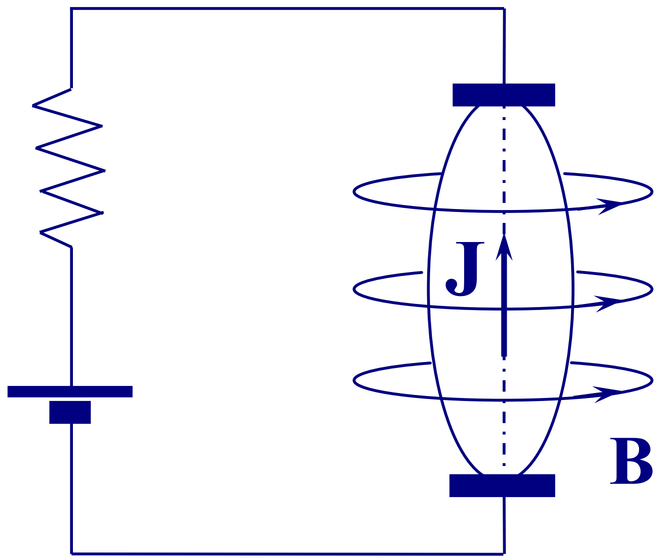
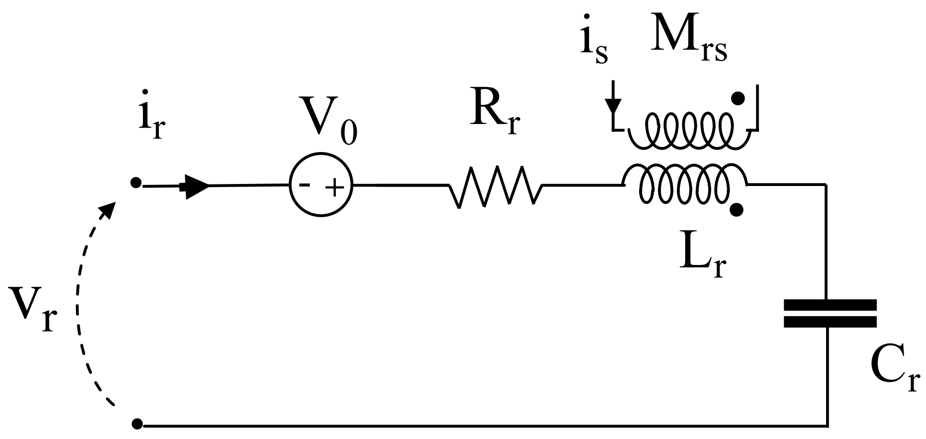
7. Transient Response



Transient Response in Electric Circuits

The element equation for the branch of the figure when the source is given by a generic function of time, is



$$v_r(t) = L_r \frac{di_r}{dt} + M_{rs}\frac{di_s}{dt} + \frac{1}{C_r}\int_{-\infty}^{t} i(t')dt' + R_r i_r(t) - V_0$$

The circuit is described by the topology equations and by the element equations of the form given by eq.1.

$$\sum_n i_r = 0$$

$$\sum_m v_r = 0$$

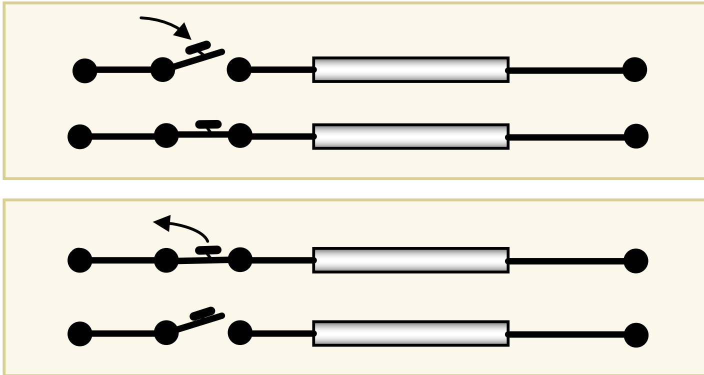
$$v_r = f(i_r)$$

This system of equation is integro-differential and can be solved by an existing conventional method of mathematics.

Transient Response in Electric Circuits

Transient Cause

A change in the circuit operating conditions is the cause of a transient before reaching the steady state operation which can be studied in the time domain by means of the set of the equations of the circuit analysis.



On-off or off-on mode changes of switches or sudden changes of the excitation (voltage or current source modeled as step functions) are the cause of transients followed by a steady state operation that is the response of the circuit to the changed condition.

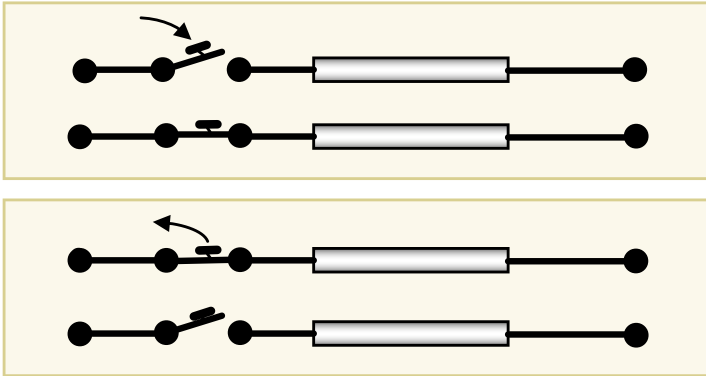
- The ***transient response*** is the circuit's temporary response that will die out with time. This is the temporary part of the response.
- The ***steady-state response*** is the behavior of the circuit a long time after the sudden change has happened. This is the permanent part of the response.

Complete response* = *transient response* + *steady-state response

Transient Response in Electric Circuits

Transient Cause

The transient response operation, which will precede the establishment of the steady-state operation, is due to the time required by the storage elements to built the new conditions under which they will operate at the final steady-state regime.



The storage elements may change their operation state with fulfilling the ***energy conservation principle.*** As a consequence of it storage elements prevent instantaneous variation of energy in the transit from $t = 0^-$ to $t = 0^+$:

$$\varepsilon(0^-) = \varepsilon(0^+)$$

An instantaneous variation of the energy would only be caused by a source of infinite power:

$$p(t=0) = \lim_{\Delta t \to 0} \frac{\varepsilon(0^-) - \varepsilon(0^+)}{\Delta t}$$

Energy Conservation Principle

In order to inhibit instantaneous variations of the stored energy, capacitors oppose any sharp variation of the tension, and inductors oppose any sharp variation of the current. The energy stored into the storage element has to be transferred following the new situation and this require times. This energy cannot vary abruptly:

$$\varepsilon_L(t) = \frac{1}{2} L i^2 \xleftarrow[\textit{magnetic energy stored by an inductor}]{\textit{in inductors } \mathrm{i} \textit{ don't vary abruptly}} \qquad v = L\frac{di}{dt}$$

$$\varepsilon_C(t) = \frac{1}{2} C v^2 = \frac{1}{2}\frac{Q^2}{C} \xleftarrow[\textit{electrostatic energy stored by a capacitor}]{\textit{in capacitors } \mathrm{v} \textit{ and } \mathrm{Q} \textit{ don't vary abruptly}} \qquad i = C\frac{dv}{dt}$$

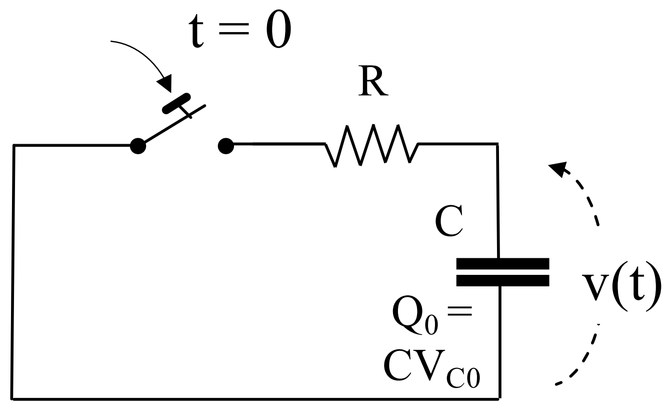
The transit from $t = 0^-$ and $t = 0^+$, representing the status of the circuit operation immediately before the change and immediately after of it respectively, has to fulfill the energy conservation principle, which has the following consequences:

$i(0^-) = i(0^+)$	*in branches with inductors*
$v(0^-) = v(0^+)$	*between the terminal of a capacitor*
$Q(0^-) = Q(0^+)$	*charge stored in each of the conducting plates of the capacitor*

First Order Circuits: *RC Circuit*

Source Free RC Circuit Response – Natural Response

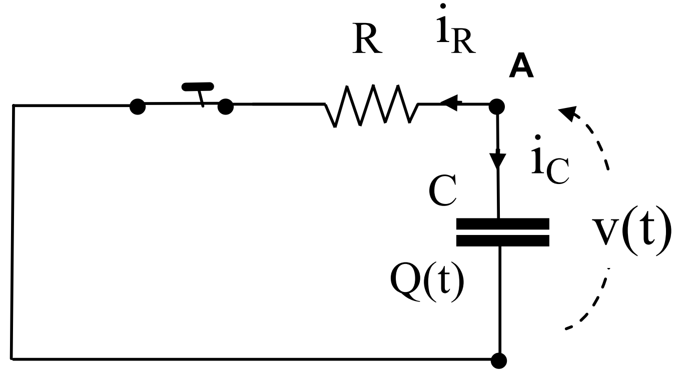
- At time $t = 0$ the switch of the RC circuit of the figure is closed. The capacitor before $t = 0$ is charged at $Q = Q_0$. A transient will start and will be extinguished when the regime at the new conditions is reached. The voltage, due to the electrostatic field of the capacitor acts on the charges stored on a capacitor plates , which flow through the resistor toward the opposite plate. This current transfers the electrostatic energy stored by the capacitor to the resistor where it is dissipated. When all the energy is dissipated the transient vanishes.



$- \infty < t \leq 0^-$

- The circuit is being excited by the energy stored in the capacitor. No external sources are present.

- The aim is to determine the circuit response that is assumed to be given by the behavior of the voltage v(t) across the capacitor.



$0^+ \leq t < +\infty$

*The **natural response** of a circuit refers to the behavior (in terms of voltages or currents) of the circuit itself, with no external sources of excitation.*

First Order Circuits: *RC Circuit*

Source Free RC Circuit Response – Natural Response

At time $-\infty < t \leq 0^-$ it is $Q = Q_0$ and $v = Q_0/C = V_{C0}$. For the energy conservation principle at $t = 0^+$ it is also $Q = Q_0$ and $v = Q_0/C = V_{C0}$.

At the $0^+ \leq t < +\infty$ from KCL at node A it is:

$$i_R + i_C = 0$$

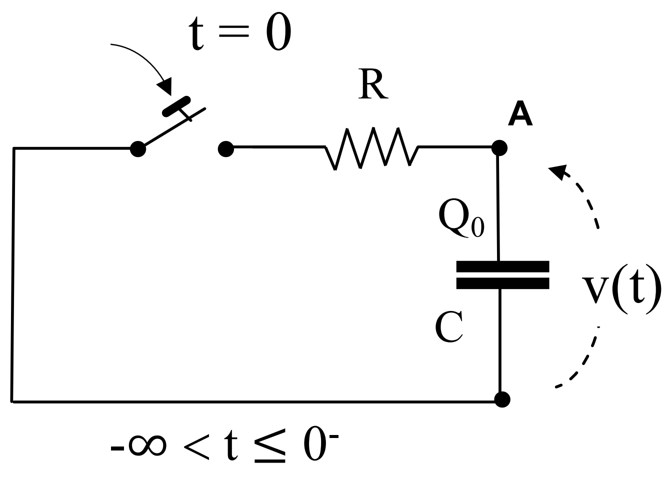
as $i_C = C\,dv/dt$ and $i_R = v/R$, it results

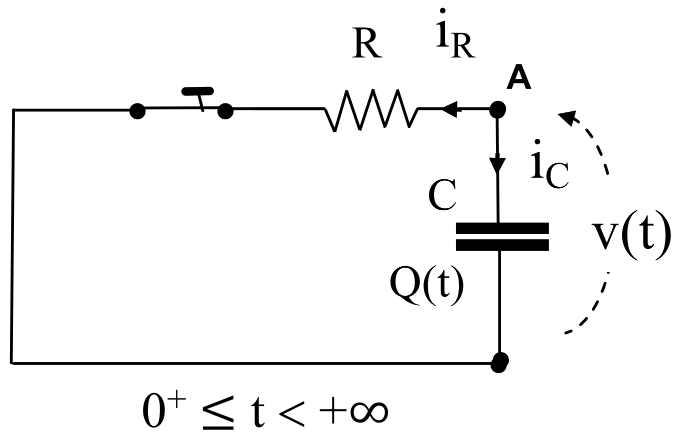
$$C\frac{dv}{dt} + \frac{v}{R} = 0 \qquad (1)$$

The solution of the transient, which represents the natural response of the RC circuit, is given by the solution of eq. 1 that is a first order homogeneous differential equation (this is the motivation of the term ***first order circuit***):

$$v(t) = A\,e^{\alpha t}$$

$$\alpha = -1/(RC) = -1/\tau$$

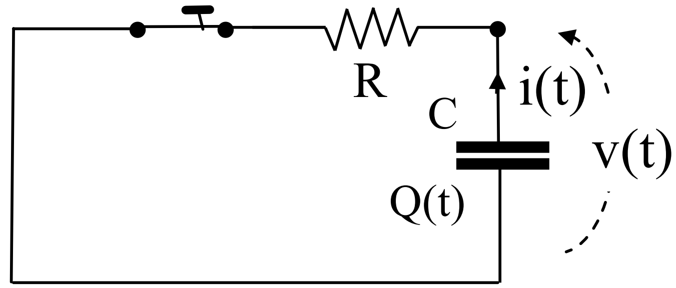




where α is the solution of the characteristic equation associated to eq. 1. The time constant is define by $\tau = -1/\alpha$.

First Order Circuits: *RC Circuit*

Source Free RC Circuit – Natural Response



$0^+ \leq t < +\infty$

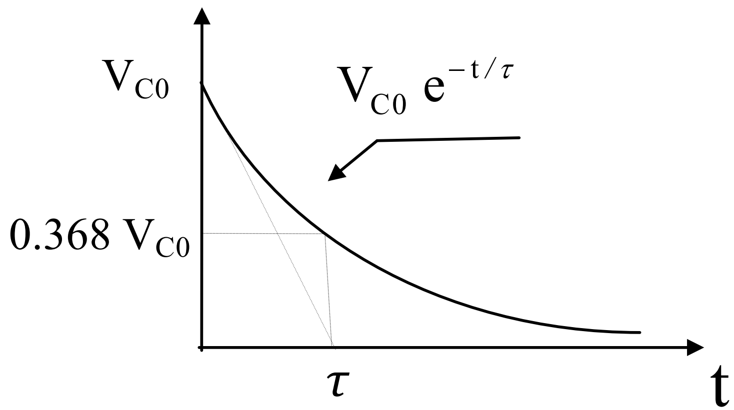
The constant A is derived from the initial conditions at $t = 0^+$:

$$v(0^+) = V_{C0}(0^-) = Q(0^-)/C$$

therefore:

$$v(t) = V_{C0}\, e^{-t/\tau}$$

where τ = RC is the time constant.



The time constant τ of a circuit is the time required to the response to decay to a factor 1/e or 36.8 % of its initial value.

t	$V(t)/V_{C0}$
τ	0.36788
2τ	0.13534
3τ	0.04979
4τ	0.01832
5τ	0.00674

Every time interval of τ the voltage is reduced by 0.368 % of its previous value: $v(t+\tau) = v(t)/e = 0.368\ v(t)$.

It takes 5τ to the circuit to reach its final state (steady state).

First Order Circuits: *RC Circuit*

Source Free RC Circuit
Natural Response

The current in the circuit is:

$$i(t) = \frac{v(t)}{R} = \frac{V_{C0}}{R}\, e^{-t/(RC)}$$

The power dissipated by the resistor is

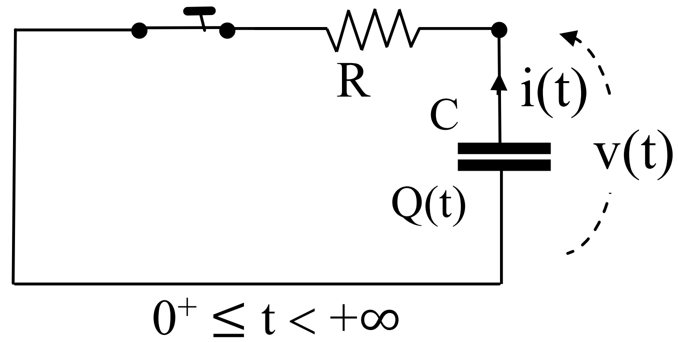
$$p(t) = v(t)\; i(t) = \frac{V_{C0}^2}{R}\, e^{-2t/(RC)}$$

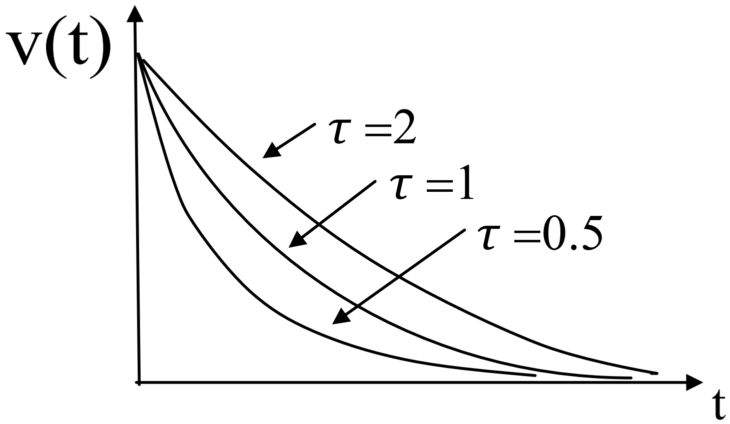
and the energy transfered from the capacitor and absorbed by the resistor up to time t is

$$\varepsilon\;(t) = \int_0^t p(t)\; dt = \int_0^t \frac{V_{C0}^2}{R}\, e^{-2t/(RC)} dt =$$

$$= \left(\frac{\tau\; V_{C0}^2}{2R}\, e^{-2t/(RC)} \right)_0^t = \frac{CV_{C0}^2}{2}\left(1 - e^{-2t/(RC)}\right)$$

and for $t \to +\infty,\; \varepsilon(+\infty) \to (CV_{C0}^2)/2.$





The RC circuit can be obtained as an equivalent circuit (Thévenin/Norton).

<u>The key quantities are:</u>

1. ***The initial capacitor voltage:*** ***$v(0^-) = v(0^+) = V_{C0}$***
2. ***The time constant $\tau = RC$***

First Order Circuits: *RC Circuit*

Step Response of an RC Circuit

The transient is caused by a step of the voltage. This can be done by a source voltage which is suddenly applied by closing a switch. In this case it is:

$-\infty < t \leq 0^- : \quad Q=Q_0, \ v = Q_0/C=V_{C0}$

At $0^+ \leq t < +\infty$ from the KTL it is

$$V_{C0} + \frac{1}{C}\int_0^t i(t')\,dt' + R\,i - V_0 = 0 \qquad (1)$$

From the time derivative of eq. 1 it is obtained:

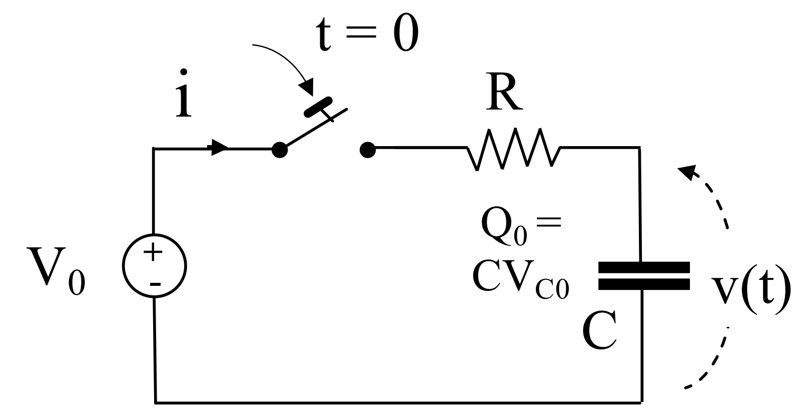
$$\frac{d\,i}{dt} + \frac{1}{RC}\,i = 0 \qquad (2)$$

The solution of the transient solution of eq. 2 (a first order homogeneous differential equation) :

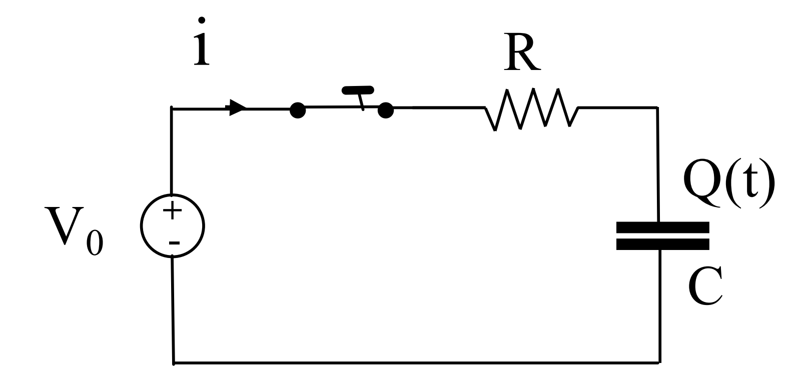
$$i(t) = A\,e^{\alpha t} \qquad (3)$$

where again $\alpha = -1/\tau = -1/(RC)$.

The value of the constant A is determined through the analysis of the initial conditions.



$-\infty < t \leq 0^-$



$0^+ \leq t < +\infty$

First Order Circuits: *RC Circuit*

Step Response of an RC Circuit

Analysis of the initial data at t = 0^-:

$v_C(0^-) = V_{C0}$; $Q(0^-) = C\,V_{C0}$ *(initial data)*

$i(0^-) = 0$

Analysis of the initial conditions at $t = 0^+$:

- Across the capacitor: $v(0^+) = v(0^-) = V_{C0}$

  and from the KTL: $Ri - V_0 + V_{C0} = 0$

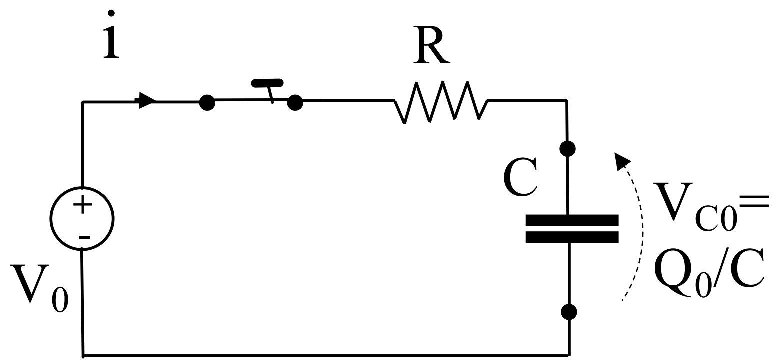
  $\rightarrow$ $i(0^+) = (V_0 - V_{C0})/R$

- From eq. 3 [$i(t)=Ae^{-t/RC}$], it is: $i(0^+) = A$

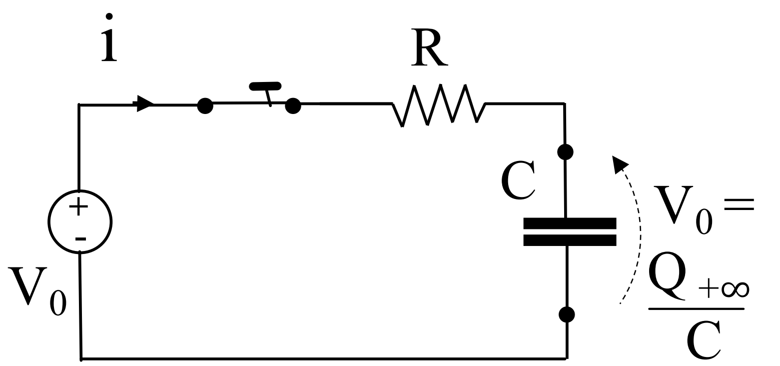
  $\rightarrow$ $A = (V_0 - V_{C0})/R$

Hence the expression of eq. 3 is

$$i(t) = \frac{V_0 - V_{C0}}{R}\, e^{-t/(RC)}$$



$t = 0^+$



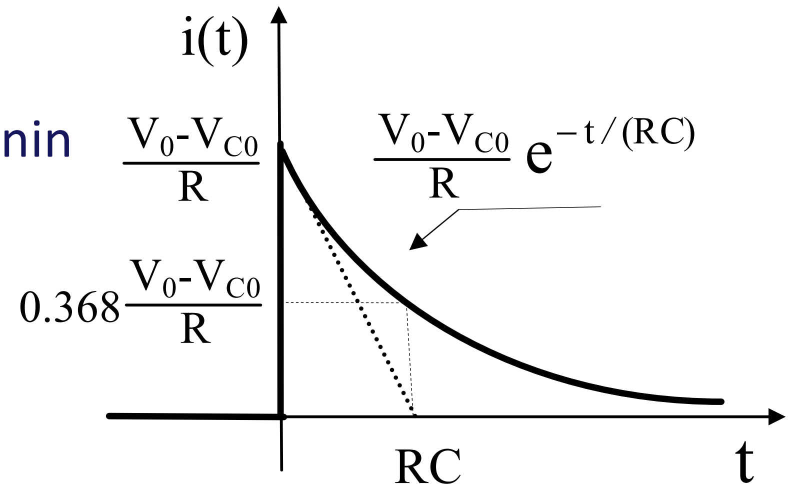
$t = +\infty$

First Order Circuits: *RC Circuit*

Step Response of an RC Circuit

The response of the RC circuit (also for a Thévenin equivalent circuit) to a sudden voltage source excitation in terms of the current i(t) is :

$$i(t) = \frac{V_0 - V_{C0}}{R} e^{-t/(RC)}$$

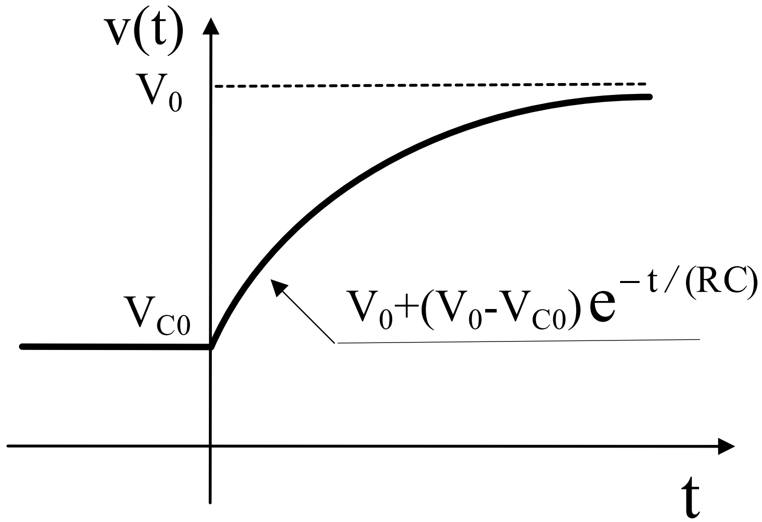


The response of the RC circuit in terms of the voltage v(t) across the capacitor is derived from the element equation of the capacitor:

$$v(t) = V_{C0} + \frac{1}{C}\int_0^t i(t')\,dt'$$

from the solution of the integral it results

$$v(t) = V_0 + \left(V_{C0}\text{-}V_0\right) e^{-t/(RC)}$$



The analysis of the regime at t = +∞ gives the steady-state response (<u>forced response</u>) [i(+∞) = 0, v(+∞) = V_0] that added to the transient response (<u>natural response</u>) gives the <u>complete response</u> of the RC circuit suddenly excited.

First Order Circuits: *RL Circuit*

Step Response of an RL Circuit

When a voltage source is suddenly applied by switching on the RL circuit it is:

$-\infty < t \leq 0^-$: $i = 0$,

At $0^+ \leq t < +\infty$ from the KTL it is

$$L\frac{di}{dt} + Ri - V_0 = 0 \qquad (1)$$

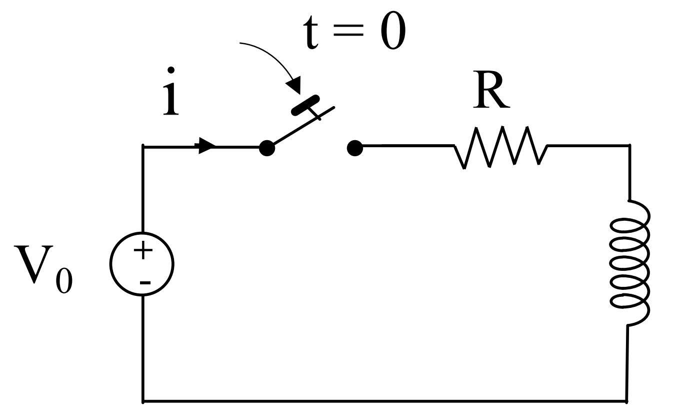
The solution of the transient is given by the solution of the homogeneous differential equation (natural component of the response):

$$i(t) = A\,e^{\alpha t}$$

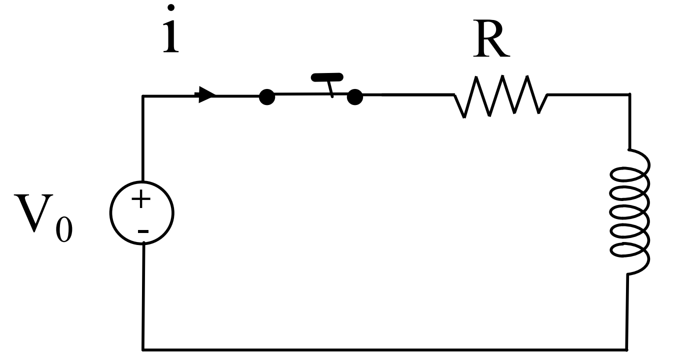
where α is given by the characteristic equation: $\alpha = -R/L = -1/\tau$. The complete response of the RL circuit excited by a voltage source is

$$i(t) = A\,e^{-\frac{R}{L}t} + f_0$$

f_0 is a particular solution of eq. 1. For it the steady-state-solution at $t = +\infty$ (***forced response***) is taken: $f_0 = i(+\infty)$. The constant A is given by the initial condition at $t = 0^+$: $i(0^+) = i(0^-) = 0$.



$-\infty < t \leq 0^-$



$0^+ \leq t < +\infty$

First Order Circuits: *RL Circuit*

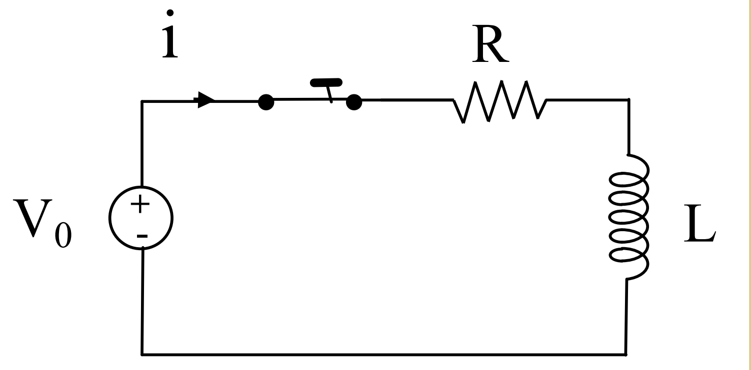
Step Response of an R Circuit

Analysis of the steady state at $t = +\infty$:

$$R\,i - V_0 = 0$$

$$\rightarrow \quad f_0 = i = V_0/R$$

$$\rightarrow \quad i(t) = A\,e^{-\frac{R}{L}t} + \frac{V_0}{R}$$



$$0^+ \leq t < +\infty$$

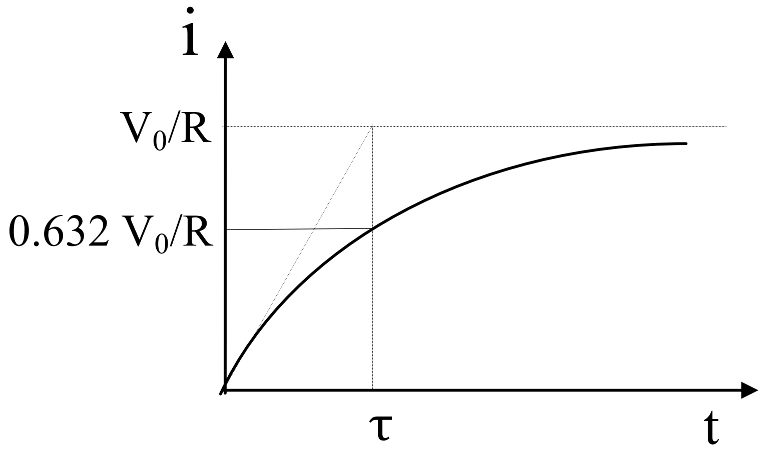
Analysis of the initial data at $t = 0^-$:

$$i(0^-) = 0$$

Analysis of the initial conditions at $t = 0^+$:

$$i(0^+) = i(0^-) = 0$$

$$\rightarrow \quad A = -V_0/R$$



Hence the complete response of the RL circuit when suddenly excited by a voltage source, is:

$$i(t) = \frac{V_0}{R}\left(1 - e^{-\,t/(L/R)}\right)$$

The <u>complete response</u> of the RL circuit is the steady-state response (<u>forced response</u>) [i(+ ∞) = 0] added to the transient response (<u>natural response</u>).

First Order Circuits

Response of First Order Circuits

➢ A first order circuit is a circuit which can be a Thévenin circuit containing a resistor and a memory element. The solution of the response of the circuit to a sudden excitation is given by the solution of a first order differential equation.

➢ The response of the circuit, when excited by an external source (***complete response***) is given by the response of the circuit when no external sources are present (***transient response*** or ***natural response***) superimposed to the steady state response that is the portion of the complete response (***steady state response – forced response***) which remains active when the transient response has died out.

Complete response	=	***transient response***	+	***steady-state response***
		natural response		***forced response***
		(temporary part)		*(permanent part)*

- *The natural response, that gives* ***the transient part of the response****, is the* ***solution of the homogeneous time differential equation.***
- *The forced steady-state response of the circuit is the* ***particular solution of the non-homogeneous time differential equation at t = + ∞****.*

Second Order Circuits: *RLC Circuit*

A second order circuit is characterized by a second order differential equation.
It consists of resistors and two energy storage elements (equivalent to).

The analysis of a second order circuit response is similar to that used for first order. The natural response is originated by the excitation/de-excitation with transfer of the energy stored in the storage elements.

t = 0^-: The ***initial data*** are essential to define the ***excitations of the transient***. They are the magnetic energy stored in inductors and the electrostatic energy in capacitors. Therefore this is determined by the currents of the inductors and the voltages (or the charges) of the capacitors at $t = 0^-$. These initial data are given by the solution of the circuit at the steady-state conditions at $t = 0^-$.

t = 0^+: The ***initial conditions*** are derived from the initial data considering that during the transition from $t = 0^-$ to $t = 0^+$ the energy continuity principle must be fulfilled: **$i(0^+) = i(0)$** ***in inductors*** and **$q(0^+) = q(0^-)$, $v(0^-) = v(0^+)$** ***in capacitors***.

t = + ∞: The ***steady state solution*** at $t = +\infty$ gives the particular solution of the non-homogeneous differential equations in the ***forced response*** case when independent sources are present.

The natural response is calculated by solving the homogeneous differential equations. The characteristic times in the exponents are the solutions of the characteristic equation, the constants of integration are derived from the initial conditions. The particular solution of the non-homogeneous differential equations at $t = +\infty$ gives the steady state forced response.

Second Order Circuits: *RLC Circuit*

Response of Second Order Circuits

- The response of the circuit, when excited by an external source (***complete response***) is given by the response of the circuit when no external sources are present (***transient response*** - ***natural response***) superimposed to the steady state response that is the portion of the complete response (***steady state response*** – ***forced response***) which remains when the transient response has been extinguished.

Complete response	=	***transient response***	+ ***steady-state response***
		natural response	***forced response***
		(temporary part)	*(permanent part)*

As a general case of second order circuits, in the following of this chapter a series RLC circuit (R, L, and C in series) excited by an independent voltage source is considered.

Second Order Circuit: *Series RLC Circuit*

At $-\infty < t \leq 0^-$: $Q = Q_0$, $v = V_{C0} = Q_0/C$, $i = 0$

At t = 0: the switch is turned on.

At $0^+ \leq t < +\infty$: from LKT it results:

$$\frac{1}{C}\int_{-\infty}^{t} i(t')\,dt' - V_0 + R\,i + L\frac{di}{dt} = 0 \qquad (1)$$

$$\frac{Q(0)}{C} + \frac{1}{C}\int_{0}^{t} i(t')\,dt' - V_0 + R\,i + L\frac{di}{dt} = 0$$

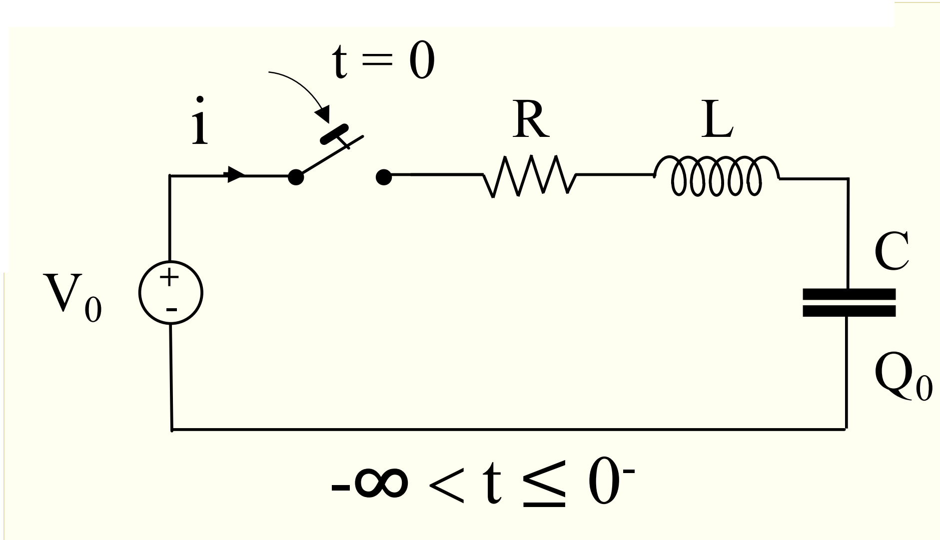
$$v_C(0) + \frac{1}{C}\int_{0}^{t} i(t')\,dt' - V_0 + R\,i + L\frac{di}{dt} = 0 \qquad (2)$$

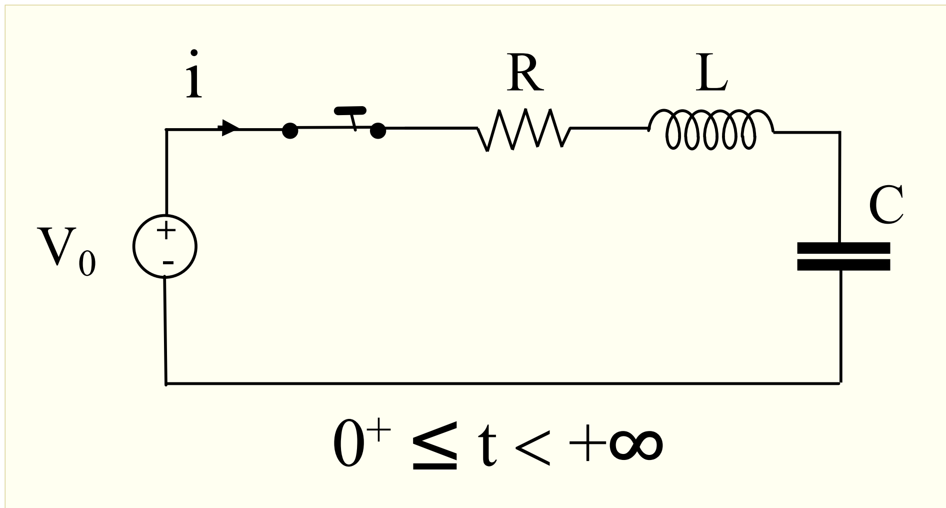
from the time derivaton it results :

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}\,i = 0 \qquad (3)$$

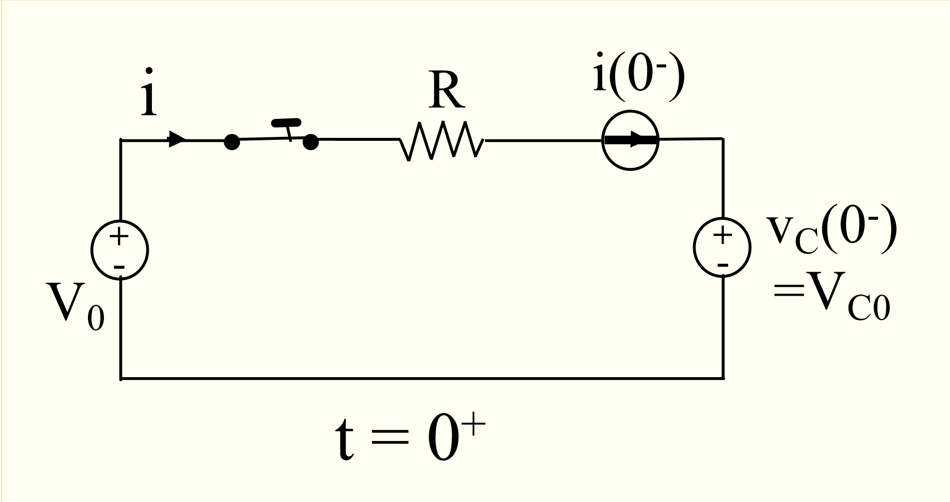
This is a ***second order differential equation*** the solution of which is:

$$i(t) = A_1 e^{x_1 t} + A_2 e^{x_2 t} + i(+\infty)$$





Second Order Circuits: *Series RLC Circuit*



Analysis of the initial data at t = 0⁻:

$v_C(0^-) = V_{C0}$; $Q(0^-) = C\,V_{C0}$

$i(0^-) = 0$

Analysis of the initial cond. at t = 0⁺:

- Across the capacitor:

$v_C(0^+) = v_C(0^-) = V_{C0}$

- Through the inductor:

$i(0^+) = i(0^-) = 0$

$$Ri + L\,\frac{di(0^+)}{dt} + V_{C0} - V_0 = 0$$

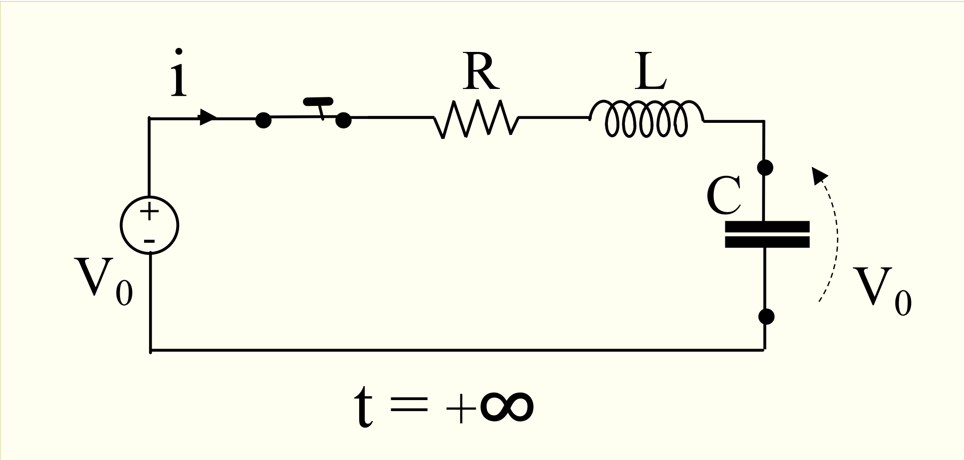
$$\rightarrow \quad \frac{di(0^+)}{dt} = (V_0 - V_{C0})/L$$

Analysis of the steady state at t = +∞ :

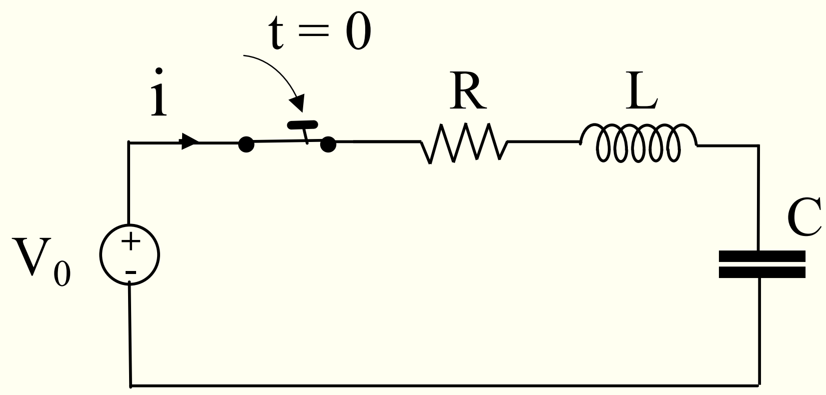
$i(+\infty) = 0$; $v_C(+\infty) = V_0$

At   t = 0⁺ :

- ***Capacitors excite the circuit as independat voltage sources: $v_C(0^+) = v_C(0^-) = Q(0^-)/C$;***
- ***Inductors excite the circuit as independat current sources: $i(0^+) = i(0^-)$.***



Second Order Circuits:
Series RLC Circuit



$$0^+ \le t < +\infty$$

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{d i}{dt} + \frac{1}{LC}\, i = 0 \qquad (3)$$

This is a ***second order differential equation*** the solution of which is:

$$i(t) = A_1 e^{x_1 t} + A_2 e^{x_2 t} + i(+\infty)$$

The characteristic equation of eq. 3 is:

$$x^2 + \frac{R}{L} x + \frac{1}{LC} = 0$$

$$x_{12} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$\rightarrow \quad x_{12} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

Case A: $\alpha > \omega_0 \rightarrow R > 2\sqrt{L/C}$
x_1 and x_2 are real and distict (negative): ***overdamped case***

Case B: $\alpha = \omega_0 \rightarrow R = 2\sqrt{L/C}$
x_1 and x_2 real double solution (negative): ***critically damped case***

Case C: $\alpha < \omega_0 \rightarrow R < 2\sqrt{L/C}$
x_1 and x_2 complex conjugate (negative real part) ***underdamped case***

x_1 , x_2 — ***natural frequencies***

$\alpha = R/(2L)$ — ***damping factor***

$\omega_0 = 1/\sqrt{LC}$ — ***the resonance frequency*** **is also the** ***underdumped natural frequency***

Second Order Circuit: *Series RLC Circuit*

Case A - ***overdamped case :*** $\alpha > \omega_0 \rightarrow R > 2\sqrt{L/C}$;
x_1 , x_2 negative, real and dinstict

The solution of eq. 3 is:

$$i(t) = A_1 e^{x_1 t} + A_2 e^{x_2 t} \quad \text{where} \quad x_{12} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

From the initial conditions at $t = 0^+$:

$$A_1 + A_2 = i(0^+) = 0$$

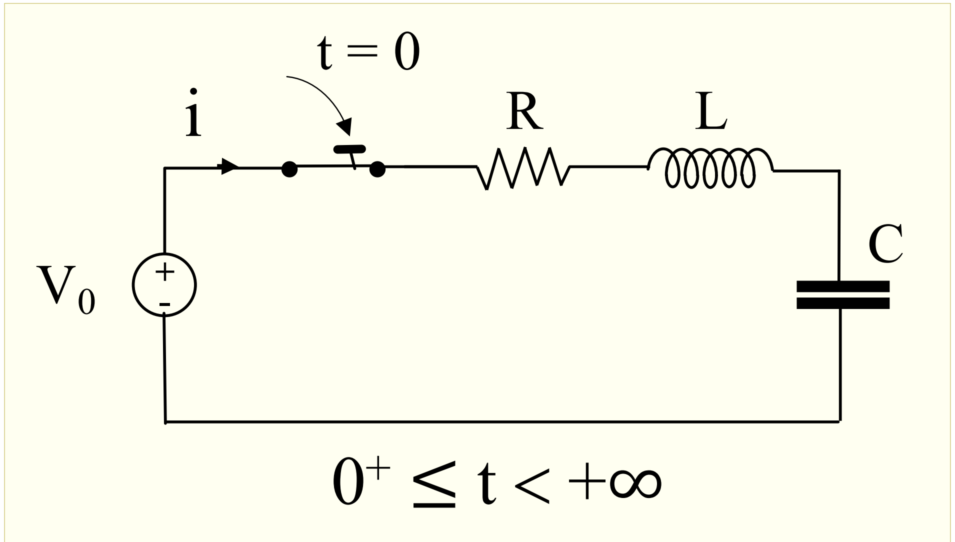
$$x_1 A_1 + x_2 A_2 = \frac{d\, i(0^+)}{dt} = \left(V_0 - V_{C0}\right)/L$$

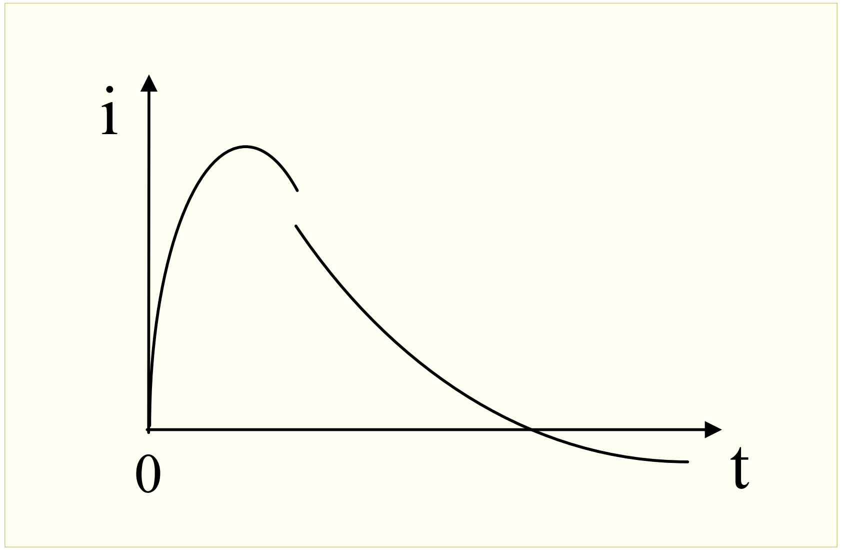
$$\Rightarrow \quad A_1 = -A_2 = \frac{V_0 - V_{C0}}{2L\sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}}$$

Second Order Circuit: *Series RLC Circuit*

Transient solution in case A – Overdamped case : $\alpha > \omega_0$, $R > 2\sqrt{L/C}$

$$i(t) = \frac{V_0 - V_{C0}}{2L\sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}}\left[e^{\left(\sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}\right)t} - e^{-\left(\sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}\right)t}\right]e^{-\frac{R}{2L}t}$$





Second Order Circuit: *Series RLC Circuit*

Case B - *critically damped case :* $\alpha = \omega_0 \rightarrow R = 2\sqrt{L/C}$;

x_1 , x_2 negative coincident solutions

The solution of eq. 4 is:

$$i(t) = A_1 e^{-\alpha t} + A_2\, t\, e^{-\alpha t} \quad \text{where} \quad \alpha = \frac{R}{2L}$$

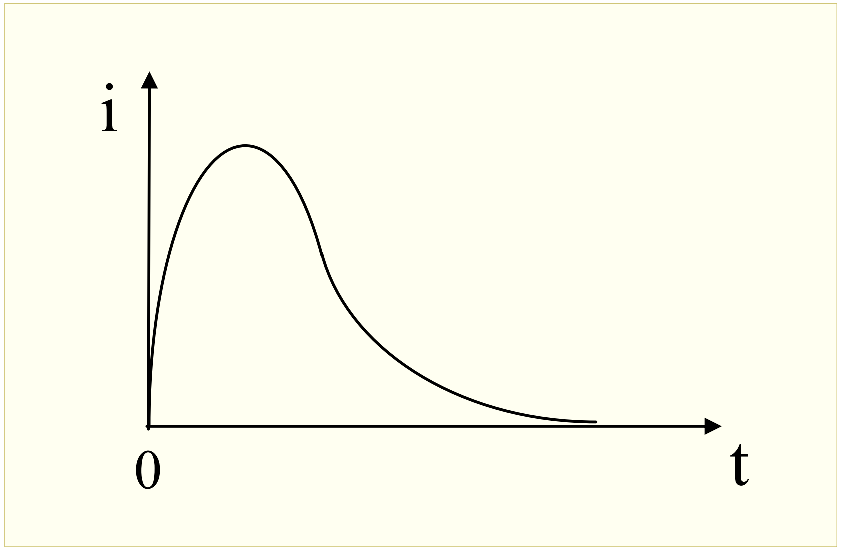
From the initial conditions at $t = 0^+$:

$$A_1 = i(0^+) = 0$$

$$A_2 = \frac{d\, i(0^+)}{dt} = (V_0 - V_{C0})/L$$

Transient solution in Case B – critically damped case:

$$i(t) = \frac{V_0 - V_{C0}}{L}\, t\, e^{-\frac{R}{2L}t}$$



Second Order Circuit: *Series RLC Circuit*

Case C - ***underdamped case :*** $\alpha < \omega_0 \quad \rightarrow \quad R < 2\sqrt{L/C}$;

x_1 , x_2 complex conjugate (negative real part)

$$x_1 = -\alpha + j\beta \qquad x_2 = -\alpha - j\beta \qquad \textit{where} \quad \begin{cases} \alpha = \dfrac{R}{2L} & \textit{(damping fact.)} \\ \beta = \sqrt{\dfrac{1}{LC} - \left(\dfrac{R}{2L}\right)^2} \end{cases}$$

The solution of eq. 3 is:

$$i(t) = \left(A_1 \cos\beta t + A_2 \sin\beta t\right)e^{-\alpha t}$$

From the initial conditions at $t = 0^+$:

$$A_1 = i(0^+) = 0$$

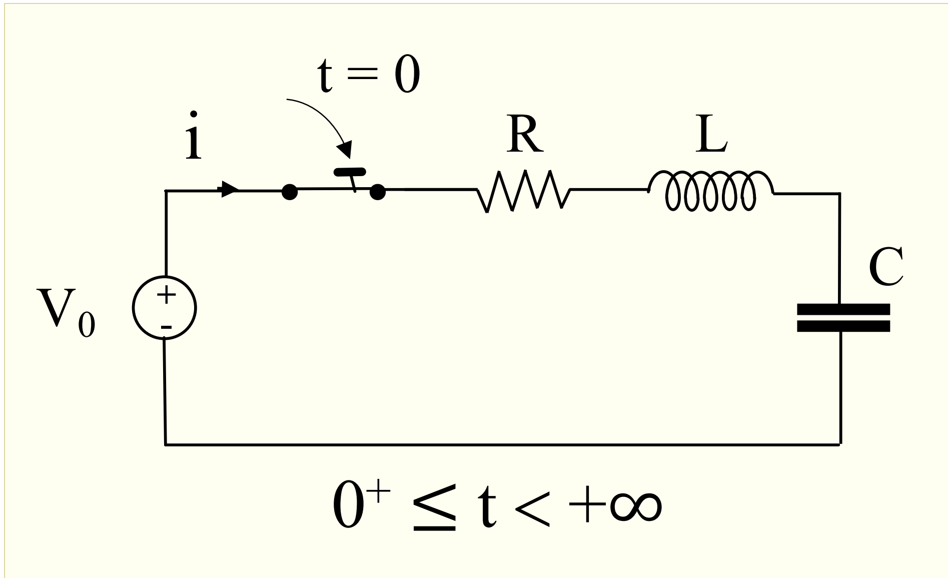
$$\alpha A_1 + \beta A_2 = \frac{d\, i(0^+)}{dt} = \left(V_0 - V_{C0}\right)/L$$

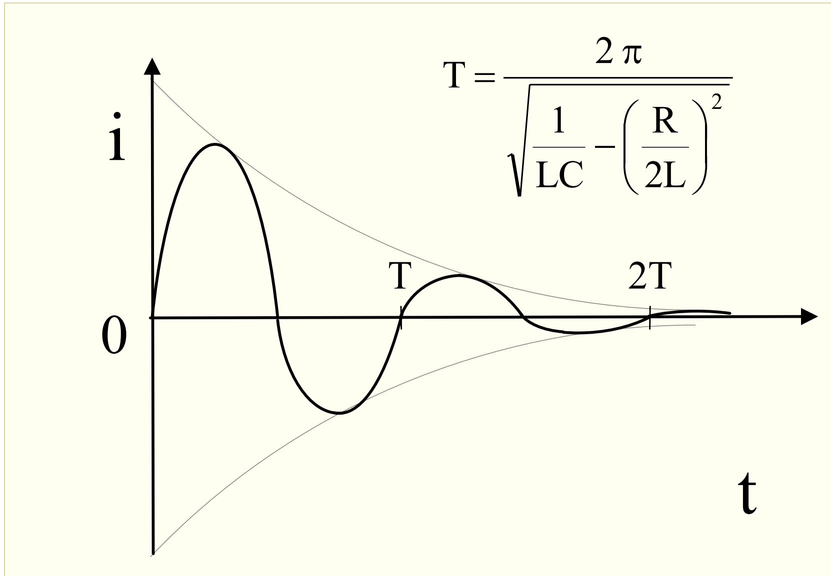
$$\begin{cases} A_1 = 0 \\ A_2 = \dfrac{V_0 - V_{C0}}{L\sqrt{\dfrac{1}{LC} - \left(\dfrac{R}{2L}\right)^2}} \end{cases}$$

Second Order Circuit: *Series RLC Circuit*

Transient solution in case C – Underdamped case: $R < 2\sqrt{L/C}$

$$i(t) = \frac{V_0 - V_{C0}}{L\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}}\, e^{-\alpha t} \sin\left\{\left[\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}\right] t\right\}$$





Transient Analysis: *General Method*

Analysis of the transient in the time domain

1. Write the set of equations (*for example by means of the voltage substitution method*)

2. Derive of an ordinary differential equation in one unknown by substitution and derivation:

$$a_n \frac{d^n i}{dt^n} + a_{n-1} \frac{d^{n-1} i}{dt^{n-1}} + \dots\dots\dots + a_1 \frac{d i}{dt} + a_0 \, i = f(t) \qquad (4)$$

the solution of which is:

$$i(t) = i_T(t) + i_S(t)$$

where

$$i_T(t) = \sum_{k=1}^{n} A_k e^{x_k t} \qquad \text{transient response} \quad \left(\lim_{t \to \infty} i_T(t) = 0 \right)$$

$$i_S(t) \;\; \text{steady state response} \;\; \left(\text{particular integral at } t = +\infty \text{ of eq. 4}\right)$$

Analysis of the transient in the time domain

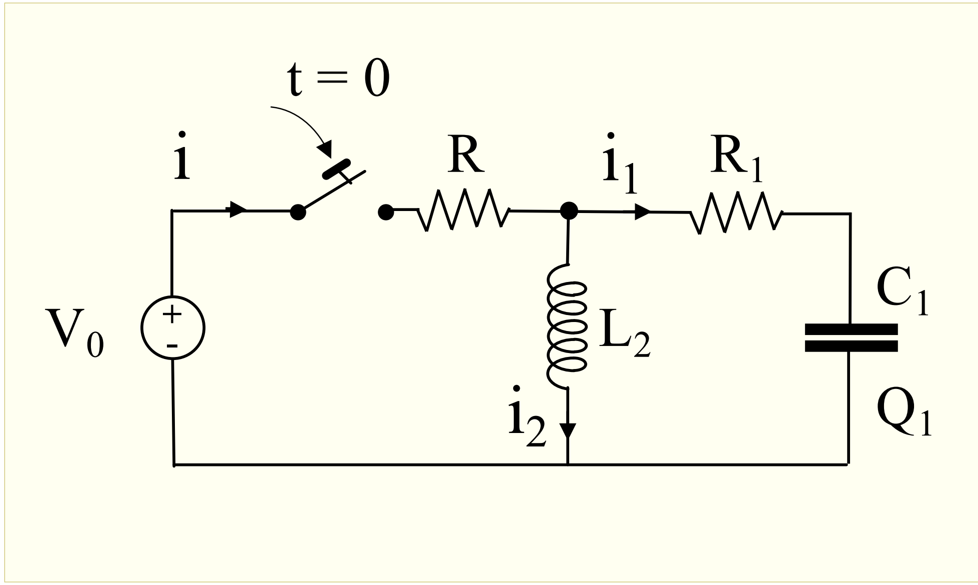
3. Derivation of the particular integral of eq. 4 at $t = +\infty$ by means of the analysis of the steady state at $t = +\infty$.
4. Derivation of the natural frequencies of $i_T(t)$ which are the solutions x_1, x_2, x_3,….,x_n of the characteristic equation of the homogeneous differential equation associated to eq. 5.
5. Derivation of the integration constants A_1, A_2, A_3,…., A_n by means of the initial conditions $i_k(0^+)$ in inductive branches and $v_k(0^+)$ in capacitive branches.
6. Derivation of the other unknowns.

Hence, in order to derive the complete response of the circuit, it is necessary to find the circuit operation at:

- **$t = +\infty$** *to determine the steady state solution, that is the particular integral of eq. 4 at* **$t = +\infty$** *;*
- **$t = 0^-$** *to determine the initial data;*
- **$t = 0^+$** *to determine the initial conditions.*

Analysis of the transient in the time domain

At t = 0 the switch is closed and the circuit response with a transient is initiated.



1. Set of simultaneous equations describing the circuit at $0^{+} < t < +\infty$

$$\begin{cases} L_2 \dfrac{d\,i_2}{dt} + R\,i - V_0 = 0 \\ \dfrac{Q_1}{C_1} + \dfrac{1}{C_1}\displaystyle\int_0^t i_1(t')\,dt' + R_1 i_1 + R\,i - V_0 = 0 \\ i - i_1 - i_2 = 0 \end{cases}$$

2. Successive derivation and substitution to obtain an ordinary differential equation in one unknown:

$$L_2 \frac{R+R_1}{R} \frac{d^2 i_2}{dt^2} + \left(R_1 + \frac{L_2}{RC_1}\right)\frac{d\,i_2}{dt} + \frac{1}{C_1}\, i_2 = \frac{V_0}{RC_1} \qquad (5)$$

Analysis of the transient in the time domain

Analysis of the initial data at t = 0⁻:

$i(0^-) = i_1(0^-) = i_2(0^-) = 0$

$v_C(0^-) = 0 \quad$ as $\; i_1(0^-) = i_2(0^-) = 0;$

$\rightarrow \quad Q_1(0^-) = 0$

Analysis of the initial cond. at t = 0⁺:

- Across the capacitor:

$v_C(0^+) = v_C(0^-) = 0$

- Through the inductor:

$i_2(0^+) = i_2(0^-) = 0$

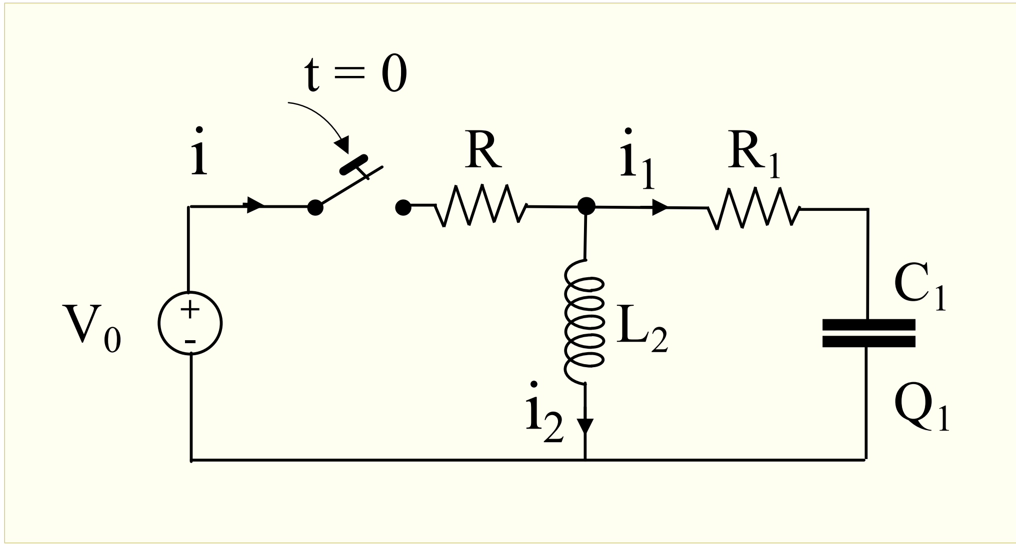
$\rightarrow \; V_0 - R\, i(0^+) - R_1\, i_1(0^+) = 0$

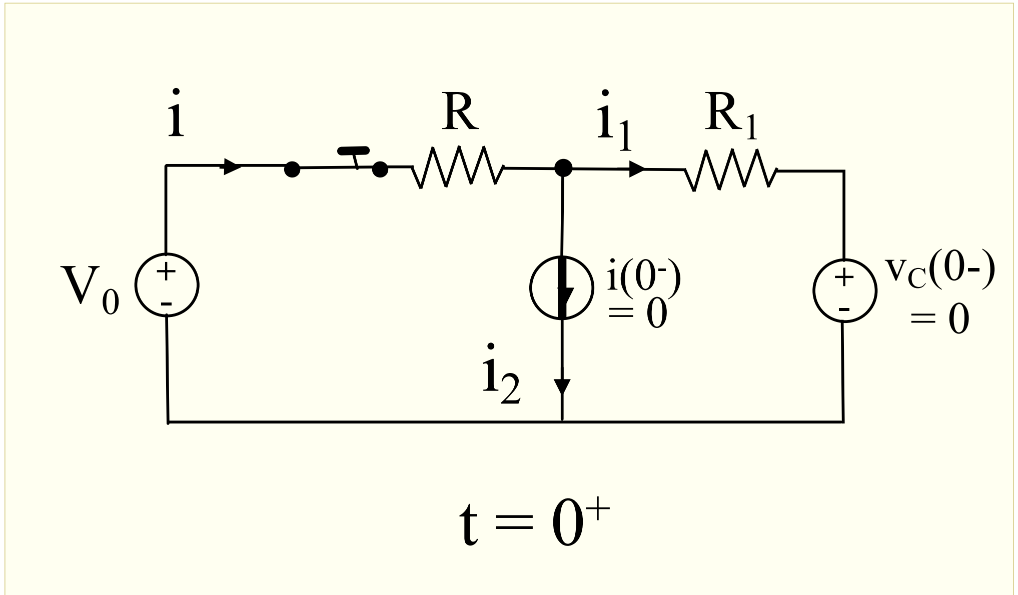
$i_1(0^+) - i(0^+) = 0$

$\rightarrow \quad i_1(0^+) = i(0^+) = V_0/(R+R_1)$

$$\rightarrow \; R_1\, i_1(0^+) - L_2 \frac{d\, i_2(0^+)}{dt} = 0$$

$$\rightarrow \quad \frac{d\, i_2(0^+)}{dt} = V_0 R_1/[L_2(R+R_1)]$$





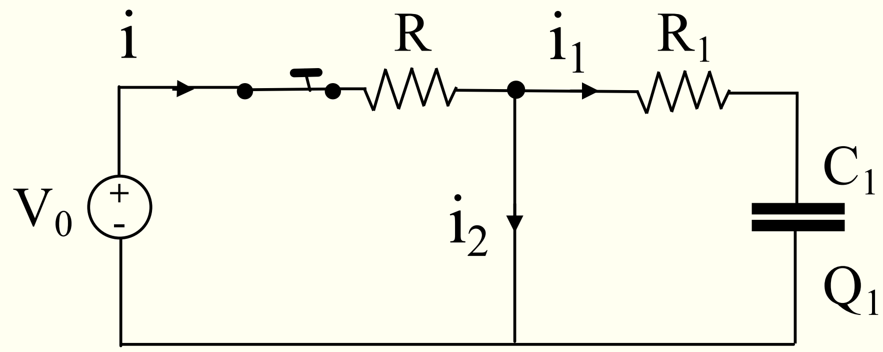
Analysis of the transient in the time domain

Analys. of the steady state at $t = +\infty$:

$$i(+\infty) = i_2(+\infty) = V_0/R;$$

$$i_1(+\infty) = 0;$$

$$v_C(+\infty) = 0.$$



$t = +\infty$

Solution of eq. 5:

$$i(t) = A_1 e^{x_1 t} + A_2 e^{x_2 t} + \frac{V_0}{R}$$

where x_1 and x_2 are the solution of the characteristic equation of eq. 5:

$$L_2 \frac{R+R_1}{R} x^2 + \left(R_1 + \frac{L_2}{RC_1} \right) x + \frac{1}{C_1} = 0$$

The integration constants A_1 and A_2 are determined by the inizial condizions:

$$i_2(0^+) = A_1 + A_2 = 0$$

$$\frac{d\, i_2(0^+)}{dt} = x_1 A_1 + x_2 A_2 = $$

$$= \frac{V_0}{L_2} \frac{R_1}{R_1 + R}$$

	$t = 0^-$	$t = 0^+$	$t = +\infty$
i	0	$V_0/(R+R_1)$	V_0/R
i_1	0	$V_0/(R+R_1)$	0
i_2	0	0	V_0/R
di_2/dt	0	$\frac{V_0}{L_2} \frac{R_1}{R_1 + R}$	0
Q_1	0	0	0
v_C	0	0	0