Abstract—A generalized multi-phase space vector theory is considered for developing the space vector modulation of a seven-phase voltage source inverter. The modulation is based on the control of the voltage vector in the first \( d-q \) plane, imposing to be zero the voltage vectors in both the second and the third \( d-q \) planes. The proposed switching pattern includes one-leg commutation at a time, with the possibility to share the zero voltage between the two null vectors. The maximum value of the modulation index for sinusoidal balanced output phase voltages is carried out. The theoretical analysis is confirmed by numerical simulations.

Index Terms—Maximum modulation index, multi-phase circuit systems, multiple \( d-q \) planes, space vector modulation, voltage source inverters.

I. INTRODUCTION

The conventional structure for variable-speed drives consists of a three-phase motor supplied by a three-phase voltage source inverter (VSI). However, when the machine is connected to an inverter supply the need for a specific number of phases, such as three, disappears. Nowadays, the development of modern power electronics, makes it possible to consider the number of phases a degree of freedom, i.e., an additional design variable.

Multi-phase motor drives have many advantages over the traditional three-phase motor drives such as reducing the amplitude and increasing the frequency of torque pulsations, reducing the rotor harmonic currents losses and lowering the dc link current harmonics. In addition, owing to their redundant structure, multi-phase motor drives improve the system reliability [1]-[4].

The increase of the number of phases is considered a possible solution to overcome the problems related to high-power applications. In the past decades, multi-level inverter-fed ac machines have emerged as a promising solution in achieving high power ratings with voltage limited devices. Similarly, the use of multi-phase inverters together with multi-phase ac machines has been recognized as a viable approach to obtain high power ratings with current limited devices.

The space vector theory can be still employed to represent the behavior of multi-phase systems as a natural extension of the traditional three-phase space vector transformation, leading to an elegant and effective vectorial approach in multiple \( d-q \) planes [5]. In particular, the space vectors can be usefully adopted for the modulation of multi-phase inverters. The Space Vector Modulation (SVM) for five-phase VSI has been developed in [6]-[9]. In [10] some general guidelines to multi-phase VSI are given, but without taking the multiple \( d-q \) planes into account.

In this paper the space vector modulation has been extended to a seven-phase voltage source inverter, considering reference space vectors in all the three \( d-q \) planes. In particular, the proposed SVM strategy univocally selects the inverter switch configurations among the \( 2^7 = 128 \) possibility by privileging the space vector on the first \( d-q \) plane, \( d_1-q_1 \), the one responsible of balanced sinusoidal output voltage waveforms. The resulting switching patterns, collected in a general switching table, include six active and two null configurations, with a single leg commutation for each configuration change. The duty cycles are calculated on the basis of a detailed analytical approach and the modulation limits are given for balanced sinusoidal voltages by introducing the maximum modulation index.

A complete set of numerical results confirm the effectiveness of the proposed space vector modulation strategy.

II. SPACE VECTOR TRANSFORMATIONS FOR SEVEN-PHASE SYSTEMS

Let us consider \( n \) homogeneous and time-dependent real quantities \( x_k(t) \) related to a \( n \)-phase system. The generalized space vector transformation [5] is defined as

\[
\bar{x}_{S_h} = \frac{1}{n} \sum_{k=1}^{n} x_k \alpha^{h(k-1)}, \quad h = 0, 1, 2, \ldots, n-1
\]

being

\[
\alpha = e^{\frac{2\pi}{n}}.
\]

The zero-sequence component \( \bar{x}_{S_0} \) is a real quantity and it is often called “homopolar component”.

With the exception for \( h = 0 \) (and \( h = n/2 \) for even number of phases), the quantity \( \bar{x}_{S_h} \) is a complex number, and it is called space vector component of sequence \( h \). Its absolute value, \( x_{S_h} \), is usually called “magnitude” of the space vector.

The inverse transformation is given by

\[
x_k = \sum_{h=0}^{n-1} \bar{x}_{S_h} \alpha^{-h(k-1)}, \quad k = 1, 2, 3, \ldots, n.
\]

It is evident that the general space vector transformation is half redundant being
Then, a reduced number of space vectors can be used to represent the \( n \)-phase system [5].

In the case of \( n = 7 \) the zero-sequence component and three opportune space vectors among the six available ones must be considered. Two of the possible choices are represented in the diagram of Fig. 1. In particular, the simplest choice (a) considers the first sequence components 1, 2, and 3, whereas the choice (b) considers the odd sequence components 1, 3, 5. Note that in the case of \( n = 5 \) the choice (a) has been assumed in [7] and [9] whereas the choice (b) has been assumed in [6] and [8].

In this paper the choice (a) is considered and the normalization factors are chosen such that the direct transformation can be written as a natural extension of the one for representing the seven-phase system [5]. The resulting inverse transformation can be written as

\[
\begin{align*}
x_0 &= x_{S_0} = \frac{1}{7} \left[ x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 \right] \\
x_1 &= 2x_{S_1} = \frac{2}{7} \left[ x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 \right] \\
x_2 &= 2x_{S_2} = \frac{2}{7} \left[ x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 \right] \\
x_3 &= 2x_{S_3} = \frac{2}{7} \left[ x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 \right] \\
\end{align*}
\]

where the symbol “\( \cdot \)” denotes the inner (scalar) product. The three space vectors \( x_1, x_2, \) and \( x_3 \) lie in the planes called \( d_1-q_1, d_2-q_2, \) and \( d_3-q_3, \) corresponding to the sequence 1, 2, and 3, respectively.

**III. REPRESENTATION OF THE INVERTER OUTPUT VOLTAGES**

The structure of a seven-phase voltage source inverter feeding a star-connected load is shown in Fig. 2. With reference to the transformations (4), the three space vectors of the line-to-neutral load voltages can be written as:

\[
\begin{align*}
x_{S_1} &= \frac{1}{n} \sum_{k=1}^{n} x_k \alpha^{(n-b)(k-1)} = \bar{x}_{S_{h-b}}, \quad h = 1, 2, \ldots, n-1. \\
x_{S_{h-b}} &= \frac{1}{n} \sum_{k=1}^{n} x_k \alpha^{-h(k-1)} = \bar{x}_{S_{h}}
\end{align*}
\]

where \( S_h \) represents the switch state \((0, 1)\) of the \( k \)-th inverter leg \((k = 1, 2, \ldots, 7)\).

It can been shown that the zero-sequence component is null if the load is balanced, i.e.,

\[
v_0 = \frac{1}{7} V_{dc} \left[ S_1 + S_2 + S_3 + S_4 + S_5 + S_6 + S_7 \right] + v_{y0} = 0.
\]

Note that (7) can be utilized to calculate the instantaneous line-to-neutral load voltages on the basis of the switch state as

\[
v_k = V_{dc} \left[ S_k - \frac{1}{7} (S_1 + S_2 + S_3 + S_4 + S_5 + S_6 + S_7) \right].
\]

IV. SPACE VECTOR MODULATION

The goal of the space vector modulation for a seven-phase VSI is to generate the three output voltage space vectors \((\bar{v}_1, \bar{v}_2, \text{ and } \bar{v}_3)\) with a given average value within the cycle period \((\bar{v}_{ref}, \bar{v}_{2ref}, \text{ and } \bar{v}_{3ref})\), corresponding to seven line-to-neutral load voltages (the zero-sequence component \(v_0\) is null with balanced loads). This condition leads to six independent scalar constraints that
can be univocally satisfied by selecting, in each cycle period, inverter configurations corresponding to six active and a null voltage vector.

In order to optimize the harmonic content and minimize the current ripple, the six active configurations should correspond to voltage vectors lying as close as possible to the reference voltage vectors. Since in seven-phase inverters there are three independent reference voltage vectors, a possible configuration selection criterion consists in privileging the voltage vector on the $d_1$ plane, $\overline{v}_{\text{ref}}$. In this way the relevant case of balanced sinusoidal output voltages, corresponding to $v_{\text{ref}} = 0$ and $v_{\text{ref}} = 0$, can be optimized.

In order to determine the six active configurations, the $d_1$ plane can be subdivided in 14 sectors with an angular size of $\pi/7$, as shown in Fig. 4. Each sector identifies three active configurations on its left border and three active configurations on its right border. These six configurations are determined so that a switching pattern requiring a single commutation for each configuration change can be defined for each sector, starting from the null configuration (0000000) up to the other null configuration (1111111), as shown in Table I. The second half of the cycle period consists in following backward the switching pattern from (1111111) to (0000000). The duty cycles of each configuration are indicated in the first column of Table I, whereas, in the last three columns the magnitudes of the corresponding voltage space vectors in the $d_1$-, $d_2$-, $d_3$-q planes are presented.

Table I shows that there are seven magnitudes in all corresponding to the switch configurations involved in the switching pattern (i.e., $V_A$, $V_B$, $V_C$, $V_D$, $V_E$, $V_F$, $V_G$, in increasing order). They can be expressed on the basis of only three coefficients $K_a$, $K_b$, and $K_c$ as

$$V_A = \frac{2}{7} \left(2 \cos \frac{3\pi}{7}\right) V_{dc} = \frac{2}{7} K_a V_{dc} = 0.127 V_{dc},$$
$$V_B = \frac{2}{7} \left(1 + 2 \cos \frac{4\pi}{7}\right) V_{dc} = \frac{2}{7} K_b V_{dc} = 0.159 V_{dc},$$
$$V_C = \frac{2}{7} \left(1 + 2 \cos \frac{6\pi}{7}\right) V_{dc} = \frac{2}{7} K_c V_{dc} = 0.229 V_{dc},$$
$$V_D = \frac{2}{7} V_{dc} = 0.286 V_{dc},$$
$$V_E = \frac{2}{7} \left(2 \cos \frac{2\pi}{7}\right) V_{dc} = \frac{2}{7} K_a V_{dc} = 0.356 V_{dc},$$
$$V_F = \frac{2}{7} \left(2 \cos \frac{\pi}{7}\right) V_{dc} = \frac{2}{7} K_b V_{dc} = 0.515 V_{dc},$$
$$V_G = \frac{2}{7} \left(1 + 2 \cos \frac{2\pi}{7}\right) V_{dc} = \frac{2}{7} K_c V_{dc} = 0.642 V_{dc},$$

being

$$K_a = \cos \frac{\pi}{14} \approx 0.975,$$
$$K_b = \cos \frac{3\pi}{14} \approx 0.782,$$
$$K_c = \cos \frac{5\pi}{14} \approx 0.434.$$

Fig. 4 shows that the 42 output voltage vectors corresponding to the active configurations utilized in the proposed modulation strategy lie on three 14-sided regular polygons on the plane $d_1$-q1 (corresponding to $V_D$, $V_F$, $V_G$). Also in the planes $d_2$-q2 and $d_3$-q3, the output voltage vectors lie on three 14-sided regular polygons (corresponding to $V_B$, $V_D$, $V_E$ and $V_A$, $V_C$, $V_D$, respectively).

### Table I

**Switching Table of the Proposed SVM Control Strategy**

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<th>$S_1$</th>
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<table>
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S8 - 8
In order to present the details of the proposed space vector modulation strategy, the case of \( \overline{v}_{1 \text{ref}} \) lying in sector \( S_1 \) is considered. Fig. 5 shows the inverter switch configurations and the corresponding output voltage vectors involved in the switching pattern in the \( d_1-q_1 \), \( d_2-q_2 \), and \( d_3-q_3 \) planes. For each reference space vector \( \overline{v}_{h \text{ref}} \) (\( h = 1, 2, 3 \)) the two components \( v_a_h \) and \( v_b_h \) along proper directions are defined, according to Fig. 5, leading to

\[
\begin{align*}
\overline{v}_{1 \text{ref}} &= v_{1 \text{ref}} e^{j \theta_1} = v_{a_1} + v_{b_1} e^{j \pi / 7}, \\
\overline{v}_{2 \text{ref}} &= v_{2 \text{ref}} e^{-j \theta_2} = v_{a_2} + v_{b_2} e^{-j 5 \pi / 7}, \\
\overline{v}_{3 \text{ref}} &= v_{3 \text{ref}} e^{j \theta_3} = v_{a_3} + v_{b_3} e^{j 3 \pi / 7},
\end{align*}
\]

where

\[
\begin{align*}
v_{a_1} &= \frac{\sin(\pi / 7 - \theta_1)}{K_c} v_{1 \text{ref}}, & v_{b_1} &= \frac{\sin \theta_1}{K_c} v_{1 \text{ref}}, \\
v_{a_2} &= \frac{\sin(5 \pi / 7 - \theta_2)}{K_c} v_{2 \text{ref}}, & v_{b_2} &= \frac{\sin \theta_2}{K_c} v_{2 \text{ref}}, \\
v_{a_3} &= \frac{\sin(3 \pi / 7 - \theta_3)}{K_c} v_{3 \text{ref}}, & v_{b_3} &= \frac{\sin \theta_3}{K_c} v_{3 \text{ref}}.
\end{align*}
\]

Then, the reference components \( v_{a_h} \) and \( v_{b_h} \) can be synthesized as weighted average of the space vector magnitudes, over the cycle period \( T \), introducing the corresponding application times \( t_1, t_2, \ldots, t_6 \), leading to

\[
\begin{align*}
v_{a_1} &= \frac{t_1}{T} V_G + \frac{t_2}{T} V_E + \frac{t_1}{T} V_D, \\
v_{b_1} &= \frac{t_1}{T} V_G + \frac{t_3}{T} V_E + \frac{t_1}{T} V_D, \\
v_{a_2} &= \frac{t_3}{T} V_B + \frac{t_3}{T} V_E + \frac{t_1}{T} V_D, \\
v_{b_2} &= \frac{t_4}{T} V_B + \frac{t_2}{T} V_E + \frac{t_1}{T} V_D, \\
v_{a_3} &= \frac{t_3}{T} V_C + \frac{t_5}{T} V_A + \frac{t_1}{T} V_D, \\
v_{b_3} &= \frac{t_4}{T} V_C + \frac{t_2}{T} V_A + \frac{t_1}{T} V_D.
\end{align*}
\]

The relationships (11) define a system of six linear equations, assuming the application times of the active configurations \( t_1, t_2, \ldots, t_6 \) as unknown variables.

The application times of the null configurations \( t_0 \) and \( t_7 \) can be determined as follows:

\[
t_0 + t_7 = T - (t_1 + t_2 + t_3 + t_4 + t_5 + t_6).
\]

It should be noted that (12) does not allow the determination of \( t_0 \) and \( t_7 \) separately, leading to a degree of freedom that can be utilized in order to modify the modulation properties in terms of switching frequency and output current distortion [11]. Introducing the duty-cycles \( \delta_h \), (12) can be rewritten as

\[
\delta_0 + \delta_7 = 1 - (\delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 + \delta_6),
\]

where

Fig. 4. Output voltage vectors corresponding to the inverter configurations in the three \( d-q \) planes.

Fig. 5. Inverter configurations and corresponding output voltage vectors in the case of \( \overline{v}_{1 \text{ref}} \) lying in sector \( S_1 \).
\[ \delta_k = \frac{t_k}{T}, \; k = 0, 1, \ldots, 7. \]

The original linear system (11) can be decomposed in two independent systems of three equations with three unknowns, characterized by the same matrix of coefficients \([M]\), leading to

\[
\begin{align*}
2V_{dc} & \begin{bmatrix} \delta_3 \\ \delta_5 \\ \delta_1 \end{bmatrix} = \begin{bmatrix} v_{a1} \\ v_{a2} \\ v_{a3} \end{bmatrix}, \\
2V_{dc} & \begin{bmatrix} \delta_4 \\ \delta_2 \\ \delta_6 \end{bmatrix} = \begin{bmatrix} v_{b1} \\ v_{b2} \\ v_{b3} \end{bmatrix},
\end{align*}
\]

where

\[
[M] = \begin{bmatrix} K_a/K_c & K_b/K_c & 1 \\ K_c/K_b & -K_a/K_c & 1 \\ K_b/K_a & K_c/K_a & 1 \end{bmatrix}.
\]

Since the matrix \([M]\) is nonsingular (\(\det[M] = -7\)), the system has the following unique solution:

\[
\begin{align*}
\begin{bmatrix} \delta_3 \\ \delta_5 \\ \delta_1 \end{bmatrix} &= \frac{7}{2V_{dc}} [M]^{-1} \begin{bmatrix} v_{a1} \\ v_{a2} \\ v_{a3} \end{bmatrix}, \\
\begin{bmatrix} \delta_4 \\ \delta_2 \\ \delta_6 \end{bmatrix} &= \frac{7}{2V_{dc}} [M]^{-1} \begin{bmatrix} v_{b1} \\ v_{b2} \\ v_{b3} \end{bmatrix},
\end{align*}
\]

where

\[
[M]^{-1} = \frac{4}{7} \begin{bmatrix} K_cK_a & K_bK_c & -K_bK_a \\ K_cK_b & -K_aK_c & K_aK_c \\ K_cK_c & K_b^2 & K_a^2 \end{bmatrix}.
\]

Introducing the coefficient \(K = 7K_c^2/V_{dc}^2\), the previous relationships can be rewritten as

\[
\begin{align*}
\delta_1 &= KV_D v_{a1}, \\
\delta_2 &= KV_F v_{b1}, \\
\delta_3 &= KV_G v_{a1}, \\
\delta_4 &= KV_G v_{b1}, \\
\delta_5 &= KV_F v_{a1}, \\
\delta_6 &= KV_D v_{b1},
\end{align*}
\]

Introducing the coefficient \(K = 7K_c^2/V_{dc}^2\), the previous relationships can be rewritten as

\[
\begin{align*}
\delta_1 &= KV_D v_{a1}, \\
\delta_2 &= KV_F v_{b1}, \\
\delta_3 &= KV_G v_{a1}, \\
\delta_4 &= KV_G v_{b1}, \\
\delta_5 &= KV_F v_{a1}, \\
\delta_6 &= KV_D v_{b1},
\end{align*}
\]

It can be noted that the duty cycle of each active configuration is proportional to the magnitude of the corresponding voltage vector on \(d_1-q_1\).

V. MAXIMUM MODULATION INDEX

The modulation index \(m\) is defined as the ratio between the amplitude of the line-to-neutral voltage and the dc-link voltage, in balanced sinusoidal operating conditions. In this case, the voltage amplitude of all phases coincides with the magnitude \(v_1\) of the space vector lying on \(d_1-q_1\) plane. Then,

\[ m = \frac{v_1}{V_{dc}}. \]

In order to determine the maximum value of the modulation index, the modulation constraints must be introduced. In particular, the application times of both active and null configurations involved in the switching pattern must be non-negative. This conditions can be written in terms of duty cycles as

\[ \delta_k \geq 0, \; k = 0, 1, \ldots, 7. \]

In the case of \(v_{2ref} = 0\) and \(v_{3ref} = 0\), as for balanced sinusoidal voltages, the duty cycles corresponding to the active configurations are always non-negative. In fact, all the terms in (17) are non-negative. In this case, the modulation constraints are represented only by the fol-
lowing inequalities
\[ \delta_0 \geq 0 \, , \, \delta_7 \geq 0. \] (19)

Introducing (19) in (13) leads to
\[ \delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_5 + \delta_6 \leq 1. \] (20)

Then, combining (20) with the expression of the duty cycles (17), leads to
\[ \frac{V_{dc}}{2K_a^2}. \]

This condition is satisfied by voltage space vectors lying within the triangle defined by the space vectors \( V_{dc}/2K_a^2 \) and \( V_{dc}/2K_a^2 e^{j\pi/7} \) on the \( d_1-q_1 \) plane. By extending this procedure to all the 14 sectors on \( d_1-q_1 \) plane the 14-sided regular polygon shown in Fig. 6 is obtained. Then, \( \tau_{ref} \) is confined inside this polygon.

If balanced sinusoidal line-to-neutral voltages are required \( \tau_{ref} \) lies on a circle. In this case the maximum voltage amplitude corresponds to the radius of the circle inscribed in the limit polygon. Then, the modulation index is
\[ m \leq \frac{1}{2K_a^2 \cos(\pi/14)} = \frac{1}{2 \cos(\pi/14)} \approx 0.513. \]

It can be noted that this limit coincides with the theoretical limit given in [12] for a multi-phase VSI with sinusoidal balanced output voltages \( (n = 7) \)

\[
\frac{1}{2 \sin \left( \frac{\pi}{2} \left( \frac{n-1}{n} \right) \right)} \leq \frac{1}{2 \cos(\pi/2n)}.
\]

VI. SIMULATION RESULTS

In order to verify the effectiveness of the proposed SVM strategy, the behavior of a system, composed by a seven-phase VSI feeding a seven-phase balanced R-L load (see Fig. 2), has been tested by numerical simulations. The values of the system parameters are shown in Table II. The numerical results are obtained in balanced and sinusoidal conditions, with an amplitude of the reference line-to-neutral output voltage of 200 V and a frequency of 50 Hz.

The choice \( \delta_0 = \delta_7 \) has been considered for the null configurations in each cycle period. This modulation strategy can be considered a generalization of the well-known “symmetrical modulation” utilized for the three-phase VSI.

The seven load currents are shown in Fig. 7. Note that the waveforms are practically sinusoidal and characterized by a small ripple due to the switching effect.

In Fig. 8 are illustrated, in the corresponding \( d-q \) planes, the trajectories of the space vectors \( \vec{\tau}_1, \vec{\tau}_2 \) and \( \vec{\tau}_3 \). As expected, the space vectors \( \vec{\tau}_2 \) and \( \vec{\tau}_3 \) are practically null, whereas \( \vec{\tau}_1 \) moves along a circular trajectory (at constant speed). These results demonstrate that the proposed SVM strategy is able to independently control the output voltage space vectors in the different \( d-q \) planes.

| TABLE II |
| SYSTEM PARAMETERS |
| Supply | \( V_a = 540 \text{ V} \) |
| Series R-L Load | \( R = 20 \text{ \Omega} \), \( L = 10 \text{ mH} \) |
| Cycle period | \( T = 200 \mu s \) |

Fig. 6. Regular polygon in \( d_1-q_1 \) plane representing the limit of \( \tau_{ref} \).

Fig. 7. Load current waveforms.

Fig. 8. Trajectories of space vectors \( \vec{\tau}_1, \vec{\tau}_2, \) and \( \vec{\tau}_3 \) in the corresponding \( d-q \) planes.
The line-to-neutral load voltage \(v_1\) and the three line-to-line load voltages \(v_{12}, v_{13}, v_{14}\), are represented in Figs. 9, and 10, 11, 12, respectively. The continuous lines correspond to the average values within the cycle period.

Note that the line-to-line load voltages have the typical 3-level waveforms \((0, \pm V_{dc})\) as for a three-phase VSI, whereas the line-to-neutral load voltage appears as a 13-levels waveform \((0, \pm \frac{1}{7} V_{dc}, \pm \frac{2}{7} V_{dc}, \ldots, \pm \frac{6}{7} V_{dc})\). In particular, the instantaneous value of \(v_1\) changes across seven adjacent levels in a voltage range of \(\frac{2}{7} V_{dc}\) within each cycle period, as expressed by (8).

In Fig. 13 the loci of the space vectors \(\bar{v}_1, \bar{v}_2\), and \(\bar{v}_3\), in the corresponding \(d-q\) planes are shown. In this figure the dots representing the output voltage space vectors involved in the modulation process are recognizable.

**VII. Conclusion**

A SVM control strategy for seven-phase VSI has been proposed in this paper. The modulation is based on the extension of the space vector approach to seven-phase circuits, leading to triple \(d-q\) planes representation.

The switching pattern includes six active and two null configurations, with a single leg commutation for each configuration change. The results obtained by proposed modulation strategy collapse in the ones obtainable with a carrier-based symmetrical PWM in the case of \(v_{2\text{ref}} = 0\) and \(v_{3\text{ref}} = 0\), as for balanced sinusoidal voltages.

The duty cycles of both active and null inverter configurations are calculated on the basis of a detailed space vector approach, leading to the analytical determination of the modulation limits.

The numerical simulations carried out with reference to a seven-phase VSI supplying a seven-phase balanced load confirm the effectiveness of the proposed SVM strategy.

**References**


