

# ANALYSIS OF THRUST PULSATION IN LINEAR INDUCTION MOTORS

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## ABSTRACT

An analysis of thrust pulsation in Linear Induction Motors (LIM) is presented with reference to a T-shaped LIM. Supplying the machine with balanced voltages or balanced currents results in large thrust pulsation. The analysis reveals that, for each speed value, exists a particular unbalanced supply condition which allows the complete elimination of thrust pulsation. A substantial reduction of the pulsating thrust is shown to be possible supplying the machine with a 3-phase PWM AC chopper subjected to unsymmetric control.

## 1. INTRODUCTION

Linear induction machines have been widely investigated for use in high speed ground transportation. Other applications including liquid metal pumps, magnetohydrodynamic, power generators, conveyors, baggage handling systems, as well as a variety of consumers applications have contributed to increase the interest in linear induction machines. A particular type of linear induction motor is the T-shaped LIM. This motor can find application particularly in drives for material dislocation along paths showing sudden direction changes in the horizontal plane and in the vertical plane. The cross-section of the machine is shown in Fig. 1.

The secondary is composed of a T-shaped iron structure topped with four sheets of conducting material (usually aluminium). The primary includes four iron stacks facing the secondary sheets. The primary windings can be individually wound in each stack, otherwise the coil conductors leaving a slot can enter the corresponding slot of the adjacent core. In this way the number of end-windings is reduced leading to a lower copper weight and to a more compact machine. The primary voltage drops are also reduced [1]. To reduce the magnetising current in LIM it is advisable to use small air-gap. In order to avoid interferences between fixed and moving parts, in T-shaped LIM it is possible to remove the horizontal secondary when the direction change takes place in a vertical plane or the vertical secondary when the direction change takes place in a horizontal plane. In this way the electromagnetic force is given by the contributions of two primaries at least.

The analysis of linear induction machines is more complicated than in conventional round-rotor induction machines owing to the so-called end-effects. In this paper the machine performance is evaluated by a mathematical model which considers only the fundamental component of the primary mmfs, taking the end effects into account [2]-

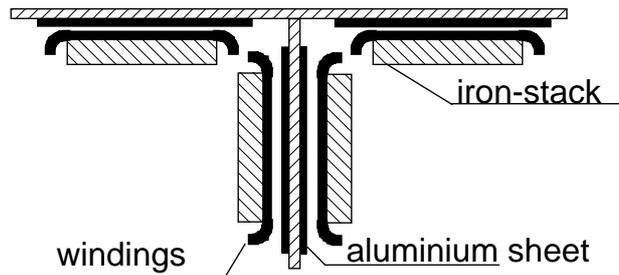


Fig. 1 - Cross-section of the T-shaped LIM

[5]. This mathematical model allows the determination of the electromagnetic force, the primary and the secondary currents corresponding to a fixed speed value, when the motor is supplied by a three-phase system of sinusoidal voltages. The geometry of the machine and the thickness of the secondary aluminium sheet determine a decreasing force versus speed curve which suggests that it is possible, in some cases, to regulate the motor speed by using a stator voltage controller. The phase-angle control of thyristors could be employed [6], but the use of this simple and reliable technique determines a high harmonic content and a low input power factor. These disadvantages may be reduced or eliminated by employing AC voltage regulators operating in chopping mode [7]-[9]. The AC PWM chopper may then be employed to drive the T-shaped LIM allowing high performance in the motor operation range as well as in the regenerative braking [1], [10], [11].

In linear induction machines, symmetric voltage operation determines unbalanced currents in the primary phases. As a consequence the copper losses increase with respect to balanced current operation. The mathematical model mentioned above permits the calculation of the thrust pulsation corresponding to unbalanced supply currents for each speed value.

It will be shown that suitable amplitude and phase displacement values of the negative sequence current component determine a practically negligible value of the thrust pulsation.

## 2. MACHINE EQUATIONS

The machine equations are derived from a simplified mathematical model [1] which is based on the following assumptions. The unbalanced primary currents are resolved into positive and negative sequence components. Each current sequence generates a diagram of sinusoidally distributed m.m.f. travelling at the air-gap. The secondary is characterised by the sheet resistance and the sheet leakage inductance per unit length of the sheet. The iron losses and saturation are neglected. The magnetic field, due to primary currents, is negligible out of the air-gap. For a given value of the space coordinate  $y$  (along the motion direction) the magnetic field due to the sheet current varies sinusoidally in time with the same angular frequency  $\omega$  of the primary winding current. The expression for the flux density distribution generated by the positive sequence component of primary current, can be expressed as

$$\mathbf{B}_p = -j B_{pM} e^{-j\alpha y} \quad (1)$$

where  $\alpha = \pi/\tau$  and  $2p\tau$  is the stack length. Under these assumptions, the field equations lead to the following expression for the magnetic field  $\mathbf{B}$  due to the sheet current

$$\mathbf{B} = \mathbf{A}_1 e^{\gamma_1 y} + \mathbf{A}_2 e^{\gamma_2 y} + \mathbf{A}_3 e^{\gamma_3 y} + \mathbf{B}_{0M} e^{-j\alpha y} + \mathbf{K}_1 \quad (2)$$

where  $\mathbf{A}_1, \mathbf{A}_2, \mathbf{A}_3, \mathbf{K}_1$  are arbitrary constants which can be determined from the boundary conditions. The sheet current density can be founded as

$$\mathbf{G} = - \frac{\delta \, d\mathbf{B}}{\mu_0 \, dy} \quad (3)$$

Similar expressions can be derived with reference to the negative sequence component of primary current, but considering negative speed values and an opportune reference frame. The effects of positive and negative sequence components are considered in the terminal voltage equations of the star-connected primary windings

$$\begin{aligned} \mathbf{V}_{ab} &= \mathbf{E}_a - \mathbf{E}_b + \mathbf{Z}_a \mathbf{I}_a - \mathbf{Z}_b \mathbf{I}_b \\ \mathbf{V}_{bc} &= \mathbf{E}_b - \mathbf{E}_c + \mathbf{Z}_b \mathbf{I}_b - \mathbf{Z}_c \mathbf{I}_c \end{aligned} \quad (4)$$

where  $\mathbf{E}_a, \mathbf{E}_b$  and  $\mathbf{E}_c$  are the back emfs in the primary windings due to the effects of both positive and negative sequence components.

Once the total magnetic field  $\mathbf{B}_t$  and the total sheet current density  $\mathbf{G}_t$ , due to both positive and negative sequence components are known, it is possible to calculate the instantaneous value of the electromagnetic force  $F$

$$F(t) = h_a \int_0^{2p\tau} \Re_e \left[ \mathbf{B}_t(y) e^{j\omega t} \right] \Re_e \left[ \mathbf{G}_t(y) e^{j\omega t} \right] dy \quad (5)$$

The electromagnetic force can be viewed as composed of two terms  $F(t) = F_a + F_p(t)$ .  $F_a$  represents the average value and can be expressed as

$$F_a = \frac{1}{2} h_a \int_0^{2p\tau} \Re_e \left[ \mathbf{B}_t(y) \mathbf{G}_t^*(y) \right] dy \quad (6)$$

where  $\mathbf{G}_t^*$  is the complex conjugate of  $\mathbf{G}_t$ .

$F_p(t)$  represents the pulsating component which is given by

$$F_p(t) = \frac{1}{2} h_a \int_0^{2p\tau} \Re_e \left[ \mathbf{B}_t(y) \mathbf{G}_t(y) e^{j2\omega t} \right] dy \quad (7)$$

It should be noted that the frequency of the pulsating component is twice the frequency of the primary currents. The amplitude of the force pulsation can be written as

$$F_{PM} = \frac{1}{2} h_a \left| \int_0^{2p\tau} \mathbf{B}_t(y) \mathbf{G}_t(y) dy \right| \quad (8)$$

Owing to the presence of this pulsating component, the primary and secondary systems experience large vibrations and consequently acoustic noise.

In conventional round rotor induction motors a pulsating torque component is present only if the motor is supplied with an unbalanced system of voltages which determines unbalanced currents. In linear induction motors a pulsating force is present even if the machine is supplied with a balanced 3-phase system of voltages owing to the open air-gap with an entry-end and an exit-end. In [1] it has been shown that in order to increase the average value of the electromagnetic force the machine should be supplied with balanced currents. However, this operating condition does not correspond to a minimum for the amplitude of the pulsating force component. It will be shown that, utilising the mathematical model above outlined, it is possible to evaluate the unbalanced voltages which completely cancel the pulsating force component for each speed value.

### 3. THRUST PULSATION

In order to show the influence of the supply conditions on the machine performance, reference is made to a T-shaped linear induction motor suitable for industrial applications. The nameplate data are given in Table I.

TABLE I

3-phase, 2-pole, 50 Hz star-connected windings	<b>Primary stacks</b>	<b>Secondary aluminium sheet</b>
Rated current = 14.5 A	stack length = 0.135m	thickness = 0.003 m
Rated speed = 3 m/s	stack width = 0.1 m	width = 0.16 m
Rated thrust = 210 N	stack height = 0.05 m	clearance = 0.0015 m

To emphasise the influence of the end-effects on the thrust pulsation, the analysis has been applied to a machine having a reduced number of poles. Utilising the above described mathematical model, it is possible to determine the machine performance when the LIM is fed with balanced voltages or balanced currents. In particular, with balanced voltages the machine shows unbalanced primary currents characterised by a negative sequence component which, at the rated speed, is about 15% of the positive sequence component. In Fig. 2, curve a) represents the average value of the force as function of the speed. This curve has been obtained supplying the T-shaped LIM with a system of balanced voltages having an amplitude which determines, at the rated speed, the same primary copper losses as for rated balanced currents. In Fig. 3, curve a) shows the amplitude of the pulsating force component for the same operating condition of Fig. 2a.

It can be noticed that the presence of the pulsating force component is not only due to the current unbalance. In fact it will be shown that similar results are obtained supplying the machine with balanced currents.

In order to supply the T-shaped LIM with a balanced system of rated currents, the required line voltages at the rated speed are:

$$V_{ab}=367V \quad V_{bc}=299V \quad V_{ca}=336V \quad (9)$$

With these line voltages the negative sequence current is negligible for all the speed values corresponding to the motor operating range.

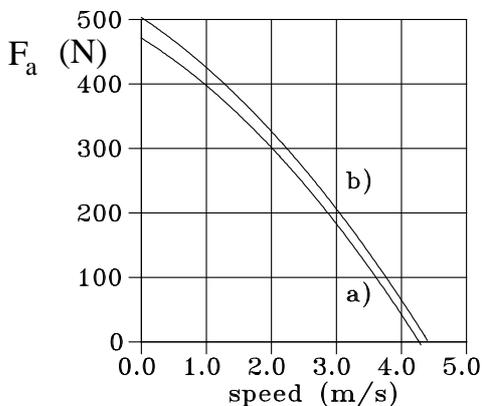


Fig. 2 -  $F_a$  versus speed.  
 a) balanced voltage supply.  
 b) balanced current supply.

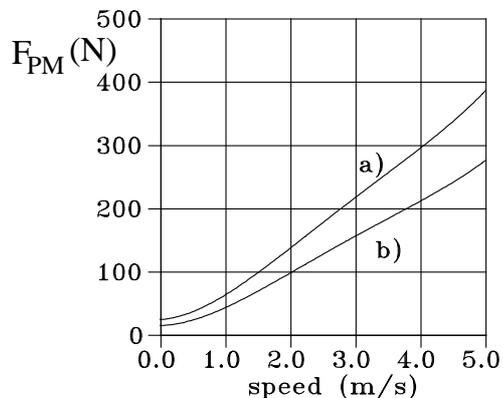


Fig. 3 -  $F_{PM}$  versus speed.  
 a) balanced voltage supply.  
 b) balanced current supply.

The average value of the thrust versus speed curve, obtained with balanced current operation, is represented in Fig. 2, curve b). It should be noted that, even with balanced currents, the machine produces force pulsation. It is then of interest to determine how the amplitude of the pulsating force component varies with the speed, supplying the machine with the line voltages (9). This has been carried out using the described mathematical model and the results are represented by curve b) in Fig. 3.

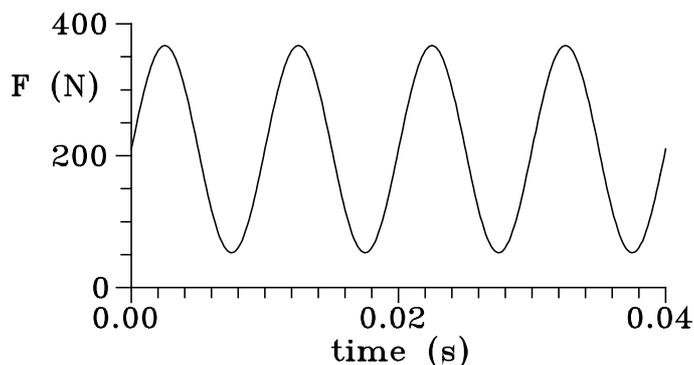


Fig. 4 - Instantaneous force variations, rated speed, balanced currents.

The instantaneous force variations, at the rated speed, are shown in Fig. 4. The amplitude of the pulsating component is about 75% of the average value and the pulsation frequency is 100 Hz, according to Eqn. 7.

Analysing Figs. 3 and 4 it becomes evident that the amplitude of the force pulsation produced by the machine is significantly high, supplying the machine either with balanced

voltage or balanced current. Hence, large vibrations and acoustic noise cannot be avoided with conventional balanced voltage source or balanced current source supply. In the next section it will be shown that it is possible to eliminate the force pulsation supplying the machine with an opportune unbalanced system of currents.

#### 4. COMPENSATION OF THE THRUST PULSATION

In order to analyse the influence of an unbalanced power supply on the thrust pulsation it is convenient to consider the primary currents as composed of a positive sequence component  $I_a^+$  and a negative sequence component  $I_a^-$ . Utilising the mathematical model it could be shown that, for low value of the negative sequence current, the average value of the thrust is basically determined by the positive sequence current, while the amplitude of the thrust pulsation is determined by phase and amplitude of the negative sequence current.

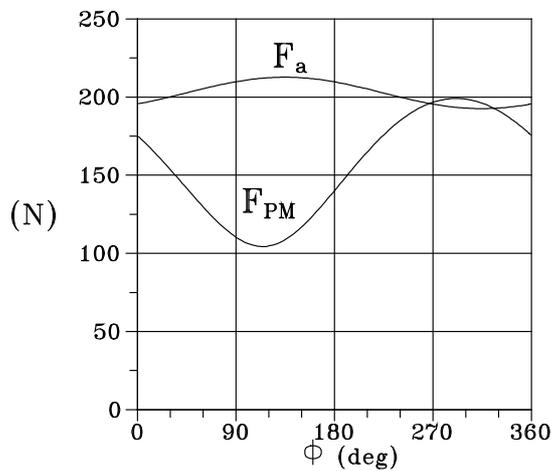


Fig. 5 -  $F_a$  and  $F_{PM}$  versus speed, 10% negative sequence current, rated speed.

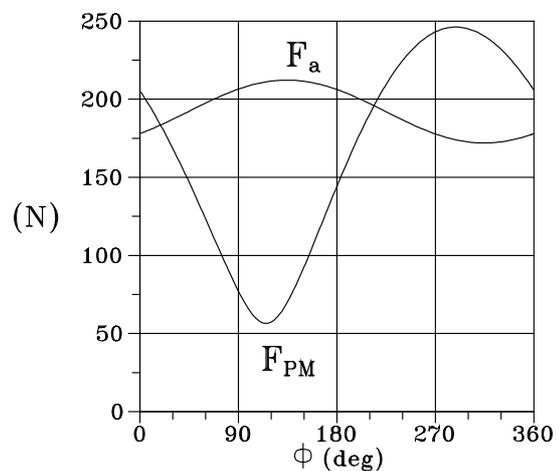


Fig. 6 -  $F_a$  and  $F_{PM}$  versus speed, 20% negative sequence current, rated speed.

In order to emphasise the influence of the negative sequence current, reference is made to the T-shaped LIM above described. In particular, considering a positive sequence current of 14.5 A (rated current) and a negative sequence current of 1.45 A (10% of positive sequence), the values of  $F_a$  and  $F_{PM}$  have been calculated as functions of the phase angle  $\phi$  between  $I_a^+$  and  $I_a^-$  ( $I_a^-$  phase lagging). The results obtained at the rated speed are shown in Fig. 5. The two curves represent the average value  $F_a$  and the pulsation amplitude  $F_{PM}$  of the electromagnetic force. Fig 6 shows the results obtained with a 20% negative sequence current.

As Figs. 5 and 6 show, for each value of the negative sequence current it is possible to determine the phase angle  $\phi$  which minimises the thrust pulsation. As a consequence, it can be expected to find the minimum value of  $F_{PM}$  varying the amplitude of the negative sequence current.

Considering the T-shaped LIM described in Table I, a practically null value of thrust pulsation has been obtained supplying the machine with unbalanced currents characterised by a negative sequence component having amplitude of 32% and a phase angle of  $116^\circ$ .

The same procedure has been applied for each speed value leading to results shown in Fig. 7. In this figure the two curves give the phase displacement angle and the per cent negative sequence current which determine null value of the thrust pulsation.

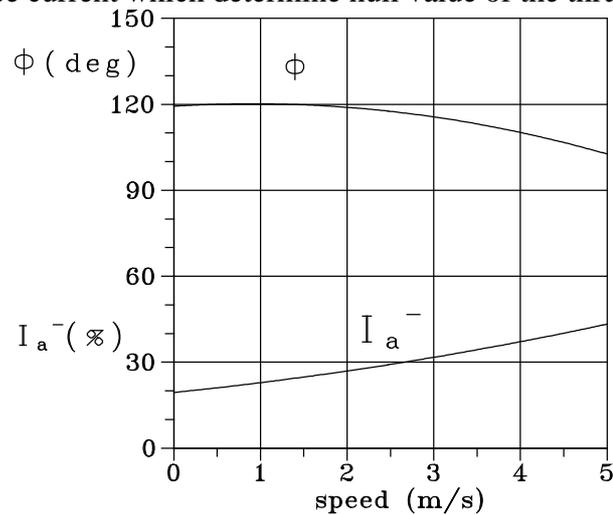


Fig. 7 - Phase angle and per cent negative sequence current corresponding to null thrust pulsation.

## 5. UNSYMMETRIC CONTROL OF THE TERMINAL VOLTAGES

In the previous section it has been shown that opportune unbalanced primary currents determine a null pulsating thrust. It is clear that, once the positive and negative sequence current are known for each speed value, the unbalanced 3-phase system of voltages required to supply the machine can be evaluated by Eqns. 4.

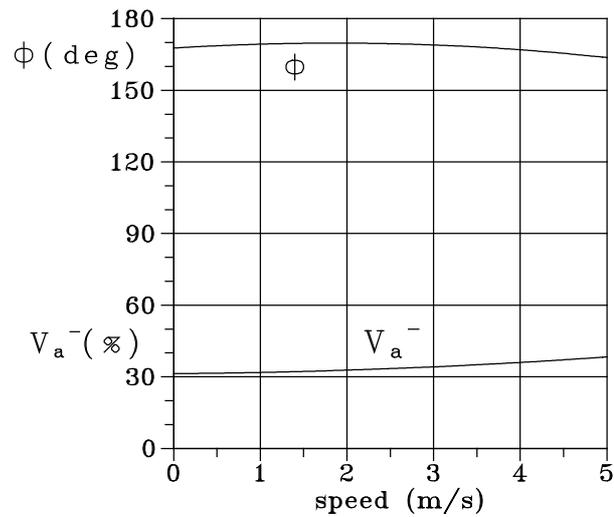


Fig. 8 - Phase angle and per cent amplitude of the negative sequence voltage corresponding to null thrust pulsation.

Fig. 8 shows the phase displacement angle and the per cent negative sequence voltage which determine null values of the thrust pulsation. As it is possible to see, the optimum values of  $V_{a^-}$  and  $\Phi$  are slightly affected by the speed. It is then of interest to evaluate how the thrust pulsation varies, maintaining the terminal voltages with the same unbalanced condition as at rated speed. These unbalanced voltages, which determine at the rated speed the same primary copper losses as for rated balanced currents, result as follows

$$V_{ab}=398 \text{ V } \underline{/0^\circ} \quad V_{bc}=212 \text{ V } \underline{/ -114^\circ} \quad V_{ca}=368 \text{ V } \underline{/ -212^\circ} \quad (10)$$

The results obtained, supplying the machine with voltages (10), are shown in Fig. 9. By comparing Figs. 2, 3 and 9 we can observe that, maintaining the same copper losses as in the case of balanced currents, the average value of the thrust is slightly reduced but the thrust pulsation is practically eliminated all over the motor operating range.

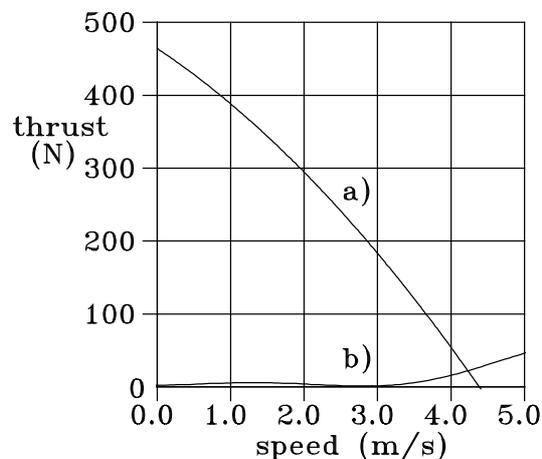


Fig. 9 - Average thrust (a) and pulsating thrust (b), with ideal supply voltages.

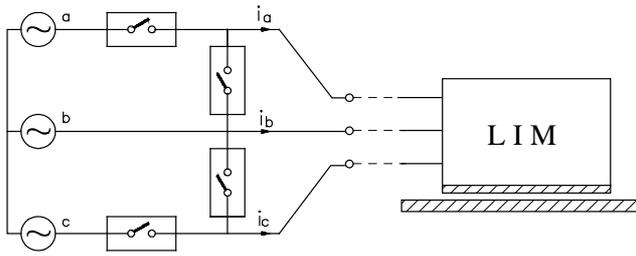


Fig. 10 - Schematic drawing of the 3-phase PWM AC chopper.

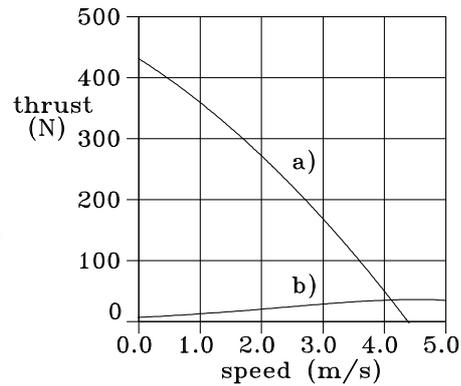


Fig. 11 - Average thrust (a) and pulsating thrust (b) with PWM AC chopper supply.

## 6. UNSYMMETRIC VARIABLE VOLTAGE CONTROL

In order to realise the supply conditions expressed by (10) a solid state control equipment could be employed. Owing to the particular thrust versus speed curve, the control of the output thrust in the T-shaped LIM can be obtained using the simple variable-voltage, constant-frequency method. The requirement of the voltage controller is to change in equal fashion the amplitude of the line voltage phasors. For this purpose a 3-phase PWM AC chopper, subjected to unsymmetric control, could be used [1]. This chopper, shown in Fig. 10, can be regarded as two single phase choppers. One of them controls the line voltage  $V_{ab}$  while the other control the line voltage  $V_{bc}$ . The line voltage  $V_{ca}$  is indirectly controlled being null the sum of the line voltages. The amplitude of the line voltages can be smoothly controlled by varying the ratio  $\gamma$  of on time to modulation period. Moreover, harmonic components in line voltages, at frequency lower than carrier frequency, do not appear [10], [11]. The chopper can be controlled unsymmetrically by using two different values of  $\gamma$  for the two single-phase choppers. As a consequence the amplitude of line voltages  $V_{ab}$  and  $V_{bc}$  can be independently regulated. However, this type of control cannot change the phase angle between the line voltages  $V_{ab}$  and  $V_{bc}$ , which remain fixed to  $120^\circ$  for balanced source voltages. Hence, analysing the ideal supply condition (10), it comes out that the 3-phase PWM AC chopper is able to produce an unbalanced 3-phase system of voltages which is very close to the 3-phase system (10). In particular with opportune values of  $\gamma_{ab}$  and  $\gamma_{bc}$  it is possible to supply the machine with the following system of voltages

$$V_{ab}=398 \text{ V } /0^\circ \quad V_{bc}=212 \text{ V } /-120^\circ \quad V_{ca}= 345 \text{ V } /-212^\circ \quad (11)$$

With this supply condition, the average thrust and the pulsating thrust versus speed curves are shown in Fig. 11. As it appears from this figure, the use of the 3-phase PWM AC chopper determines average thrust values close to that shown in Fig. 9 and pulsating

thrust values greater than that of Fig. 9. However, the machine behaviour is characterised by a substantial reduction of pulsating thrust with respect to both balanced voltage and balanced current operation (Figs. 2 and 3).

Furthermore, the control of the average thrust can be obtained varying the terminal voltages from zero to full value. This can be easily obtained employing the proposed PWM AC chopper and varying proportionally the values of  $\gamma_{ab}$  and  $\gamma_{bc}$  maintaining the ratio  $\gamma_{ab}/\gamma_{bc}$  fixed to the value 398/212.

## CONCLUSIONS

The study of linear induction motors, based on a one-dimensional field analysis, has pointed out that balanced voltage operation, as well as balanced current operation, produce a component of pulsating thrust. The thrust pulsate at a frequency which is twice the fundamental frequency. To deal with the analysis of unbalanced condition the concept of positive and negative sequence component has been used. Firstly, with reference to a T-shaped LIM, the machine performance has been analysed under balanced voltages and balanced currents. In particular the average developed force and the pulsating component have been compared in the motor operating range. Furthermore, the analysis has pointed out that, for each speed value, it is possible to determine the amplitude and the phase angle of the negative sequence component which completely eliminate the pulsating force. Then, it has been investigated the constant voltage operation when supplying the machine with the unbalanced voltages which determine, at the rated speed, a null value of the pulsating force. In these conditions, the force pulsation results strongly reduced all over the motor operating range in spite of a small reduction of the average value of the thrust. At the end it has been shown that the unsymmetric control of a 3-phase PWM AC chopper allows to obtain unbalanced output voltages which are very close to the calculated ideal voltages.

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