# Adaptive Control of a Slotless PM Linear DC Actuator

D. Casadei \*, C. Cecati \*\*, G. Grandi \*, G. Serra \*

(\*) Istituto di Elettrotecnica, Università di Bologna, Viale Risorgimento 2 - 40136 Bologna - Italy phone: +39-51-6443567; fax: +39-51-6443588; e-mail: elettrot5@ingbol.cineca.it

(\*\*) Dipartimento di Ingegneria Elettrica, Università di L'Aquila, Loc. Monteluco di Roio, 67040 L'Aquila - Italy phone: +39-862-432546; fax: +39-862-432543; e-mail: carlo@cecati.ing.univaq.it

Abstract - Linear actuators offer some advantages in respect of rotating machines coupled with mechanical converters in many applications requiring linear motion. For such reason a novel slotless PM linear DC motor has been designed and realised. Since often the drive dynamics is not well identified and/or it is subject to unknown payloads and/or parameters variations, adaptive control strategies could be used to overcome these problems. In this paper after a description of the mathematical model of the actuator, which can be utilised even with stroke length much higher than slider length, and of the adopted model reference adaptive control algorithm, the performance of the actuator under speed control in presence of some typical disturbances are evaluated and discussed. Finally, some shortcomings regarding the proposed system and its future developments are reported.

#### INTRODUCTION

There are many applications in the real world requiring linear motion which are generally realised using electromechanical systems constituted by a rotating motor and some mechanical devices which convert the revolution of the motor shaft in a translatory motion.

Obviously, such systems are effective but they often present some limitations such as reduced efficiency, mechanical complexity, limited precision, high cost, necessity of maintenance and some others one. These limitations appear very pressing in those applications in which the cost and the simplicity of the drive are very important factors.

In the last years a lot of researches have been done in the field of linear electrical machines but, most of them have focused their attention to some specific areas mainly related to transportation (e.g. trains) and, more in general, to high power applications. There are many other applications, often in the low power range, in which the substitution of traditional systems with direct linear motors, can introduce sensible benefits in terms of reliability, efficiency and cost.

In fact, the elimination of gearing backlash, the reduction of friction and joint wear, the increase of the mechanical stiffness and, more in general, of the mechanical behaviour are important factors which can significantly improve system performance and, perhaps, reduce their cost

For such reasons, several machine tool manufacturers are now assessing the possibility of using linear motors as direct-drive actuators and for the same reasons we have studied, designed and realised a novel linear DC motor for low power applications [1],[2]. Figure 1 illustrates a cross-section of the complete actuator in which the stator is constructed from standard laminations and the moving part (slider) is constituted by magnets which are mounted in a light weight holder, in order to reduce the overall inertia of the slider.

The use of such kind of permanent magnet linear DC actuator offers several advantages in respect of rotating ones, due to the very simple mechanical design, which doesn't require electrical connections to the moving member, the absence of rotating parts and brushes etc. In addition, heating effects as a result of copper losses are not

present in the moving part simplifying the overall design and improving the robustness of the complete system.

In [3] different machine configurations have been presented and compared showing that with such type of linear DC actuator it is possible to obtain a double movement also. In particular, adopting the same machine length and copper weight, two sliders moving either in the same or in the opposite directions can be successfully utilised.

In most of linear devices the armature excitation can be either single-sided or double-sided. This paper deals with a single-sided excitation, in which the large attractive forces between the fixed and the moving members are overcome by placing the magnets centrally between the iron blocks. The two coils placed at the ends of the working length are used to limit the flux density in the iron core. In this way the magnets can drive flux through a magnetic circuit of moderate reluctance.

As with any DC machine, the electromagnetic force is produced from the interaction of the magnet flux with the Ampere-turns in the armature winding. From the study described in [1] it was emphasised that with short stroke length the actuator produces a force proportional to the current and independent of the position, making it easily controllable. On the contrary, when a long stroke is required, the mmf drops in the iron core determine a thrust-displacement curve which decreases from the ends to the middle of the working length. Another critical situation may occur when the load force of the payload varies during the operation due to frictions and/or obstacles. In all situations adaptive control techniques can be employed in order to reduce the effects of the disturbances.

In this paper, after a description of the motor model and of the adopted control algorithm, some results obtained by simulations are presented and discussed. Finally some comments regarding the system and its future development are reported.

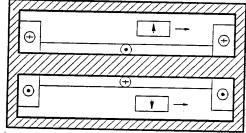


Fig. 1 - Simplified drawing of the slotless PM linear DC actuator

# MACHINE EQUATIONS

In a previous paper [2], the actuator has been modelled in order to emphasise the influence of the machine parameters on the thrust decrease and to determine a mathematical model for the analysis of the dynamic behaviour. The analytical model allows the flux density distribution and the output force of a given configuration to be determined. Equations for the flux density distributions in the air-gap

and in the iron core were derived from Ampere's and Gauss's law. The output force can be evaluated by calculating the rate of coenergy variation due to the magnet displacement under constant current:

$$F = \frac{\partial W_{co}(i, \chi_m)}{\partial \chi_m} \tag{1}$$

In the linear actuator under description, the total flux  $\lambda$  linking the armature winding depends on current and magnet position. Hence the voltage equation is:

$$y = ri + \frac{\partial \lambda}{\partial i} \frac{di}{dt} + \frac{\partial \lambda}{\partial \chi_m} \cdot \frac{d\chi_m}{dt}$$
 (2)

where  $\chi_m$  represents the magnet position. The mechanical system is described by:

$$M\frac{dv}{dt} = F - bv - FL \qquad (3), \qquad v = \frac{d\chi_m}{dt} \qquad (4)$$

The transformer and dynamic emf coefficients in Eq. 2 can be derived in terms of energy variation as:

$$\frac{\partial \lambda}{\partial \hat{i}} = \frac{1}{i} \frac{\partial W_f}{\partial \hat{i}} = L_d \tag{5}$$

$$\frac{\partial \lambda}{\partial \chi_m} = \frac{1}{i} \frac{\partial W_f}{\partial \chi_m} + \frac{F}{i} = K_v \tag{6}$$

Assuming a linear single-valued B-H characteristic for both the iron and the magnet leads to transformer and dynamic emf coefficients which are function of the magnet position only. Otherwise these coefficients are functions of both magnet position and current values. In order to simulate the transient behaviour the force and the transformer and dynamic emf coefficients should be determined in advance.

With reference to a linear DC actuator having a stroke length of 0.7 m, a continuos force of 70 N and a rated voltage of 48 V, a design optimised for maximum force to total mass ratio has been carried out. The moving member consists of two anisotropic ceramic magnets with a remanent flux density of 0.38 T, a coercive force of 250 kA/m and a recoil permeability of 1.1. According to the design produced a prototype of the linear actuator proposed has been realised.

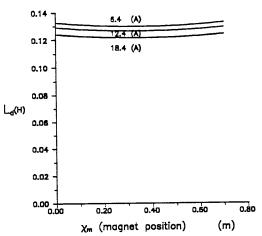


Fig. 2 - Transformer emf coefficient versus magnet position curves

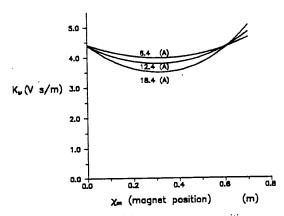


Fig. 3 - Dynamic emf coefficient versus magnet position curves

Utilising equations (1), (5) and (6) the electromagnetic force and the coefficients  $L_d$  and  $K_\nu$  can be determined with the same procedure evaluating coenergy and energy variations due to magnet displacement and current variation. Figures. 2, 3 and 4 illustrate respectively the transformer emf coefficient, the dynamic emf coefficient and the force versus slider position with the winding current as parameter. Figure 4 clearly shows the effects of saturation on the electromagnetic force as the winding current increases. The numerical results represented in figures 2, 3 and 4 have been confirmed from experimental tests on the prototype. The curves obtained can be easily modelled using analytical functions, then a step-by-step solution can be obtained from equations 2, 3 and 4.

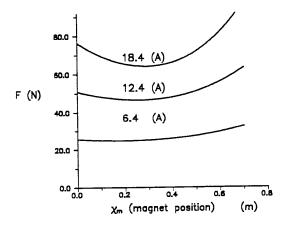


Fig. 4 - Force versus magnet position curves

#### CONTROL ALGORITHM

When a linear direct-drive actuator is employed the motion control problems primarily arise from a substantial increase in the load inertia reflected on the actuators. Changes in load inertia and parameter variations may cause significant tracking errors particularly if simple control algorithms are implemented. Other problems arise from the non-linear transformer emf coefficient and dynamic emf coefficient as well. Traditional PI and/or PID regulators often appear not suitable due to their inherent impossibility to compensate parameter variations and non linearities. In fact, these controllers are mainly oriented towards the elimination of the effect of state perturbations leaving the system sensible to structural perturbations such those discussed.

Adaptive control systems, instead, are appropriate in many cases allowing the compensation of parameter variations and leaving the system unaffected, or thereabouts, by structural perturbations. Furthermore, if the model reference approach is adopted, there are some additional features, such as the assignment of a different dynamic allowing the modification of system performance (e.g. the time response, the sensibility in respect of a particular parameter and so on) and then the design of feasible linear drives [4], [5], [6]. For such reasons this paper deals with the study of the application of an adaptive control scheme applied to the linear motor previously described. The employed control scheme is based on a state augmentation of the system and requires the real-time estimation of the parameters for a subsequent adaptation of the control input [7]. Parameter variations are taken under control by the chosen algorithm according to the algorithm itself and the chosen reference model.

The drive including both the linear actuator and the controlled power supply is represented in a space state by:

$$\frac{d}{dt}x(t) = A(t)x(t) + B(t)u(t)$$

$$x(0) = x_0$$
(7)

where x(t) represents the state variable vector constituted by three elements  $(i, \nu, \chi_m)$ , u(t) is the input vector, A(t) and B(t) are the time varying matrices containing the system parameters derived by equations (1)...(6).

The chosen reference model is described by:

$$\frac{d}{dt}x_M(t) = A_M x_M(t) + B_M u_M(t)$$

$$x_M(0) = x_{M0}$$
(8)

In this formula,  $A_M$ ,  $B_M$  are constant matrices which represent the state and the input matrices for the reference model. These matrices have the same dimensions of the system and can be chosen according to the desired system's behaviour.

At each instant, the adaptive control algorithm generates the control input signal in such a way the error between state variables vectors of the two systems is minimised:

$$\lim_{t \to \infty} \left[ x_M(t) - x(t) \right] = 0 \tag{9}$$

The model reference adaptive control algorithm requires that some hypotheses are satisfied:

- rank  $B = rank (|B, (A_M-A)|) = rank (|B, B_M|)$
- A and B are constant during the adaptation process and (together with A<sub>M</sub> and B<sub>M</sub> are stabilizable),
- the reference input  $u_M$  should be sufficiently exiting (this hypothesis is required by the estimation algorithm).

In this case the control signal is the following:

$$u(k-1) = D_{-}^{+} [x_{M}(k) - Qx_{M}(k-1)] + D_{-}^{+} [Q-E] x(k-1) (10)$$

where

- Q is a diagonal matrix which determines the control law behaviour: its coefficients should be less than 1: small values increase the weight of the corresponding state variable on the control action:
- E and D are the estimates of the submatrices of the plant augmented state matrix which represent, respectively the estimates of the state parameters and input parameters;
- $D^+=[D^TD]^{-1}D^T$  is the left pseudo-inverse of D

Parameters are estimated row by row by using the recursive least square estimation algorithm given by the following formula:

$$\hat{\theta}(k) = \hat{\theta}(k-1) + \Gamma(k) \left( x(k) - \psi^{T}(k) \hat{\theta}(k-1) \right)$$
where:

 $\hat{\theta}(k) = \left[\hat{a}_i \cdot \hat{b}_i\right]^T$  is the vector of the estimated parameters and  $\psi(k) = \left[x(k) \ u(k)\right]^T$  is the vector of output-input measurements

$$\Gamma(k) = P(k)\psi(k) =$$

$$= P(k-1)\psi(k) \left(\rho(k)I + \psi^{T}(k)P(k-1)\psi(k)\right)^{-1}$$

$$P(k) = \frac{\left(I - \Gamma(k) \psi^{T}(k)\right) P(k-1)}{\rho(k)}$$

$$\rho(k) = \Theta_0 \; \rho(k-1) + \Theta_1(1-\Theta_0)$$

 $\Theta_0 = 0.999, \Theta_1 = 0.970$ 

The coefficients  $\rho$  represent the forgetting factor. In this application they change dynamically improving the precision of the estimated values. Constants  $\Theta_{\varrho}$   $\Theta_{r}$  have been found through experiments. Fig. 5 shows a schematic block diagram of the control system.

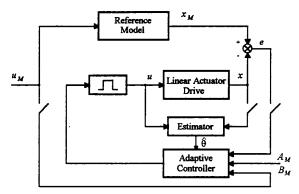


Fig. 5 - Basic block diagram of the adaptive control scheme.

## SYSTEM DESCRIPTION

This paper mainly concerns with a theoretical study of the problem, and it is carried out by means of simulations. For such reason, in order to obtain results which matches as closely as possible with those of the real systems an accurate model of the physical phenomena is mandatory. In order to accurately model the dynamic behaviour of the linear actuator it is necessary to analytically represent the variation of the electromagnetic force and the transformer and dynamic emf coefficients as function of both magnet position and winding current. Figure 4 shows that the force curves may be well described by an analytical function containing of two terms [1]. The first term is a parabola and represents the force contribution due to the interaction between the winding current and the magnet flux. The second term is a straight line and represents a reluctance force which is proportional to the squared current and the magnet position. With reference to the linear actuator considered in this paper, the force curves of Fig. 4 are well described by the following equation:

$$F(\chi_m, i) = c_1 \left( \chi_m - \frac{1}{2} \right) i^2 + \left[ c_2 + c_3 \left( \chi_m - \frac{1}{2} \right)^2 \right] i^{\frac{3}{2}} + c_4 i =$$

$$= f_1 \left( \chi_m, i \right) \qquad \text{(for } i > 0 \text{)}$$

where l is the working length.

With reference to the transformer and dynamic emf coefficients, they can be represented by

$$L_d(\chi_m, i) = c_5 i^2 + c_6 \left(\chi_m - \frac{1}{2}\right)^2 + c_7 \tag{13}$$

$$K_{\nu}(\chi_{m}, i) = c_{8}(\chi_{m} - \frac{1}{2}) i + c_{9}(\chi_{m} - \frac{1}{2})^{2} i + c_{10} i + c_{11} =$$

$$= f_{2}(\chi_{m}, i) \qquad (\text{for } i > 0) \qquad (14)$$

Note that for negative values of the winding current the force value can be evaluated taking into account the following relationship:

$$F(\chi_m, i) = -f_1(1-\chi_m, -i)$$
 (for  $i < 0$ )

Similarly, the dynamic emf coefficient, for negative value of the winding current, is given by:

$$K_{\nu}(\chi_m, i) = f_2(l-\chi_m, -i)$$
 (for  $i < 0$ )

Coefficients  $c_1$ ,  $c_{11}$  in equations (12), (13) and (14), are expressed using the international system of units SI, and assume the following values:

Expressions (12), (13) and (14) as well as coefficients  $c_1$ ...  $c_{II}$  have been chosen in order to approximate as closed as possible the curves in Figs. 2, 3 and 4.

The reference model for the control algorithm has been chosen in order to give rise to a behaviour of the system similar to that of a motor without the previously discussed phenomena. So, in the reference model the following values for  $K_f = F/i$ ,  $K_\nu$ ,  $L_d$ , have been assumed:

$$K_f = 4 \text{ N/A}$$
  $K_v = 4 \text{ Vs/m}$   $L_d = 0.13 \text{ H}$ 

The winding resistance is  $r=2.4 \Omega$  With these values the maximum current reached at the rated voltage (equal to 48V) is 20 A.

Note that the large electrical time constant of the considered actuator ( $\tau_e \cong 54 \text{ ms}$ ) limits the rate of the current build-up in the winding and hence limits the dynamic response. In order to improve the system dynamic performance, the supply voltage can be chosen greater than that necessary to deliver the maximum current. This fact doesn't constitute a relevant problem in the real system both from the technical and economical points of view, due to the low voltage and power of the motor.

### SIMULATION RESULTS

As previously described this paper only deals with simulations. Results concerning with the real system will be presented in future papers. The numerical simulations have been carried out basically considering variations of winding resistance, moving mass, load force and friction.

Figure 6 shows the response of the system without control for a reference speed alternating between +/-0.65 m/s. This speed gives rise to a stroke of about 0.6 m, which is quite near to the stroke limit (0.7 m) of the actuator. In this first case the system has initial

parameter values as in the reference model ( $M=2.5 \text{ Kg}, r=2.4\Omega$ ),  $F_L = 40 \text{ N}$ ) but it is subject to a sudden load variation (+30 N), at the instant t=2.5 s, a resistance variation  $(+2\Omega)$  at the instant t=4.5 s, a friction variation at instant t=6.5 s and a mass variation (+.5 Kg)at the instant t=8.5 s. As it could be expected, as some as the force disturbance is applied, the speed (Fig. 6a), the position (Fig. 6b) and the current (Fig. 6c) assume values very different in comparison with those of the reference model. The numerical results related to the system have been plotted only for t < 3.5 sbecause after this time interval the slider runs outside the limits for the working length, so the force and the transformer and the dynamic emf coefficients are not correctly represented by eqn. (12), (13) and (14). The reference model has been chosen with fixed coefficients (force coefficient  $K_r = 4.0$ , speed coefficient  $K_{rr} = 4.0$ , inductance coefficient  $L_1=0.13 \ H$ ). Figure 7 shows the response for the same reference speed as in Fig. 6 but with the system under control. Also in this case the system is subjected to the same disturbances of the previous experiment. As it can be seen, all disturbances are quickly eliminated so that the system and the reference model are in good agreement between them. Note that when new disturbances are introduced the previous ones are not removed so that the final part of the curves are referred to a situation with all the disturbance at the same time.

#### CONCLUSIONS

The linear DC actuator described appears an attractive solution for many applications in which linear motion is required. The theoretical principles of the machine have been described and the influence of saturation effects on force curve, dynamic and transformer emf coefficients have been shown; a model reference adaptive control has also been described. Some results obtained by means of simulations have been presented showing the validity of the proposed system. Further works are in progress in order to compare these results with those of other control algorithms looking at a reduction of the algorithm complexity in view of a low cost implementation and to improve system performance. In parallel, we are working in order to verify the effectiveness of the results on the actual system.

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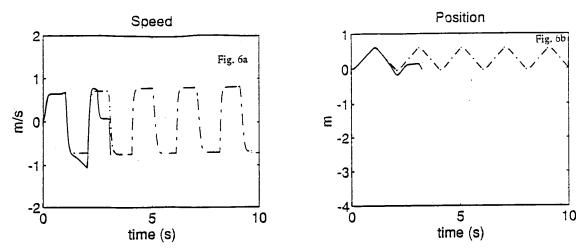


Fig.6 - System response without control (continuous line), in comparison with that of the reference model (pointed/dashed line)

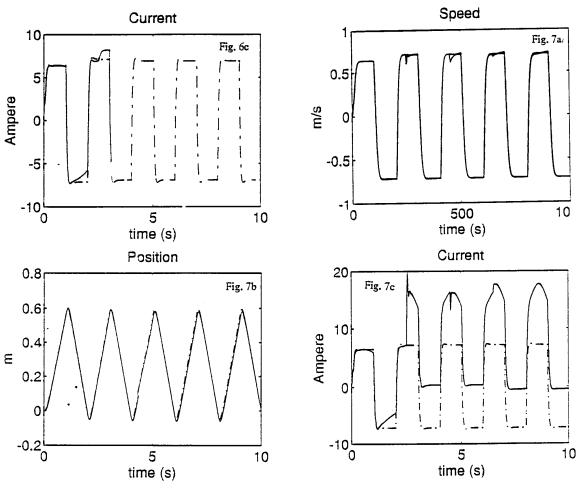


Fig.7 - System response with control (continuous line), in comparison with that of the reference model (pointed/dashed line)