SPACE VECTOR CONTROL OF MATRIX CONVERTERS WITH UNITY INPUT POWER FACTOR AND SINUSOIDAL INPUT/OUTPUT WAVEFORMS

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Abstract. In this paper the control of a three-phase ac-ac matrix converter with sinusoidal input and output waveforms and unity power factor at the input side is presented. The control strategy has been developed utilizing the space vector modulation technique. This approach has the advantage of an immediate comprehension of the switching strategies. The maximum value $\sqrt{3}/2$ of the voltage transfer ratio is achieved without considering any third harmonic voltage component. The number of switch commutations within a cycle period can be limited utilizing an opportune commutation sequence. Furthermore, safe commutation is achieved by current sensing. A numerical simulation of the overall system has been carried out and the numerical results are presented to verify the operating principle.

Keywords. Matrix Converter, Space Vector Modulation.

INTRODUCTION

Direct ac-ac converters have received considerable attention with the progress of power devices. The matrix converters have advantages over traditional rectifier-inverter type frequency changers such as

* four-quadrant operation capability
* sinusoidal input and output waveforms with minimum harmonic distortion
* controllable power factor
* minimal energy storage requirements.

The fundamentals of the modulation strategy of matrix converters have been well established in [1] and [2]. The maximum voltage transfer ratio has been found to be $\sqrt{3}/2$ for sinusoidal input and output waveforms, and can be obtained by adding third harmonic voltage components to the desired ac output voltage [2]. Other researchers have also investigated the operation of matrix converters using PWM converter and inverter switching strategies [3].

In a recent paper [4] the Space-Vector Modulation (SVM) technique has been employed to control a 3-phase/3-phase matrix converter with input phase-current spectrum very similar to the corresponding spectrum of a 3-phase full bridge diode rectifier.

This paper proposes a new control algorithm based on space vector modulation theory which allows sinusoidal input and output waveforms with unity input power factor. The space vector modulation approach has advantages with respect to the traditional modulation technique

* immediate comprehension of the required commutation processes
* simplified control algorithm
* maximum voltage transfer without adding third harmonic components
* no synchronisation requirements with input voltage waveforms.

Moreover, the proposed switching algorithm allows to reduce the number of switching devices involved in a commutation process with respect to conventional switching strategies [1], [2]. It should be noted that, because the modulation strategy is defined directly in terms of the output waveform amplitude and phase angle, it is readily adapted to high-performance motor drive applications such as field-oriented control.

The control scheme analysed requires the measurement of two input line voltages to determine the on-time to modulation period ratios of the permitted switching combinations. Furthermore, the measurement of two output line currents allows safe commutation to be achieved.

These characteristics allow good performance to be obtained even under unbalanced supply voltages, avoiding the introduction of low order harmonics in the output voltages.

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### Table I - Switching combinations and their voltage and current vector representations.

<table>
<thead>
<tr>
<th>Group</th>
<th>#</th>
<th>Name</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>V_{AB}</th>
<th>V_{BC}</th>
<th>V_{CA}</th>
<th>I_{a}</th>
<th>I_{b}</th>
<th>I_{c}</th>
<th>V_{o}</th>
<th>\alpha_{o}</th>
<th>I_{i}</th>
<th>\beta_{i}</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>1</td>
<td>a b c</td>
<td>c</td>
<td>a b</td>
<td>c</td>
<td>V_{ab}</td>
<td>V_{bc}</td>
<td>V_{ca}</td>
<td>I_{a}</td>
<td>I_{b}</td>
<td>I_{c}</td>
<td>V_{o}</td>
<td>\alpha_{o}</td>
<td>I_{i}</td>
<td>\beta_{i}</td>
</tr>
<tr>
<td>IIa</td>
<td>7</td>
<td>a b b</td>
<td>b</td>
<td>a b</td>
<td>b</td>
<td>V_{ab}</td>
<td>0</td>
<td>V_{o}</td>
<td>I_{b}</td>
<td>I_{a}</td>
<td>0</td>
<td>2V/3 V_{ab}</td>
<td>2V/3 I_{a}</td>
<td>-x/6</td>
<td></td>
</tr>
<tr>
<td>IIb</td>
<td>13</td>
<td>a b b</td>
<td>b</td>
<td>a b</td>
<td>b</td>
<td>V_{ab}</td>
<td>0</td>
<td>I_{b}</td>
<td>0</td>
<td>I_{a}</td>
<td>0</td>
<td>2V/3 V_{ab}</td>
<td>2V/3 I_{b}</td>
<td>-x/6</td>
<td></td>
</tr>
<tr>
<td>IIc</td>
<td>19</td>
<td>a b b</td>
<td>b</td>
<td>a b</td>
<td>b</td>
<td>V_{ab}</td>
<td>0</td>
<td>I_{b}</td>
<td>0</td>
<td>I_{a}</td>
<td>0</td>
<td>2V/3 V_{ab}</td>
<td>2V/3 I_{b}</td>
<td>-x/6</td>
<td></td>
</tr>
</tbody>
</table>

**VECTOR ANALYSIS OF THE SWITCHING COMBINATIONS**

The general topology of a three-phase ac-ac matrix converter is shown in Fig. 1. It consists of nine bi-directional switches which allow any output phase to be connected to any input phase. Being the converter supplied by voltage source, the input phases should never be short-circuited and, owing to the presence of inductive loads, the load currents should not be interrupted. With these constraints in 3-phase/3-phase matrix converter there are 27 permitted switching combinations.

For each combination, the input and output line voltages can be expressed in terms of space vectors as

\[
\vec{V}_{i} = \frac{2}{3} \left( V_{ab} e^{j2\pi/3} + V_{bc} e^{j4\pi/3} \right) = V_{o} e^{j\alpha_{o}} \tag{1}
\]

\[
\vec{V}_{o} = \frac{2}{3} \left( V_{AB} e^{j2\pi/3} + V_{CA} e^{j4\pi/3} \right) = V_{o} e^{j\alpha_{o}} \tag{2}
\]

In the same way, the input and output line currents result as follows

\[
\vec{i}_{i} = \frac{2}{3} \left( I_{a} e^{j2\pi/3} + I_{b} e^{j4\pi/3} \right) = I_{o} e^{j\beta_{i}} \tag{3}
\]

\[
\vec{i}_{o} = \frac{2}{3} \left( I_{A} e^{j2\pi/3} + I_{B} e^{j4\pi/3} \right) = I_{o} e^{j\beta_{o}} \tag{4}
\]

**Fig. 2** - Output voltage space vectors corresponding to the permitted switching combinations.

**Fig. 3** - Input current space vectors corresponding to the permitted switching combinations.
In Tab. 1 all the permitted switching combinations are shown. For each of them the resulting output line voltage is given as function of the input line voltages as well as the resulting input current is expressed as function of the output line currents.

As it is possible to see in Tab. 1, for the 6 combinations of group I each output phase is connected to a different input phase. In the 18 combinations of group II, two output phases are short-circuited. In the 3 combinations of group III all the output phases are short-circuited.

Each combination of group I determines an output voltage vector having a phase angle \( \alpha_0 \), which is dependent on the phase angle \( \alpha_i \) of the corresponding input voltage vector. In the same way, the input current vector has a phase angle \( \beta_i \) which is related to the phase angle \( \beta_o \) of the output current vector. Hence, in order to apply the SVM technique, these combinations cannot be usefully employed.

On the contrary, the 18 configurations of group II determine six prefixed positions of the output voltage space vector (Fig. 2) which are not dependent on \( \alpha_i \). The 18 configurations of group II determine also 6 prefixed positions of the input current space vector (Fig. 3) which are not dependent on \( \beta_o \).

Finally, the 3 configurations of group III determine zero output voltage and zero input current vectors.

**SPACE VECTOR CONTROL STRATEGY**

The aim of the proposed control strategy is to generate the desired output voltage vector with the constraint of unity input power factor. For this purpose, let \( v_o \) be the desired output line voltage space vector and \( v_i \) the input line voltage space vector at a given time instant (Figs. 2 and 3).

The input line-to-neutral voltage vector \( \bar{v}_i \) is defined according to the following relationship:

\[
\bar{v}_i = \frac{1}{\sqrt{3}} v_i e^{j\pi/6}
\]

In order to obtain unity input power factor, the direction of the input current space vector \( \bar{i}_k \) has to be the same of \( \bar{v}_i \).

In Fig. 2, \( v_{o'} \) and \( v_{o''} \) represent the components of \( v_o \) along the two adjacent vector directions. Owing to the small variation of the input voltage during the switching cycle period, the desired \( v_{o'} \) component can be approximated utilising two switching configurations corresponding to two space vectors in the same direction of \( v_o \) and one zero voltage configuration. Among the six possible switching configurations, the two giving the higher voltage values and corresponding to vectors with the same sense of \( v_{o'} \) are chosen. In the same way, two different switching configurations and one zero voltage configuration are utilised to define \( v_{o''} \).

With reference to the example shown in Figs. 2 and 3, the input voltage \( v_i \) has a phase angle \( 0 \leq \alpha_i \leq \pi/3 \). In this case the line voltages \( v_{ab} \) and \( -v_{ca} \) assume the higher values. Then, according to Tab. I, the configurations used to obtain \( v_{o'} \) are \( +1 \) and \( -3 \), while for \( v_{o''} \) are \( -4 \) and \( +6 \). These four configurations can be utilised to determine the input current vector direction also. In fact, as it is shown in Fig. 3, these configurations are associated to vector directions adjacent to the input current vector position.

Applying the space vector modulation technique the current ratio \( \delta \) of each configuration can be obtained solving two systems of algebraic equations. In particular, utilising the configurations \(+1, -3\) to generate \( v_{o'} \) and to set the input current vector direction, we can write:

\[
\delta_1 = \frac{2}{\sqrt{3}} v_{ab} - \delta_3 \frac{2}{\sqrt{3}} v_{ca} = v_{o'} = \frac{|v_o|}{\sqrt{3}} \sin (\frac{\pi}{6} + \alpha_0)
\]

\[
\delta_3 = \frac{2}{\sqrt{3}} i_A = \bar{i}_1 = \frac{|v_i|}{\sqrt{3}} \sin (\frac{\pi}{6} - \alpha_i)
\]

\[
\delta_1 = \frac{2}{\sqrt{3}} i_A = \bar{i}_1 = \frac{|v_i|}{\sqrt{3}} \sin (\frac{\pi}{6} + \alpha_i)
\]

Considering a balanced system of sinusoidal supply voltages expressed as:

\[
v_{ab} = |v_i| \cos \alpha_i
\]

\[
v_{bc} = |v_i| \cos (\alpha_i - 2\pi/3)
\]

\[
v_{ca} = |v_i| \cos (\alpha_i - 4\pi/3)
\]

Eq. 6 can be written as follows:

\[
\delta_1 \cos \alpha_i - \delta_3 \cos (\alpha_i - \frac{\pi}{3}) = \frac{|v_o|}{|v_i|} \sin (\alpha_o + \frac{\pi}{6})
\]

Combining Eqs. 7 and 8 yields:

\[
\delta_1 \cos (\alpha_i - \frac{\pi}{3}) - \delta_3 \cos (\alpha_i + \frac{\pi}{6}) = 0
\]

The solution of the system of Eqs. 9 and 10 gives:

\[
\delta_1 = q \frac{2}{\sqrt{3}} \sin (\alpha_o + \frac{\pi}{6}) \cos (\alpha_i + \frac{\pi}{6})
\]

\[
\delta_3 = q \frac{2}{\sqrt{3}} \sin (\alpha_o + \frac{\pi}{6}) \cos (\alpha_i - \frac{\pi}{6})
\]

where \( q = \frac{|v_o|}{|v_i|} \) is the voltage transfer ratio.

Eqs. 11 and 12 can be rewritten in terms of input phase voltage, giving:

\[
\delta_1 = q \frac{2}{\sqrt{3}} \sin (\alpha_o + \frac{\pi}{6}) \frac{v_b}{E_i}
\]

\[
\delta_3 = q \frac{2}{\sqrt{3}} \sin (\alpha_o + \frac{\pi}{6}) \frac{v_a}{E_i}
\]
Eqs. 13 and 14 are the solving equations which allow the determination of the on-time ratios when \( q \) and \( \alpha_0 \) are known.

With the same procedure, utilising the configurations

\[-4, +6\]

and the input current vector direction, yields

\[
\delta_4 = \frac{q}{\sqrt{3}} \sin \left( -\alpha_0 - \frac{3\pi}{6} \right) \frac{\phi}{E_i}
\]

\[\delta_6^* = \frac{q}{\sqrt{3}} \sin \left( -\alpha_0 + \frac{3\pi}{6} \right) \frac{\phi}{E_i}\]

The results obtained are valid for \(-\pi/6 \leq \alpha_0 \leq \pi/6\) and for \(0\leq\alpha_1\leq\pi/3\).

Applying a similar procedure for the other possible pairs of angular sectors, the required switching configurations and the on-time ratio of each configuration can be determined.

Note that the values of the on-time ratios result greater than zero as required for the feasibility of the control strategy. Furthermore, the sum of these ratios must be lower than unity. By adding (13), (14), (15) and (16) with the constraint

\[
\delta_4^* + \delta_3 + \delta_4 + \delta_6^* \leq 1
\]

it is possible to determine the maximum value of the voltage transfer ratio which results \( q=\sqrt{3}/2 \), as determined in [2].

**COMMUTATION SEQUENCES AND REQUIREMENTS**

Once the phase angle of the input and output line voltages are known, the four switching configurations required by the proposed control algorithm are readily determined. These configurations are utilised until \( \alpha_1 \) or \( \alpha_0 \) will change the angular sector. One of the zero voltage configurations must be used to complete the switching cycle period. The sequence of the resulting five configurations should be defined in order to minimise the number of bidirectional switch commutations.

With reference to \( \alpha_1 \) and \( \alpha_0 \) values considered in Figs. 2 and 3, the available switching combinations are summarised in Tab. II. A simple method which allows the number of commutations to be limited is based on sorting the switching combinations so that

- the first and the fourth have two equal letters in the same position
- the second and the third have only one letter change with respect to the first and fourth respectively
- the last zero voltage configuration has only one letter change with respect to first and fourth configuration.

Applying this method to the example considered leads to the switching sequence shown in Tab. III.

<table>
<thead>
<tr>
<th>Config</th>
<th>ABC</th>
<th>Config</th>
<th>ABC</th>
</tr>
</thead>
<tbody>
<tr>
<td>+1</td>
<td>abb</td>
<td>I</td>
<td>ab</td>
</tr>
<tr>
<td>-3</td>
<td>acc</td>
<td>II</td>
<td>ab</td>
</tr>
<tr>
<td>-4</td>
<td>aba</td>
<td>III</td>
<td>ac</td>
</tr>
<tr>
<td>+6</td>
<td>aca</td>
<td>IV</td>
<td>ac</td>
</tr>
<tr>
<td>0</td>
<td>aaa</td>
<td>V</td>
<td>a</td>
</tr>
<tr>
<td>0</td>
<td>bbb</td>
<td>I</td>
<td>ab</td>
</tr>
<tr>
<td>0</td>
<td>ccc</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

It should be noted that in this way six commutations only are required in each cycle period. Once the configurations are selected and sequenced, the on-time ratio of each configuration is calculated using Eqs. 13-16 written for the appropriate sector.

For matrix converters a fundamental commutation requirement is that output current continuity is maintained at all times. Furthermore short-circuits at the input terminals must be avoided. To achieve this a multistep switching procedure is usually employed. In [2] and [5] the switching sequence is based on the sign of the voltage between the incoming and the outgoing input lines. Safe commutation can be also achieved by detecting the direction of the output current in [6], [7], [8].

In particular in [6] a switching technique has been presented which is based on the presence of a current band centred on zero value to ensure a correct commutation. This technique has been successfully verified experimentally for a 3-phase AC PWM chopper and could be applied to 3-phase/3-phase matrix converters also.

**NUMERICAL SIMULATIONS AND RESULTS**

A numerical simulation of the matrix converter shown in Fig. 1 has been carried out utilising the ATP general purpose program. The converter has been modelled as a set of ideal bidirectional switches. The switching algorithm has been implemented in ATP according to the principles outlined in the previous sections. In order to validate the control algorithm the simulation has been carried out under various operating conditions. The ATP simulation results, showing the switched voltage and current waveforms and their harmonic contents, are represented in Figs. 4-9. In all cases, the switching rate is set to 2 kHz in order to clearly show the converter operation. For display purposes the voltage and current waveforms are smoothed by averaging the instantaneous values over each switching cycle period and by linear interpolation of the discrete values.

Figs. 4 and 5 illustrate the results in steady state conditions for an input frequency of 50 Hz, output frequency 60 Hz and voltage transfer ratio \( q=0.866 \). Fig. 5 clearly shows that the input power factor seen by the supply is unity.
STEADY-STATE OPERATION

Input frequency 50 Hz, output frequency 60 Hz, \( q = 0.866 \).

Fig. 4 - Output line voltage \( v_{AB} \), averaged value \( v_{AB}^* \) and output line current \( i_e \).

Fig. 5 - Input line-to-neutral voltage \( e_A \), input line current \( i_a \) and averaged value \( i_a^* \).

Fig. 6 - Frequency spectrum of the output line voltage.

Fig. 7 - Frequency spectrum of the input line current.

In Figs. 6 and 7 the frequency spectrum of the output line voltage and input line current are represented. The only significant harmonic components are centred around the switching frequency.

Figs. 8 and 9 illustrate the numerical results for a step change in output frequency and output voltage amplitude. The frequency changes instantaneously from 60 Hz to 30 Hz, while the voltage transfer ratio changes from 0.74 to 0.37. It should be noted that the initial deviation of the input current waveform from sinusoidal is due to the initial condition perturbation of the simulation.

CONCLUSIONS

In this paper the space vector modulation technique has been utilised to control a 3-phase/3-phase matrix converter. This technique allows the control of the output voltage amplitude up to \( \sqrt{3}/2 \) of the input voltage amplitude and the control of the output frequency from zero to a maximum value limited by the switching frequency, at unity input power factor. Regenerative operation is also possible. Time- and frequency-domain analysis of the output voltage and input current have been presented. The input current and output voltage are sinusoidal except for high order switching harmonics. The control strategy presented is very simple and can be implemented in a microprocessor system with reduced real time computation. Finally, the knowledge of the input line voltages and the use of a suitable commutation law permit a complete compensation of unbalanced supply voltages avoiding the presence of low order harmonics. The converter behaviour under unbalanced conditions will be investigated in a future work.
TRANSIENT OPERATION

Step change in output frequency (60→30 Hz) and in voltage transfer ratio \( q \) (0.74→0.37).

Fig. 8 - Output line voltage and output line current waveforms for a step change in output frequency and voltage transfer ratio.

Fig. 9 - Input line-to-neutral voltage and input line current waveforms for a step change in output frequency and voltage transfer ratio.

REFERENCES


