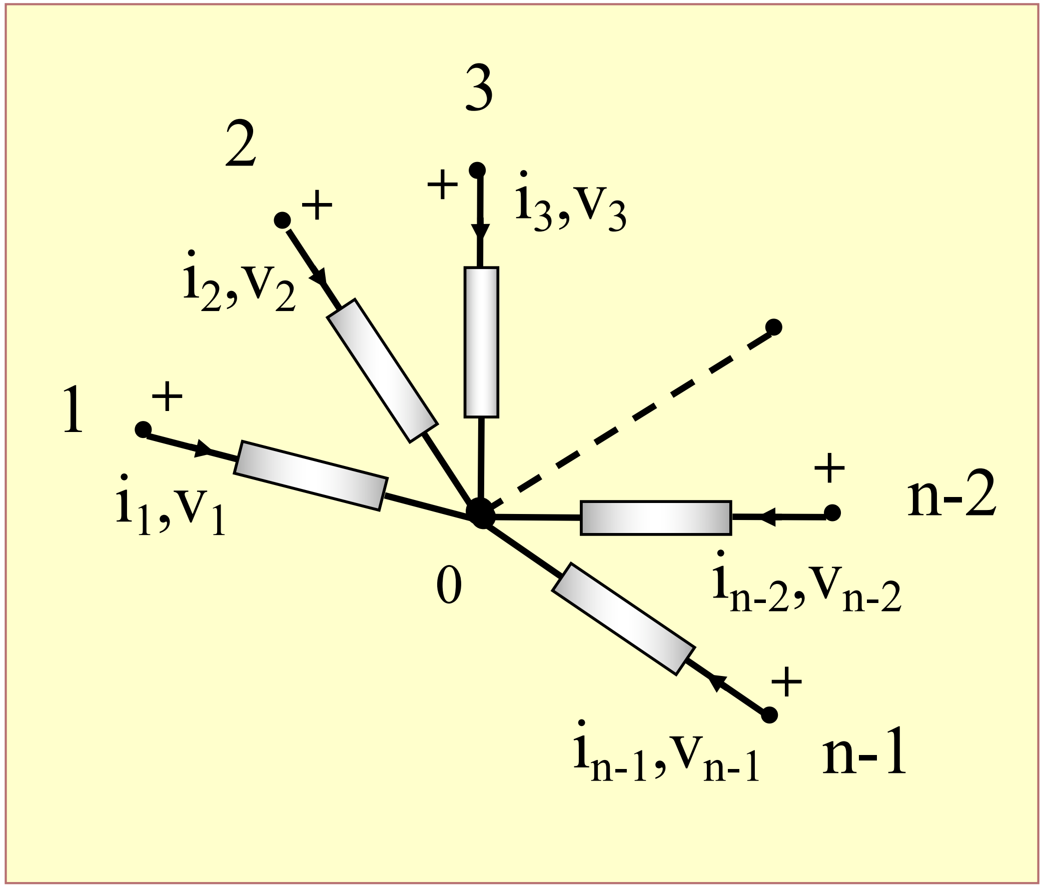


n-Terminal Circuit Element

A n-terminal is described by n-1 pairs of values, n-1 currents and n-1 voltages. Hence it is equivalent to n-1 two terminal elements with a common terminal.

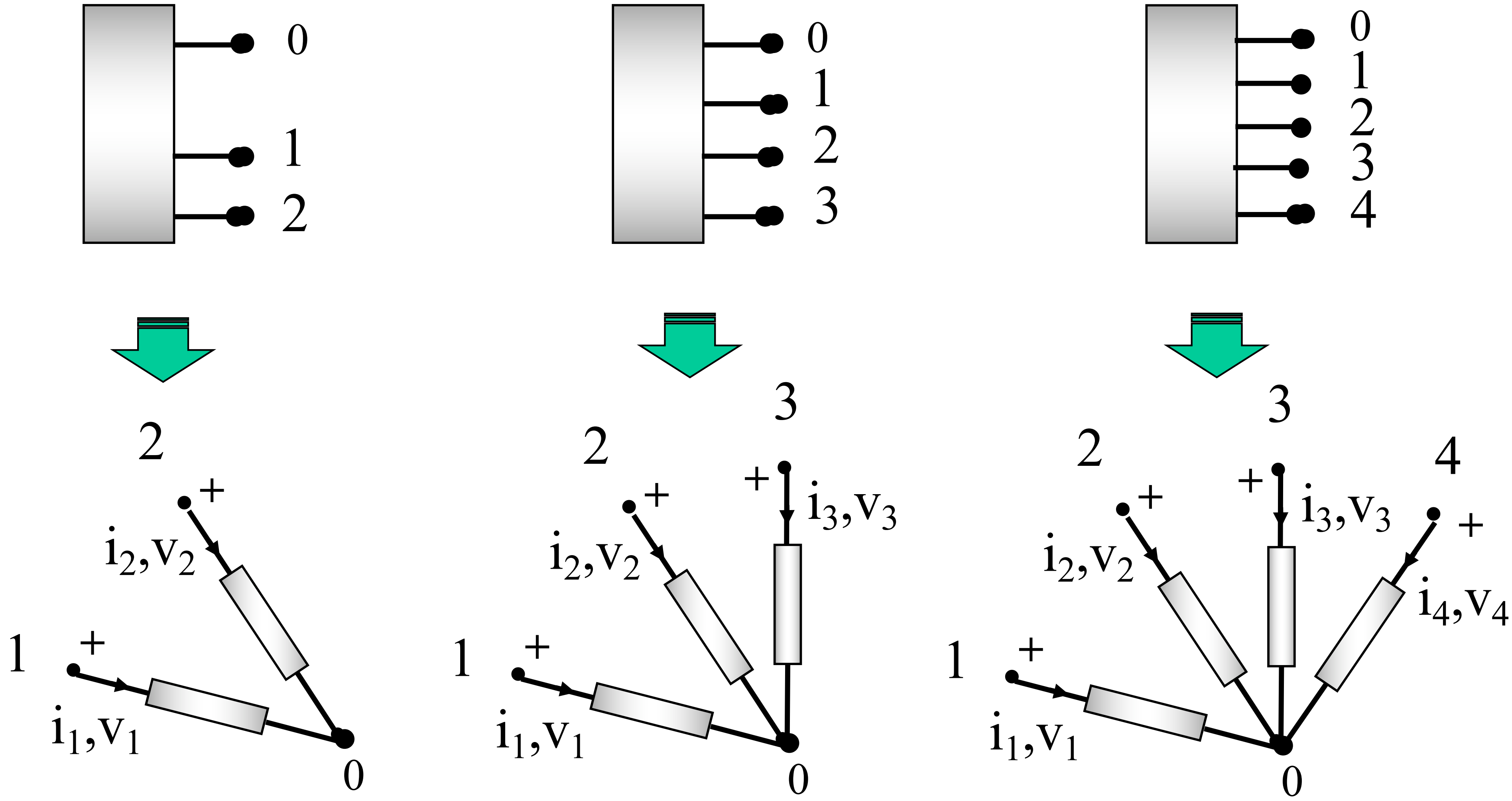
$i_1, i_2, i_3, \ldots\ldots, i_{n-1}$

$v_1, v_2, v_3, \ldots\ldots, v_{n-1}$



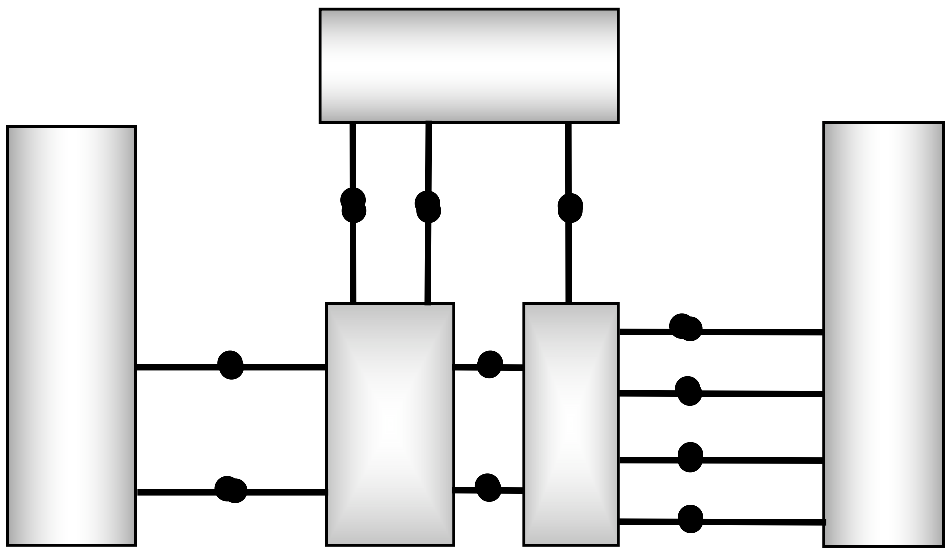
n-Terminal Circuit Element

A three terminal element corresponds to two dipoles with two common terminals. A four terminal element correspond to three dipoles with three common terminals etc.



Electric Circuit

In a circuit the interconnections among the circuit elements are realized through conductive cables (assumed as ideal conductors, $\sigma = \infty$). In the circuit theory the elements are the ***branches*** of the circuit. The interconnections are the ***nodes*** of the circuit. A circuit is characterized by ***n*** nodes and ***r*** branches.



Circuit with 11 nodes (n = 11) and 17 branches (r = 17)

Electric Circuit

❖ A circuit is made of circuit elements connected through conductive cables (a conductor). Dipoles (two terminal circuit elements) are the ***branches*** of the circuit. The interconnections are the ***nodes***.

❖ The EM phenomena are confined inside the circuit elements where **B** or **D** can vary in time. Outside of the circuit elements **D** and **B** are constant in time:

Outside c.elements: $\partial \mathbf{D}/\partial t = 0 \rightarrow dq/dt = 0 \rightarrow \nabla \cdot \mathbf{J} = 0 \rightarrow$ KCL

Outside c.elements: $\partial \mathbf{B}/\partial t = 0 \rightarrow d\Phi/dt = 0 \rightarrow \nabla \times \mathbf{E} = 0 \rightarrow$ KTL

In any cross section of a conductor the current is constant.

❖ A dipole is described by branch current i and branch tension v. The branch current i is assumed to be positive if it enters into the dipole from the positive terminal (terminal of the dipole at higher electrical potential). The relation between v and i, given by $v = f(i)$, is the characteristic of the circuit element.

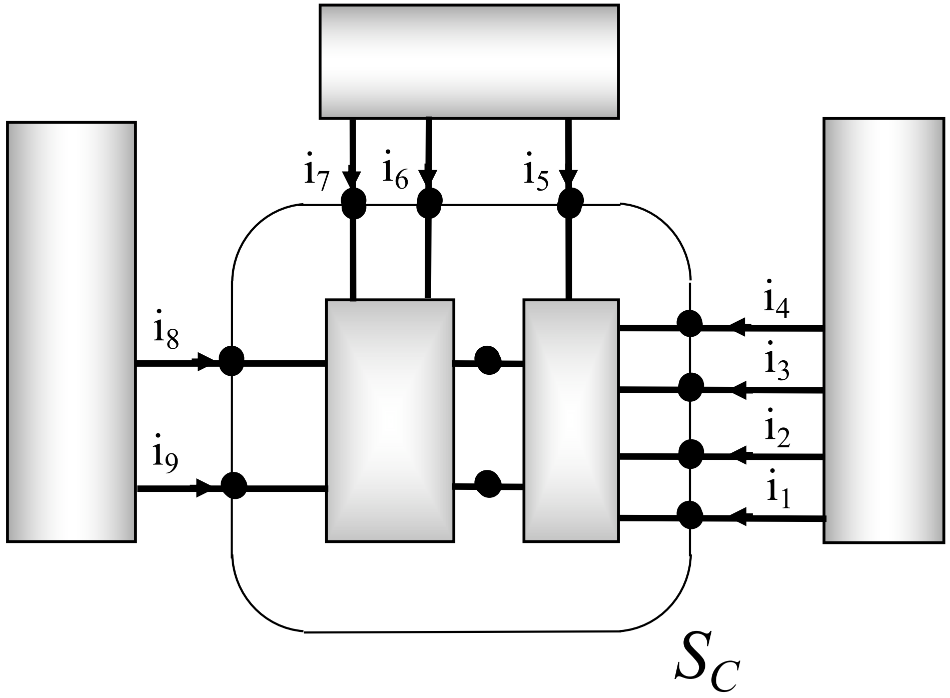
❖ The conductors connecting circuit element are assumed to be ideal ($\sigma = \infty$), The propagation time between the connected elements is zero. As a consequence the change of any electrical quantity in an element is immediately transferred to the other elements connected to it.

Kirchhoff's Laws

Kirchhoff's Current Law (KCL)

A closed surface S_C, which passes through some nodes of the circuit but not through circuit elements, is considered. No charge variation is assumed inside conductors and **J** here is solenoidal. Hence for the charge conservation law the total currents entering S_C is equal to zero:

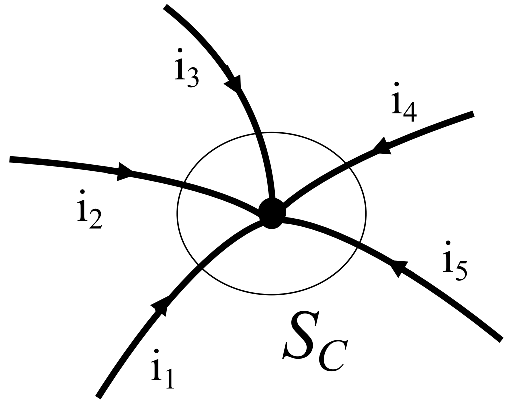
$$i_1 + i_2 + i_3 + \ldots + i_9 = 0$$



When S_C contains only one node, the following corollary is obtained:

$$\sum_{k=1}^{n} i_k = 0$$

Node equation

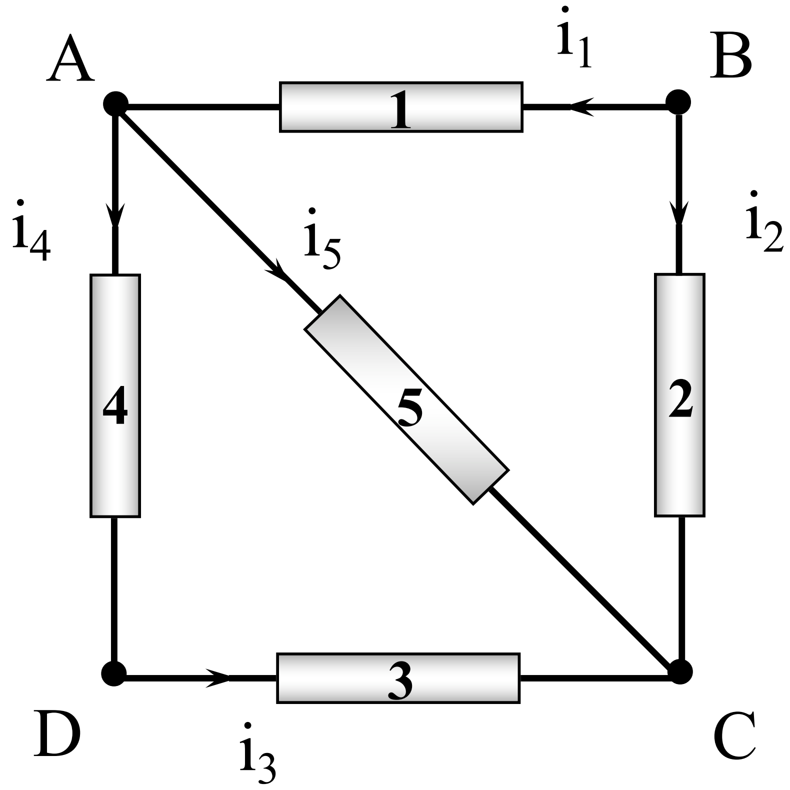


The algebraic sum of the currents entering a node is equal to zero.

Kirchhoff's Laws - Node Equations

For the nodes A, B, and C from the KCL it is:

$$i_1 - i_4 - i_5 = 0 \qquad (A)$$
$$i_1 + i_2 = 0 \qquad (B)$$
$$i_2 + i_3 + i_5 = 0 \qquad (C)$$



Each node equation takes into account at least a new current which does not appear in the other equations. In the equation derived from the KCL for node D only currents appear which are present in the equations of the other three nodes. This equation is a linear combination of the others. For node D it is:

$$i_3 - i_4 = 0 \qquad (\textit{this is eq. C + eq. A – eq. B})$$

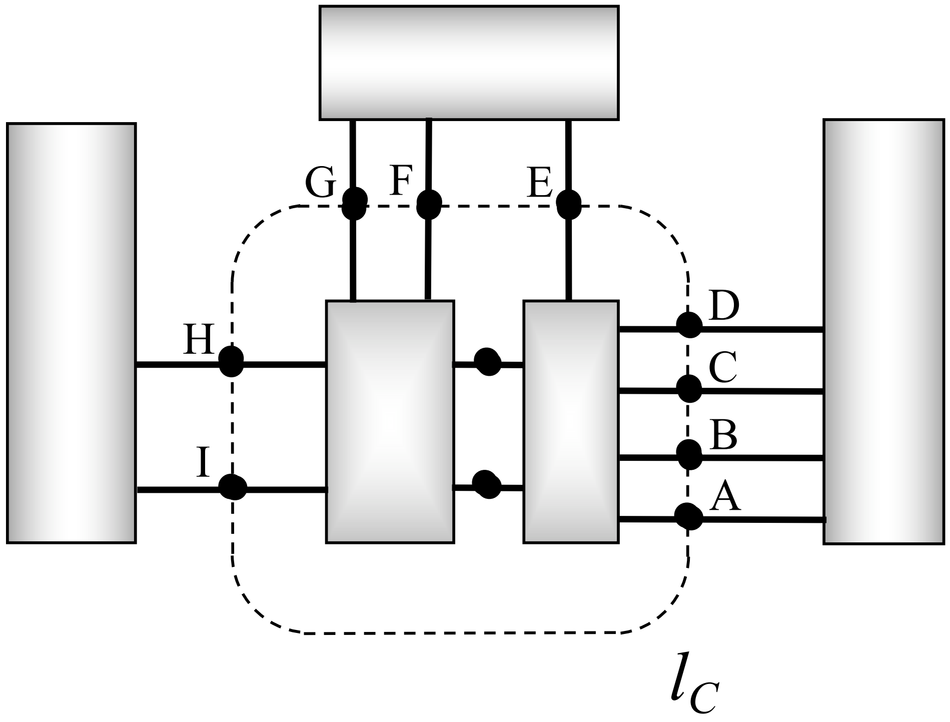
This is stated as follows:

In a circuit n-1 node equations are linearly independent.

Kirchhoff's Laws

Kirchhoff's Tension Law (KTL)

The field **E** is conservative in the region outside the circuit elements. Therefore the sum of the potential differences on a closed path l_C, which connects nodes and does not intersect circuit elements, is equal to zero.



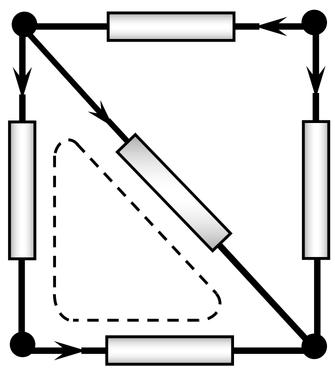
$$v_{AB} + v_{BC} + \dots + v_{HI} + v_{IA} = 0$$

When l_C connects the nodes of a circuit loop, the following corollary is obtained. :

The algebraic sum of the voltages of the branches of a loop is equal to zero.

$$\sum_{k=1}^{m} v_k = 0$$

Loop equation



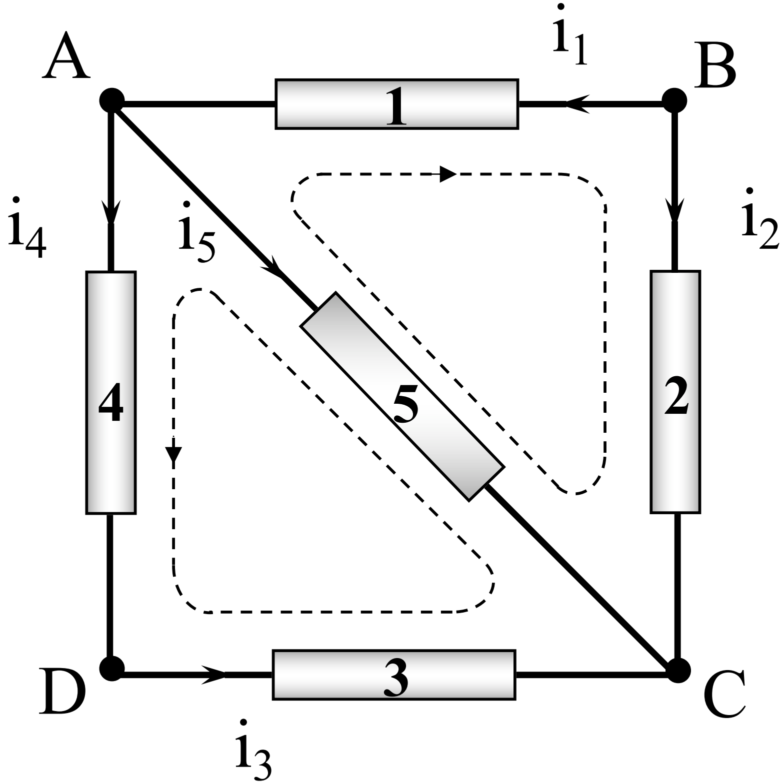
In an electric circuit a loop is defined as a closed path passing only once through every node in the path.

Kirchhoff's Laws - Loop Equations

The loops ADCA and ABCA sre considered. From the KTL it follows

$$- v_4 \; - v_3 + v_5 = 0 \qquad \text{(a)}$$

$$v_1 - v_2 + v_5 = 0 \qquad \text{(b)}$$



Each equation takes into account at least a new voltage which does not appear in the other equations. In the equation derived from the KTL for loop ABCDA only voltages appear, which are present in the above equations. This equation is a linear combination of the two above mentioned equations. This is stated as follows:

In a circuit r-n+1 loop equations are linearly independent.

Node and Loop Equations

In a electric circuit of r dipoles, hence of r branches and n nodes there are r-n+1 independent loop eq.s and n-1 node eq.s.

In the circuit of the figure for each node and each loop it is:

KCL for each node: $\Sigma_n i_n = 0$

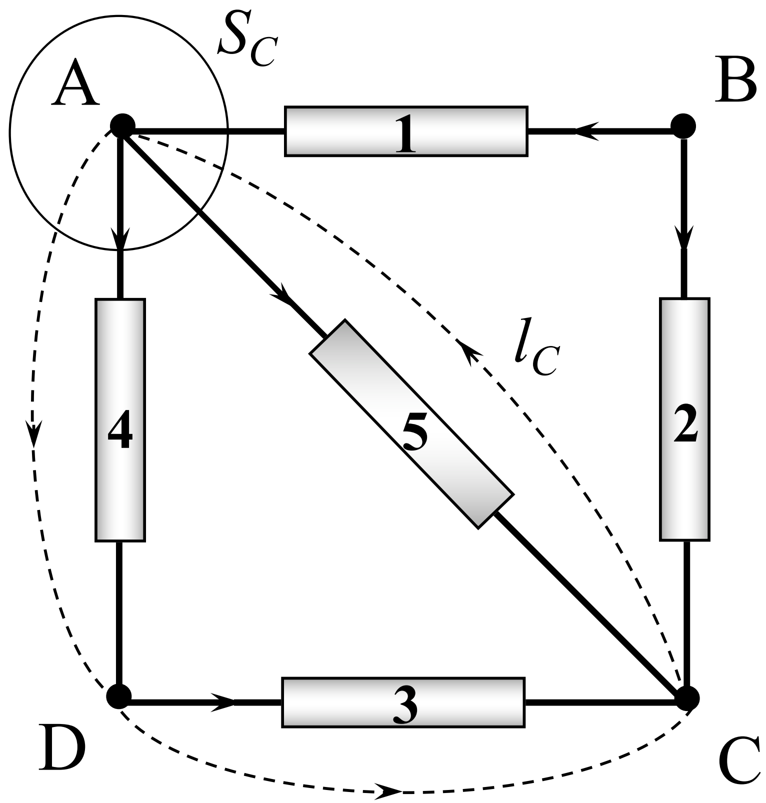
KTL for each loop: $\Sigma_m v_m = 0$

In the figure r = 5 and n = 4:

- n – 1 = 3 linearly independent node equations,
- r – n + 1 = 2 linearly independent loop equations

The equations are of the following type:

- for loop ADCA: $v_5 - v_3 - v_4 = 0$
- for node A: $i_4 + i_5 - i_1 = 0$



Topology Equations

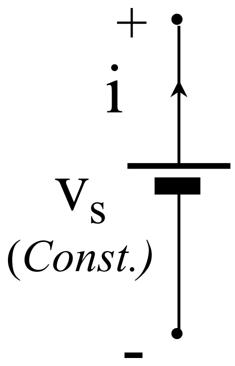
The status of a circuit of r branches and n nodes is described by *r* branch currents and *r* branch voltages, which are 2*r* quantities. Therefore 2*r* (*r* branch currents and *r* branch voltages) are the unknown quantities of the analysis problem.

From the topology of the circuit, given by the number of branches, the number of nodes, and their connections, *r* linearly independent topology equations are obtained. *n-1* are the node equations derived from the KCL. *r-n+1* are the loop equations derived from the KTL.

In order to have an unique solution of the analysis problem, other r equations are necessary. They are given by the element equations which state the relation between current and voltage for each branch.

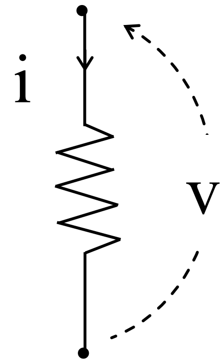
Element Equations

Independent Voltage Source



$$v = v_s\,, \quad \forall i$$

Ideal Resistor

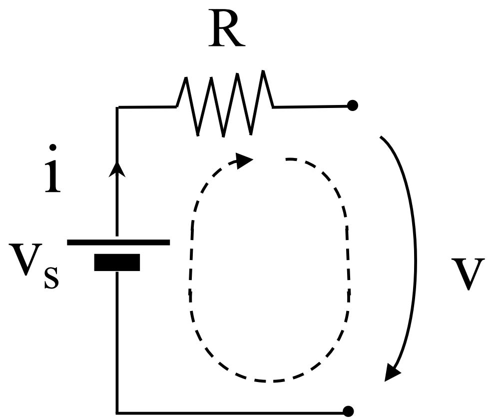


v(t) = R i(t)

[Ohm's law]

R [Ω (Ohm)]

electric resistance



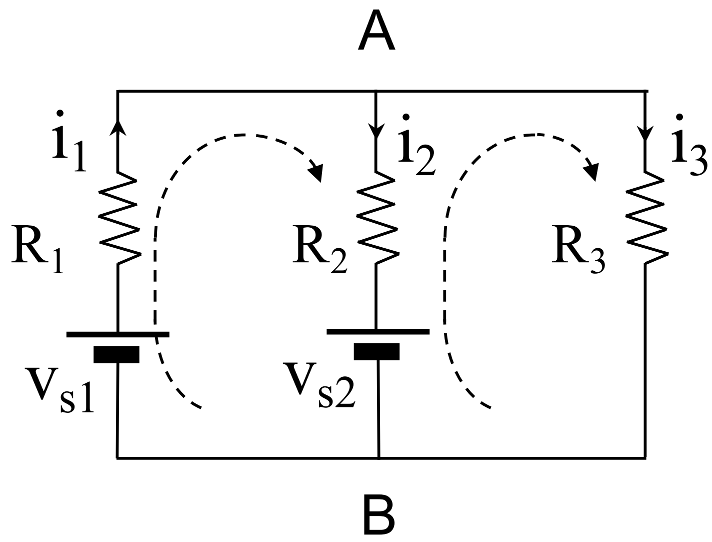
$$v + v_s - R\,i = 0$$

$$\Rightarrow \quad v = R\,i - v_s$$

Circuit analysis: an example

The problem of analysis

The topology of the circuit and the independent voltage sources (v_{s1}, v_{s2}) and the resistances (R_1, R_2, R_3) are the problem input. The branch voltages and the branch currents (v_1, v_2, v_3, i_1, i_2, i_3) are the problem output.



1. Direct the branches (define the positive direction of the currents i_1, i_2, i_3).
2. Define the nodes and the loops for the independent topology equations (in the circuit of the figure: n-1=1; r-n+1=2).
3. Define the direction of each loop.
4. Write the topology equations (KCL and KTL) and the element equations:

$i_1 - i_2 - i_3 = 0$
$-v_1 - v_2 = 0$
$v_2 - v_3 = 0$

$v_1 = R_1 i_1 - v_{s1}$
$v_2 = R_2 i_2 + v_{s2}$
$v_3 = R_3 i_3$

(these are 6 eq.s in 6 unknown with an unique solution)

⇨ By substituting the element eq.s into the topology eq.s:

$i_1 - i_2 - i_3 = 0$
$-R_1 i_1 - R_2 i_2 = -v_{s1} + v_{s2}$
$R_2 i_2 - R_3 i_3 = -v_{s2}$

⇨ From which the branch currents are obtained (i_1, i_2, i_3).

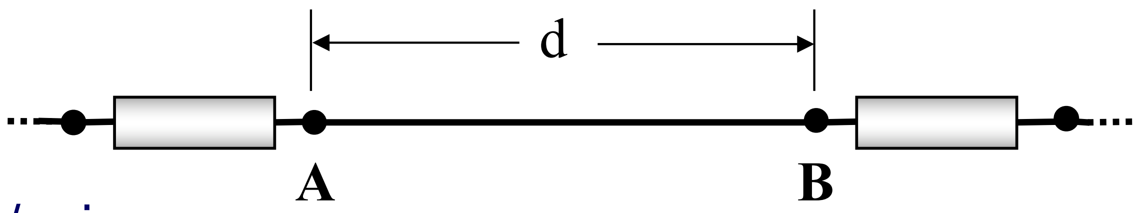
By substituting the currents i_1, i_2, i_3 into the element eq.s the branch voltages v_1, v_2, v_3 are derived.

Lumped Circuit Approximations

The lumped electric quantities in a circuit can have rapid or slow time variations in comparison with the ***propagation times*** within the circuit.

- The assumption of ***lumped circuit*** is that the propagation times within the circuit are much smaller then the time variation of the electric quantities. When treating waves (electric quantities expressed by sinusoidal functions in time), the lumped circuit assumption considers them propagating instantaneously within the circuit.

- The transit time for the propagation of a signal from A to B is t_{AB} = d/v = = d/c (v = c = 2,998×10^8 m/s is the speed of light). For a sinusoidal signal with a frequency f and period T with f = 1/T, and wave length λ = c/f, it is T = λ/c and and the time interval of an EM signal to go from A to B is t_{AB} = d/c. The assumption of lumped circuits to be verified needs that:



$$t_{AB} << T \iff d << \lambda$$

Lumped Circuit Approximations

✓ Electrical power:	f = 50/60 Hz →	λ = 6000/5000 km
✓ Microwaves:	f = 100 MHz →	λ = 3 m
✓ Computer clock:	f = 3 GHz →	λ = 10 cm

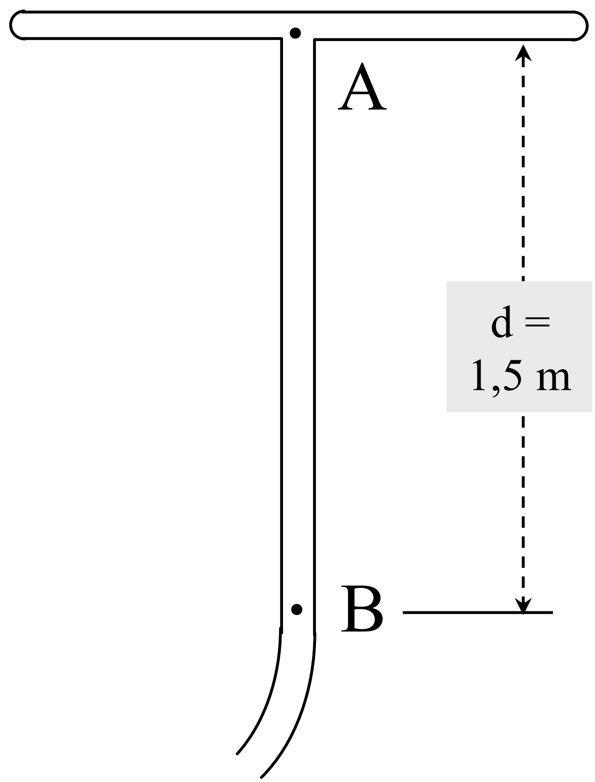
The antenna of the figure receives a signal of a frequency of 100 MHz corresponding an angular frequency $\omega = 2\pi f = 2\pi \times 10^8$.

In A at the time t there is a tension of:

$v_A(t) = V_0 \sin \omega t = V_0 \sin(2\pi \times 10^8 t)$

In B the signal arrives after a time $\Delta t = d/c = 1{,}5/3\times10^8 = 0{,}5\times10^{-8}$ s. Therefore in B at the time t there is the signa that left A at the time t- Δt:

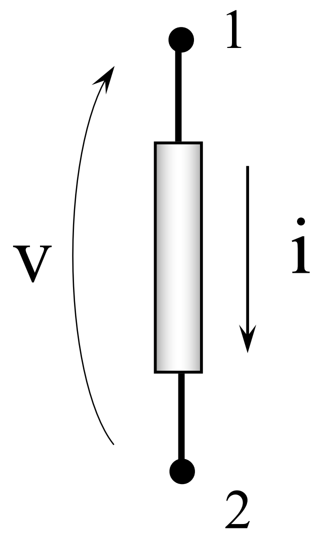
$$v_B(t) = v_A(t - \Delta t) =$$
$$= V_0 \sin[2\pi\times10^8 (t - \Delta t)] =$$
$$= V_0 \sin[2\pi\times10^8 (t - 0{,}5\times10^{-8})] =$$
$$= V_0 \sin(2\pi\times10^8 t - \pi) =$$
$$= - V_0 \sin(2\pi\times10^8 t) = - v_A(t)$$



Circuit Elements
Two Terminal Element - Dipole

- **Element Equation**

The relation between the branch current i flowing through the element and the branch voltage v, which is the potential difference between the terminals of the circuit element, defines the behavior of that element within the circuit. This relation is the ***element equation*** (said also the ***i-v characteristic***).



Current controlled element

$$v = f(i)$$

the current is the independent variable.

Voltage controlled element

$$i = g(v)$$

the voltage is the independent variable.

Circuit Elements
Two Terminal Elements

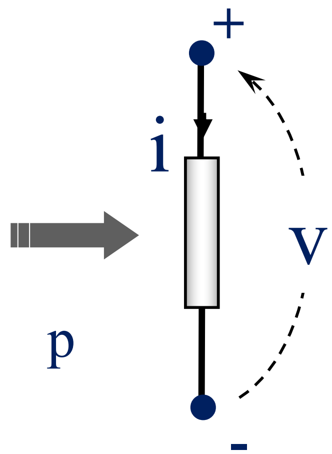
- **Element Equation**

The ***element equation***, which defines the relation between the branch current i flowing through the element and the branch v between the terminals of the circuit element, is determined by the physical phenomena caused by of the element.

- **Passive Two Terminal Elements**

In the ***passive element convention*** the current enters into the element from the positive terminal. In the element, as in resistors, the charge is displaced from the higher potential to the lower potential due to the positive potential difference. Therefore the energy results to be dissipated. *In passive elements the energy is always positive or equal to zero.*

Passive dipole convention



$$w(t) = \int_{-\infty}^{t} v(t')\, i(t')\, dt' \geq 0$$

Circuit Elements

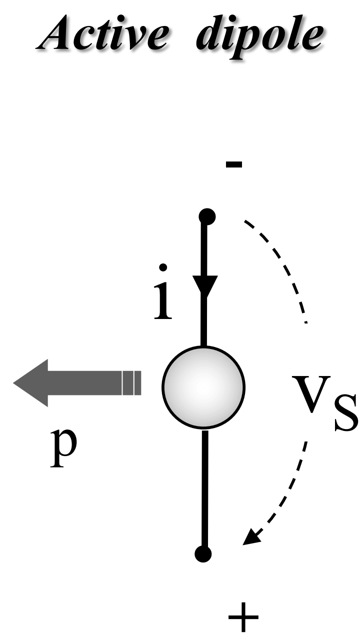
Two Terminal Elements

- **Active Two Terminal Elements**

In the ***active element*** the current enters into the element from the negative terminal. The current flows from the negative to the positive terminal. The circuit element is doing work in moving charge from a lower potential to a higher potential.

*Electric power sources (**tension sources** and **current sources**) are active elements.*

(As stated by the passive element convention the branch voltage $v = -V_S$*)*



Element Equations

- **Linear and non-linear two terminal elements**
  - **Linear element:** the element equation consists of linear operators.

example: $$v(t) = a + b\,i(t) + c\frac{di}{dt} + d\int_{t_0}^{t} i(t)dt \qquad (1)$$

  - **non-linear element:** the element equation is non-linear

example: $$v(t) = a' + b'\,i^2(t) \qquad (2)$$

- **Time-independent and time-dependent elements:**
  - **time-independent elements:** the element equations do not depend on time (in eq.s 1 and 2 a, b, c, d, a’ and b’ are constant).
  - **time-dependent elements:** the element equations are time-dependent (in eq.s 1 and 2 a, b, c, d, a’ and b’ depend on time).

Element Equations

- **Dissipative and Storage Elements**
  - **Elements without memory – dissipative elements:** the element equation expresses the relation between i and v at the same time t (In this case the passive elements are dissipative).

$$v(t) = a + b\, i^2(t) + c \sin\left[i(t)\right] \quad \text{(non-linear without memory}$$

  - **Elements with memory – storage elements:** the element equation expresses the relation between i and v at different times.

$$v(t) = a + b\, i(t) + c\,\frac{di}{dt} \qquad \text{(linear dipole with memory)}$$

$$v(t) = \int_{-\infty}^{t} a\, i(t')\, dt' \qquad \text{(linear dipole with memory)}$$

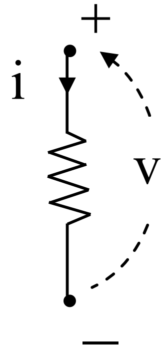
The elements with memory store energy, which can be retrieved at a later time. These elements are also called ***storage elements*.**

Ideal Elementary Elements

- **Resistors, capacitors, inductors, tension sources, and current sources.**

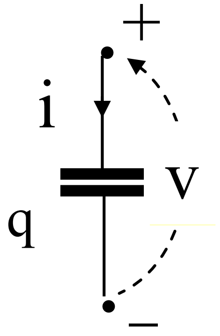
*The **Ideal Elementary Passive Elements** are the ideal resistor, the ideal capacitor and the ideal inductor described by the linear expressions given below. Each of these ideal elementary two terminal elements represents a single elementary EM process. In a real element a single elementary process is never present.*

Ideal resistor



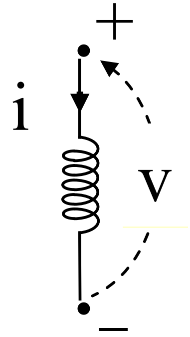
$$v(t) = R\, i(t)$$

Ideal Capacitor



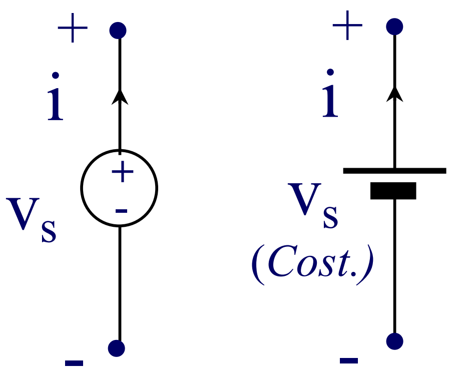
$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(t')\, dt'$$

Ideal inductor



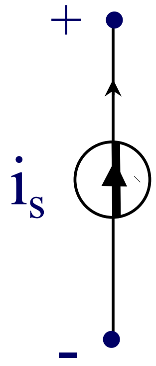
$$v(t) = L\,\frac{di}{dt}$$

Ideal Tension Source
(Indipendent Tension Source)



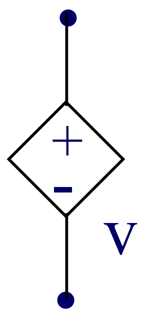
$v = v_s\,,\quad \forall i$

Ideal Current Source
(Independent Current Source)



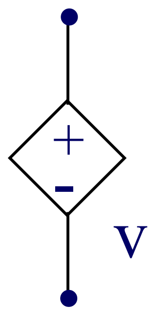
$i = i_s\,,\quad \forall v$

Controlled Sources (Dependent Sources)



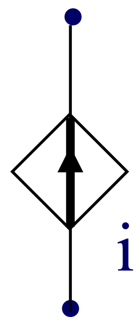
$v = \mu\, v_r$

Tension controlled tension source



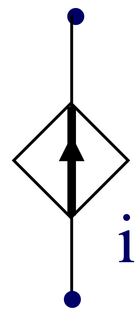
$v = r_m\, i_r$

Current controlled tension source



$i = g_m\, v_r$

Tension controlled current source



$i = \alpha\, i_r$

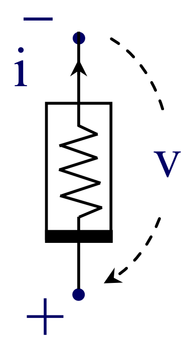
Current controlled current source

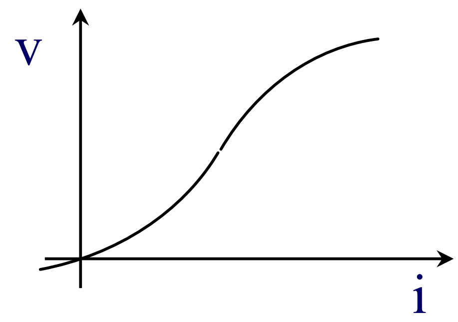
Elementary Ideal Dipoles

The Resistor

The resistor is a passive element which dissipates energy.

$$v = f(i)$$

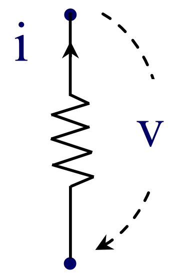


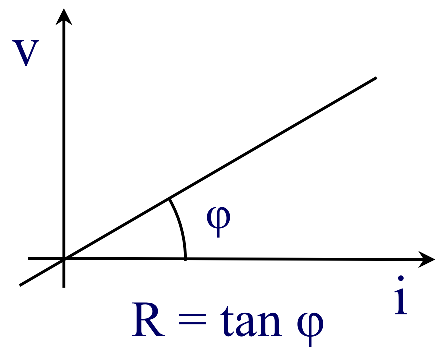


- ***Linear time independent resistor*** (dissipative passive element):

$$v(t) = R\,i(t) \quad \text{[Ohm's law]}$$

$$p(t) = v(t)\,i(t) = R\,i(t)^2$$



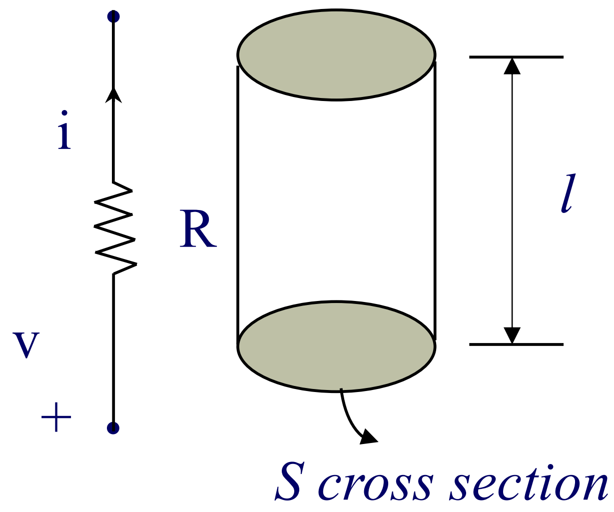


- R ***resistance*** (SI unit ***ohm*** [Ω]),
- G = 1/R ***conductance*** (SI unit ***siemens*** [S]),

The Linear Resistor

The ***electrical resistance*** of a circuit element is the parameter that quantifies its property to oppose the current:

$$v = R\,i \quad \Longleftrightarrow \quad R = v/i$$



The ***resistivity*** ρ [Ωm] of a material quantifies its property to oppose the flow of electrical charges:

$$\rho = \frac{1}{\sigma}$$

where σ [S/m] is the ***electrical conductivity*** ($\mathbf{J} = \sigma\,\mathbf{E}$)

In a cylindrical circuit element with a constant cross section S and a length l, and ρ uniform in the whole volume, the element resistance for a current flowing parallelly to the cylinder axis is:

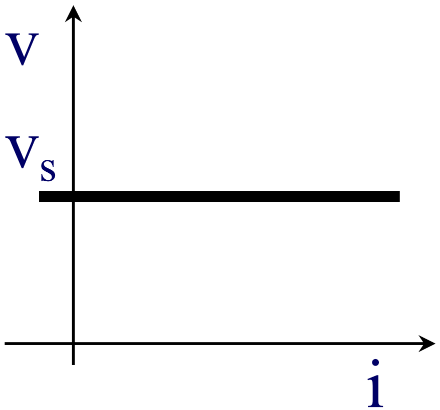
$$R = \rho\frac{l}{S} = \frac{l}{\sigma S}$$

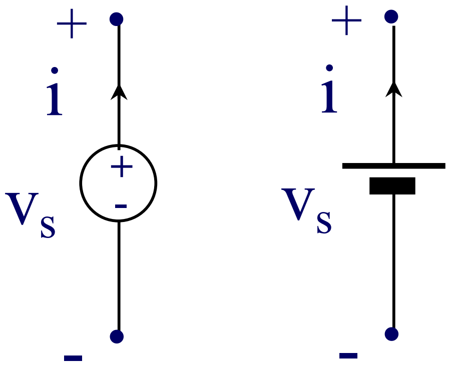
Elementary Ideal Dipoles

The Ideal Voltage Source

The ***ideal voltage source*** is an active element. It keeps the tension v_s between its terminals independently from the current flowing through it.

$$v = v_s \,, \quad \forall i$$



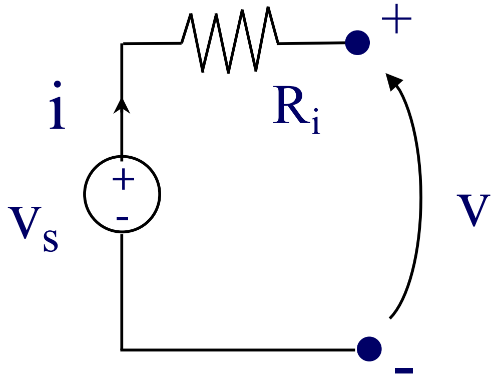


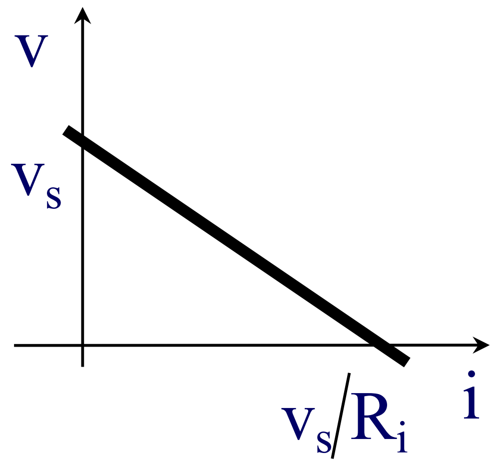
The symbol at the right hand side is used for DC voltages.

Real Voltage Source

To simulate a real voltage source a resistor R_i in series with the ideal source is considered.

$$v = v_s - R_i i$$



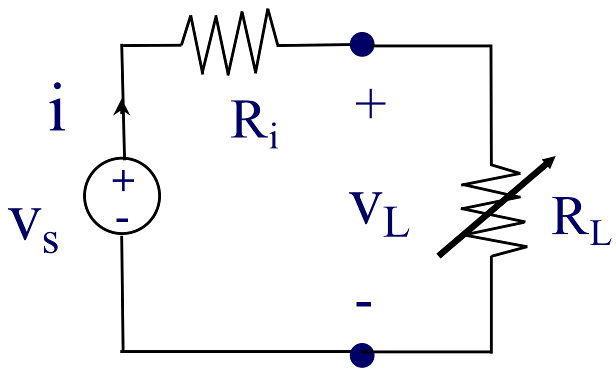


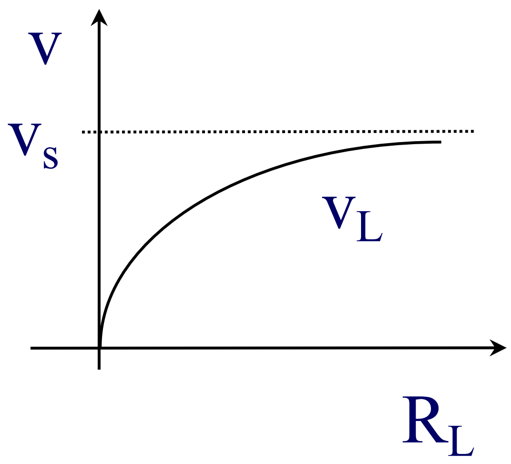
When a variable load R_L connected to a real source, from the KTL it follows:

$$R_L i + R_i i - v_s = 0$$

$$R_L i - v_L = 0$$

$$\Rightarrow \quad v_L = \frac{R_L}{R_L + R_i} v_s$$

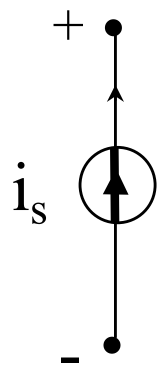


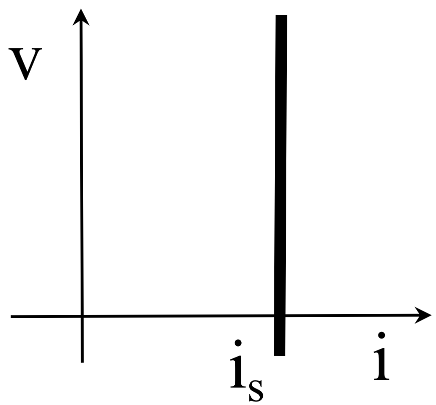


The Independent Ideal Current Source

The ***independent ideal* current *source*** is an active element. It keeps a current i_s flowing through it independently from the voltage .

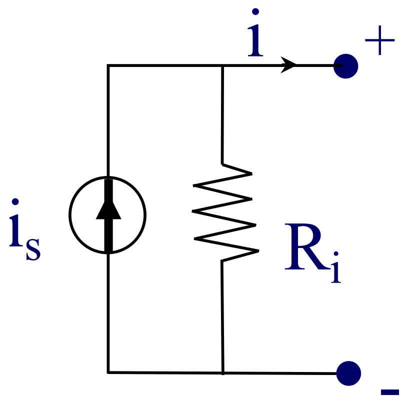
$$i = i_s\ , \quad \forall v$$

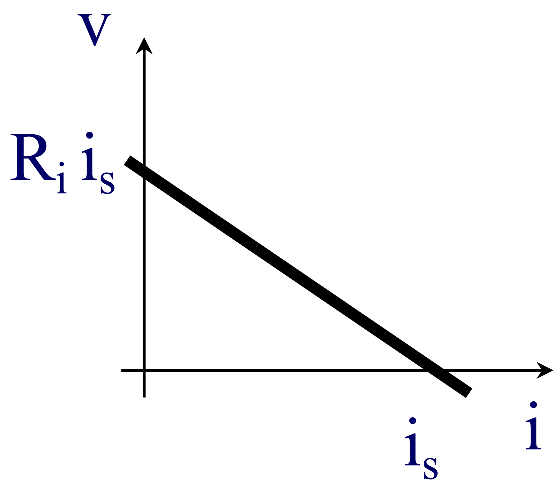




Real Current Source

To simulate a real current source a resistor R_i in parallel with the ideal source is considered.





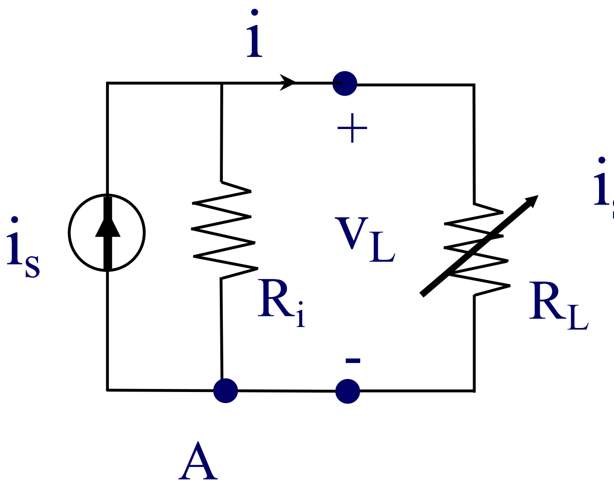
$$i = i_s - v/R_i$$

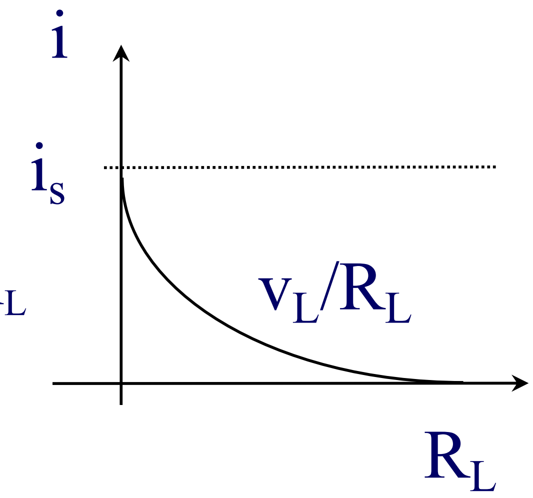
When a variable load R_L is connected to a real source, from the KCL it follows :

$$i - i_s + v_L/R_i = 0$$

$$\textbf{\textit{and}} \quad v_L = R_L\, i$$

$$\Rightarrow \quad i = \frac{R_i}{R_L + R_i} i_s$$

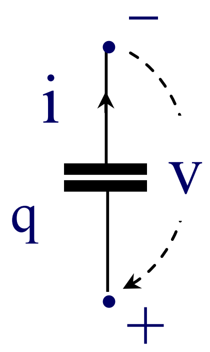


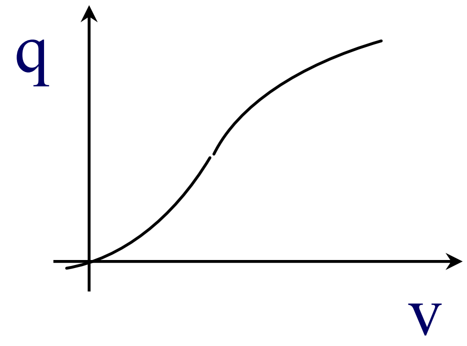


Elementary Ideal Dipoles

The Capacitor

It consists of two conducting plates separated by an insulating material





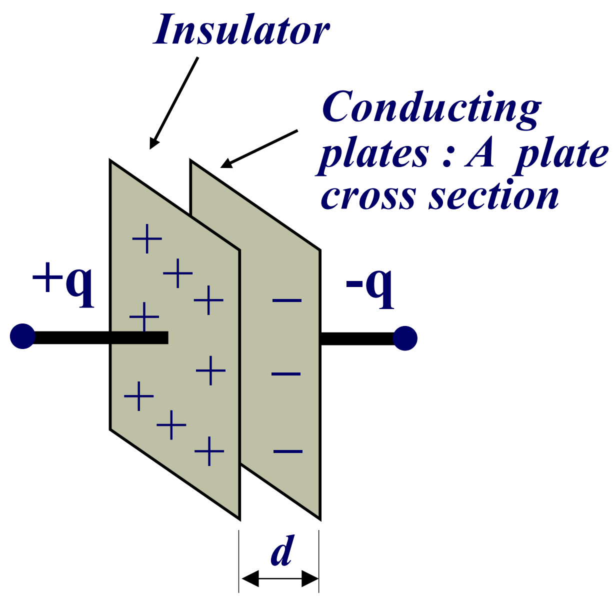
$$q = f(v)$$

$$i = \frac{dq}{dt} \quad \Rightarrow \quad i = \frac{d}{dt} f(v)$$

- For the ***linear time independent capacitor*** it is:

$$q(t) = C\,v(t) \quad \Rightarrow \quad i(t) = C\frac{dv(t)}{dt}$$

C is the ***capacitance*** [SI unit: F (***farad***)]. It is given by the ratio between the absolute value of the charge on one of the capacitor ***conducting plates*** and the voltage between them. When the insulator (mostly it is a ***dielectric)*** of a thickness d, placed between the two armature is homogeneous, it is:



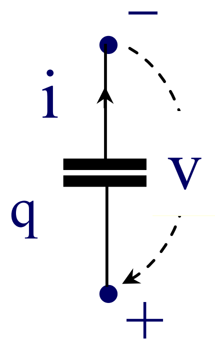
$$C = \varepsilon \frac{A}{d}$$

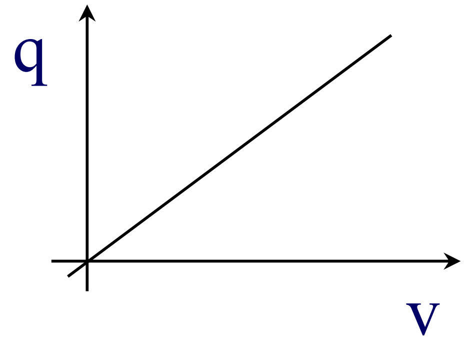
ε is the ***dielectric constant*** of the insulating material.

Elementary Ideal Dipoles

The linear time independent capacitor

$$q(t) = C\,v(t),$$





$$dq = C dv \quad \rightarrow \quad i(t) = \frac{dq}{dt} = C\,\frac{dv}{dt}$$

$$i(t) = C\,\frac{dv}{dt} \quad \rightarrow \quad dv = \frac{1}{C}\,i(t)\,dt \quad \rightarrow \quad v(t) = \frac{1}{C}\int_{-\infty}^{t} i(t')\,dt',$$

$$v(t) = v(t_0) + \frac{1}{C}\int_{t_0}^{t} i(t')\,dt', \qquad \forall\ t > t_0$$

[v(-∞) = 0 is assumed]

Energy stored in the capacitor, electrostatic energy, at the time t:

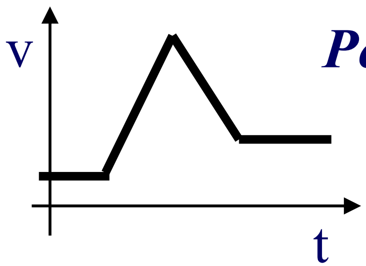
$$\varepsilon_C = \int_{-\infty}^{t} v(t')\,i(t')\,dt' = \int_{0}^{q(t)} v(q')\,dq' = \frac{1}{C}\int_{0}^{q(t)} q'\,dq' = \frac{1}{2}\,\frac{q(t)^2}{C} = \frac{1}{2}\,C\,v(t)^2$$

[At $t = -\infty$ q is assumed to be zero]

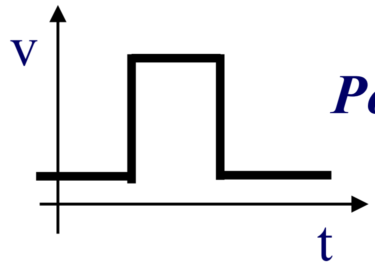
The Linear Capacitor $\quad q = C\,v,\;\; i = C\dfrac{dv}{dt},\;\; \varepsilon_C = \dfrac{1}{2}\dfrac{q^2}{C} = \dfrac{1}{2}C\,v^2$

- ***For a DC current a capacitor is an open circuit (resistor with R = + ∞):***
  - When the voltage is constant in time, the current is equal to zero ($i = C\,dv/dt$).
  - When a capacitor is connected to a battery it will be charged ($q = C\,v$).

- ***The tension and the charge of a capacitor cannot vary instantaneously.***
  - As from the capacitor $i = dq/dt = C\,dv/dt$. Hence a charge and a voltage discontinuities (dv and dq finite, dt infinitesimal) imply an infinite current . This is not physically possible. ***Therefore the capacitor opposes instantaneous charge and voltage variations***.
  - As stated by the capacitor energy relation $\varepsilon_C = q^2/(2C)$,$= Cv^2/2$ any instantaneous variation of the charge and the voltage implies an instantaneous variation of the energy and therefore an infinite power ($p_C = d\varepsilon_C/dt$). This is not physically possible.



Possible



Not Possible

- ***The capacitor is a passive element. It does not dissipate energy. The energy is stored in the form of electrostatic energy. The energy is used to create an electric field due to the deposition of the charges carried by the current on both conducting plates. This energy is given back when the current changes its direction.***

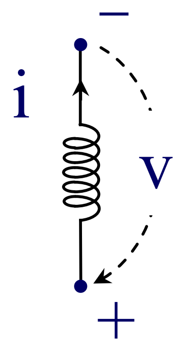
Elementary Ideal Dipoles

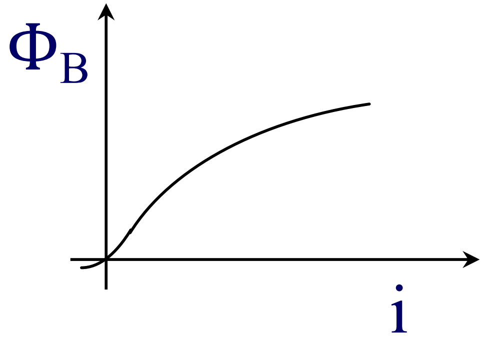
The Inductor

It consists of windings around a core of ferromagnetic material. The current flows through the windings and generates a magnetic flux. A time variation of this flux induces a voltage.

$$\Phi_B = f(\mathrm{i})$$

$$\mathrm{v} = \frac{d\Phi_B}{dt} \Rightarrow \mathrm{v} = \frac{d}{dt} f(\mathrm{i})$$





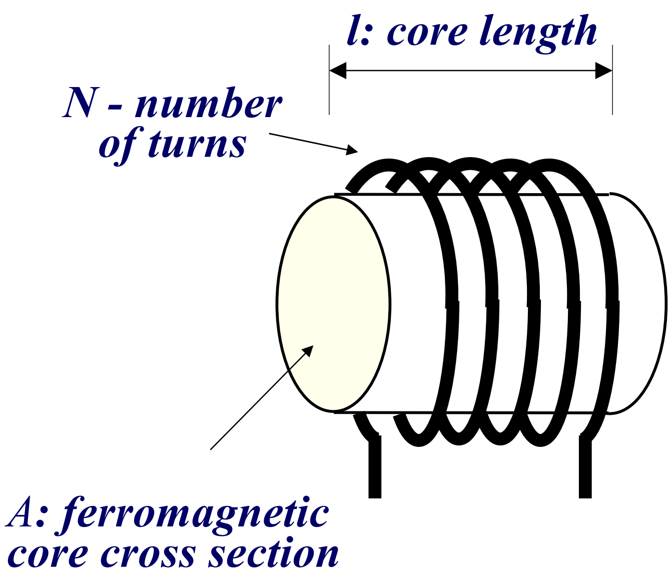
- For the ***linear time independent inductor*** it is:

$$\Phi_B(t) = L\,\mathrm{i}(t) \Rightarrow \mathrm{v} = L\frac{d\mathrm{i}}{dt}$$

L is the ***inductance*** [SI unit: H (***henry***)]. It is given by the ratio between the magnetic flux generated by the current and the current. In an inductor as in the figure with homogeneous material, L is given by:

$$L = \mu\frac{N^2 A}{l}$$

μ is the magnetic permeability [SI unit: H/m].

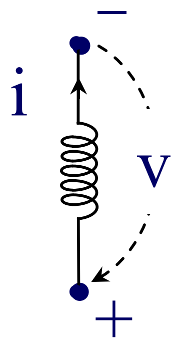


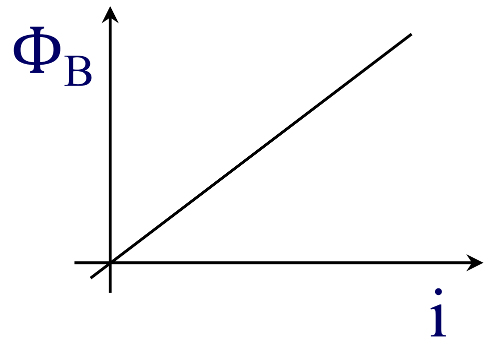
Elementary Ideal Dipoles

The linear time independent inductor

$$\Phi_B(t) = L\, i(t)$$

$$v(t) = \frac{d\Phi_B}{dt} = L\frac{di}{dt}$$





$$v(t) = L\frac{di}{dt} \;\rightarrow\; di = \frac{1}{L}v(t)\,dt \;\rightarrow\; i(t) = \frac{1}{L}\int_{-\infty}^{t} v(t')\,dt',$$

[i(-∞) = 0 is assumed]

$$i(t) = i(t_0) + \frac{1}{L}\int_{t_0}^{t} v(t')\,dt', \qquad \forall\, t > t_0$$

Energy stored by the inductor – electromagnetic energy - at the time t:

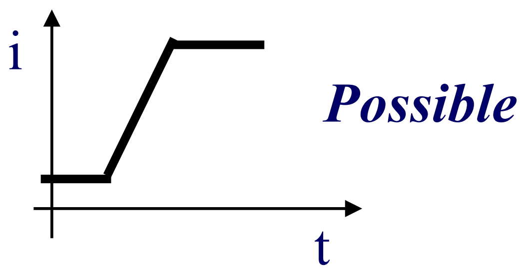
$$\varepsilon_L = \int_{-\infty}^{t} v(t')\,i(t')\,dt' = \int_{0}^{i} L\,i'\,di' = \frac{1}{2}L\,i^2$$

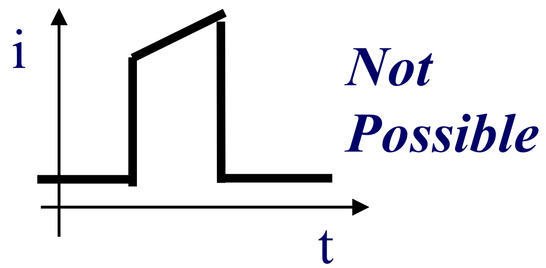
← *[i(-∞) = 0 is assumed]*

The Linear Inductor

$$\Phi_B = L\,i,\quad v = L\frac{di}{dt},\quad \varepsilon_L = \frac{1}{2}L\,i^2$$

- ***For a DC current an inductor is a closed circuit (Resistor with R = 0):***
  - When the current is constant in time, the voltage is equal to zero ($v = L\,di/dt$).
- ***The current in an inductor cannot vary instantaneously:***
  - Due to the inductor equation $v = L\,di/dt$, a current discontinuity implies an infinite tension, that is not physically possible. ***Therefore the inductor opposes any sharp variation of the current.***
  - From the inductor energy relation, any instantaneous variation of the current implies an instantaneous variation of the energy stored into it. Hence in order to have this variation an infinite power ($p_L = d\varepsilon_L/dt$) is necessary. This is not physically possible.





- ***The inductor is a passive element. It does not dissipate energy. The energy is stored in the form of magnetic energy. The energy is used for the creation of the magnetic field by means of the current flowing in the windings. This energy is given back when the current changes its direction.***

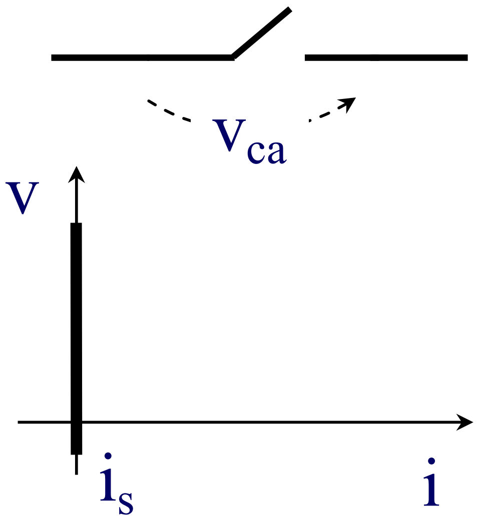
Two Terminal Elements

Open Circuit

It can be considered as either of the following elements:

- a current source with $i_s = 0$
- a resistor with $R = \infty$

Element eq.: $i = 0, \quad \forall\, v$

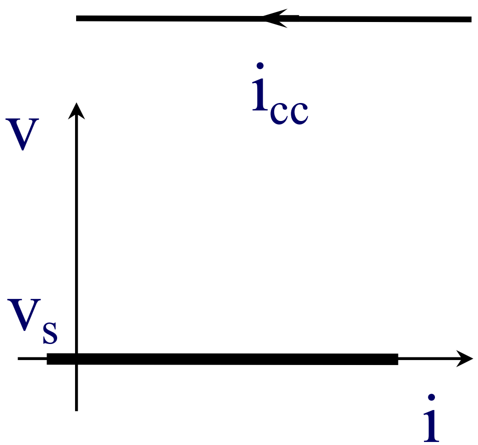


Closed Circuit

It can be considered as either of the following elements:

- a voltage source with $v_s = 0$
- a resistor with $R = 0$

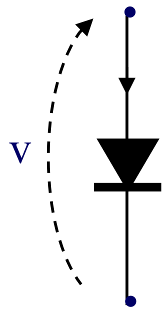
Element eq.: $v = 0, \quad \forall\, i$

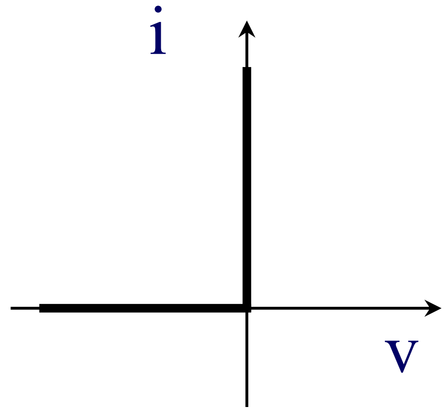


Two Terminal Elements

Ideal Diode

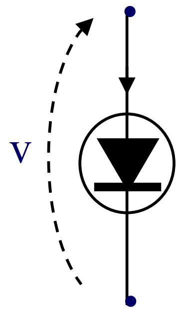
$$v\,i = 0 \quad \Longleftrightarrow \quad \begin{cases} i = 0 & \text{per } v < 0 \\ v = 0 & \text{per } i > 0 \end{cases}$$

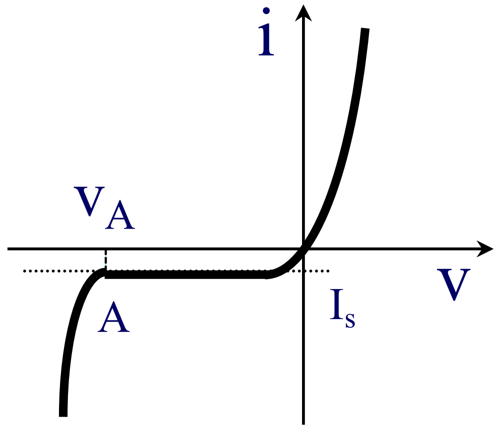




pn-Junction Diode

The operation region is for $v > v_A$. For $v < v_A$ the diode burns out.





$$i = I_s \left[\exp\left(\frac{v}{V_T} \right) - 1 \right]$$

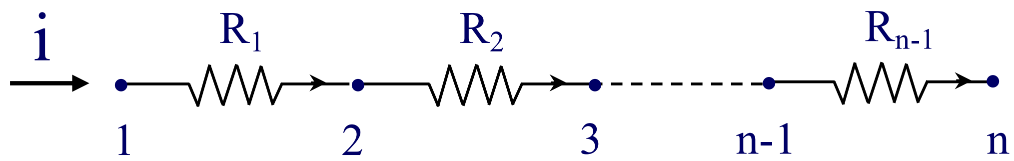
I_s ($\approx \mu A$) *saturation current*

$V_T = kT/e$ ($\approx$ 0.026 V) *thermal tension*

Series Resistors

Two or more dipoles are said to be ***in series*** when the current from onedipole exclusively flows into the next one. Therefore the same current flows through each element one after another. For a series of resistors it is:

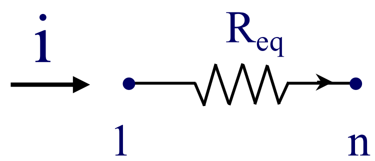
$$v_{12} = R_1\, i$$
$$v_{23} = R_2\, i$$
$$\cdots\cdots\cdots\cdots$$
$$v_{n-1,n} = R_{n-1}\, i$$



$$v_{1,n} = i \sum_{k=1}^{n-1} R_k \Rightarrow$$

$$R_{eq} = \sum_{k=1}^{n-1} R_k$$

$$\frac{1}{G_{eq}} = \sum_{k=1}^{n-1} \frac{1}{G_k}$$



$$\left(G_k = R_k^{-1},\ G_{eq} = R_{eq}^{-1}\right)$$

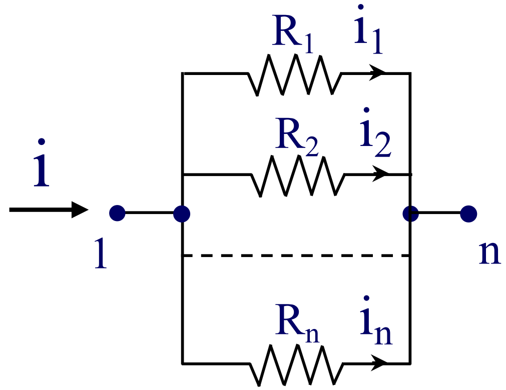
Parallel Resistors

Two or more dipoles are said to be ***in parallel*** when they share the same two terminals. Therefore the dipoless will be under the same voltage. For resistors in parallel it is:

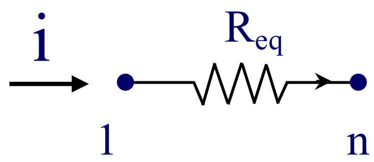
$$i = i_1 + i_2 + \ldots\ldots + i_n =$$

$$= \frac{v_{1,n}}{R_1} + \frac{v_{1,n}}{R_2} + \ldots\ldots\ldots + \frac{v_{1,n}}{R_n} =$$

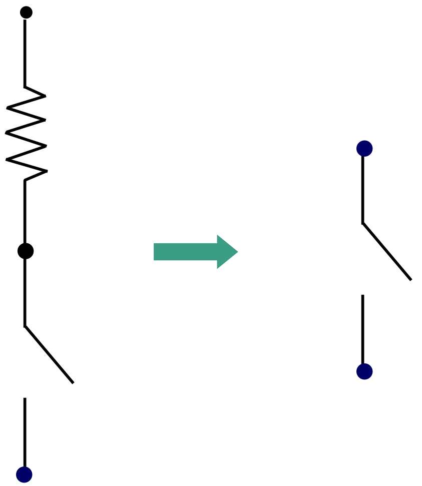
$$= v_{1,n}\left(\frac{1}{R_1} + \frac{1}{R_2} + \ldots\ldots\ldots + \frac{1}{R_n}\right) = \frac{v_{1,n}}{R_{eq}}$$



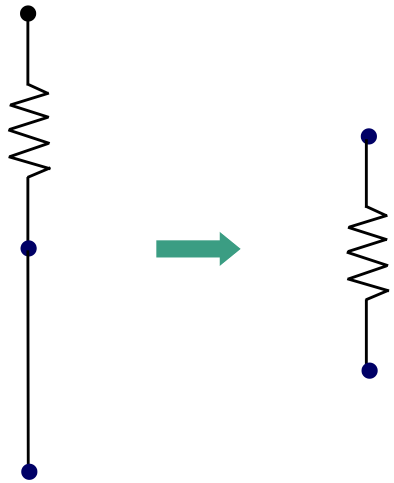
$$i = \frac{1}{R_{eq}} v_{1,n} \quad \Rightarrow \quad \boxed{\frac{1}{R_{eq}} = \sum_k \frac{1}{R_k} \qquad G_{eq} = \sum_K G_k}$$



$$\left(G_k = R_k^{-1},\ G_{eq} = R_{eq}^{-1}\right)$$



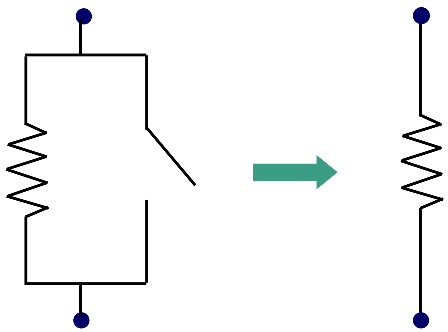
Resistor in series with an open circuit



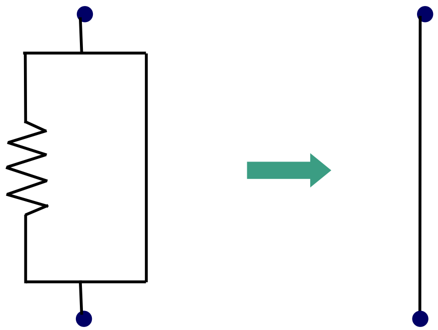
Resistor in series with a closed circuit

$$R_{eq} = n\,R$$

n resistors in series with equal R



Resistor in parallel with an open circuit



Resistor in parallel with a closed circuit

$$R_{eq} = R/n$$

n resistors in parallel with equal R

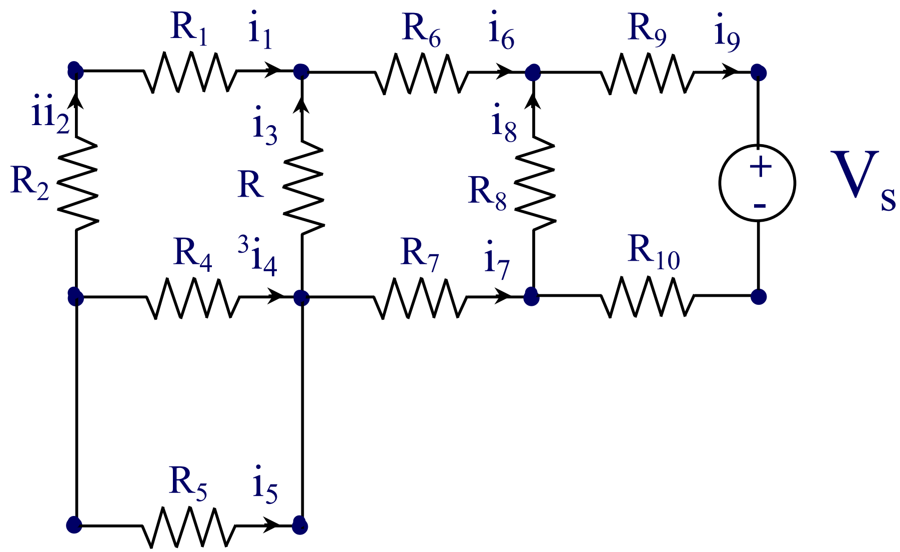
Analys of a circuit

Known quantities:

$R_1 = 1\ \Omega$, $R_2 = 1\ \Omega$, $R_3 = 3\ \Omega$,
$R_4 = 2\ \Omega$, $R_5 = 2\ \Omega$, $R_6 = 0.5\ \Omega$,
$R_7 = 1\ \Omega$, $R_8 = 6\ \Omega$, $R_9 = 1\ \Omega$,
$R_{10} = 1\ \Omega$.

$V_s = 40$ V.

Determine the branch currents.



$v_{1eq} = i_{1eq}R_{1eq} = 3.33$ V
$i_4 = v_{1eq}/ R_4$; $i_5 = v_{1eq}/R_5$
$i_4 = 1.67$ A
$i_5 = 1.67$ A

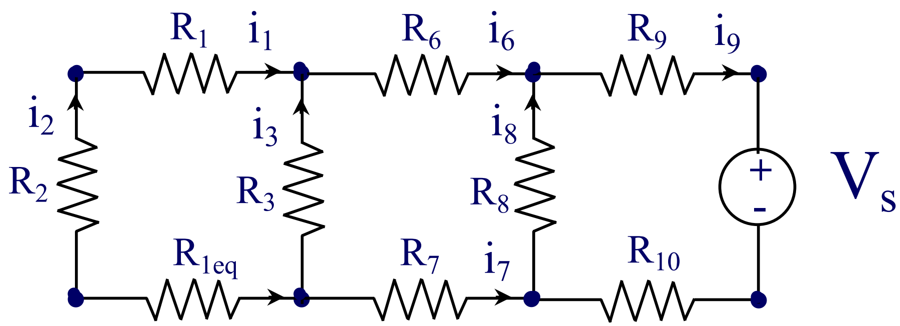
$$R_{1eq} = \frac{R_4 R_5}{R_4 + R_5}$$

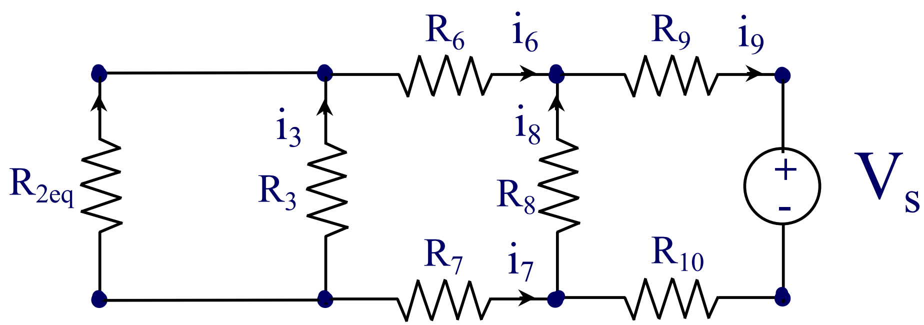
$$R_{1eq} = 1\,\Omega$$

$i_3 = v_{3eq}/ R_3 = -3.33$ A
$i_1 = i_2 = -\ i_{1eq} = v_3/R_{2eq}$
$i_1 = i_2 = -\ i_{1eq} = -3.33$ A

$$R_{2eq} = R_1 + R_2 + R_{1eq}$$

$$R_{2eq} = 3\,\Omega$$





$i_6 = - i_7 = i_{4eq} = - 6.67$ A

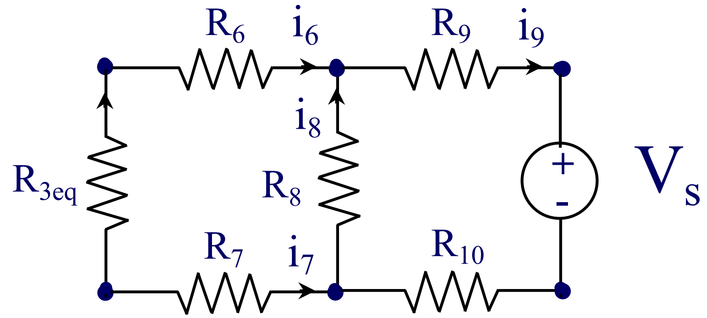
$v_{4eq} = v_{3eq} + v_6 - v_7$

$v_{3eq} = v_{4eq} - R_6\, i_6 + R_7\, i_7$

$v_{3eq} = -10$ V

$$R_{3eq} = \frac{R_{2eq} R_3}{R_{2eq} + R_3}$$

$$R_{3eq} = 1.5\,\Omega$$



$v_{5eq} = v_{4eq} = v_8 = -20$ V

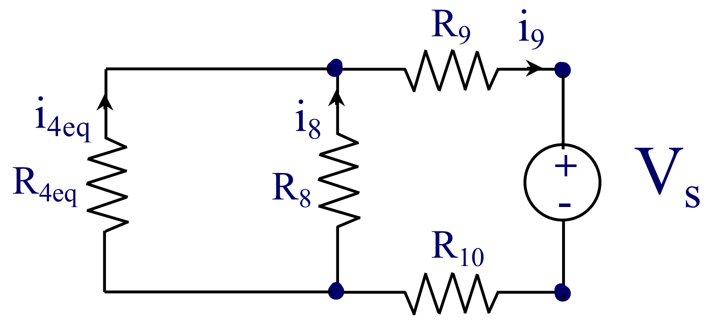
$i_{4eq} = v_{4eq}/ R_{4eq}$; $i_8 = v_8/R_8$

$i_{4eq} = -6.67$ A;

$i_8 = -3.33$ A

$$R_{4eq} = R_6 + R_7 + R_{3eq}$$

$$R_{4eq} = 3\,\Omega$$



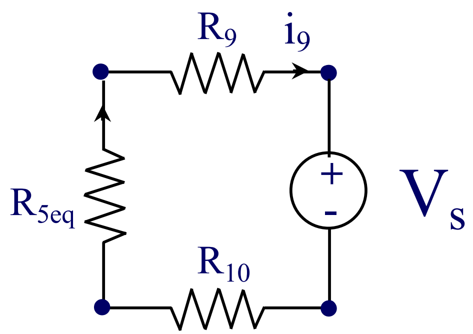
$v_9 + v_{5eq} + v_{10} = -V_s$

$v_{5eq} + R_9\, i_9 + R_{10}\, i_9 = -V_s$

$v_{5eq} = -20$ V

$$R_{5eq} = \frac{R_{4eq} R_8}{R_{4eq} + R_8}$$

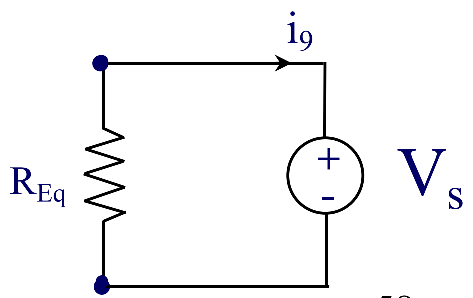
$$R_{5eq} = 2\,\Omega$$



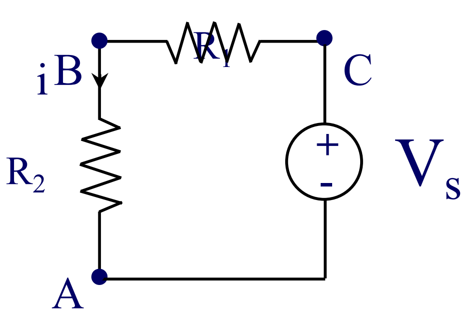
$i_9 = -V_s/R_{Eq} = -10$ A

$$R_{Eq} = R_9 + R_{10} + R_{5eq}$$

$$R_{Eq} = 4\,\Omega$$



Tension Divider



$$R_{eq} = R_1 + R_2$$

$$i = \frac{V_s}{R_{eq}} = \frac{V_S}{R_1 + R_2}$$

From the KTL *it is*

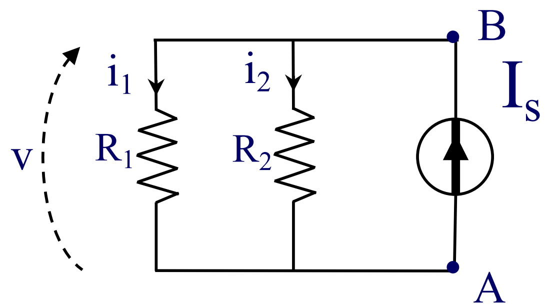
$$v_1 + v_2 - V_s = 0$$

$$\Rightarrow \quad v_1 = V_s - v_2 = V_s - R_2\, i$$
$$v_2 = V_s - v_1 = V_s - R_1\, i$$

$$v_1 = \frac{R_1}{R_1 + R_2} V_s$$
$$v_2 = \frac{R_2}{R_1 + R_2} V_s$$

As $p = V_s^2/R_{eq}$, to reduce the power dissipated, R_{eq} has to be large.

Current Divider



$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

$$v = R_{eq}\, I_s = \frac{R_1 R_2}{R_1 + R_2} I_S$$

From the KCL *it is*

$$i_1 + i_2 - I_s = 0$$

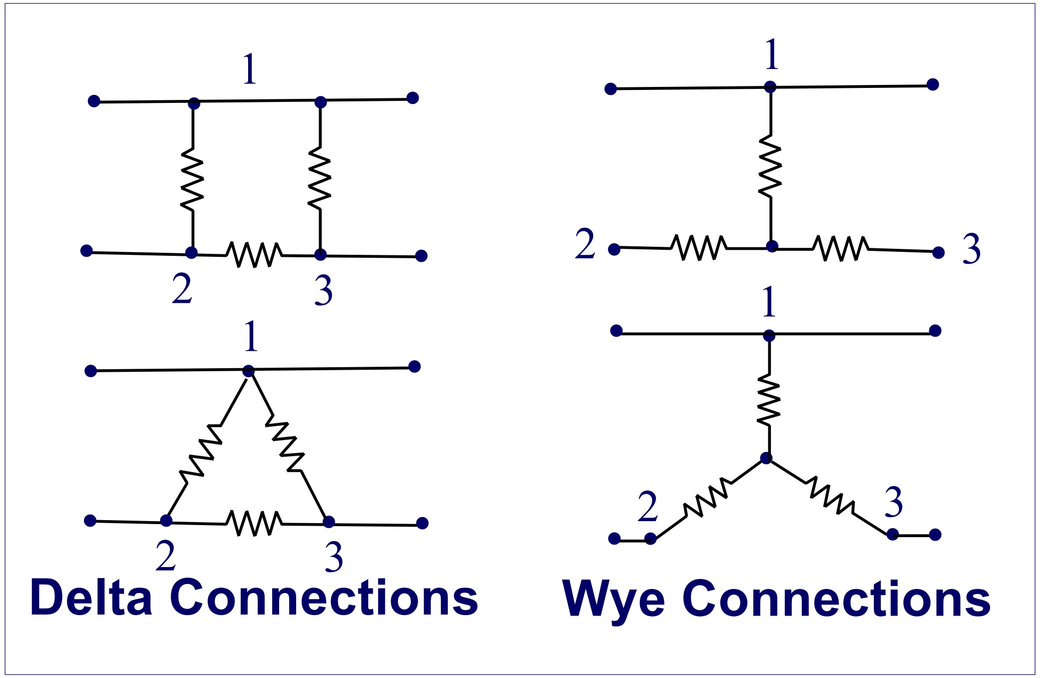
$$\Rightarrow \quad i_1 = I_s - i_2 = I_s - \frac{v}{R_2}$$
$$i_2 = I_s - i_1 = I_s - \frac{v}{R_1}$$

$$i_1 = \frac{R_2}{R_1 + R_2} I_s$$
$$i_2 = \frac{R_1}{R_1 + R_2} I_s$$

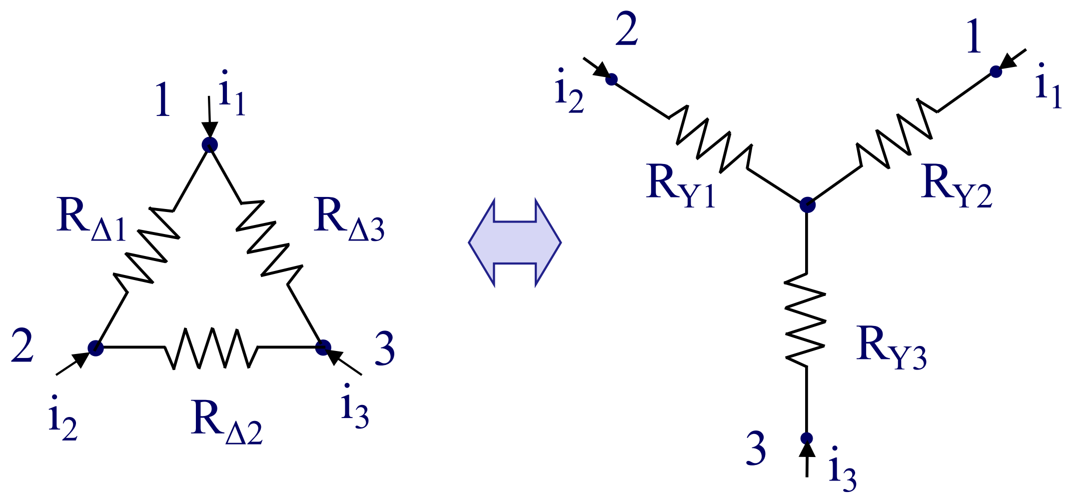
As $p = R_{eq} I_s^2$, to reduce the power dissipated, R_{eq} has to be low.

Wye and Delta Resistor Connections

A system of three resistances may be delta connected or wye connected. It can be more convenient to work with a wye network in a place where the circuit contains a delta configuration. ***A wye network can operate in an equivalent way as a delta network and in the other way around.***



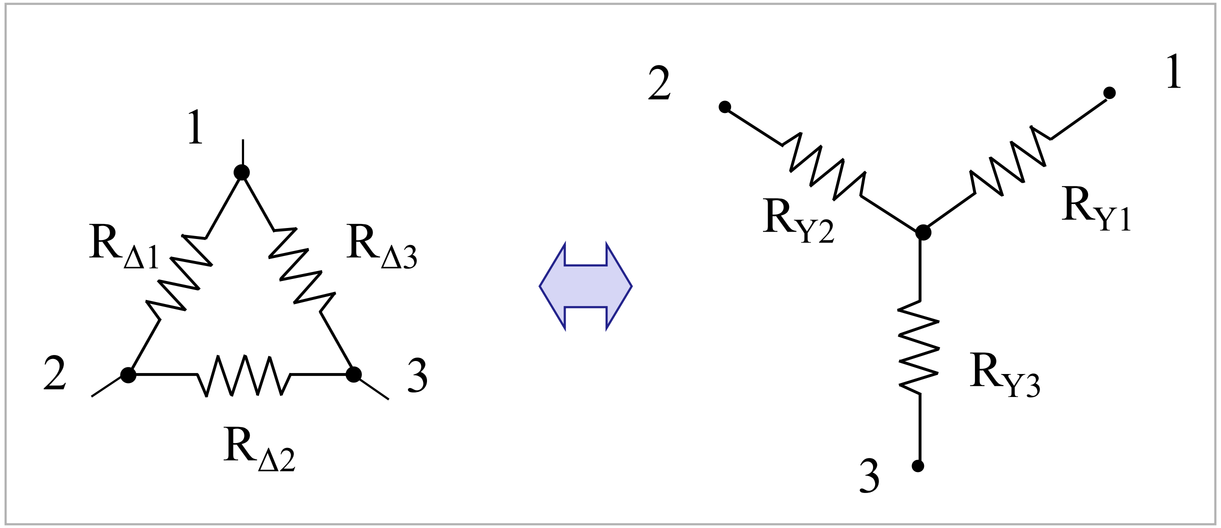
This means that the same tensions v_{12}, v_{23} and v_{31} between nodes 1 and 2, nodes 2 and 3, and nodes 3 and 1 induce the same currents to node 1, node 2 and node 3.



Wye and Delta Resistor Connections

Between node 1 and node 2, if node 3 is not connected, in the ***wye*** and the ***delta*** connection there are the following resistances

$$R_{12}(Y) = R_{Y1} + R_{Y2}$$
$$R_{12}(\Delta) = R_{\Delta 1} // (R_{\Delta 2} + R_{\Delta 3})$$



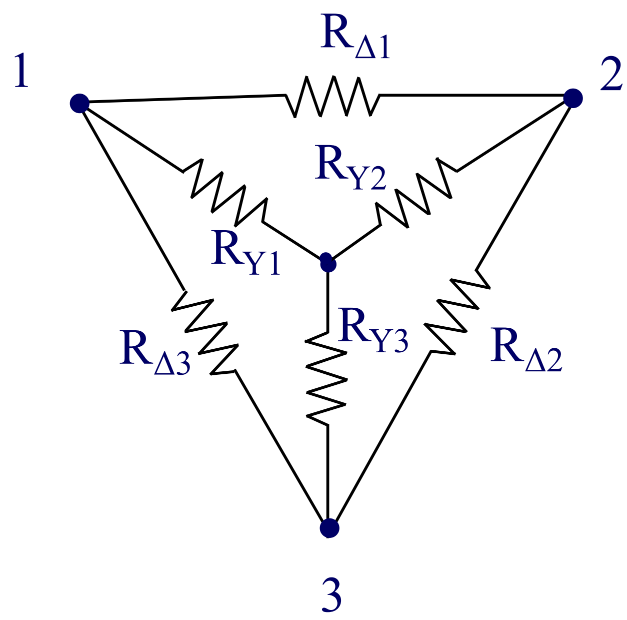
If node 3 is not connected the same current has to correspond to the same voltage. This as to hold for the brances 1-3 and 2-3 when node 2 and are not connected. Therefore:

$$R_{12}(Y) = R_{12}(\Delta) \qquad R_{Y1}+R_{Y2} = \frac{R_{\Delta1}\,(R_{\Delta2}+R_{\Delta3})}{R_{\Delta1}+R_{\Delta2}+R_{\Delta3}}$$

$$R_{13}(Y) = R_{13}(\Delta) \quad\Rightarrow\quad R_{Y1}+R_{Y3} = \frac{R_{\Delta3}\,(R_{\Delta1}+R_{\Delta2})}{R_{\Delta1}+R_{\Delta2}+R_{\Delta3}} \quad\Rightarrow$$

$$R_{23}(Y) = R_{23}(\Delta) \qquad R_{Y2}+R_{Y3} = \frac{R_{\Delta2}\,(R_{\Delta1}+R_{\Delta3})}{R_{\Delta1}+R_{\Delta2}+R_{\Delta3}}$$

Y-Δ Transformation



$$R_{Y1} = \frac{R_{\Delta 1} R_{\Delta 3}}{R_{\Delta 1} + R_{\Delta 2} + R_{\Delta 3}}; \quad R_{Y2} = \frac{R_{\Delta 1} R_{\Delta 2}}{R_{\Delta 1} + R_{\Delta 2} + R_{\Delta 3}}; \quad R_{Y3} = \frac{R_{\Delta 2} R_{\Delta 3}}{R_{\Delta 1} + R_{\Delta 2} + R_{\Delta 3}}$$

Each resistance of the ***wye*** connection is the product of the two resistance of the ***delta*** connection connected to the same node, divided by the sum of the three resistances of the delta connection.

$$R_{\Delta 1} = \frac{R_{Y1} R_{Y2} + R_{Y2} R_{Y3} + R_{Y3} R_{Y1}}{R_{Y3}}; \quad R_{\Delta 2} = \frac{R_{Y1} R_{Y2} + R_{Y2} R_{Y3} + R_{Y3} R_{Y1}}{R_{Y1}}; \quad R_{\Delta 3} = \frac{R_{Y1} R_{Y2} + R_{Y2} R_{Y3} + R_{Y3} R_{Y1}}{R_{Y2}}$$

Each resistance of the ***delta*** connection is the sum of all the products of the resistances of the ***wye*** connection two by two, divided by the resistance in the opposite branch of the wye connection.

For $R_{Y1} = R_{Y2} = R_{Y3} = R_Y$ and $R_{\Delta 1} = R_{\Delta 2} = R_{\Delta 3} = R_\Delta$ it results:

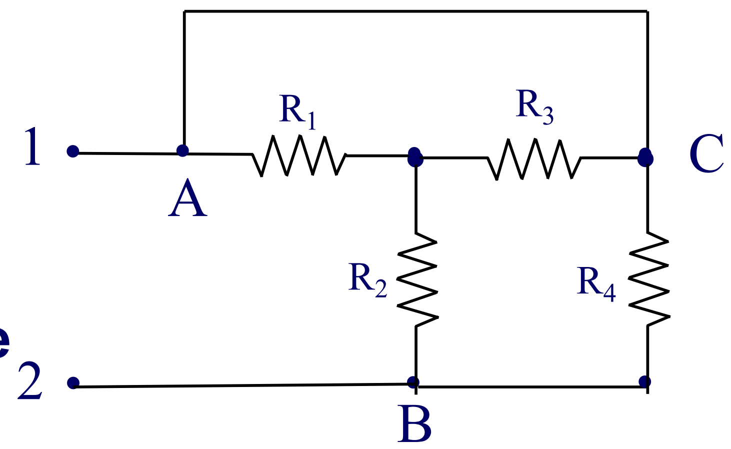
$$R_Y = R_\Delta/3 \quad \textit{and} \quad R_\Delta = 3\, R_Y$$

Y-Δ Transformation

Known quantities:

$R_1 = 3\ \Omega$, $R_2 = 3\ \Omega$, $R_3 = 3\ \Omega$,
$R_4 = 2\ \Omega$,

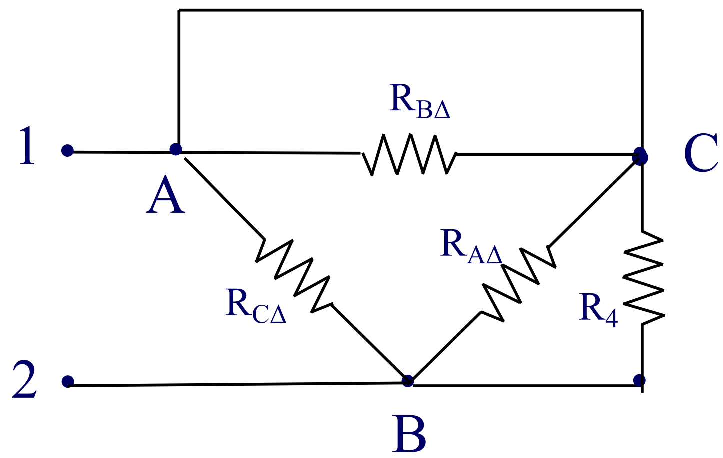
Determine the equivalent resistance between the nodes 1 and 2.



$$R_{A\Delta} = \frac{R_1R_2 + R_2R_3 + R_1R_3}{R_1} = 9\ \Omega$$

$$R_{B\Delta} = \frac{R_1R_2 + R_2R_3 + R_1R_3}{R_2} = 9\ \Omega$$

$$R_{C\Delta} = \frac{R_1R_2 + R_2R_3 + R_1R_3}{R_3} = 9\ \Omega$$



Y-Δ Transformation

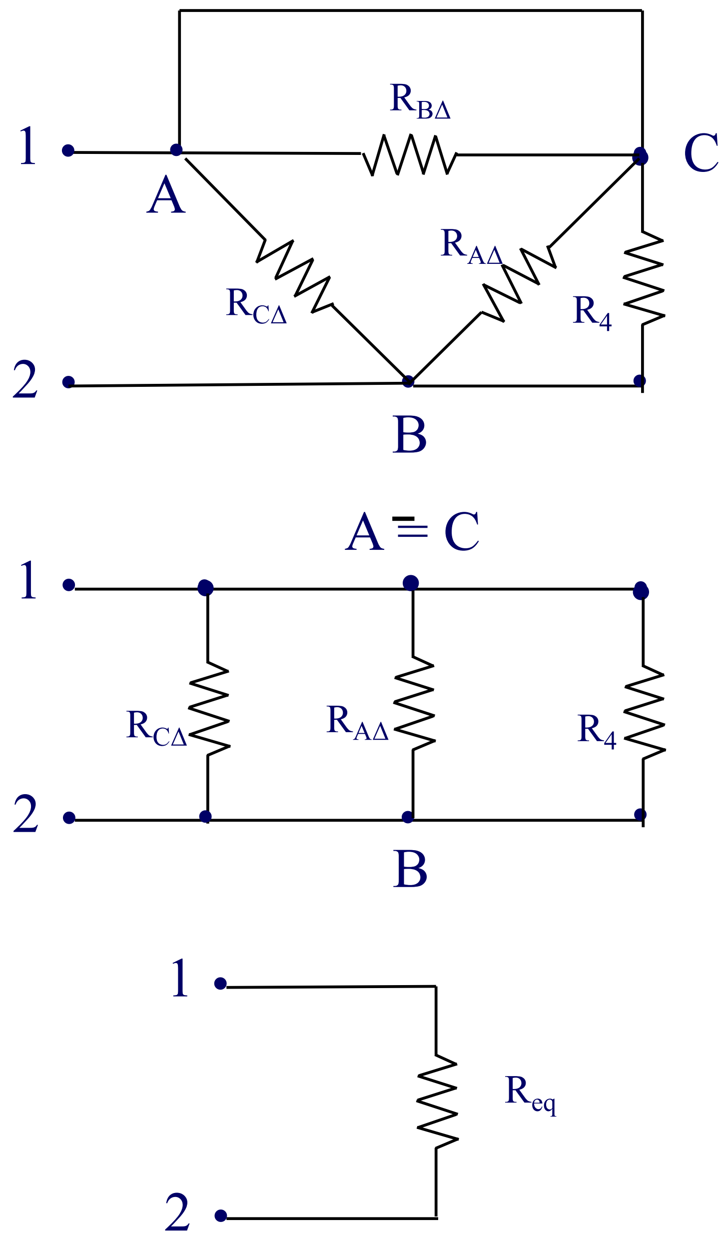
Known quantities:

$R_1 = 3\ \Omega$, $R_2 = 3\ \Omega$, $R_3 = 3\ \Omega$,
$R_4 = 2\ \Omega$,

$$R_{A\Delta} = R_{B\Delta} = R_{C\Delta} = 9\ \Omega$$

$$R_{eq} = \frac{R_4 R_{A\Delta} R_{C\Delta}}{R_4 R_{A\Delta} + R_{A\Delta} R_{C\Delta} + R_4 R_{C\Delta}} =$$

$$= 1{,}385\ \Omega$$

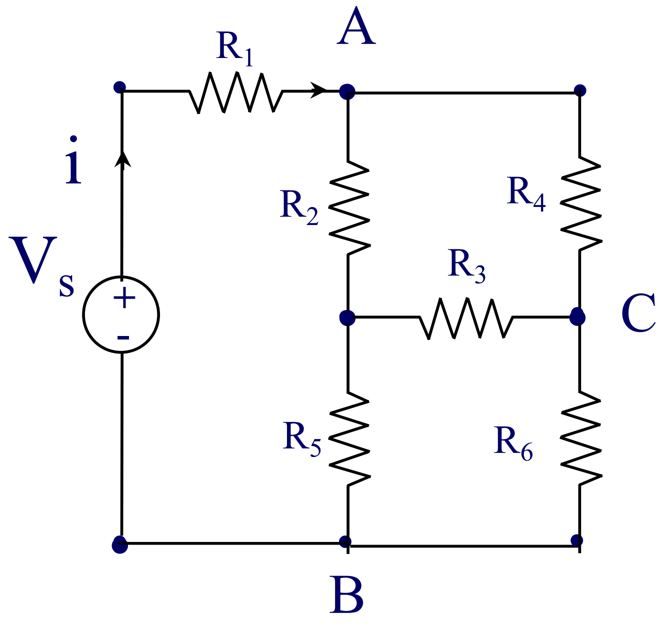


Y-Δ Transformation

Known quantities:

$R_1 = 1\ \Omega$, $R_2 = 2\ \Omega$, $R_3 = 2\ \Omega$,
$R_4 = 3\ \Omega$, $R_5 = 2\ \Omega$, $R_6 = 0.5\ \Omega$,
$V_s = 40$ V.

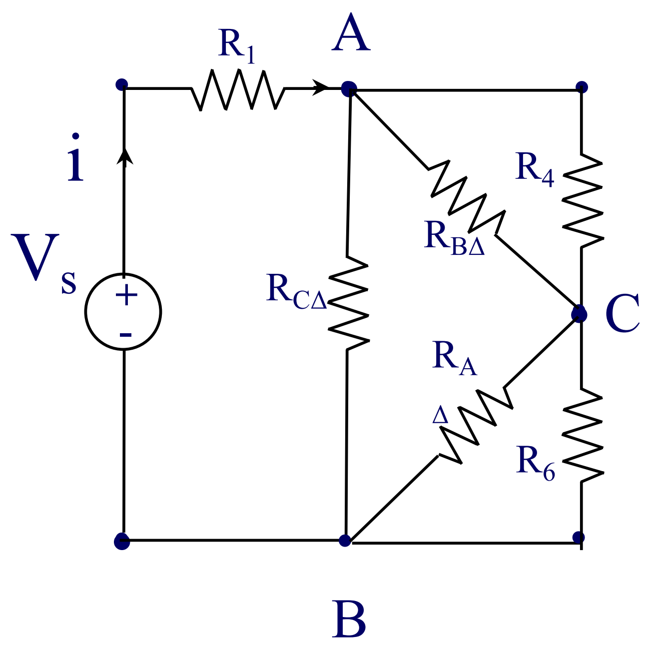
Determine the power delivered by the voltage generator.



$$R_{A\Delta} = \frac{R_2R_5 + R_2R_3 + R_5R_3}{R_2} = 6\ \Omega$$

$$R_{B\Delta} = \frac{R_2R_5 + R_2R_3 + R_5R_3}{R_5} = 6\ \Omega$$

$$R_{C\Delta} = \frac{R_2R_5 + R_2R_3 + R_5R_3}{R_3} = 6\ \Omega$$

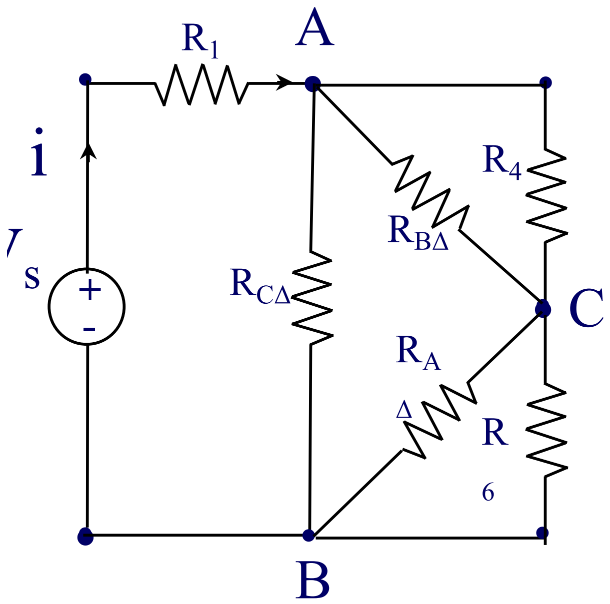


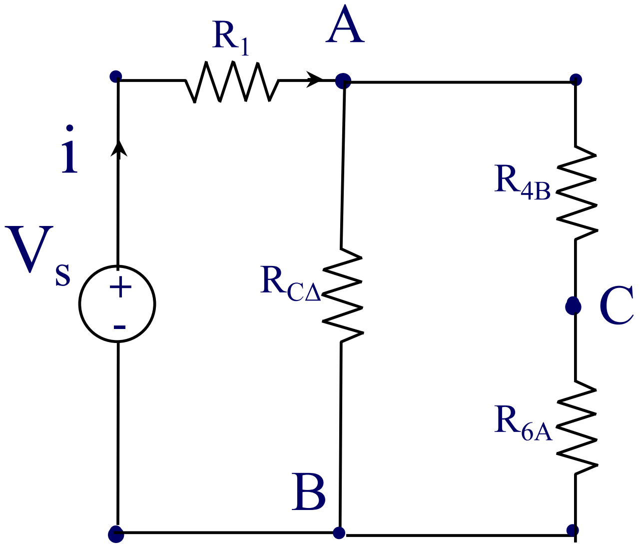
Y-Δ Transformation

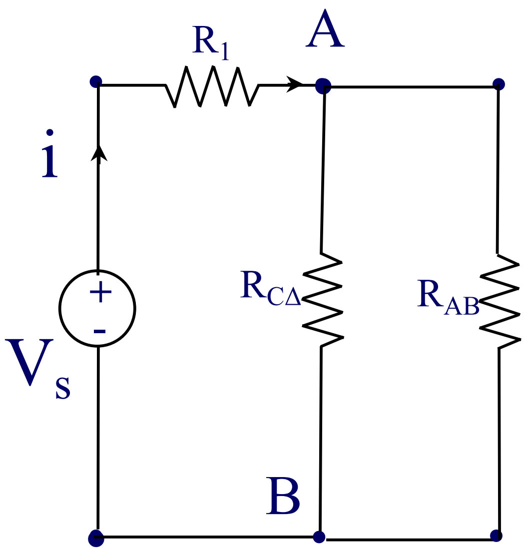
Known quantities:

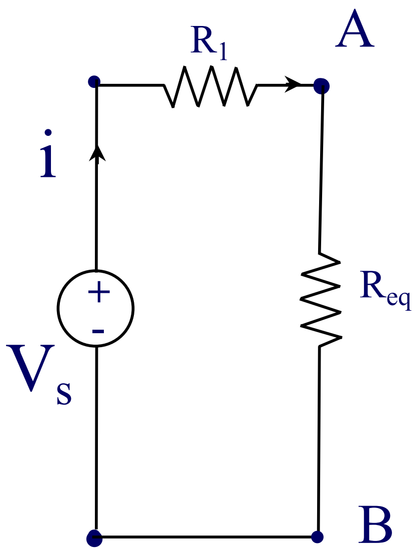
$R_1 = 1\ \Omega$, $R_2 = 2\ \Omega$, $R_3 = 2\ \Omega$,
$R_4 = 3\ \Omega$, $R_5 = 2\ \Omega$, $R_6 = 0.5\ \Omega$,
$V_s = 40$ V.

$$R_{A\Delta} = R_{B\Delta} = R_{C\Delta} = 6\ \Omega$$









$$R_{4B} = \frac{R_4 R_{B\Delta}}{R_4 + R_{B\Delta}} = 2\ \Omega$$

$$R_{6A} = \frac{R_6 R_{A\Delta}}{R_6 + R_{A\Delta}} = 0{,}46\ \Omega$$

$$R_{AB} = R_{4B} + R_{6A} = 2{,}46\ \Omega$$

$$R_{eq} = \frac{R_{C\Delta} R_{AB}}{R_{AB} + R_{C\Delta}} = 1{,}745\ \Omega$$

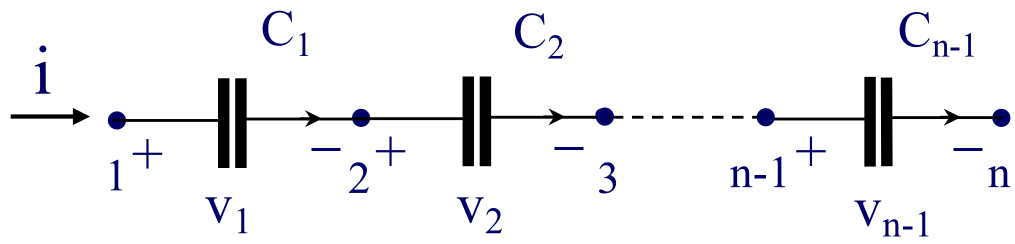
$$i = V_s / \left(R_1 + R_{eq}\right) = 14{,}572\ \text{A}$$

$$p = i\ V_s = 582{,}88\ \text{W}$$

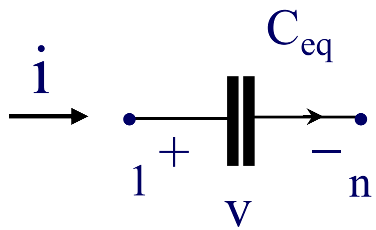
Capacitors in series

Two or more two capacitors are ***in series*** when the current from one element exclusively flows into the next one. Therefore the same current flows through each element one after another:

$$v = v_1 + v_2 + \dots + v_{n\text{-}1}$$



$$\textit{dove:}\quad v_k = \frac{1}{C_k}\int_{t_0}^{t} i(t')dt' + v_k(t_0)$$



$$v = \frac{1}{C_1}\int_{t_0}^{t} i(t')dt' + \frac{1}{C_2}\int_{t_0}^{t} i(t')dt' + \dots + \frac{1}{C_{n\text{-}1}}\int_{t_0}^{t} i(t')dt'$$

$$+ v_1(t_0) + v_2(t_0) + \dots + v_{n\text{-}1}(t_0)$$

$$v = \left(\frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_{n\text{-}1}}\right)\int_{t_0}^{t} i(t')dt' + v_1(t_0) + v_2(t_0) + \dots + v_{n\text{-}1}(t_0)$$

$$v = \frac{1}{C_{eq}}\int_{t_0}^{t} i(t')dt' + v(t_0) \qquad \textit{dove:}$$

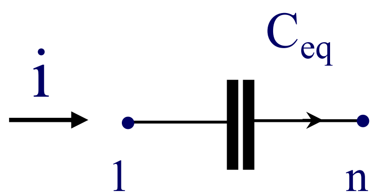
$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_{n\text{-}1}}}$$

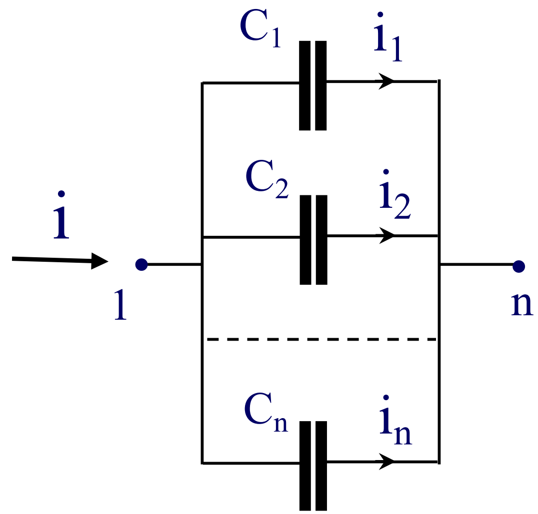
Capacitors in parallel

Two or more capacitors are ***in parallel*** when they share the same two terminals. Therefore the elements will be under the same voltage:

$$i = i_1 + i_2 + ... + i_n$$

$$dove: \quad i_k = C_k \frac{dv}{dt}$$





$$i = C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt} + ... + C_n \frac{dv}{dt}$$

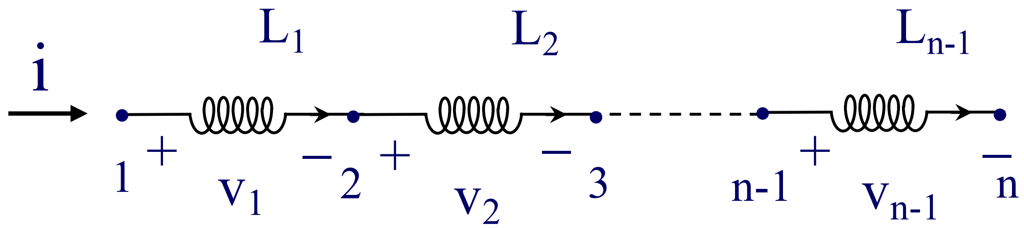
$$i = \left(C_1 + C_2 + ... + C_n\right) \frac{dv}{dt}$$

$$i = C_{eq} \frac{dv}{dt} \qquad dove:$$

$$C_{eq} = C_1 + C_2 + ... + C_n$$

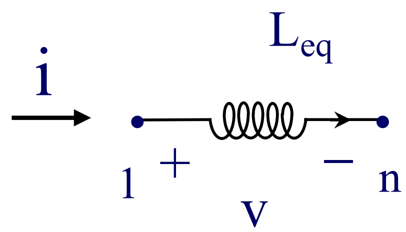
Indictors in series

Two or more two inductors are ***in series*** when the current from one element exclusively flows into the next one. Therefore the same current flows through each element one after another:



$$v = v_1 + v_2 + ... + v_{n\text{-}1}$$

$$\textit{dove :} \quad v_k = L_k \frac{di}{dt}$$



$$v = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + ... + L_{n\text{-}1} \frac{di}{dt}$$

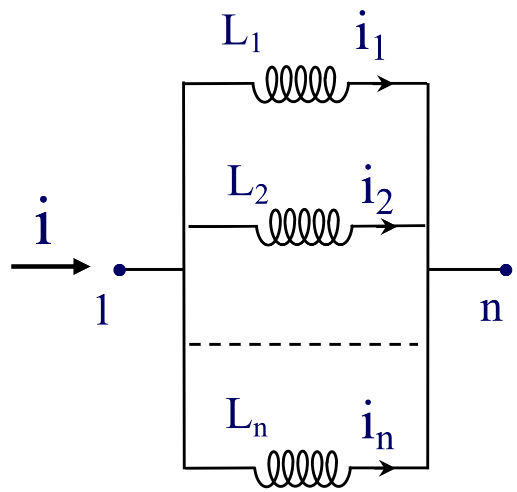
$$v = \left(L_1 + L_2 + ... + L_{n\text{-}1}\right)\frac{di}{dt}$$

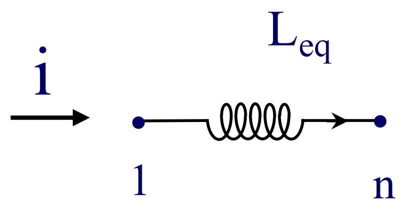
$$v = L_{eq} \frac{di}{dt} \qquad \textit{dove :}$$

$$L_{eq} = L_1 + L_2 + ... + L_{n\text{-}1}$$

Inductors in parallel

Two or more inductors are ***in parallel*** when they share the same two terminals. Therefore the elements will be under the same voltage:





$$i = i_1 + i_2 + ... + i_n$$

$$\textit{dove:} \quad i_k = \frac{1}{L_k}\int_{t_0}^{t} v(t')\,dt' + i_k(t_0)$$

$$i = \frac{1}{L_1}\int_{t_0}^{t} v(t')\,dt' + \frac{1}{L_2}\int_{t_0}^{t} v(t')\,dt' + ... + \frac{1}{L_n}\int_{t_0}^{t} v(t')\,dt'$$
$$+ i_1(t_0) + i_2(t_0) + ... + i_n(t_0)$$

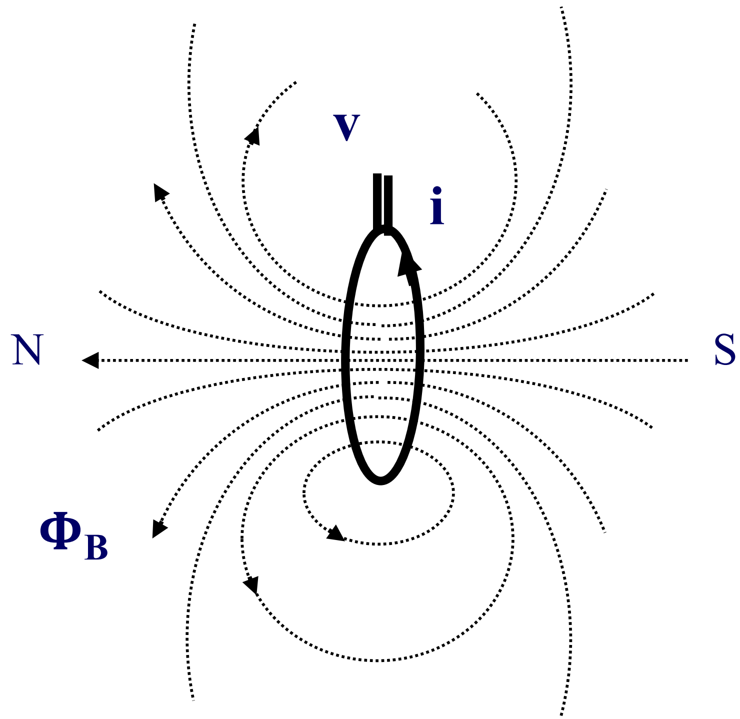
$$i = \left(\frac{1}{L_1} + \frac{1}{L_2} + ... + \frac{1}{L_n}\right)\int_{t_0}^{t} v(t')\,dt' + i_1(t_0) + i_2(t_0) + ... + i_n(t_0)$$

$$i = \frac{1}{L_{eq}}\int_{t_0}^{t} v(t')\,dt' + i(t_0) \quad \textit{dove:}$$

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + ... + \frac{1}{L_n}$$

Magnetic Flux in Inductors

The Magnetic flux Φ_B, ***linked*** with the winding, through which the current i flows, is **generated** by i (1° Maxwell's law or Ampere's law):



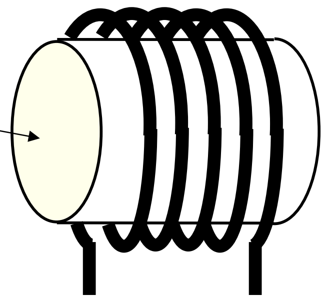
$$\Phi_B = f(i)$$

For a linear inductor it is

$$\Phi_B = L\,i$$

For the 2° Maxwell's law (also Faraday's law or inductance law) it is

$$v = \frac{d\Phi_B}{dt} \Rightarrow v = L\frac{di}{dt}$$



Linear Time Independent Coupled Inductors

The magnetic flux linked with the circuit 1 is generated by two currents: the flux component generated by i_1 and the flux component generated by i_2:

$$\Phi_{C1} = \Phi_1(i_1) + \Phi_1(i_2)$$

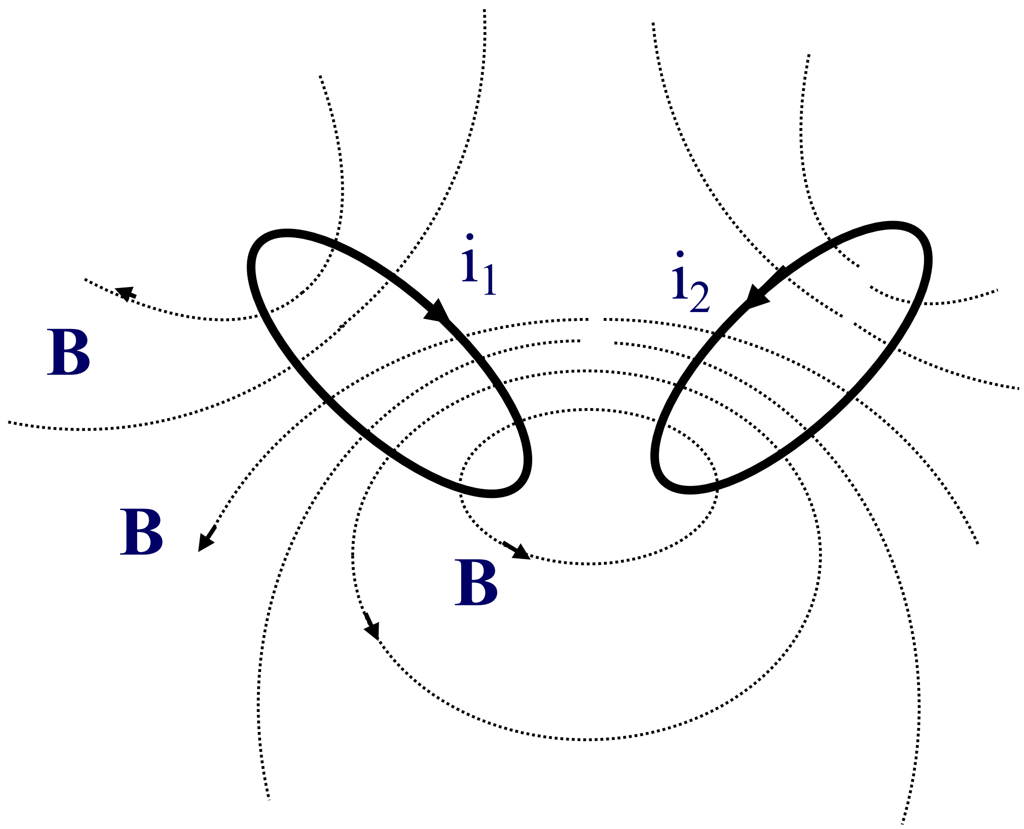
For the <u>linear</u> case it is

$$\Phi_{C1} = L_1 i_1 + M_{12} i_2$$

Here L_1 and M_{12} are the ***self inductance*** and the ***mutual inductance*** respectively.

For <u>time independent</u> circuit elements it results to be

$$v_1 = \frac{d\Phi_{C1}}{dt} = L_1 \frac{d i_1}{dt} + M_{12} \frac{d i_2}{dt}$$



M_{12} depends on each of the two circuits and on their relative positions. M_{12} can be positive or negative.

Linear Time Independent Coupled Inductors

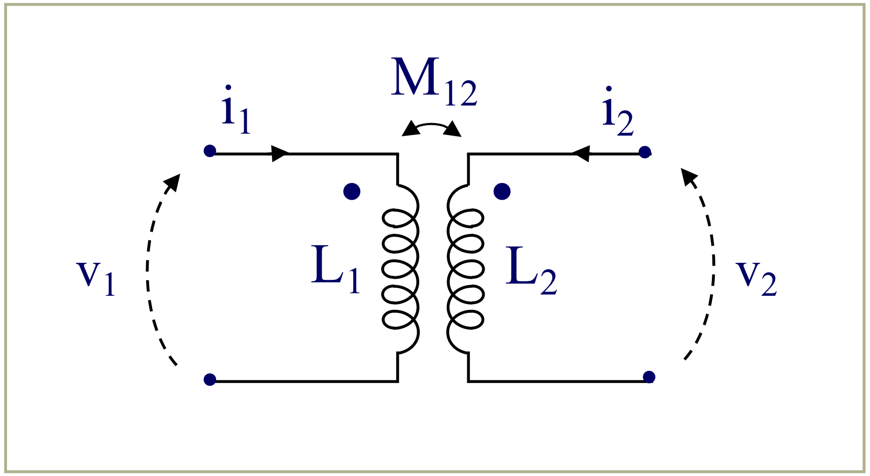
- The two circuits, with or without contacts between them, affect each other by means of the magnetic field generated by the currents flowing through them. They are said to be ***magnetically coupled***.

For two magnetically coupled inductors it is

$$\Phi_{C1} = L_1 i_1 + M_{12} i_2$$
$$\Phi_{C2} = L_2 i_2 + M_{21} i_1$$

It can be demonstrated that $\mathbf{M_{12} = M_{21}}$. Hence it results:

$$v_1 = L_1 \frac{d i_1}{dt} + M_{12} \frac{d i_2}{dt}$$
$$v_2 = M_{12} \frac{d i_1}{dt} + L_2 \frac{d i_2}{dt}$$

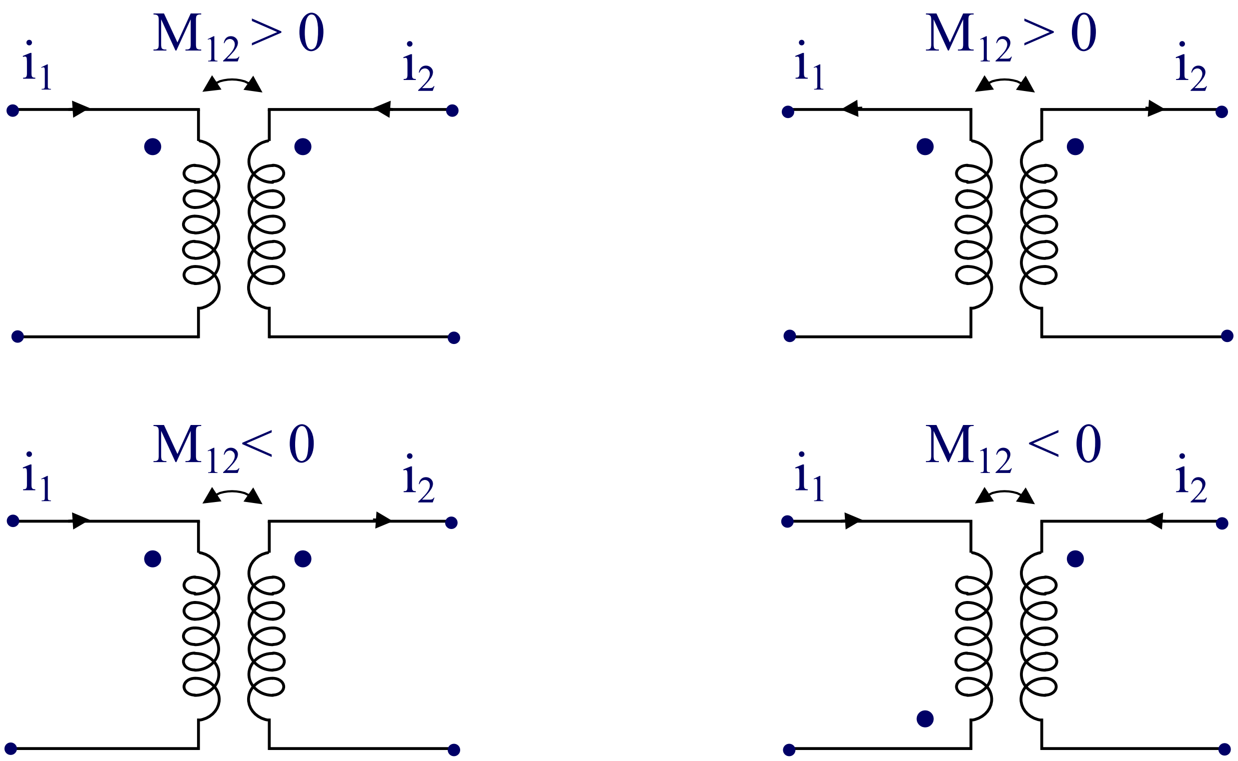


Spot convention: *$M_{12} > 0$ when the spots correspond to terminals with currents flowing inside both of them or terminals with currents flowing outside both of them.*

Linear Time Independent Coupled Inductors

Spot Convention

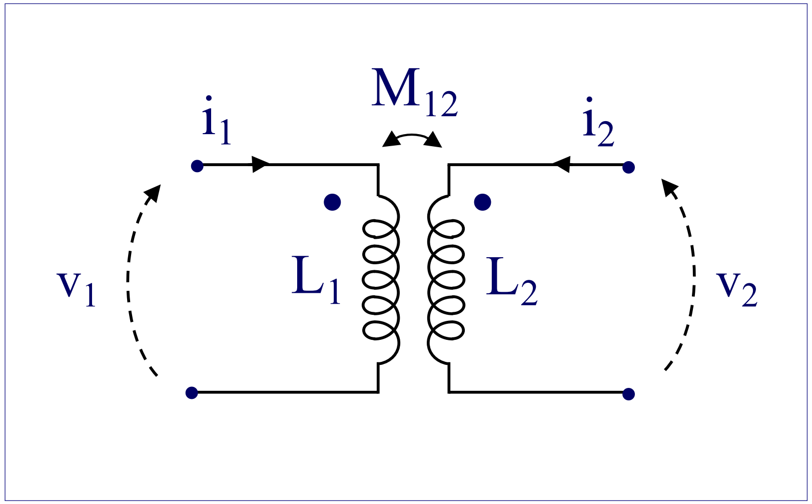
When the spots correspond to the two terminals with the currents flowing inside both or terminals with currents flowing outside both, the mutual inductance M_{12} is assumed to be positive. Otherwise M_{12} is negative.



Linear Time Independent Coupled Inductors

Magnetic Energy Stored in Coupled Inductors

The magnetic energy e(-∞, t) stored in the magnetically coupled inductors in the time interval from -∞ to t, assuming that $i_1(-\infty) = i_2(-\infty) = 0$, $i_1(t) = I_1$ and $i_2(t)=I_2$, is given by:



$$\varepsilon(-\infty, t) = \int_{-\infty}^{t} [i_1 v_1 + i_2 v_2] dt =$$

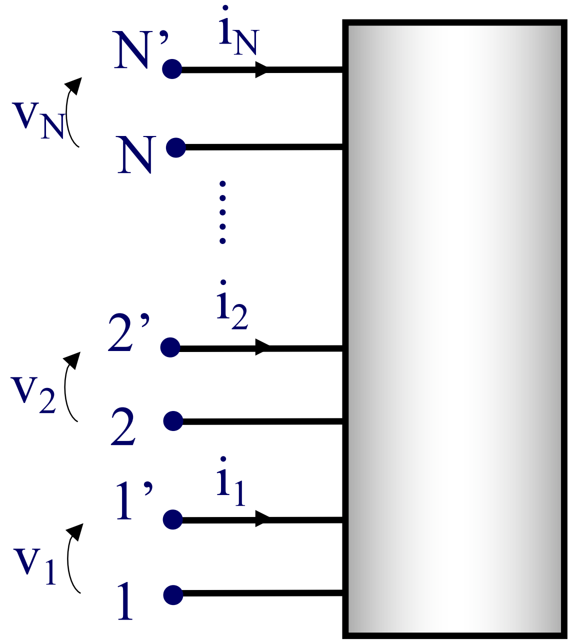
$$= \int_{-\infty}^{t} \left\{ L_1 i_1 \frac{di_1}{dt} + M_{12}\left[i_1 \frac{di_2}{dt} + i_2 \frac{di_1}{dt} \right] + L_2 i_2 \frac{di_2}{dt} \right\} dt$$

$$= \int_{-\infty}^{t} d\left[½\, L_1 i_1^2 + M_{12}\, i_1 i_2 + ½\, L_2 i_2^2 \right]$$

$$\varepsilon(-\infty, t) = ½\, L_1 I_1^2 + M_{12}\, I_1 I_2 + ½\, L_2 I_2^2$$

N-Port Elements

- In a p-terminal circuit element with p even, the terminals can be organized in pairs. When the current flowing inside the first terminal of each pair is equal to the current flowing outside the second terminal of it, the element is an N-port element with N = p/2..



- The quantities defining a N-port element are:

$$\mathbf{i} = \begin{bmatrix} i_1 \\ i_2 \\ . \\ i_N \end{bmatrix} ; \qquad \mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ . \\ v_N \end{bmatrix}$$

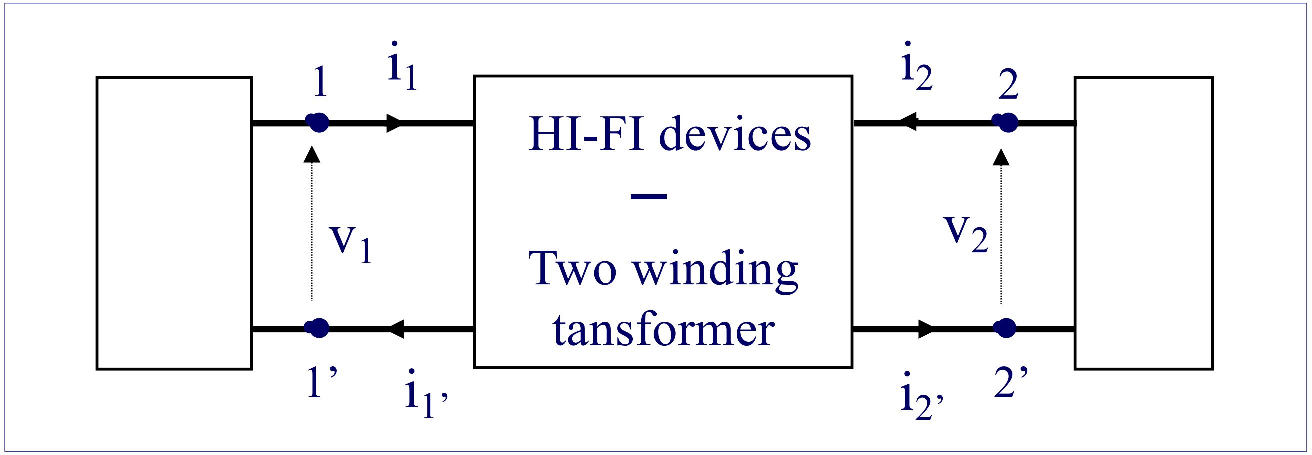
$$p(t) = v_1 \, i_1 + v_2 \, i_2 + ... + v_N \, i_N = \mathbf{v}^T \, \mathbf{i}$$

$$\varepsilon = \int_{-\infty}^{t} p(t') \, dt'$$

When $\varepsilon \geq 0$ the N-port element is passive.

- The N-port element equations are N equations: $v_k = f(i_1, i_2, i_3, \ldots, i_N)$
  where: $k = 1, 2, 3, \ldots, N$.

Two Port Elements



- ***Input port***: port 11'
- ***Output port***: port 22'
- ***Port tension***: v_1, v_2
- ***Port current***: i_1, i_2 ($i_1 = i_{1'}$, $i_2 = i_{2'}$)
- ***Linear two port element***

$$\mathbf{i} = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}; \qquad \mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$\mathbf{A}\,\mathbf{v} + \mathbf{B}\,\mathbf{i} + \mathbf{c} = \mathbf{0}$$

$$p = \mathbf{v}^T\,\mathbf{i}$$

$$a_{11}v_1 + a_{12}v_2 + b_{11}i_1 + b_{12}i_2 + c_1 = 0$$

$$a_{12}v_1 + a_{22}v_2 + b_{21}i_1 + b_{22}i_2 + c_2 = 0$$

$$p = v_1\,i_1 + v_2\,i_2$$

Terminology: English – Chinese

active element	有源元件
branch	支路
branch current	支路电流
branch tension	支路电压
capacitor	电容器
capacitance	容量
circuit component	电路元件
circuit element	电路元件
closed circuit	闭合电路
conductance	电导
conducting plates, armatures	电导板, 电枢
conductivity	电导率
controlled source	受控源
current controlled element	电流控制元件
current divider	分流

coupled inductors	耦合电感
delta connection	三角形连接
dielectric constant	介电常数
dielectric, insulator	电介质,绝缘体
diode	二极管
dissipative element	耗散元件
distributed parameters	分布参数
electric circuit	电路
electric power	电力
electric resistance	电阻
electromagnetic energy	电磁能
electrostatic energy	静电能
element equation	元件方程
independent current source	独立电流源

Terminology: English – Chinese

independent voltage source	独立电压源
inductance	电感
inductor	电感
linear (non-linear) element	线性(非线性)元件
lumped parameters	集总参数
Kirchhoff's current law	基尔霍夫电流定律
Kirchhoff's tension law	基尔霍夫电压定律
monolithic chip	单芯片
mutual inductance	互感
n-terminal element	N端口元件
n-port element	N端口元件
two-port element	两端元件
node	节点

open circuit	开路
parallel connection	并联
passive element	被动元件
pn-junction	Pn结
resistor	电阻
resistance	电阻
resistivity	电阻率
self inductance	自感
series connection	串联
storage element	储能元件
tension divider	分压
two terminal circuit element	两端电路元件
voltage controlled element	电压控制元件
wye connection	星形连结

Terminology
English – Italian

active element	elemento attivo
branch	ramo, lato
branch current	corrente di ramo
branch tension	tensione di ramo
capacitor	condensatore
capacitance	capacità
circuit component	componente circuitale
circuit element	elemento circuitale
closed circuit	circuito chiuso
conductance	conduttanza
conducting plates, armatures	armature di condensatore
conductivity	Conducibilità
controlled source	generatore controllato
current controlled element	elemento controllato in corrente
current divider	partitore di corrente

coupled inductors	induttori accoppiati
delta connection	connessione a triangolo
dielectric constant	costante dielettrica
dielectric, insulator	dielettrico, isolante
diode	diodo
dissipative element	elemento dissipativo
distributed parameters	parametri distribuiti
electric circuit	circuito elettrico
electric power	potenza elettrica
electric resistance	resistenza elettrica
electromagnetic energy	energia elettromagnetica
electrostatic energy	energia elettrostatica
element equation	equazione caratteristica, equazione costitutiva
independent current source	generatore di current indipendente

Terminology
English – Italian

independent voltage source	*generatore di tensione indipendente*
inductance	*induttanza*
inductor	*induttore*
linear (non-linear) element	*elemento lineare (non lineare)*
lumped parameters	*parametri concentrati*
Kirchhoff's current law	*legge di Kirchhoff delle correnti*
Kirchhoff's tension law	*legge di Kirchhoff delle tensioni*
monolithic chip	*chip monolitico*
mutual inductance	*mutua iduttanza*
n-terminal element	*eemento ad n poli*
n-port element	*elemento a n porte*
two-port element	*elemento a dueporte*
node	*nodo, polo*

open circuit	*circuito aperto*
parallel connection	*connessione in parallelo*
passive element	*elemento passivo*
pn-junction	*giunzione pn*
resistor	*resistore*
resistance	*resistenza*
resistivity	*resistività*
self inductance	*autoinduttanza*
series connection	*connessione in serie*
storage element	*elemento con memoria*
tension divider	*partitore di tensione*
two terminal circuit element	*elemento circuitale a due terminali, bopolo*
voltage controlled element	*elemento controllato in tensione*
wye connection	*connessione a stella*