General Analysis of Multi-Phase Systems Based on Space Vector Approach

Gabriele Grandi, Giovanni Serra, and Angelo Tani
Dept. of Electrical Engineering, University of Bologna, Italy

Abstract – **In this paper the space vector approach is applied to the analysis of multi-phase electric systems. This approach builds on an existing but non systematic knowledge base, partially available in literature. In particular, power and RMS are expressed in terms of space vectors, and a comparison is carried out with respect to the Fortescue's symmetrical components in the case of *n*-phase circuits with sinusoidal waveforms. The use of space vectors allows real time analysis and regulation of both multi-phase converters and multi-phase machines with an elegant and effective vectorial approach.**

I. Introduction

In 1918 Fortescue published a milestone paper on the AIEE entitled "Method of symmetrical coordinates applied to the solution of polyphase Networks" [1]. The proposed transformation soon became generally known as the method of symmetrical components, and it greatly facilitated the analysis of unbalanced three-phase systems.

The Fortescue's approach considers phasors, i.e., constant complex numbers representing, in a compact form, the amplitude and the phase angle of sinusoidal quantities. In past decades a similar transformation has been widely applied to three-phase instantaneous variables, leading to the definition of space vectors, i.e., complex numbers moving in a plane usually called *d-q* plane [2], [3].

The space vector approach allows to simplify modeling and regulation of both the converter and the machine in traditional three-phase motor drive applications. However, when the machine is connected to an inverter supply, the need for a specific number of phases, such as three, disappears. On the other hand, the development of modern power electronic devices, makes it possible to consider the number of phases as a degree of freedom, i.e., as one of the design variables.

Multi-phase motor drives have many advantages over the traditional three-phase motor drives such as reducing the amplitude and increasing the frequency of torque pulsations, reducing the rotor harmonic currents losses and lowering the dc link current harmonics.

In addition, owing to their redundant structure, multi-phase motor drives improve the system reliability. In particular, unlike in traditional three-phase systems, the failure of one or more phases in a multi-phase drive system does not prevent the machine from starting and running, even if with reduced performance.

The use of multi-phase drives is considered also a possible solution to overcame the problems related to high-power applications. In fact, in the past decades, multi-level inverter-fed ac machines have emerged as a promising solution in achieving high power ratings with voltage limited devices. Similarly, the use of multi-phase inverters together with multi-phase ac machines has been recognized as a viable approach to obtain high power ratings with current limited devices [4]-[6].

Furthermore, the spatial harmonic components of the air gap flux density can be usefully utilized in order to increase the torque production capability of a multi-phase machine [7]-[10].

In order to analyze multi-phase systems, Fortescue's and space vector transformations can be still adopted. As in the case of three phase systems, the Fortescue's method applied to multi-phase systems considers only steady-state conditions, whereas the space vector theory can be referred to arbitrary time-dependent variables. Then, by the space vector approach, real time analysis and regulation of both the multi-phase converter and the multi-phase machine can be performed with an elegant and effective vectorial approach.

In this paper the space vector approach for multi-phase systems is developed as a natural extension of the Fortescue's transformation, leading to the definition of multiple space vectors lying in different *d-q* planes. The proposed analysis builds on an existing but non systematic knowledge base, partially available in literature.

In particular, the paper is organized as follows: in Section II the basic definitions for multi-phase systems are given. The symmetrical components resulting by the Fortescue's transformations are presented in Section III. In Section IV the multi-phase space vector transformations are introduced for arbitrary waveforms, whereas in Section V the space vectors are considered for sinusoidal waveforms and a comparison with the symmetrical components is carried out.

II. Basic Definitions for Multi-Phase Systems

Let us consider n homogeneous and time-dependent real quantities $x_k(t)$ related to the n-phase system shown in Fig. 1,

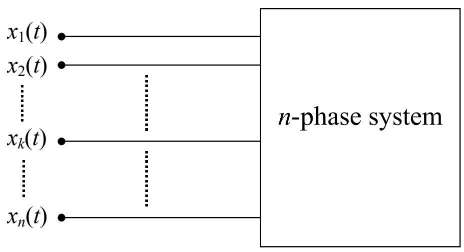

Fig. 1. Schematic drawing of a generic multi-phase system.

$$[x] = (x_1, x_2, \ldots, x_n). \quad (1)$$

The multi-phase RMS value X of the n-dimensional vector $[x]$ over a given time interval Δt can be defined as the RMS of its norm

$$X^2 = \frac{1}{\Delta t}\int_0^{\Delta t} |x|^2 dt\,, \quad (2)$$

being

$$|x|^2 = \sum_{k=1}^{n} x_k^2\,. \quad (3)$$

If all the quantities $x_k(t)$ have sinusoidal waveforms with the same frequency, the n phasors $\overline{X}_k$ are introduced as

$$[\overline{X}] = (\overline{X}_1, \overline{X}_2, \ldots, \overline{X}_n)\,, \quad (4)$$

according to

$$x_k(t) = \sqrt{2}\, X_k \cos(\omega t + \varphi_k) = Re\left[\sqrt{2}\,\overline{X}_k e^{j\omega t}\right]\,, \quad (5)$$

being X_k the RMS value of $x_k(t)$ over the fundamental period $T = 2\pi/\omega$. In this case, the multi-phase RMS value is given by

$$X^2 = \sum_{k=1}^{n} X_k^2 = \frac{1}{T}\int_0^{T} |x|^2 dt\,. \quad (6)$$

With reference to a multi-phase electric circuit, the more general configuration consists in a n-port network, as shown in Fig. 2. For arbitrary voltage and current waveforms, the input instantaneous power is expressed as

$$p = \sum_{k=1}^{n} v_k\, i_k = [v]^{\mathrm{T}}[i]\,. \quad (7)$$

In the case of sinusoidal steady-state conditions, the complex power $\overline{S}$ can be defined as the Hermitian inner product between voltage and current phasors, as follows

$$\overline{S} = \sum_{k=1}^{n} \overline{V}_k\, \overline{I}_k^{*} = [\overline{V}]^{\mathrm{T}}\, [\overline{I}]^{*}\,. \quad (8)$$

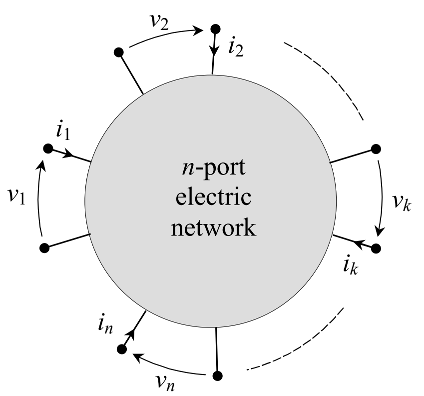

Fig. 2. General representation of a n-port electric network.

The active power P is the average value of the instantaneous power over the fundamental period T. It can be expressed as the real part of the complex power or by the inner products between voltage and current phasors

$$P = Re(\overline{S}) = \sum_{k=1}^{n} \overline{V}_k \cdot \overline{I}_k\,. \quad (9)$$

III. Review of Symmetrical Components

A. Transformations

The Fortescue's transformation [1] applied to the n phasors (4) leads to the following n constant complex numbers, called symmetrical sequence components

$$[\overline{X}_S] = (\overline{X}_{S_0}, \overline{X}_{S_1}, \overline{X}_{S_2}, \ldots, \overline{X}_{S_{n-1}})\,, \quad (10)$$

where

$$\overline{X}_{S_h} = \frac{1}{n}\sum_{k=1}^{n} \overline{X}_k\, \alpha^{h(k-1)}\,,\ h = 0, 1, 2, \ldots, n-1 \quad (11)$$

being $\alpha = e^{j\frac{2\pi}{n}}$.

The inverse transformation is given by

$$\overline{X}_k = \sum_{h=0}^{n-1} \overline{X}_{S_h}\, \alpha^{-h(k-1)}\ ,\ k = 1, 2, \ldots, n\,. \quad (12)$$

The previous transformations can be expressed in a compact matrix form as follows

$$[\overline{X}_S] = [\alpha]^{-1}[\overline{X}]\,, \quad (13)$$

$$[\overline{X}] = [\alpha][\overline{X}_S]\,, \quad (14)$$

where

$$[\alpha]^{-1} = \frac{1}{n}\begin{bmatrix} 1 & 1 & 1 & 1 & \cdots & 1 \\ 1 & \alpha & \alpha^2 & \alpha^3 & \cdots & \alpha^{n-1} \\ 1 & \alpha^2 & \alpha^4 & \alpha^6 & \cdots & \alpha^{n-2} \\ 1 & \alpha^3 & \alpha^6 & \alpha^9 & \cdots & \alpha^{n-3} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \alpha^{n-1} & \alpha^{n-2} & \alpha^{n-3} & \cdots & \alpha \end{bmatrix}, \quad (15)$$

$$[\alpha] = \begin{bmatrix} 1 & 1 & 1 & 1 & \cdots & 1 \\ 1 & \alpha^{-1} & \alpha^{-2} & \alpha^{-3} & \cdots & \alpha \\ 1 & \alpha^{-2} & \alpha^{-4} & \alpha^{-6} & \cdots & \alpha^2 \\ 1 & \alpha^{-3} & \alpha^{-6} & \alpha^{-9} & \cdots & \alpha^3 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \alpha & \alpha^2 & \alpha^3 & \cdots & \alpha^{n-1} \end{bmatrix}. \quad (16)$$

Note that $[\alpha]$ is symmetric and its complex conjugate is

$$[\alpha]^{*} = n\,[\alpha]^{-1}\,. \quad (17)$$

Then, with the exception of the factor n, $[\alpha]$ is a unitary matrix, i.e. $[\alpha]^{\mathrm{H}} = n\,[\alpha]^{-1}$. It is useful to point out that the determinant of a unitary matrix has absolute value 1, and a linear transformation with a unitary matrix preserves Hermitian inner products.

The similarity between (11) and the Discrete Fourier Transform (DFT) of a list of n complex numbers should be noted. As for the DFT, the normalization factors multiplying direct and inverse Fortescue's transformation ($1/n$ and n) and the signs of the exponents are merely conventions. The only requirements are that direct and inverse transformations have opposite-sign exponents and that the product of their normalization factors be $1/n$. A normalization of $1/\sqrt{n}$ for both makes the transforms unitary, which has some theoretical advantages, but it is often more practical in numerical computation to perform the scaling all at once as above.

The symmetrical sequence components defined in (11) can be reorganized introducing the concept of positive and negative sequence component. In particular, the component $n–h$ can be redefined as the component $–h$, according to

$$\begin{aligned} \overline{X}_{S_{n-h}} &= \frac{1}{n}\sum_{k=1}^{n}\overline{X}_k\,\alpha^{(n-h)(k-1)} = \\ &= \frac{1}{n}\sum_{k=1}^{n}\overline{X}_k\,\alpha^{-h(k-1)} = \overline{X}_{S_{-h}} \end{aligned} \quad . \qquad (18)$$

On the basis of (18), the n sequence components expressed in (10) are rewritten as

$$\left[\overline{X}_S\right] = \left(\overline{X}_{S_0}, \overline{X}_{S_{+1}}, ..., \overline{X}_{S_{+r}}, (\overline{X}_{S_{n/2}}), \overline{X}_{S_{-r}}, ..., \overline{X}_{S_{-1}}\right), \qquad (19)$$

being $r = (n–1)/2$ for odd number of phases and $r = n/2–1$ for even number of phases. Then, it is possible to consider sequences from 1 to r only, introducing for each one the positive and the negative component, as shown in (19). Note that the zero-sequence component, also called "homopolar" component, must be always considered, whereas the $n/2$ sequence component, that could be called "Nyquist" component, appears only for an even number of phases (for this reason it will be enclosed within round brackets).

On the basis of the previous considerations, direct and inverse Fortescue's transformations can be rewritten as

$$\overline{X}_{S_0} = \frac{1}{n}\sum_{k=1}^{n}\overline{X}_k \ , \quad \left(\overline{X}_{S_{n/2}} = \frac{1}{n}\sum_{k=1}^{n}\overline{X}_k(-1)^{k-1}\right), \qquad (20)$$

$$\overline{X}_{S_{\pm h}} = \frac{1}{n}\sum_{k=1}^{n}\overline{X}_k\,\alpha^{\pm h(k-1)}\ , \quad h = 1, 2, \ldots, r \qquad (21)$$

$$\overline{X}_k = \left(\overline{X}_{S_{n/2}}(-1)^{k-1}\right) + \sum_{h=-r}^{r}\overline{X}_{S_h}\alpha^{-h(k-1)}\ , \ k = 1, \ldots, n \qquad (22)$$

In can be shown that the positive (negative) sequence components corresponding to given phasors, are equal to the complex conjugate of the negative (positive) sequence components, corresponding to the complex conjugate of those phasors, that is

$$\overline{X}_{S_{\pm h}} = \left(\frac{1}{n}\sum_{k=1}^{n}\overline{X}_k^*\,\alpha^{\mp h(k-1)}\right)^*, \quad h = 1, 2, \ldots, r. \qquad (23)$$

B. Powers and RMS

Applying the Fortescue's transformations to voltage and current phasors in (8) and introducing (17), the complex power can be easily rewritten in terms of symmetrical components as (Plancherel's theorem)

$$\overline{S} = \left([\alpha][\overline{V}_S]\right)^{\mathrm{T}}\left([\alpha][\overline{I}_S]\right)^* = [\overline{V}_S]^{\mathrm{T}}[\alpha]^{\mathrm{T}}[\alpha]^*[\overline{I}_S]^*\ ,$$

$$\overline{S} = n\,[\overline{V}_S]^{\mathrm{T}}[\overline{I}_S]^*\ . \qquad (24)$$

Note that only voltage and current symmetrical components of the same sequence interact and take part in the complex power. Then, by introducing the concept of positive and negative sequence component, the complex power can be written as the sum of the complex power of each sequence as

$$\overline{S} = \left(\overline{S}_{n/2}\right) + \sum_{h=-r}^{r}\overline{S}_h\ , \qquad (25)$$

being

$$\overline{S}_0 = n\overline{V}_{S_0}\overline{I}_{S_0}^*\ , \ \left(\overline{S}_{n/2} = n\,\overline{V}_{S_{n/2}}\overline{I}_{S_{n/2}}^*\right), \qquad (26)$$

$$\overline{S}_{\pm h} = n\,\overline{V}_{S_{\pm h}}\overline{I}_{S_{\pm h}}^*\ , \quad h = 1, 2, \ldots, r\ . \qquad (27)$$

The active power P is the real part of the complex power. It can be expressed by inner products rather than real operators, leading to

$$P = Re(\overline{S}) = \left(n\,\overline{V}_{S_{n/2}}\cdot\overline{I}_{S_{n/2}}\right) + n\sum_{h=-r}^{r}\overline{V}_{S_h}\cdot\overline{I}_{S_h}\ . \qquad (28)$$

Then, also the active power can be written as the sum of the active power of each sequence as

$$P = \left(P_{n/2}\right) + \sum_{h=-r}^{r}P_h\ , \qquad (29)$$

being

$$P_0 = Re\left(\overline{S}_0\right) = n\,\overline{V}_{S_0}\cdot\overline{I}_{S_0}\ , \qquad (30)$$

$$\left(P_{n/2} = Re\left(\overline{S}_{n/2}\right) = n\,\overline{V}_{S_{n/2}}\cdot\overline{I}_{S_{n/2}}\right), \qquad (31)$$

$$P_{\pm h} = Re\left(\overline{S}_{\pm h}\right) = n\,\overline{V}_{S_{\pm h}}\cdot\overline{I}_{S_{\pm h}}\ , \quad h = 1, 2, \ldots, r\ . \qquad (32)$$

The multi-phase RMS value expressed in (6) for the generic phasorial quantities $[\overline{X}]$ can be rewritten as

$$X^2 = [\overline{X}]^{\mathrm{T}}[\overline{X}]^*\ . \qquad (33)$$

By applying to (33) the same procedure followed for complex power, it can be shown that (Parseval's theorem)

$$X^2 = \left(n\,X_{S_{n/2}}^2\right) + n\sum_{h=-r}^{r}X_{S_h}^2\ . \qquad (34)$$

Then, the multi-phase RMS value of voltage and current phasors can be expressed by the square root of (34) as function of the RMS values of their symmetrical sequence components.

IV. Space Vector Approach

A. Transformations

Let us consider again the n real quantities $x_k(t)$ introduced in (1). The Fortescue's transformation can be directly applied to these real quantities with arbitrary waveforms rather than the phasors representing only sinusoidal waveforms, leading to

$$[\bar{x}_S] = (\bar{x}_{S_0}, \bar{x}_{S_1}, \bar{x}_{S_2}, ..., \bar{x}_{S_{n-1}}), \tag{35}$$

where

$$\bar{x}_{S_h} = \frac{1}{n}\sum_{k=1}^{n} x_k\, \alpha^{h(k-1)}, \quad h = 0, 1, 2, \ldots, n-1. \tag{36}$$

With the exception for $h = 0$ (and $h = n/2$), the quantity $\bar{x}_{S_h}$ is a complex number. It is called *space vector component of sequence h*, or *space vector on the d_h-q_h plane*. Its absolute value x_{S_h} is usually called "magnitude" of the space vector.

The inverse transformation is given by

$$x_k = \sum_{h=0}^{n-1} \bar{x}_{S_h}\, \alpha^{-h(k-1)}, \quad k = 1, 2, 3, \ldots, n. \tag{37}$$

Also the space vector transformations can be expressed in a compact matrix form as follows

$$[\bar{x}_S] = [\alpha]^{-1}[x], \tag{38}$$

$$[x] = [\alpha][\bar{x}_S]. \tag{39}$$

The space vector sequence components defined in (36) can be reorganized introducing the concept of positive and negative sequence component. Also in this case, the component $n–h$ can be redefined as the component $–h$, according to

$$\begin{aligned}\bar{x}_{S_{n-h}} &= \frac{1}{n}\sum_{k=1}^{n} x_k\, \alpha^{(n-h)(k-1)} = \\ &= \frac{1}{n}\sum_{k=1}^{n} x_k\, \alpha^{-h(k-1)} = \bar{x}_{S_{-h}}\end{aligned}. \tag{40}$$

Then, the n space vector sequence components are rewritten as

$$[\bar{x}_S] = (x_{S_0}, \bar{x}_{S_{+1}}, ..., \bar{x}_{S_{+r}}, (x_{S_{n/2}}), \bar{x}_{S_{-r}}, ..., \bar{x}_{S_{-1}}). \tag{41}$$

The zero-sequence space vector component must be always considered, whereas the additional space vector component of sequence $n/2$ appears for an even number of phases only.

On the basis of the previous considerations, the space vector transformations can be rewritten as

$$x_{S_0} = \frac{1}{n}\sum_{k=1}^{n} x_k, \left(x_{S_{n/2}} = \frac{1}{n}\sum_{k=1}^{n} x_k (-1)^{k-1} \right), \tag{42}$$

$$\bar{x}_{S_{\pm h}} = \frac{1}{n}\sum_{k=1}^{n} x_k\, \alpha^{\pm h(k-1)}, \quad h = 1, 2, \ldots, r. \tag{43}$$

It can be shown that positive space vector sequence components are complex conjugates of the corresponding negative sequence components, that is

$$\bar{x}_{S_{+h}} = \bar{x}^*_{S_{-h}}, \qquad h = 1, 2, \ldots, r. \tag{44}$$

Then, the space vector sequence components defined by (36) have Hermitian symmetry. Note that (44) expressed in terms of space vector sequence components corresponds to (23) expressed in terms of symmetrical sequence components.

Owing to (44) it is possible to consider r space vectors only, then, different choices can be made. In this paper the space vector positive sequence components are considered, leading to

$$\bar{x}_{S_{+h}} = \frac{1}{n}\sum_{k=1}^{n} x_k\, \alpha^{h(k-1)}, \quad h = 0, 1, 2, \ldots, r \tag{45}$$

$$x_k = x_{S_0} + \left(x_{S_{n/2}} (-1)^{k-1} \right) + 2\sum_{h=1}^{r} \bar{x}_{S_{+h}} \cdot \alpha^{h(k-1)}, \tag{46}$$

$$k = 1, 2, \ldots, n.$$

For the reasons that will be discussed in Section V, the factor 2 that appears in the last term of (46) is usually included in the definition of space vectors [2]-[13]. In this way, the space vector transformations can be redefined as

$$x_0 = x_{S_0} = \frac{1}{n}\sum_{k=1}^{n} x_k, \left(x_{n/2} = x_{S_{n/2}} = \frac{1}{n}\sum_{k=1}^{n} x_k (-1)^{k-1} \right), \tag{47}$$

$$\bar{x}_h = 2\bar{x}_{S_h} = \frac{2}{n}\sum_{k=1}^{n} x_k\, \alpha^{h(k-1)}, \quad h = 1, 2, \ldots, r \tag{48}$$

$$x_k = x_0 + \left(x_{n/2} (-1)^{k-1} \right) + \sum_{h=1}^{r} \bar{x}_h \cdot \alpha^{h(k-1)}, \tag{49}$$

$$k = 1, 2, \ldots, n.$$

Note that the subscript "S" disappears in the redefinition of the space vectors (47)-(48).

The choice of considering the positive sequence components has been also assumed in [11], [12] for $n = 5$ (leading to x_0, $\bar{x}_1$, $\bar{x}_2$), and in [13] for $n = 7$ (leading to x_0, $\bar{x}_1$, $\bar{x}_2$, $\bar{x}_3$). A different solution has been considered in [5], [6] and [10], where a five-phase system has been described by the space vectors x_0, $\bar{x}_1$, $\bar{x}_3$, i.e., the odd-order space vector sequence components.

B. Power and RMS

With reference to n-phase electric circuits with arbitrary voltage and current waveforms, the instantaneous power expressed by (7) can be rewritten in terms of space vector sequence components as

$$p=[v]^{\mathrm{T}}[i]=\left([\alpha][\bar{v}]\right)^{\mathrm{T}}[\alpha][\bar{i}]=[\bar{v}]^{\mathrm{T}}[\alpha]^2[\bar{i}]\,. \tag{50}$$

It can be shown that

$$[\alpha]^2=n\begin{bmatrix}1&0&0&\cdots&0&0\\0&0&0&\cdots&0&1\\0&0&0&0&1&0\\\vdots&\vdots&0&\iddots&0&\vdots\\0&0&1&0&0&0\\0&1&0&\cdots&0&0\end{bmatrix}. \tag{51}$$

Then, (50) yields

$$p=n\left[v_{S_0}i_{S_0}+\sum_{h=1}^{n-1}\bar{v}_{S_h}\,\bar{i}_{S_{n-h}}\right]. \tag{52}$$

With reference to the space vector definition (48), the instantaneous power becomes

$$p=n\,v_0 i_0+\left(n\,v_{n/2}i_{n/2}\right)+\frac{n}{2}\sum_{h=1}^{r}\bar{v}_h\cdot\bar{i}_h\,. \tag{53}$$

Note that only voltage and current components of the same sequence interact and take part in the instantaneous power. Then, the instantaneous power can be written as the sum of contributions of each sequence

$$p=\left(p_{n/2}\right)+\sum_{h=0}^{r}p_h\,, \tag{54}$$

being

$$p_0=n\,v_0 i_0\,,\quad \left(p_{n/2}=n\,v_{n/2}\,i_{n/2}\right), \tag{55}$$

$$p_h=\frac{n}{2}\bar{v}_h\cdot\bar{i}_h\quad,\quad h=1,2,\ldots,r\,. \tag{56}$$

For the generic multi-phase system $[x]$, considered as a n-dimensional real vector, the square norm introduced in (3) can be rewritten as

$$|x|^2=[x]^{\mathrm{T}}[x]\,. \tag{57}$$

By applying to (57) the same procedure followed for the instantaneous power, it can be shown that

$$|x|^2=n\,x_0^2+\left(n\,x_{n/2}^2\right)+\frac{n}{2}\sum_{h=1}^{r}x_h^2\,. \tag{58}$$

Then, the square norm of a multi-phase system can be expressed as a linear combination of the squared magnitudes of its space vector sequence components. In this way, the multi-phase RMS value X over a given time interval Δt expressed by (2) can be calculated by

$$X^2=n\,X_0^2+\left(n\,X_{n/2}^2\right)+\frac{n}{2}\sum_{h=1}^{r}X_h^2\,, \tag{59}$$

being X_0, $(X_{n/2})$, and X_h the RMS values of the space vector sequence components defined by

$$X_0^2=\frac{1}{\Delta t}\int_0^{\Delta t}x_0^2dt\,, \tag{60}$$

$$\left(X_{n/2}^2=\frac{1}{\Delta t}\int_0^{\Delta t}x_{n/2}^2dt\right), \tag{61}$$

$$X_h^2=\frac{1}{\Delta t}\int_0^{\Delta t}x_h^2dt\quad,\quad h=1,2,\ldots,r\,. \tag{62}$$

V. Space Vectors in the Case of Sinusoidal Waveforms

In order to highlight the relationship between symmetrical and space vector sequence components, the case of sinusoidal waveforms with the same frequency is considered. In particular, the quantities expressed in (5) are rewritten as

$$x_k=\frac{\sqrt{2}}{2}\left(\overline{X}_k e^{j\omega t}+\overline{X}_k^{*}e^{-j\omega t}\right),\ k=1,2,3,\ldots,n. \tag{63}$$

A. Space Vector components

Applying the space vector transformations (47) and (48) to (63), and taking (21) into account, yields

$$x_0=Re\left[\sqrt{2}\,\overline{X}_{S_0}e^{j\omega t}\right],\ \left(x_{n/2}=Re\left[\sqrt{2}\,\overline{X}_{S_{n/2}}e^{j\omega t}\right]\right)\,, \tag{64}$$

$$\bar{x}_h=\sqrt{2}\,\overline{X}_{S_{+h}}\,e^{j\omega t}+\sqrt{2}\,\overline{X}_{S_{-h}}^{*}\,e^{-j\omega t}\,,\ h=1,2,\ldots,r. \tag{65}$$

In (65) is shown that each space vector sequence component $\bar{x}_h$ can be decomposed in a direct and an inverse rotating vector, as represented in Fig. 3. These counter-rotating vectors have constant magnitude and opposite angular speed $\pm\omega$. In particular, their magnitudes are $\sqrt{2}$ times the magnitudes of positive and negative symmetrical components of the same sequence.

Note that for a multi-phase system the definition “sinusoidal and symmetrical” usually corresponds to the presence of only the direct rotating component of the space vector of sequence 1. In this case, the amplitude of sinusoids, $\sqrt{2}\,X_k$, equals the magnitude of the space vector, $|\bar{x}_1|$. This equality is a consequence of the presence of factor 2 in (48).

B. Power and RMS

Introducing (63) in (47) and (48), the instantaneous power sequence components expressed by (55) and (56) become

$$p_0=\ P_0+Re\left[n\overline{V}_{S_0}\overline{I}_{S_0}e^{j2\omega t}\right], \tag{66}$$

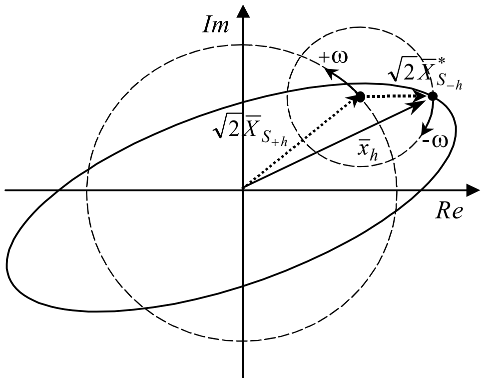

Fig. 3. Space vector decomposition in counter rotating components.

$$\left(p_{n/2} = P_{n/2} + Re\left[n\overline{V}_{S_{n/2}}\overline{I}_{S_{n/2}}e^{j2\omega t}\right]\right) , \quad (67)$$

$$p_h = P_{+h} + P_{-h} + Re\left[n\left(\overline{V}_{S_{+h}}\overline{I}_{S_{-h}} + \overline{V}_{S_{-h}}\overline{I}_{S_{+h}}\right)e^{j2\omega t}\right] . \quad (68)$$

It can be noted that the active power related to each symmetrical sequence component represents the average value, over the fundamental period, of the instantaneous power related to the corresponding sequence component of space vectors.

The instantaneous contributions of the space vector sequence components to their RMS values (60)-(62) become

$$x_0^2 = X_{S_0}^2 + Re\left[\overline{X}_{S_0}^2 e^{j2\omega t}\right] , \quad (69)$$

$$\left(x_{n/2}^2 = X_{S_{n/2}}^2 + Re\left[\overline{X}_{S_{n/2}}^2 e^{j2\omega t}\right]\right), \quad (70)$$

$$x_h^2 = \overline{x}_h\overline{x}_h^* = 2X_{S_{+h}}^2 + 2X_{S_{-h}}^2 + 4\,Re\left[\overline{X}_{S_{+h}}\overline{X}_{S_{-h}}e^{j2\omega t}\right] , \quad (71)$$

$$h = 1, 2, \ldots, r.$$

Then, the RMS of these contributions over the fundamental period T are given by

$$X_0^2 = \frac{1}{T}\int_0^T x_0^2 dt = X_{S_0}^2 , \quad (72)$$

$$\left(X_{n/2}^2 = \frac{1}{T}\int_0^T x_{n/2}^2 dt = X_{S_{n/2}}^2\right), \quad (73)$$

$$X_h^2 = \frac{1}{T}\int_0^T x_h^2\, dt = 2X_{S_{+h}}^2 + 2X_{S_{-h}}^2 , \quad h = 1, 2, \ldots, r. \quad (74)$$

As for the power, the RMS value of each symmetrical sequence component is strictly related to the RMS of the corresponding space vector sequence component.

VI. Conclusions

In this paper a multi-phase electric system has been analyzed in terms of both Fortescue's symmetrical components and multiple space vector components.

With reference to sinusoidal waveforms, the symmetrical components have been reorganized introducing the concept of positive and negative sequences. Complex and active powers have been written as the sum of the powers related to each sequence component. The multi-phase RMS value of voltages and currents have been expressed by the RMS values of their symmetrical sequence components.

The multiple space vectors are introduced as an extension of the Fortescue's transformations, with reference to arbitrary time dependent waveforms, leading to the definition of space vectors of different sequences lying in different *d-q* planes. Instantaneous power and RMS in terms of space vectors have been introduced as well.

The space vectors have been considered with reference to sinusoidal waveforms in order to emphasize the relationship with the symmetrical components. In particular, it has been shown that, for each *d-q* plane, the space vector can be decomposed in two counter-rotating vectors, having constant both the magnitude and the angular speed. Their magnitudes are √2 times the magnitudes of positive and negative symmetrical components of the same sequence. Furthermore, the active power related to each symmetrical sequence component represents the average value, over the fundamental period, of the instantaneous power related to the space vector on the corresponding *d-q* plane. Finally, the relationship between the RMS value of each symmetrical sequence component and the RMS value of the corresponding space vector sequence component has been carried out.

The proposed analysis could be further extended to periodic non-sinusoidal waveforms by Fourier series expansion. In addition, it could been shown that, for a non prime number of phases, the multi-phase system can be seen as a combination of basic sub-systems having a prime number of phases.

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