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Modelling of bulk superconductor magnetisation

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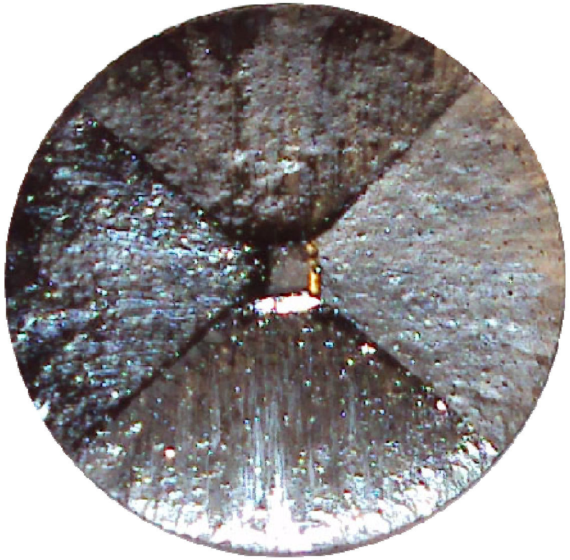
Presentation Outline

- **Overview of numerical modelling of bulk superconductor magnetisation**
- **Case study #1: Field cooling magnetisation of bulk MgB_2**
- **Case study #2: Split coil, pulsed field magnetisation of bulk high-temperature superconductors with an iron yoke**

Bulk Superconductors

- **Bulk superconducting materials trap magnetic flux via macroscopic electrical currents**
  - Magnetisation increases with sample volume



A large, single grain bulk superconductor

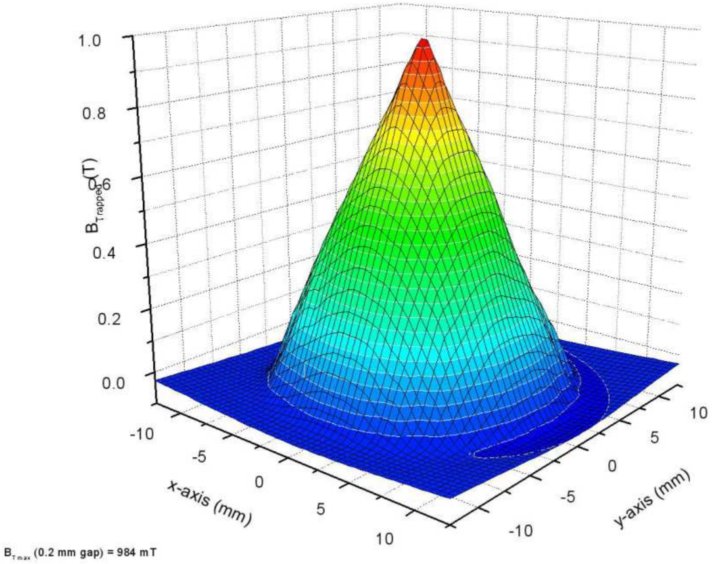
Bulk Superconductors

- **Bulk superconducting materials trap magnetic flux via macroscopic electrical currents**
  - Magnetisation increases with sample volume
- **Trapped field given by**

$$B_{\mathrm{trap}} = k\,\mu_0\,J_c\,R$$

$$k = \frac{t_B}{2R} \cdot \ln\left(\frac{R + \sqrt{R^2 + t_B{}^2}}{t_B}\right)$$

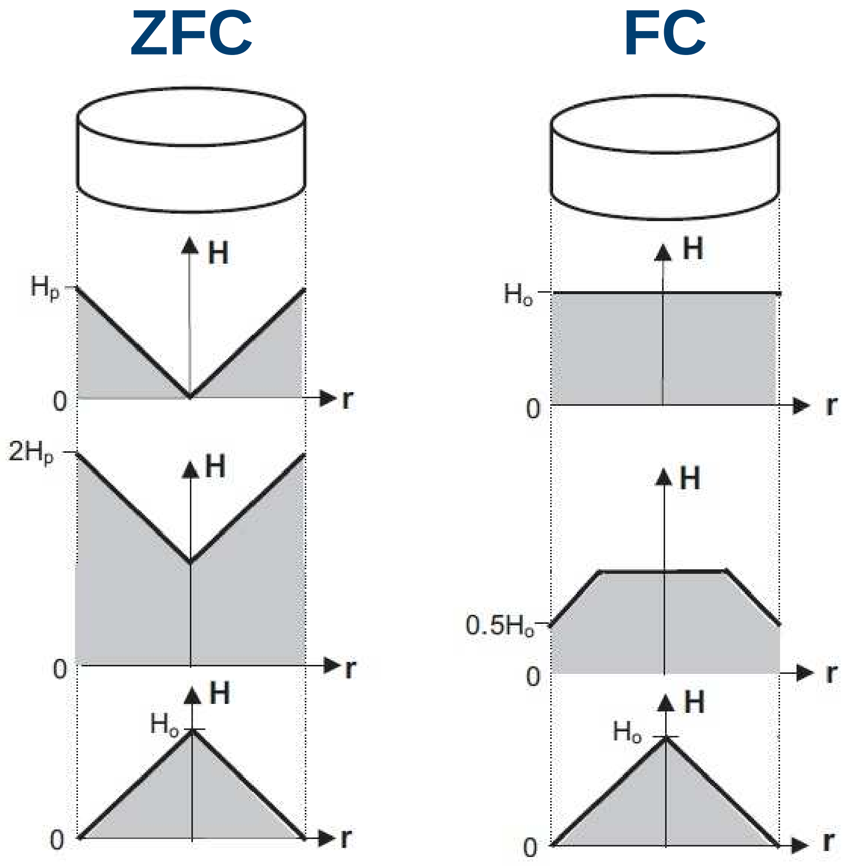
from Biot-Savart law + Bean model (infinite slab) = constant J_c



Typical trapped magnetic field profile of a bulk superconductor

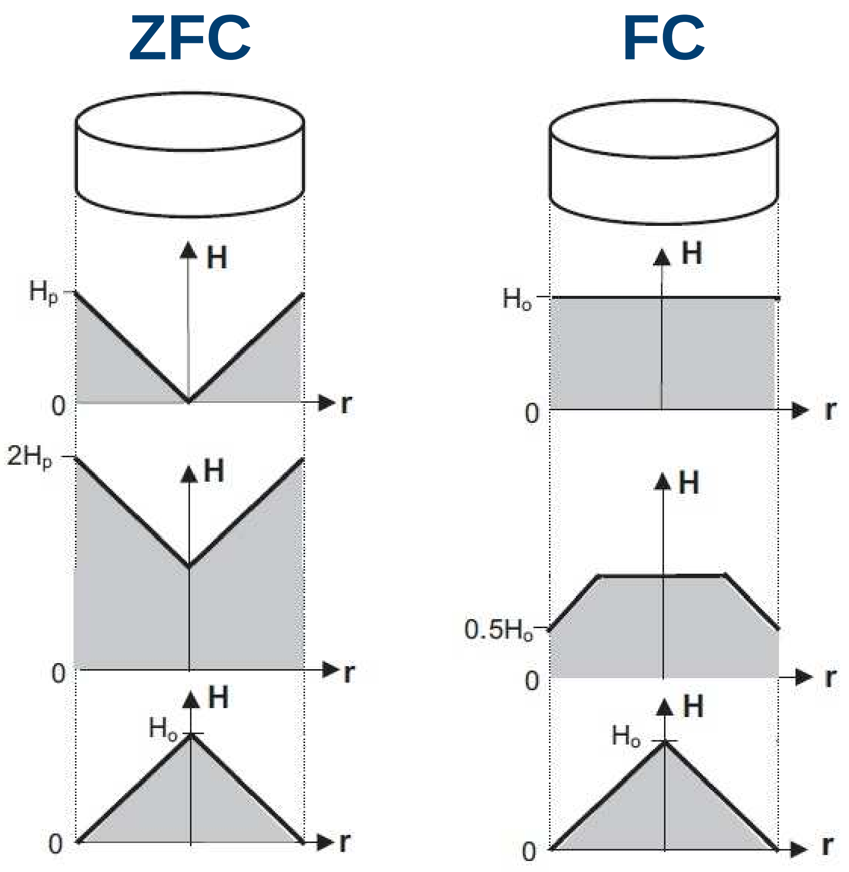
Magnetisation of Bulk Superconductors

- **Three magnetisation techniques:**
  - Field Cooling (FC)
  - Zero Field Cooling (ZFC)
  - Pulsed Field Magnetisation (PFM)



Magnetisation of Bulk Superconductors

- **Three magnetisation techniques:**
  - Field Cooling (FC)
  - Zero Field Cooling (ZFC)
  - Pulsed Field Magnetisation (PFM)
- **To trap *B*$_{trap}$, need at least *B*$_{trap}$ or higher**
  - FC and ZFC require large magnetising coils
  - Impractical for applications/devices



Pulsed Field Magnetisation

- **PFM technique: compact, mobile, relatively inexpensive**
- **Issues: B_{trap} [PFM] < B_{trap} [FC], [ZFC]**
  - Temperature rise ΔT due to rapid movement of magnetic flux
- **Record PFM trapped field: 5.2 T @ 29 K** (45 mm diameter Gd-Ba-Cu-O) Fujishiro et al. *Physica C* 2006
  - Record trapped field by FC: 17.6 T @ 26 K (2 x 25 mm dia Gd-Ba-Cu-O) Durrell et al. *Supercond. Sci. Technol.* 2014
- **Many considerations:**
  - Pulse magnitude, pulse duration, temperature, number of pulses, shape of magnetising coil(s)
  - Dynamics of magnetic flux during PFM process

Numerical Modelling of Magnetisation

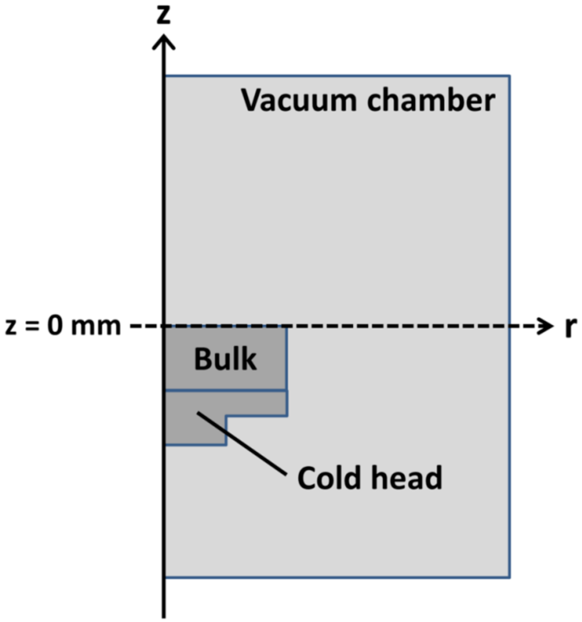
BULK GEOMETRY & MAGNETISATION FIXTURE

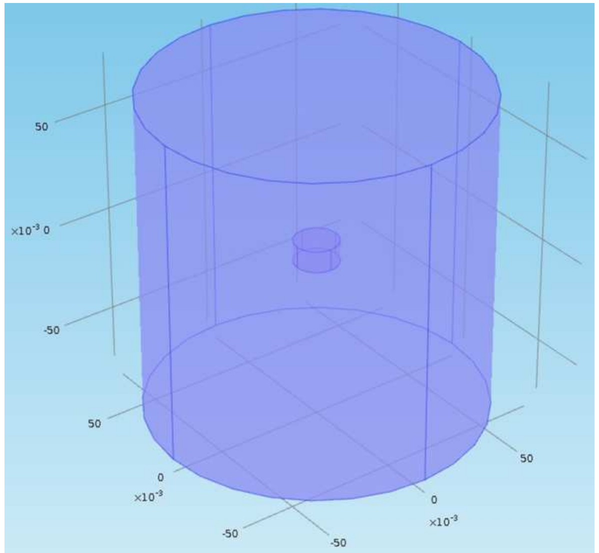
ELECTROMAGNETIC FORMULATION

THERMAL EQUATIONS & PROPERTIES

$J_c(B,T)$

E-J POWER LAW





- 2D axisymmetric generally sufficient for cylindrical bulks with a homogeneous J_c distribution
- 3D required for an inhomogeneous J_c distribution around the *ab*-plane; for non-symmetric shapes

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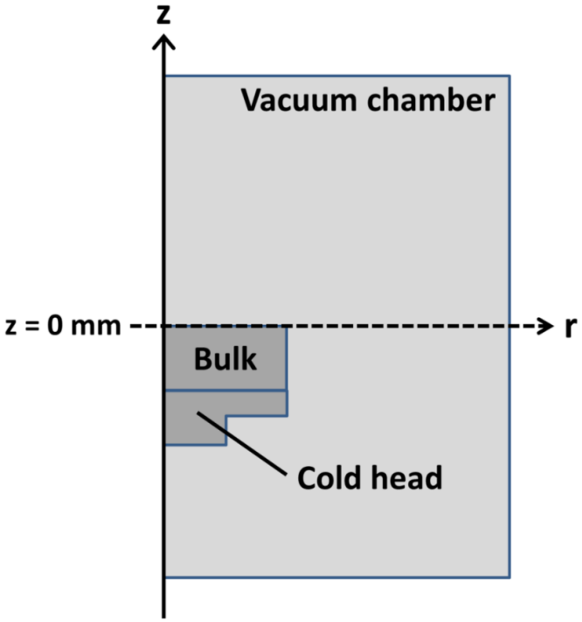

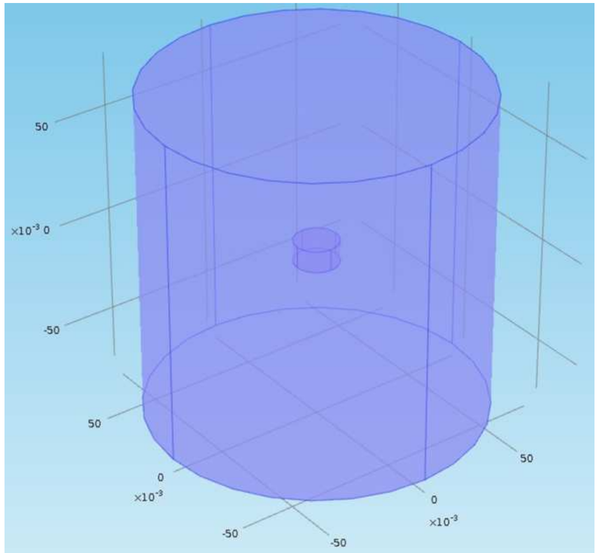

- Magnetising fixture can be simulated by using uniform boundary conditions or by inserting a copper coil sub-domain
- Cooling can be simulated using a cold head / vacuum chamber (left) or by submersion in liquid cryogen (right)

Numerical Modelling of Magnetisation

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E-J POWER LAW

Finite element method is commonly used & well developed (other techniques do exist)

Governing equations:

Maxwell's equations (***H*** formulation)

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} = -\frac{\partial(\mu_0 \mu_\mathrm{r} \mathbf{H})}{\partial t}$$

Faraday's Law

$$\nabla \times \mathbf{H} = \mathbf{J}$$

Ampere's Law

Other formulations also exist (***A***-*V*, ***T***-*Ω*, *Campbell's equation*)

Numerical Modelling of Magnetisation

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Thermal behaviour needs to be modelled when the bulk experiences a significant change in temperature, e.g., during PFM or modelling complete FC magnetisation process

Governing equations:

$$\rho \cdot C \frac{\mathrm{d}T}{\mathrm{d}t} = \nabla \cdot (k \nabla T) + Q$$

$$Q = E_{\mathrm{norm}} \cdot J_{\mathrm{norm}}$$

ρ = mass density, *C* = specific heat, *κ* = thermal conductivity, *Q* = heat source

Numerical Modelling of Magnetisation

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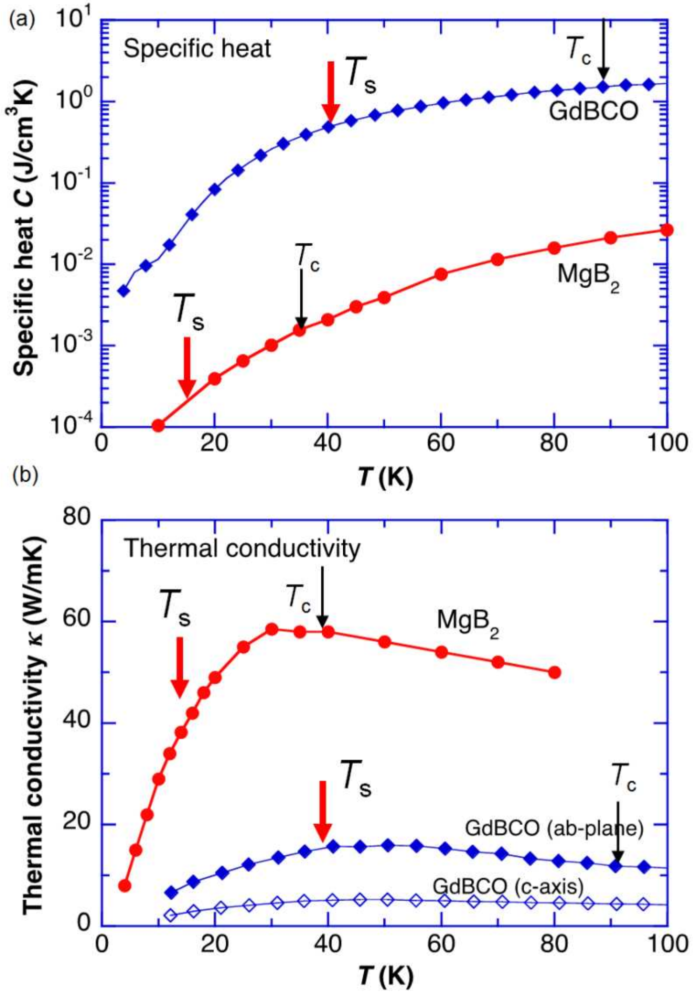
E-J POWER LAW

Can choose constant parameters for *C*, *κ* for $T = T_{op}$ as a reasonable approximation

Can use measured experimental data/fitting function over a specific temperature range



Numerical Modelling of Magnetisation

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J_c(B,T)

E-J POWER LAW

(RE)BCO materials
Kim-like model:

$$J_c = \frac{J_{c0}}{\left(1+\frac{B}{B_0}\right)^{\alpha}}$$

Fishtail effect: $J_c(B) = J_{c1}\exp\left(-\frac{B}{B_L}\right) + J_{c2}\frac{B}{B_{\max}}\exp\left[\frac{1}{y}\left(1-\left(\frac{B}{B_{\max}}\right)^{y}\right)\right]$

MgB$_2$ materials

$$J_c(B, T) = J_{c0}(T)\exp\left(-\frac{B}{B_0}\right)^{a}$$

- Some software packages, e.g., COMSOL, allow direct interpolation of experimental data without need for data fitting
e.g., Hu et al. *Supercond. Sci. Technol.* **28** (2015) 065011
- Important for coated conductor modelling where in-field behaviour can be quite complex: $J_c(B,\theta,T)$

Numerical Modelling of Magnetisation

BULK GEOMETRY & MAGNETISATION FIXTURE

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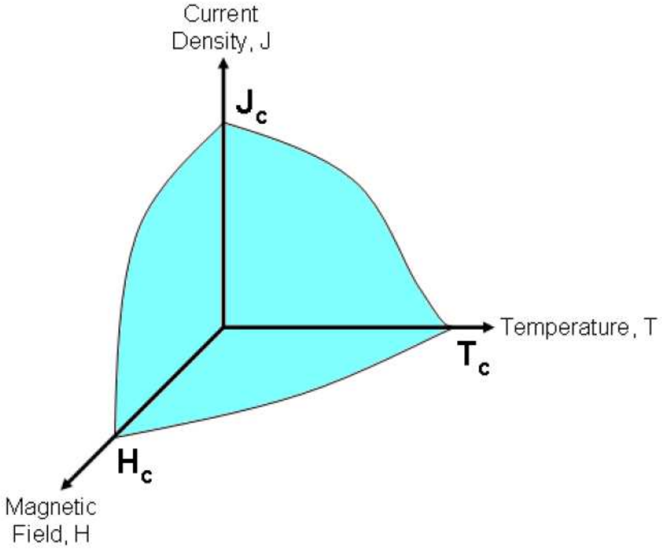
E-J POWER LAW

Linear $J_{c0}(T)$ relationship has also been used in the literature

$$J_{c0}(T) = \alpha \left[1 - \left(\frac{T}{T_c} \right)^2 \right]^{1.5}$$

Assumption made is that in-field behaviour, $J_c(B,T=T_{op})$, doesn't change for variations around the operating temperature, T_{op}



Numerical Modelling of Magnetisation

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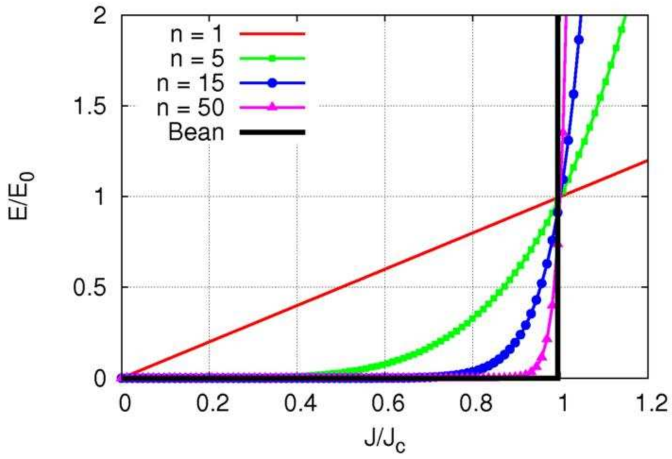
E-J POWER LAW

E-*J* power law

- Conventional conductors → non-linear permeability, linear resistivity
- Superconductors → linear permeability (μ_0), non-linear resistivity
- Non-linearity is extreme: power law with $n > 20$



$$\mathbf{E} = E_0 \left(\frac{J}{J_c}\right)^{n-1} \frac{\mathbf{J}}{J_c}$$

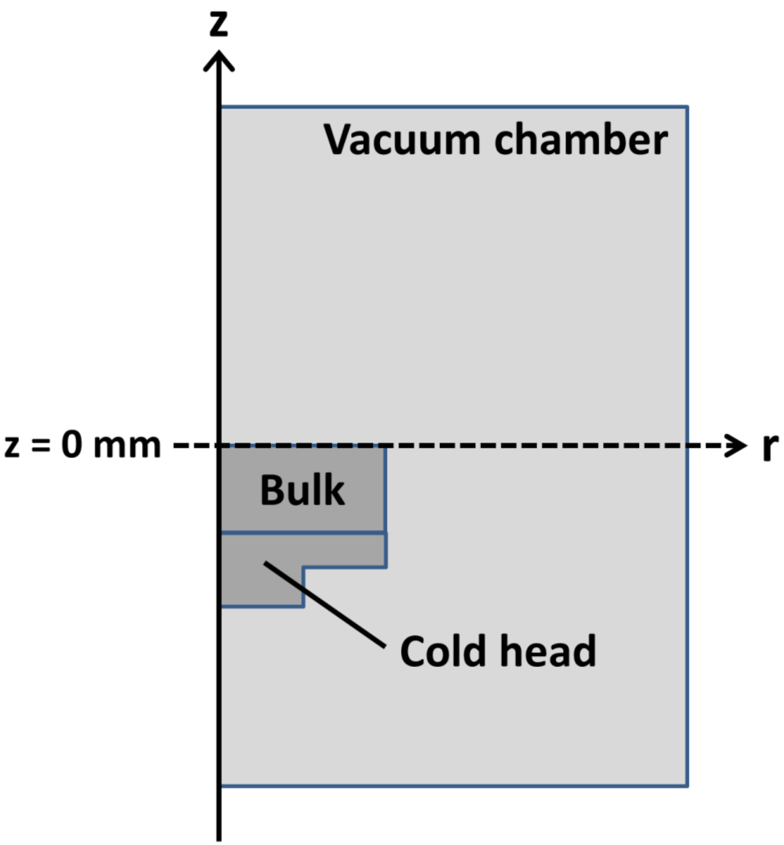
Case Study:

Field Cooling Magnetisation of

MgB_2 Bulk Superconductors

Bulk MgB_2 Modelling

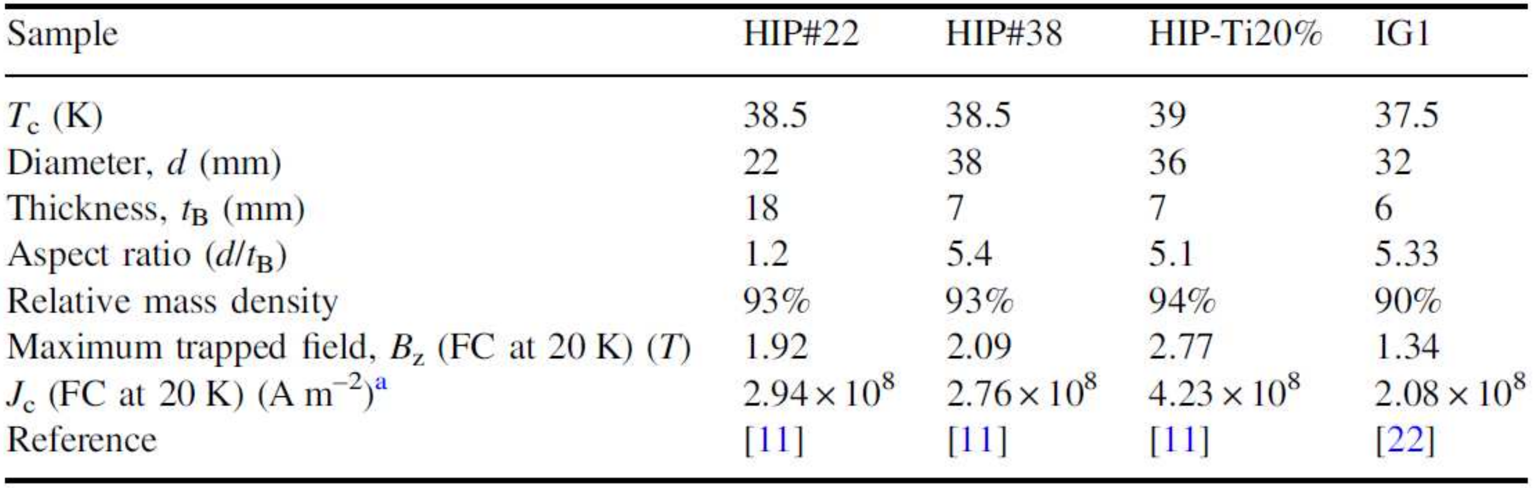
- **Numerical modelling of bulk MgB_2 is simpler than (RE)BCO**
  - J_c distribution generally quite homogeneous
  - 2D axisymmetric geometry, so $\boldsymbol{H} = [H_r, H_z]$, $\boldsymbol{J} = [J_\varphi]$, $\boldsymbol{E} = [E_\varphi]$
  - Can use measured $J_c(B,T)$ characteristics of a single, small specimen



Bulk MgB_2 Modelling – $J_c(B,T)$ Data Fitting

- **Four samples measured:**
  - Trapped field (FC) between ~ 5-15 K and 40 K
  - $J_c(B,T)$ of single, small specimen

Bulk MgB_2 sample information.



Sample	HIP#22	HIP#38	HIP-Ti20%	IG1
T_c (K)	38.5	38.5	39	37.5
Diameter, d (mm)	22	38	36	32
Thickness, t_B (mm)	18	7	7	6
Aspect ratio (d/t_B)	1.2	5.4	5.1	5.33
Relative mass density	93%	93%	94%	90%
Maximum trapped field, B_z (FC at 20 K) (T)	1.92	2.09	2.77	1.34
J_c (FC at 20 K) (A m^{-2})[a]	2.94×10^8	2.76×10^8	4.23×10^8	2.08×10^8
Reference	[11]	[11]	[11]	[22]

Zou, Ainslie, Fujishiro et al. *Supercond. Sci. Technol.* **28** (2015) 075009

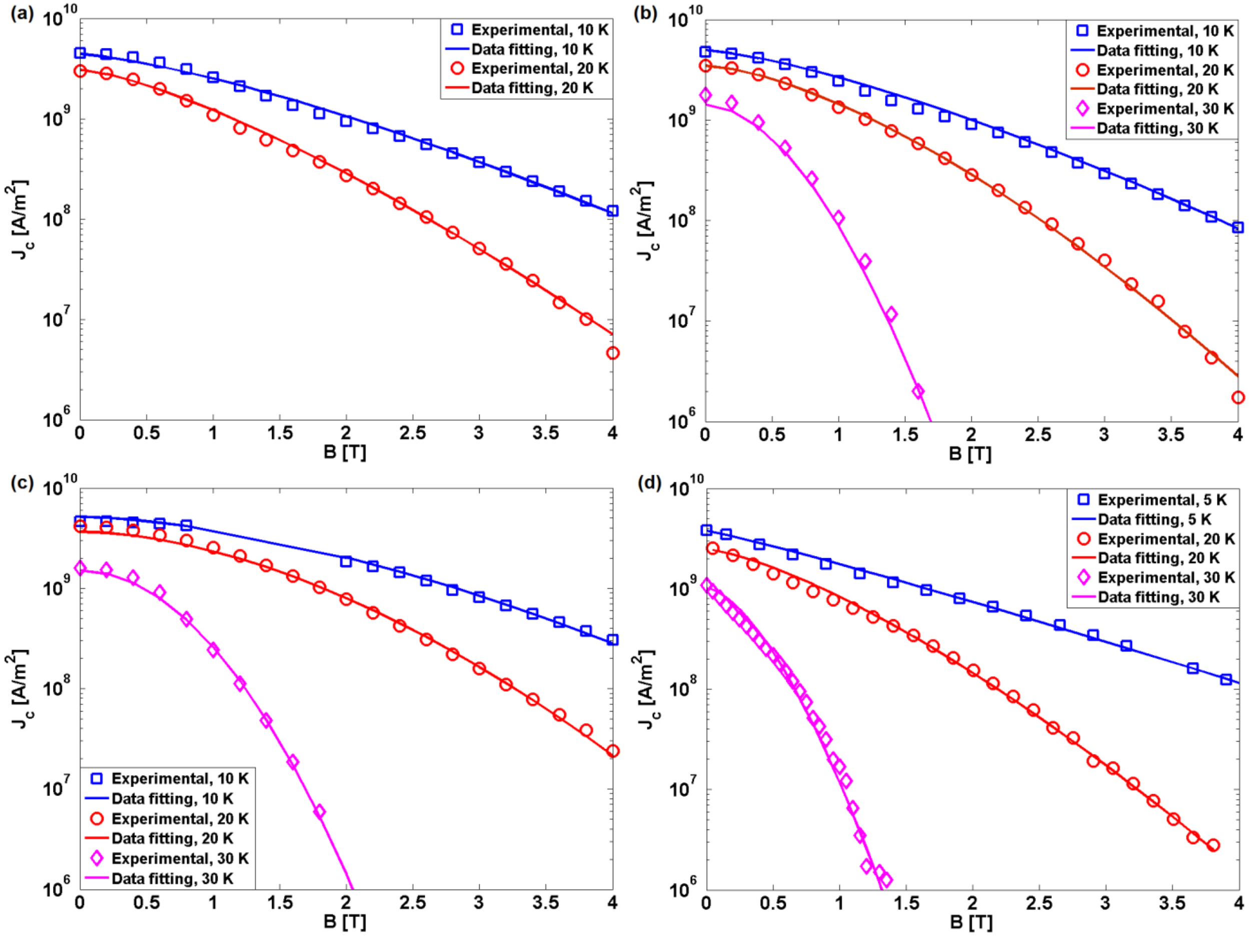
Bulk MgB_2 Modelling – Field Cooling Magnetisation

- **Simulating FC magnetisation process:**
  - FC with $B_{app} = B_{trap}$:
    1. $0 \leq t \leq x_1$ Apply ramped field to $B_{app} = B_{trap}$ at $T = T_{ex} > T_c$
    2. $x_1 \leq t \leq x_2$ Slow cooling of bulk to operating temperature, $T = T_{op}$
    3. $x_2 \leq t \leq x_3$ Slowly ramp applied field $B_{ex} \rightarrow 0$
    - In electromagnetic model, need to define ρ for all temperatures:
      - For $T > T_c$, need to define ρ_{normal} ($\rho_{normal} = 3 \times 10^{-8}$ Ωm)
      - For $T < T_c$, ρ_{sc} defined from E-J power law, where $\boldsymbol{E} = \rho \boldsymbol{J}$: $\mathbf{E} = \boxed{E_0 \left(\frac{J}{J_c}\right)^{n-1}} \frac{\mathbf{J}}{J_c}$
      - To avoid non-convergence at T_c: $\rho(T) = \frac{\rho_{sc} \cdot \rho_{normal}}{\rho_{sc} + \rho_{normal}}$

Zou, Ainslie, Fujishiro et al. *Supercond. Sci. Technol.* **28** (2015) 075009

Bulk MgB_2 Modelling – $J_c(B,T)$ Data Fitting

$$J_c(B,\, T) = J_{c0}(T)\exp\left(-\frac{B}{B_0}\right)^a$$



Zou, Ainslie, Fujishiro et al. *Supercond. Sci. Technol.* **28** (2015) 075009

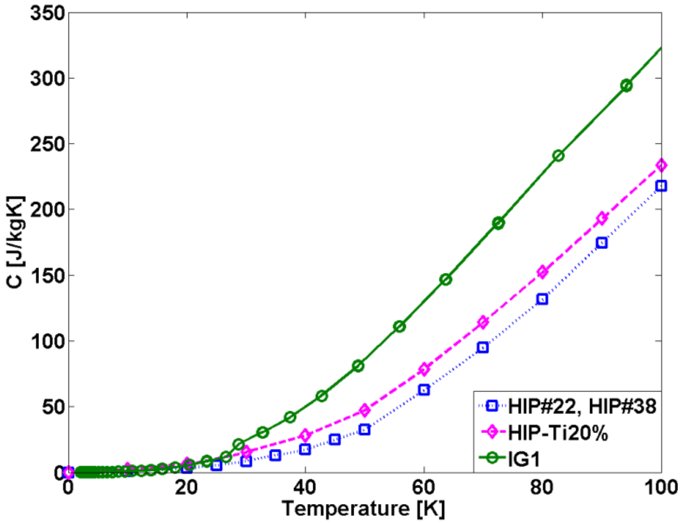
Bulk MgB_2 Modelling – Thermal Properties

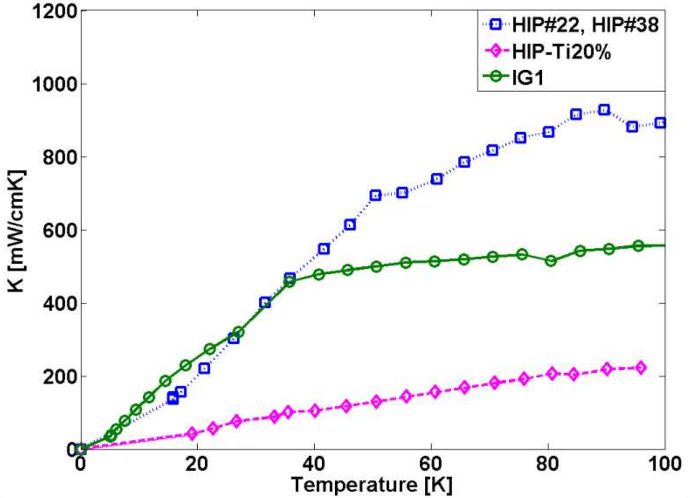
Thermal equation: $\rho \cdot C \dfrac{\mathrm{d}T}{\mathrm{d}t} = \nabla \cdot (k \nabla T) + Q$

- Here T changes from T_{ex} = 100 K (> T_c) to T_{op} = 5 – 30 K
  - Measured experimental data from 0 – 100 K for each sample input directly into model (direct interpolation)

$$J_c(B,\, T) = J_{c0}(T)\exp\left(-\frac{B}{B_0}\right)^{a}$$

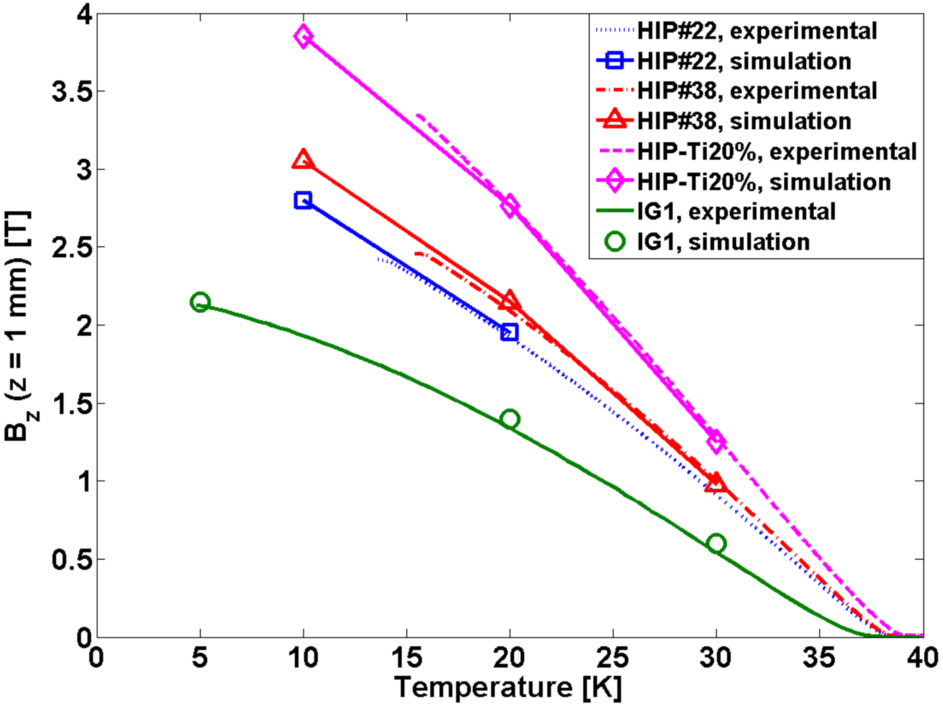
$$J_{c0}(T) = \alpha\left[1 - \left(\frac{T}{T_c}\right)^2\right]^{1.5}$$





Bulk MgB_2 Modelling – Comparison of Results

- **Simulation reproduces experimental trapped field measurements extremely well**
  - Samples have excellent homogeneity
  - Model is validated as a fast & accurate tool to predict trapped field performance
    - Any size of bulk MgB_2 disc & any specific operating conditions
    - Can investigate current distribution / trapped field in partially magnetised situation



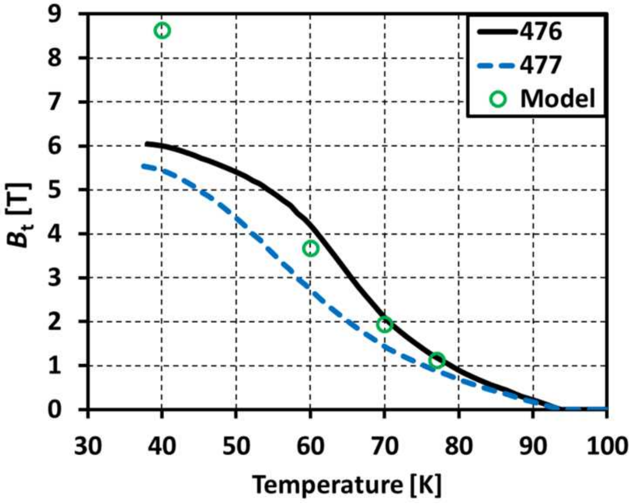
Comparison of simulation & experimental results for trapped field in different bulk MgB_2 samples

Zou, Ainslie, Fujishiro et al. *Supercond. Sci. Technol.* **28** (2015) 075009

Case Study #2:

Split Coil, Pulsed Field Magnetisation of Bulk High-Temperature Superconductors With an Iron Yoke

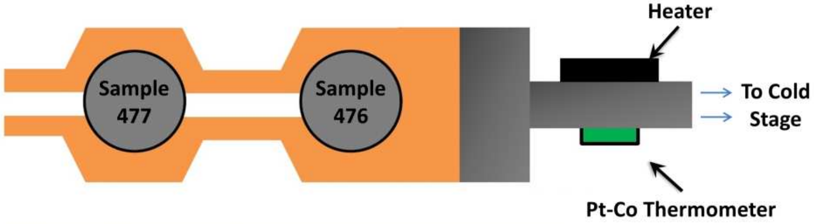
Split Coil PFM with an Iron Yoke

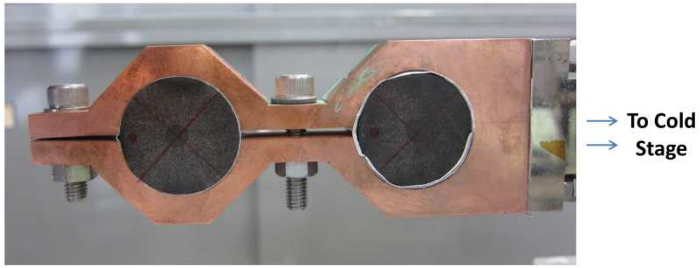


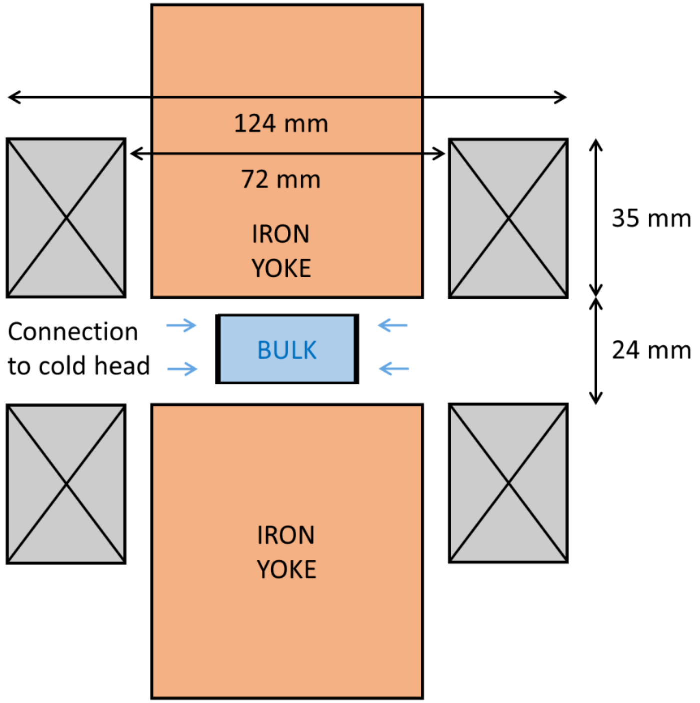
Two Ag-containing (15 wt%) Gd-Ba-Cu-O samples: 30 mm diameter, 15 mm thickness

#476 sample B_t = 6 T (40 K) 3.11 T (65 K)

#477 sample B_t = 5.44 T (40 K) 2.02 T (65 K)







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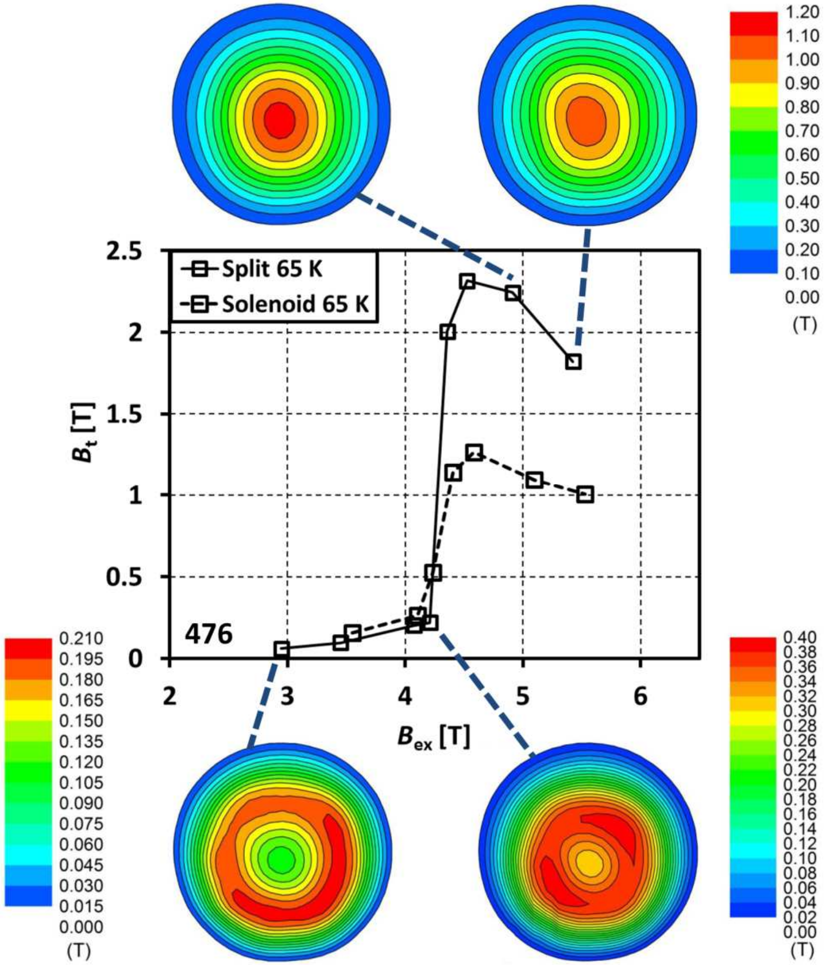
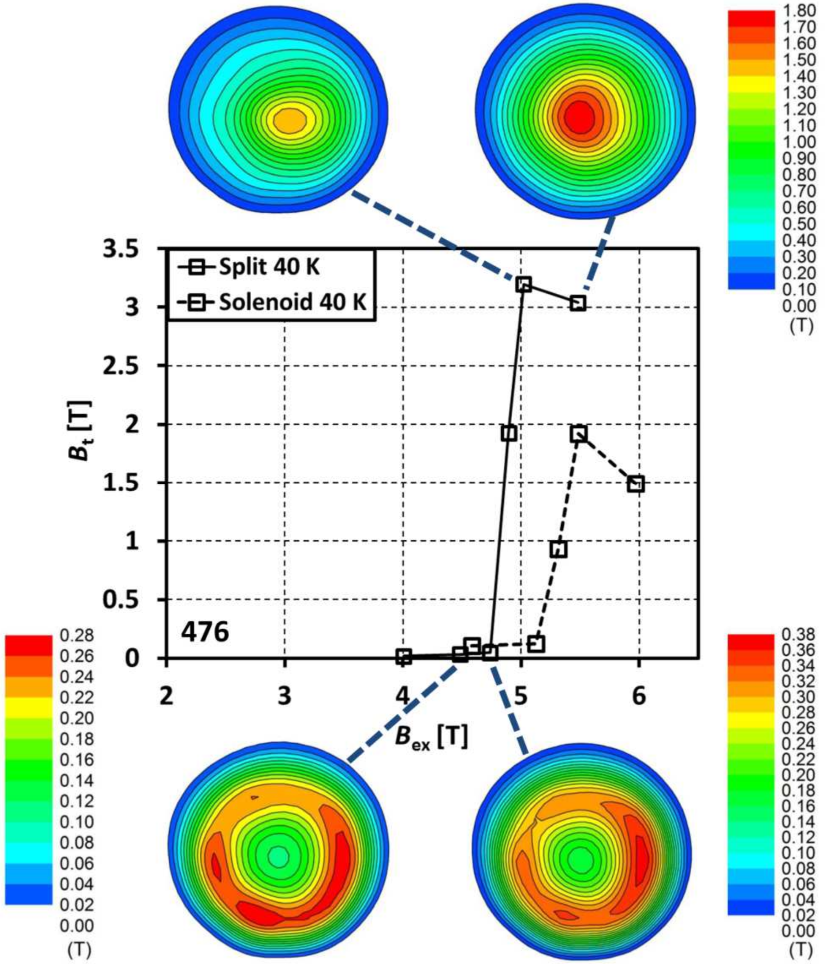
Split Coil PFM with an Iron Yoke





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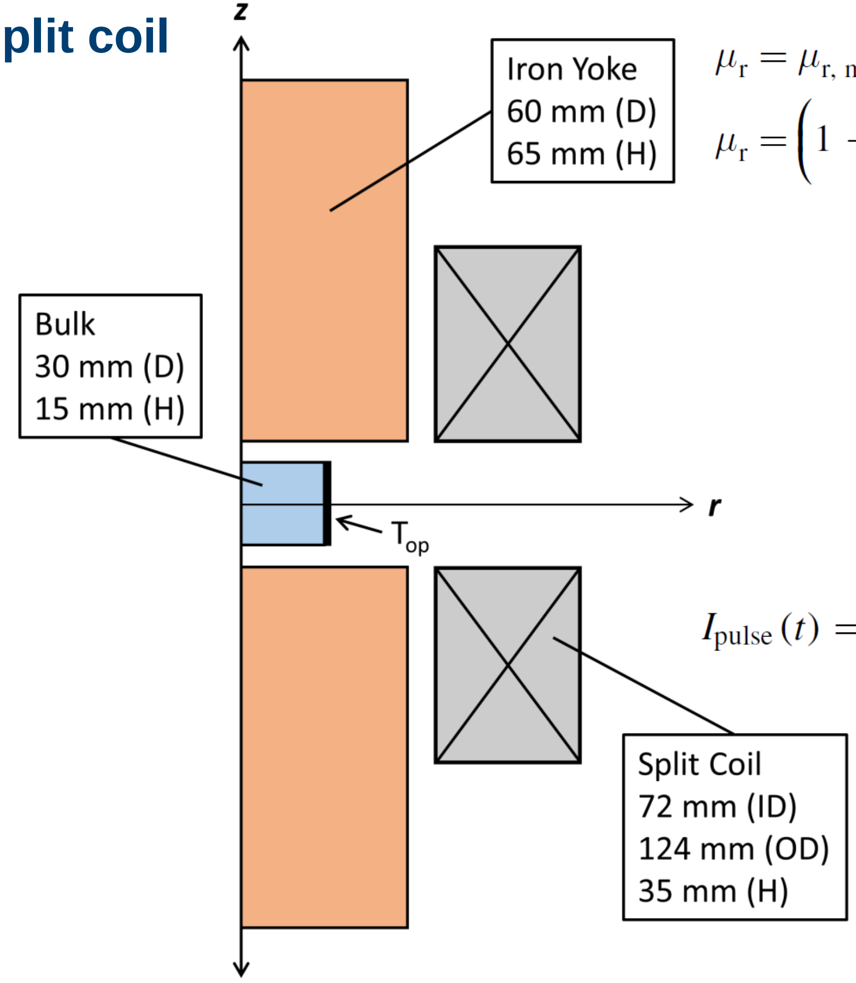
Split Coil PFM with an Iron Yoke

Split coil



Bulk
30 mm (D)
15 mm (H)

Iron Yoke
60 mm (D)
65 mm (H)

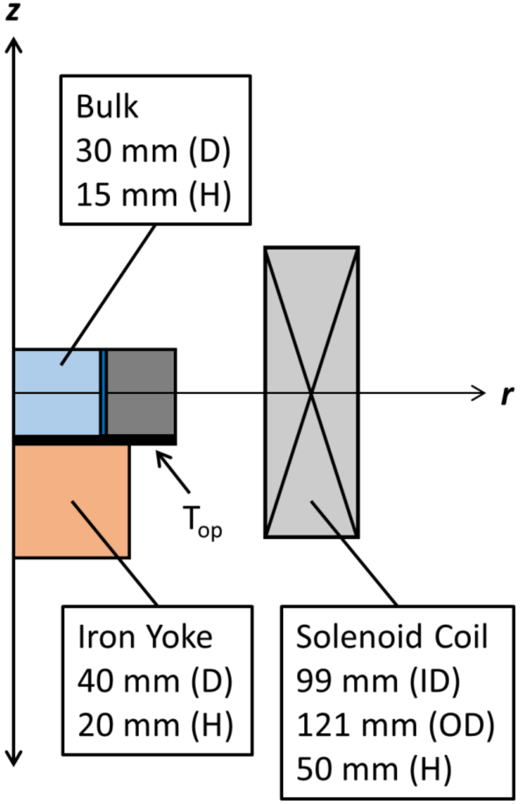
Split Coil
72 mm (ID)
124 mm (OD)
35 mm (H)

$$\mu_r = \mu_{r,\,\max} \text{ for } B < \mu_0 M_{sat}$$

$$\mu_r = \left(1 + \frac{M_{sat}}{H}\right) \text{ for } B \geqslant \mu_0 M_{sat}$$

$$I_{pulse}(t) = N \cdot I_0 \frac{t}{\tau} \exp\left(1 - \frac{t}{\tau}\right)$$

Solenoid coil



Bulk
30 mm (D)
15 mm (H)

Iron Yoke
40 mm (D)
20 mm (H)

Solenoid Coil
99 mm (ID)
121 mm (OD)
50 mm (H)

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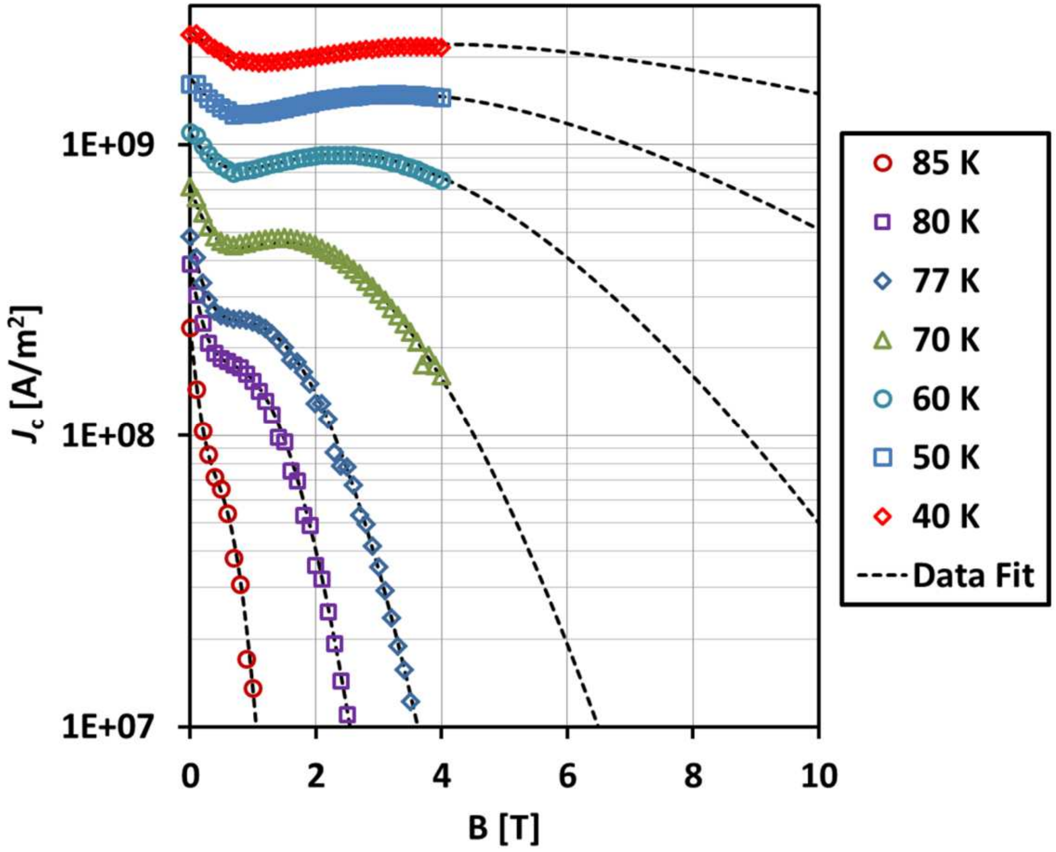
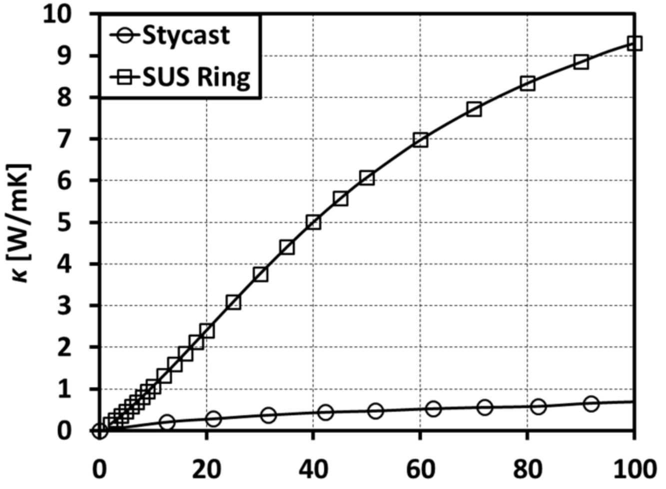
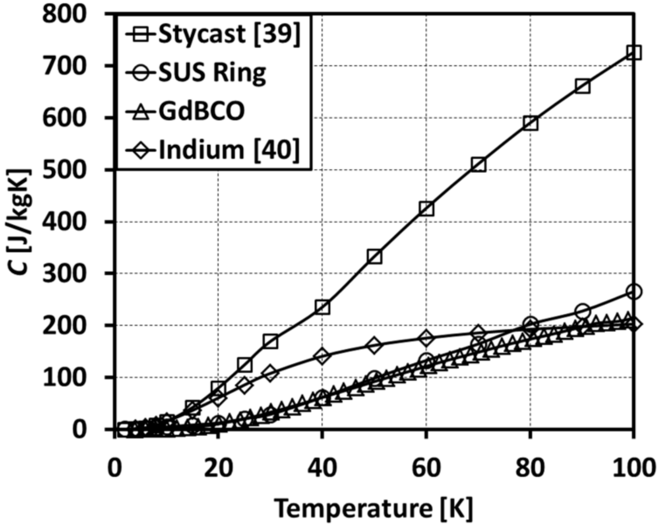
Split Coil PFM with an Iron Yoke

$$J_c(B) = J_{c1}\exp\left(-\frac{B}{B_L}\right) + J_{c2}\frac{B}{B_{\max}}\exp\left[\frac{1}{y}\left(1-\left(\frac{B}{B_{\max}}\right)^{y}\right)\right]$$

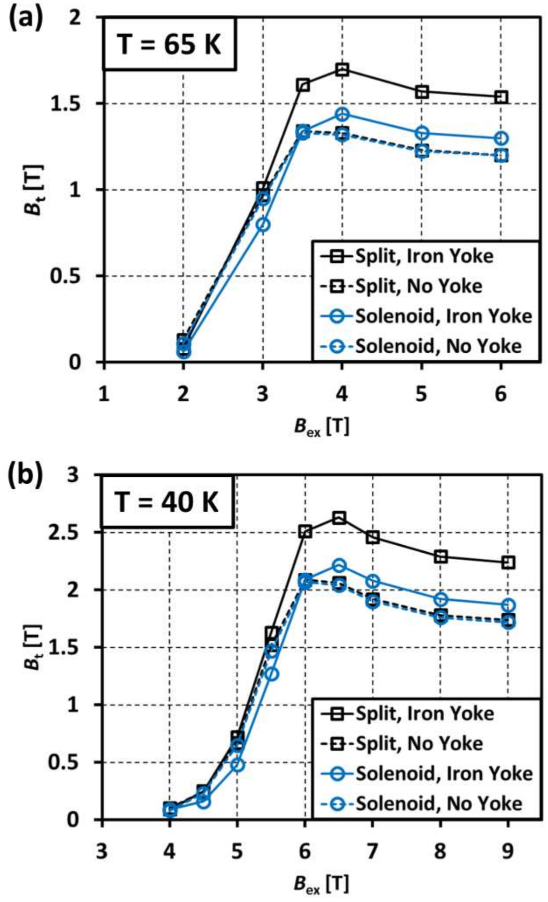




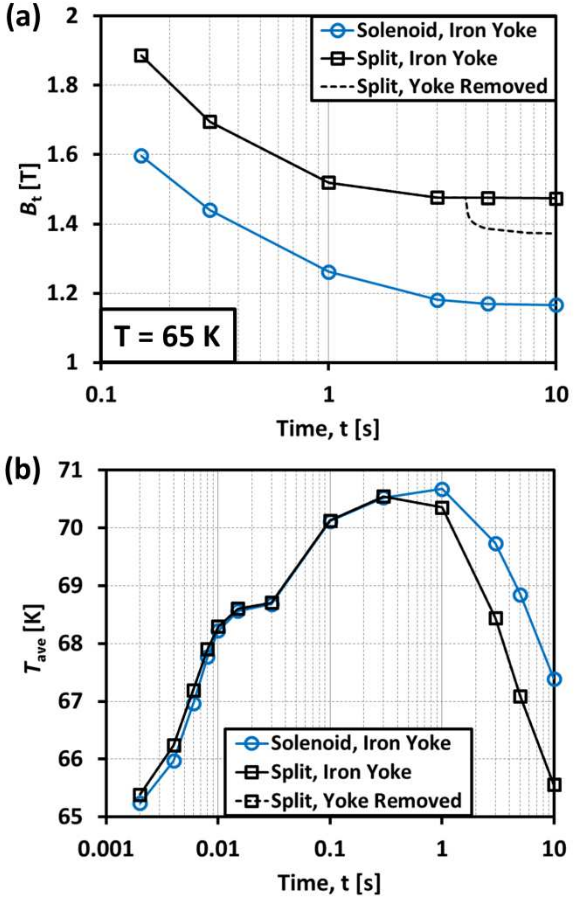


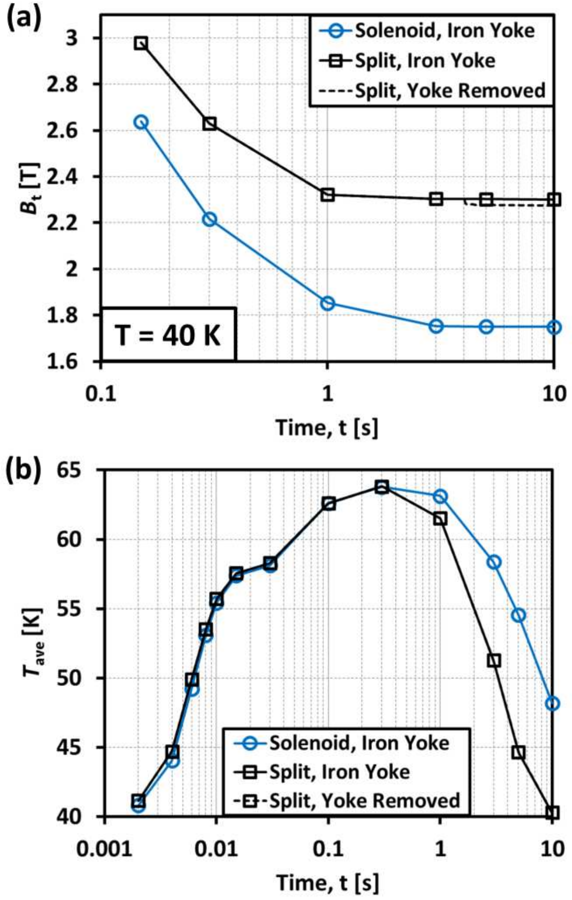
Split Coil PFM with an Iron Yoke

Trapped field @ *t* = 300 ms



Trapped field & T_{ave} with time, incl. yoke removed





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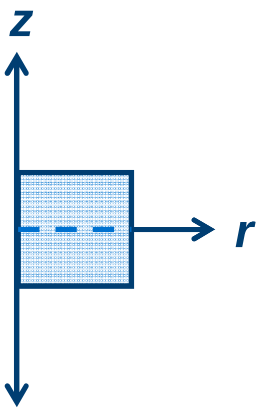
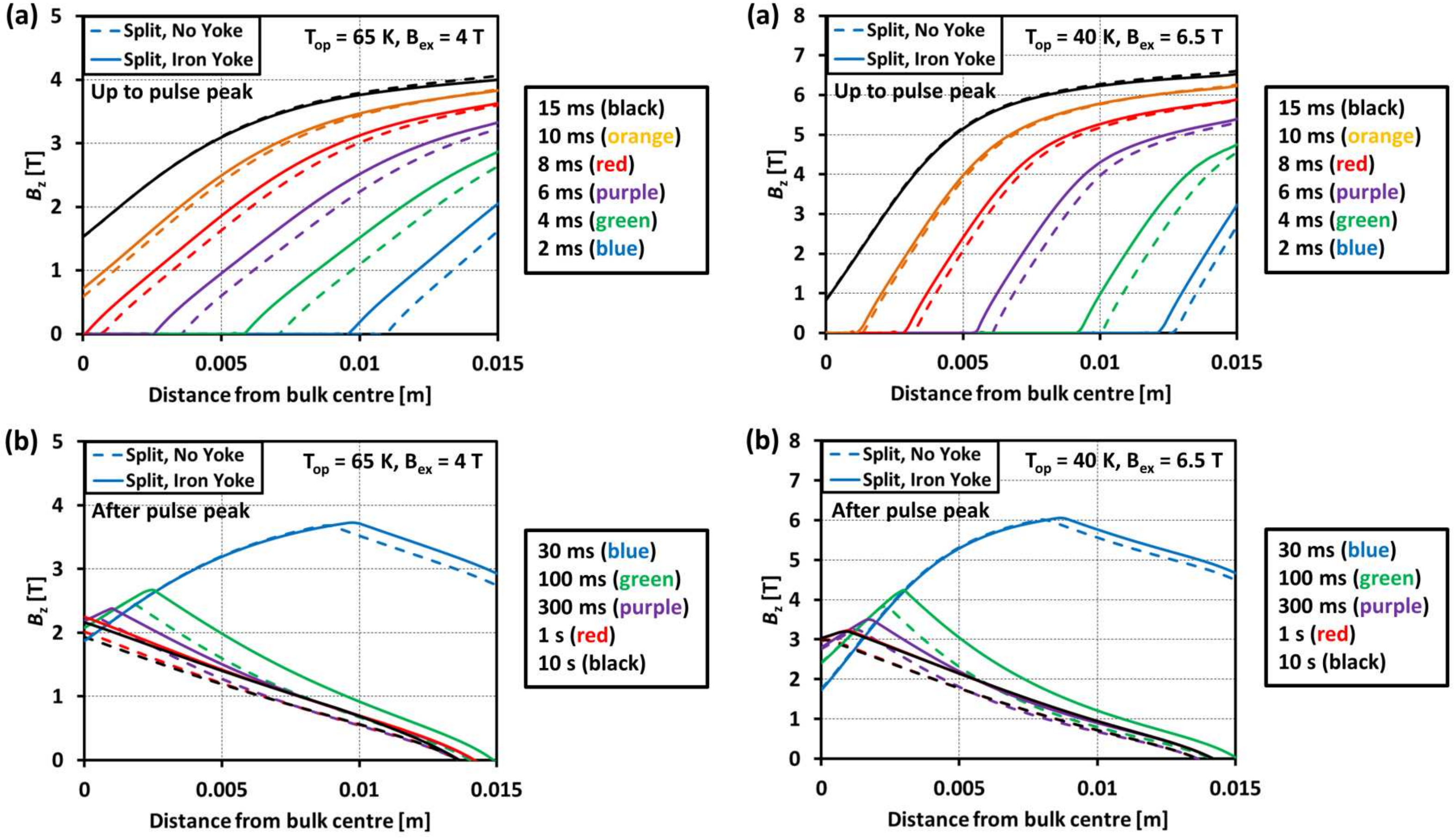
Split Coil PFM with an Iron Yoke

Magnetic flux entry & exit during & after pulse – split coil with & without iron yoke





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- **Case study #1: Field cooling magnetisation of bulk MgB_2**
- **Case study #2: Split coil, pulsed field magnetisation of bulk high-temperature superconductors with an iron yoke**

Thank you for listening

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