Optimal Design of Single-Layer Solenoid Air-Core Inductors for High Frequency Applications

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Abstract - This paper deals with the optimized design of single-layer solenoid air-core inductors for HF applications. The presence of shields or other conductive materials in the neighborhood of the inductor is neglected. Optimum design is obtained with respect to weight, resistance, and losses of inductors. The highest operating frequency is affected by turn-to-turn stray capacitances and it is determined by the first self-resonant frequency. The optimization procedure is formulated as a nonlinear programming problem. A Sequential Quadratic Programming (SQP) algorithm is used for the solution. The influence of different geometrical parameters on the optimum design is highlighted and guidelines for the design procedure are given.

I. INTRODUCTION

The increasing interest in HF electrical applications, such as switching power converters, EMC filters, RF power amplifiers, and radio transmitters, poses unusual problems that are to be solved for the design and effective operation of electrical devices at high frequencies. An important issue concerns the reliable prediction of effects related to the behavior of inductors at high frequencies, which is very different from the low-frequency behavior. Consequently, special design criteria must be employed when considering inductors for HF applications. In particular, the stray capacitances between the turns of the winding significantly affect the inductor HF response and are responsible for resonant frequencies. Since these inductors must usually operate at frequencies above several hundred kilohertz, their first (parallel) self-resonance must be at a reasonably higher frequency.

In this paper, a procedure for the optimum design of single-layer solenoid air-core inductors is presented. The aim of the design procedure is to match the correct value of the coil inductance in a fixed frequency range and with a rated current, which is also fixed. In the next section, it will be shown that this problem has some degrees of freedom and can be formulated as an optimization problem choosing a proper objective function. In Section II, the optimization is formulated as a nonlinear programming problem in which the design electrical parameters of the inductor, expressed in terms of its geometry, give the constraints. In Section III, a Sequential Quadratic Programming (SQP) algorithm is introduced to determine the optimal geometric dimensions of the coil. The results are discussed and guidelines for the design are given in Section IV.

II. FORMULATION OF THE PROBLEM

The design of optimized inductors can be formulated as a general nonlinear programming problem [1-2] as follows:

$$\text{find} \quad \boldsymbol{x} = (x_1, x_2, \ldots, x_n)$$

such that

$$F(\boldsymbol{k},\boldsymbol{x}) \text{ is minimum (maximum)} \tag{1}$$

subject to:

$$g_i(\boldsymbol{k},\boldsymbol{x}) = 0, \quad i = 1, 2, \ldots, r, \tag{2a}$$

$$h_j(\boldsymbol{k},\boldsymbol{x}) \le 0, \quad j = 1, 2, \ldots, s \tag{2b}$$

$$\boldsymbol{x}_l \le \boldsymbol{x} \le \boldsymbol{x}_u \tag{2c}$$

where $F(\boldsymbol{k}, \boldsymbol{x})$, $g_i(\boldsymbol{k}, \boldsymbol{x})$, and $h_j(\boldsymbol{k}, \boldsymbol{x})$ are real-valued scalar functions; $\boldsymbol{k}$ is a set of constant parameters; $\boldsymbol{x}$ is a vector of n variables for which the optimization is to be accomplished. The distinction between the variables and the constants depends on the design philosophy and the manufacturing constraints. $\boldsymbol{x}_l$ and $\boldsymbol{x}_u$ are vectors whose entries are the lower bounds, and upper bounds of $\boldsymbol{x}$ variables, respectively. The function $F(\boldsymbol{k}, \boldsymbol{x})$ is called the "objective function," for which the optimal values of $\boldsymbol{x}$ result in the minimum (maximum) of $F(\boldsymbol{k}, \boldsymbol{x})$. Usually, the objective function may be identified with the cost, weight, losses, quality factor, etc. of the inductor. Additional requirements and/or dimensional limits of the inductor are referred to as "constraints," and can be of two types: equality (2a), and inequality (2b). Any design $\boldsymbol{x}$ that satisfies (2a) and (2b) is called a "feasible design," and the region generated by these constraints is called a "feasible design region". The values of $\boldsymbol{x}$ in this region comply with the design requirements, and among these points is the optimal solution. For the considered problem, both the objective function and the constraints are nonlinear multivariable functions. The input data for the HF inductor design are:

1. The rated inductance L_r,
2. The maximum operating frequency f_{max},
3. The rated current.

The maximum operating frequency can be expressed as a fraction of the first self-resonant frequency f_o of the inductor, $f_{max} = k_f f_o$, with $k_f \le 1$. The first self-resonant frequency f_o can be calculated as a function of the overall stray capacitance C_o of the inductor

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$$f_o \cong \frac{1}{2\pi\sqrt{L_r C_o}}\,. \qquad (3)$$

This means that the overall stray capacitance C_o can be assumed as an input parameter instead of f_{max}. With reference to Fig. 1, the geometric parameters for which the optimization must be achieved are

1. The diameter of the coil D.
2. The number of turns N.
3. The winding pitch p.
4. The wire diameter d.

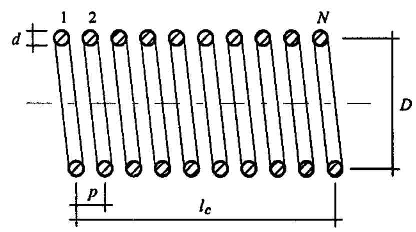

Fig. 1. Inductor cross section with its essential dimensions.

A single-layer solenoid air-core inductor having N turns is assumed. Each turn consists of a single wire with a circular cross section. The wire diameter d is normally chosen among the standardized ones on the basis of the rated current of the inductor. Thus, if we want to optimize the inductor with respect to the weight and cost of the winding, we choose the wire diameter equal to the smallest possible diameter compatible with the expected values of the current density. Obviously, this condition does not imply minimum coil resistance and losses. Hence, at the beginning of the optimization procedure, the wire diameter d can be considered an already fixed quantity and treated as a constant parameter. Therefore, we have $\boldsymbol{k} = d$ and $\boldsymbol{x} = (D, N, p)$.

We can adopt as an objective function the total length l_w of the copper wire. Therefore, we have

$$F(\boldsymbol{x}) = l_w(D,N,p) = N\sqrt{(\pi D)^2 + p^2}\,. \qquad (4)$$

For a fixed wire diameter d, the minimization of the proposed objective function l_w reduces the copper weight (cost) to a minimum value. Neglecting the influence of the design variables on skin and proximity effects, also the winding resistance and losses are minimized.

In some cases, the design must comply with some other geometrical constraints. In these cases, an alternative objective function could be the overall coil volume V_c

$$F(\boldsymbol{x}) = V_c(D,N,p) = (N-1)p\cdot\frac{\pi}{4}D^2\,. \qquad (5)$$

On the basis of previous considerations, the constraints of the problem under consideration can be written as

$$g(\boldsymbol{x}) = L(d,D,N,p) - L_r = 0 \qquad (6a)$$

$$h(\boldsymbol{x}) = C(d,D,N,p) - C_o \le 0 \qquad (6b)$$

where L and C are the inductance and the overall stray capacitance of the coil, respectively.

The ranges of variation of variables D, N, and p are normally limited. In particular, the lower bounds are assigned by geometrical limits as follows:

$D \ge d$, $N \ge 1$ with integer N, $p \ge d$.

The upper bounds could be limited by other technical constraints. These could be input data for the design.

A. Inductance and Stray Capacitance

The overall coil inductance can be evaluated as a linear combination of self- and mutual inductances of the turns

$$L = \sum_{i=1}^{N}\sum_{j=1}^{N} M_{i,j}\,. \qquad (7)$$

Exploiting the geometric symmetry and neglecting the influence of skin and proximity effects [3] yields

$$L = NL_1 + 2\sum_{k=1}^{N-1}(N-k)M_{1,k+1}\,, \qquad (7a)$$

where the self-inductance L_1 of each turn and the N-1 different mutual-inductances between turns can be calculated as

$$L_1 = \mu_o\frac{D}{2}\left(\ln\frac{8D}{d} - \frac{7}{4}\right), \text{ and} \qquad (7b)$$

$$M_{1,k+1} = \frac{\mu_o}{4}\int_0^{2\pi}\frac{D^2\cos\vartheta}{\sqrt{4(kp)^2 + 2D^2(1-\cos\vartheta)}}\,d\vartheta\,. \qquad (7c)$$

The evaluation of the mutual inductances can be carried out introducing the elliptic integrals K and E of the first and second kinds, respectively,

$$M_{1,k+1} = \frac{\mu_o D}{2}\left[\left(\frac{2}{c} - c\right)K(c) - \frac{2}{c}E(c)\right], \qquad (7d)$$

where $c^2 = D^2/(D^2 + (kp)^2)$.

Simpler formulae, valid in a limited geometric range, can be found in [4]. More general expressions of the coil inductance are given in [5], but they contain tabulated coefficients. In [6], surface currents are considered to approximately take skin and proximity effects into account. In this case, the improvement in the final result seems to be not adequate enough to justify the higher computational effort. A method for calculating the capacitance between adjacent turns is also provided in [6]. It is based on an averaging process that involves the capacitance formula between two coaxial circular filaments. Although the averaging process is approximated, in this case also elliptical integrals must be solved. In [3],[7], simpler and reliable formulae for the prediction of parasitic capacitances of coils are given as a function of the winding geometry. Following such an approach, the turn-to-turn capacitance between adjacent turns can be calculated as

$$C_{tt} = \frac{\pi^2 D \varepsilon_o}{\ln\left(p/d + \sqrt{(p/d)^2 - 1}\right)}. \quad (8)$$

Neglecting the direct capacitive coupling between non-adjacent turns, the overall stray capacitance at the coil terminals can be evaluated as the series connection of (N-1) capacitances C_{tt}, leading to

$$C = \frac{C_{tt}}{N-1} = \frac{1}{N-1} \cdot \frac{\pi^2 D \varepsilon_o}{\ln\left(p/d + \sqrt{(p/d)^2 - 1}\right)}. \quad (9)$$

As explained in [7], (9) gives the overall stray capacitance with a good approximation when the adjacent turns are close to each other (small values of the ratio p/d) and for a reduced number of turns (i.e., tens of turns).

Under the above assumptions, we can employ (7a) and (9) in the constraints (6a) and (6b), respectively. Due to the nonlinearity of (4) or (5), (7a), and (9), a numerical approach is required to reach the optimal solution.

III. Optimization Procedure

In general, the solution to an optimization problem may be obtained analytically or numerically. However, due to the nonlinearity of the problem under study, a numerical technique was chosen. Many available and reliable numerical algorithms, which are not difficult to implement in the present case, can perform the task. We used a Successive Quadratic Programming (SQP) algorithm, which can handle a fairly wide range of nonlinear programming problems. This algorithm uses iterations that minimize a quadratic approximation of the Lagrangian function subject to linear approximation of the constraints [8],[9]. A finite-difference method can be used to estimate the gradients of both the objective function and the constraints. In this case, for some single precision calculations, an inaccurate estimate of the objective function gradient may cause the algorithm to terminate at a noncritical point. Therefore, high precision arithmetic is recommended. Also, if an analytical form is available, the exact gradients should be used instead.

IV. Optimal Design Examples

In order to highlight the influence of various geometrical parameters on the optimum inductor design, some examples are presented. In all the considered cases, the wire diameter has been fixed at d = 1 mm.

As a first case, the optimal design of an inductor is obtained without introducing any frequency constraint (i.e., the constraint (6b) on the overall stray capacitance is not considered). In this case, assuming the total winding length as an objective function, the optimization process leads to a minimum at the lower limit for the winding pitch, $p=d$.

Curves of the winding length as a function of the coil diameter for different values of the coil inductance are shown in Fig. 2. The straight lines represent the locus of the points corresponding to a same turn number, being in (4) $p \ll D$.

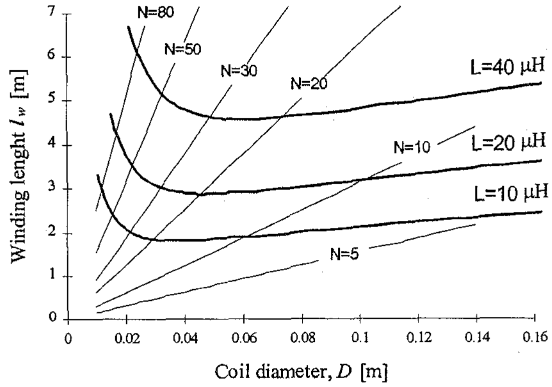

Fig. 2. Winding length l_w vs. coil diameter D for different values of coil inductance L and $p=d=1$ mm.

As Fig. 2 clearly shows, a minimum value of the winding length is obtained in all the considered cases. The corresponding optimal design parameters are given in Tab. I.

Table I. Optimal Parameter Values Referred to Fig. 2

Inductance L_r [μH]	Winding length l_w [m]	Coil diameter D [cm]	Turn number N
10	1.82	3.88	15
20	2.89	4.84	19
40	4.57	6.06	24

In the following examples, the optimal design of an inductor is obtained with a frequency constraint (i.e., the constraint on the overall stray capacitance is considered). In the diagrams represented in Fig. 3, the winding length l_w (objective function) and the coil axial length l_c are depicted as functions of the coil diameter D, for a coil inductance of 20 μH and for two values of the overall stray capacitance (i.e., of the self-resonant frequency). Fig. 4 shows the same curves for a coil inductance of 40 μH. The coil axial length l_c is defined as

$$l_c(x) = (N-1)\,p. \quad (10)$$

These figures clearly show a minimum for the total wire length. The corresponding geometrical dimensions are all in a feasible technical range. It means that an optimum design of the inductor is possible in the considered cases. The corresponding values of the design parameters are given in Tab. II.

It could be noted that the objective function l_w shows a wide flat region around the point of minimum. This leaves a further small degree of freedom in the inductor design. For example, additional requirements for the coil diameter or the coil axial length could be satisfied as long as the winding length remains very close to its minimum.

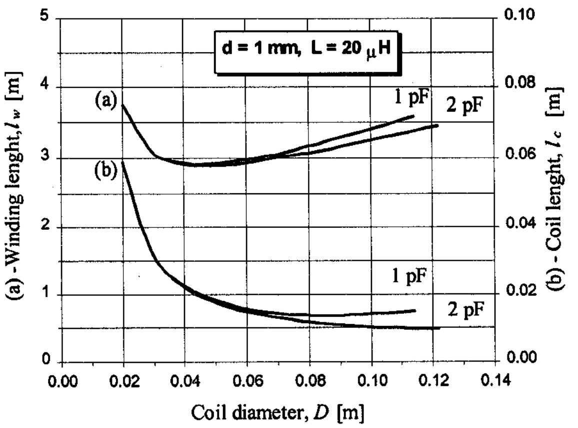

Fig. 3. Winding length l_w (a) and coil axial length l_c (b) vs. coil diameter D for $L_r = 20$ μH and two values of C_o.

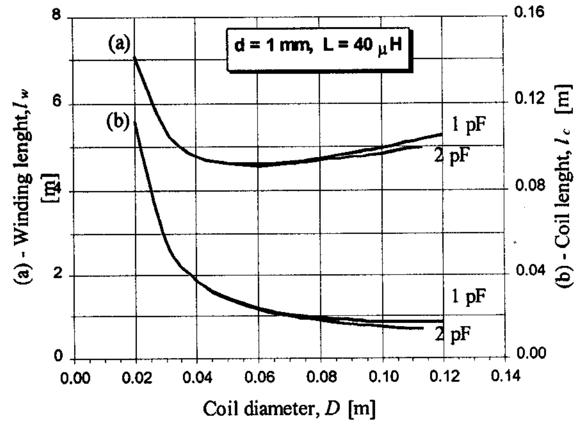

Fig. 4. Winding length l_w (a) and coil axial length l_c (b) vs. coil diameter D for $L_r = 40$ μH and two values of C_o.

Table II. Optimal Parameter Values Referred to Figs. 3 and 4

L_r [μH]	C_o [pF]	l_w [m]	D [cm]	N	l_c [cm]
20	1	2.91	4.62	20	1.94
20	2	2.89	4.60	20	1.91
40	1	4.60	5.64	26	2.55
40	2	4.58	6.06	24	2.32

V. Conclusions

In this paper, a procedure for the optimum design of single-layer solenoid air-core inductors for HF applications has been presented. The design procedure matches the desired values of the coil inductance and the first self-resonant frequency expressed in terms of overall stray capacitance. The remaining degrees of freedom are employed to minimize the winding length that was chosen as the objective function of the optimization problem. The optimization is formulated as a nonlinear programming problem in which the design electrical parameters of the inductor, expressed in terms of its geometry, give the constraints. A SQP algorithm has been introduced to determine the optimal geometric dimensions of the coil. Numerical results have been discussed and some guidelines for the inductor design have been given.

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