Modelling pancake coils and solenoid coils wound with coated conductors for electromagnetic field analyses: using exact coil geometry or using axisymmetric approximation

N. Amemiya, Y. Sogabe, N. Tominaga (Kyoto University)

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Non-uniform current distributions in HTS tapes affect the field qualities of magnets wound with HTS tapes: shielding (screening) current issue.

Electromagnetic field analyses are the popular approach to this issue.
A concern has been raised over the axisymmetric approximation for shielding current calculation (by Ueda et al.).

- Cross-sectional analyses
- Reduction of CPU time and memory
Objective

Comparison between the model based on the exact coil geometry and the model based on the axisymmetric approximation

- Focus on shielding-current-induced field (SCIF)
- Detailed results on stacked-pancake coils
- Some results on a multilayered solenoid coil: the difference from the pancake coils
Outline

- Stacked pancake coils
  - Key issues of the concern on the axisymmetric approximation
  - Models for analyses
  - Results

- Multilayered solenoid coil
  - Model for analyses
  - Results

- Conclusion
Pancake coils
Key issues of the concern on the axisymmetric approximation
Key issues of concern on axisymmetric approximation

1. Different current path: neglecting transverse $J \rightarrow$ SCIF
2. Different inductance $\rightarrow$ decay time constant of shielding current
3. Different tape geometry (even if assuming uniform current distribution)
Models for analyses
\[ E = E_0 \left( \frac{J}{J_c} \right)^n \]

\[ J_c = J_{c0} \frac{B_0}{B_0 + |B_n|} \]

**Parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_0 )</td>
<td>( 1 \times 10^{-4} ) V/m</td>
</tr>
<tr>
<td>( n )</td>
<td>32.6</td>
</tr>
<tr>
<td>( J_{c0} )</td>
<td>( 1.65 \times 10^{11} ) A/m²</td>
</tr>
<tr>
<td>( B_0 )</td>
<td>0.69 T</td>
</tr>
</tbody>
</table>
# Stacked pancake coils for analyses

<table>
<thead>
<tr>
<th>Parameter</th>
<th>PC-S</th>
<th>PC-L</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conductor width</td>
<td>5 mm</td>
<td></td>
</tr>
<tr>
<td>Conductor / SC layer thickness</td>
<td>0.2 mm / 2 µm</td>
<td></td>
</tr>
<tr>
<td>Inner / outer radius</td>
<td>50 mm / 62.5 mm</td>
<td>250 mm / 262.5 mm</td>
</tr>
<tr>
<td>Number of turn per PC</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>Turn separation</td>
<td>0.05 mm</td>
<td>0.05 mm</td>
</tr>
<tr>
<td>Conductor length per PC</td>
<td>18 m</td>
<td>80.4 m</td>
</tr>
<tr>
<td>Number of PCs</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>Separation between PCs</td>
<td></td>
<td>0.5 mm</td>
</tr>
<tr>
<td>Magnetic field at coil center</td>
<td>2.83 T @ 300 A</td>
<td>1.05 T @ 300 A</td>
</tr>
<tr>
<td>Maximum normal field component</td>
<td>1.8 T @ 300 A</td>
<td>1.97 T @ 300 A</td>
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<tr>
<td>Load ratio of coil @ 300 A</td>
<td>0.66</td>
<td>0.70</td>
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Operating point on $I - B$ plots and excitation pattern

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<td>0.70</td>
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Two models for analyses

Model SP  (Exact spiral geometry)
Model AX  (Nested axisymmetric one-turn loops)

Analyzing one PC

Other PCs are represented by line currents in order to apply the field to the analyzed PC

Pancake by pancake analysis
Repeated for all PCs

N. Amemiya, HTS MODELLING 2016
Governing equation and constitutive equation

(Faraday’s law)
\[ \nabla \times E + \frac{\partial B}{\partial t} = 0 \]

(Faraday’s law)
\[ B = \frac{\mu_0}{4\pi} \int \frac{J \times r}{r^3} dV \]

(Faraday’s law)
\[ J = \nabla \times T \]

(Biot-Savart’s law)
\[ E = J / \sigma(J) \]

(Extended Ohm’s law)
\[ \nabla \times \left( \frac{1}{\sigma} \nabla \times T_f \right) + \frac{\mu_0}{4\pi} \frac{\partial}{\partial t} \int \frac{(\nabla \times T_s) \times r}{r^3} dV = 0 \]

(Definition of current vector potential)
\[ \int_{S_t} \nabla \times \left( \frac{1}{\sigma} \nabla \times T_f \right) + \frac{\mu_0 t_s}{4\pi} \frac{\partial}{\partial t} \int_s \frac{(\nabla \times T_s) \times r}{r^3} dS = 0 \]

Thin strip approximation
High cross-sectional aspect ratio of coated-conductor allows its use.

N. Amemiya, ICSM 2016
Results
Model SP: normal field component and current profile with induced shielding current

Current lines on the entire length of coated conductor of the top pancake coil ($t = 0 \text{ s}$)

N. Amemiya, HTS MODELLING 2016
Lateral current distributions (top pancake)

Model SP ($t = 0$ s)

Center of the 25th turn of top pancake

Model SP

Model AX

$\Delta J/J \sim 10^{-4}$

N. Amemiya, HTS MODELLING 2016
Electric field and change in current distribution

$t = 0 \text{ s}$

$t = 1000 \text{ s}$

$t = 16000 \text{ s}$
Magnetization distribution on coated conductor

\[-\mu_0 M = B - \mu_0 H\]

- \(\mu_0 M\): magnetization
- \(B\): Magnetic flux density
- \(\mu_0 H\): Field generated by other PCs

\[t = 0\ s, \text{Model SP, top PC}\]

\[-\mu_0 M_{\text{tape}} = \frac{1}{S_{\text{tape}}} \int_{S_{\text{tape}}} -\mu_0 M dS = 345.9 \text{ mT}\]

\[t = 0\ s, \text{Model AX, top PC}\]

\[-\mu_0 M_{\text{tape}} = \frac{1}{S_{\text{tape}}} \int_{S_{\text{tape}}} -\mu_0 M dS = 346.3 \text{ mT}\]

N. Amemiya, HTS MODELLING 2016
Temporal evolution of magnetic field at coil center

Is this difference caused by
• different current path: neglecting transverse $J$?
• different geometry?

N. Amemiya, HTS MODELLING 2016
Compensation of influence of different geometry

Model AX
1. Calculating current distributions by using model AX
2. Projecting the current distributions on the spiral coated conductor

AX_d

Model SP
Influence of SCIF

Difference between $AX_d$ and SP normalized by the field calculated with uniform current distribution

\[
(AX_d - SP) / SP_u \approx 4 \text{ ppm}
\]
Comparison between PC-S and PC-L

PC-S

PC-L

\[ \frac{(AX_d - SP)}{SP_u} \approx 4 \text{ ppm} \]

\[ \frac{(AX_d - SP)}{SP_u} \approx -1.5 \text{ ppm} \]

N. Amemiya, HTS MODELLING 2016
Solenoid coil
Models for analyses
### Multilayered solenoid coil

<table>
<thead>
<tr>
<th>Parameter</th>
<th>SL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conductor width</td>
<td>5 mm</td>
</tr>
<tr>
<td>Conductor / SC layer thickness</td>
<td>0.2 mm / 2 µm</td>
</tr>
<tr>
<td>Inner / outer radius</td>
<td>50 mm / 62.5 mm</td>
</tr>
<tr>
<td>Number of turn per layer</td>
<td>30</td>
</tr>
<tr>
<td>Turn separation</td>
<td>0.5 mm</td>
</tr>
<tr>
<td>Conductor length per layer</td>
<td>12 m</td>
</tr>
<tr>
<td>Number of layers</td>
<td>50</td>
</tr>
<tr>
<td>Separation between layer</td>
<td>0.05 mm</td>
</tr>
</tbody>
</table>

![Diagram of the multilayered solenoid coil](image)
Models for analyses

Model HX
(Exact spiral geometry)

- The layer-by-layer analysis applied: not analyzed layers represented by sets of line currents in order to apply the field to the analyzed layer
Results
Model HX: normal field component, load ratio and current profile with induced shielding current

Current lines on the entire length of coated conductor of the 25th layer
$(t = 0 \text{ s})$

60 A / line

300 A

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Magnetization distribution on coated conductor

\[-\mu_0 M = B - \mu_0 H\]

- \(\mu_0 M\): magnetization
- \(B\): Magnetic flux density
- \(\mu_0 H\): Field generated by other PCs

\[t = 0 \text{ s, Model HX, the 25th layer}\]

Low load ratio

High load ratio

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The following scheme enables us to calculate the magnetic field of a stacked pancake coils accurately and with less CPU time and memory.

- The approximation of nested one-turn loops is also useful for non-axisymmetric coils such as saddle-shape coils.
Back-up slides
Consideration of three-dimensionally-curved coated conductors

\[ \nabla \times \left( \frac{1}{\sigma} \nabla \times nT \right) \cdot n + \frac{\partial}{\partial t} \left( \frac{\mu_0 t_s}{4\pi} \int_{S'} \frac{(\nabla \times n'T') \times r \cdot n}{r^3} \, dS' + B_{\text{ext}} \cdot n \right) = 0 \]

This term representing \( B \) in Faraday’s law is calculated by Biot-Savart’s law based on currents on arbitrary 3D-shaped conductors.
Magnetization measurements to determine $E$–$J$

\[ I = \frac{6m}{w^3d} \quad E = -\frac{\mu_0 G}{4wd} \frac{dm}{dt} \]

$d$: thickness of superconductor
$G$: geometrical factor $7.17 \times 10^{-4}$
$m$: magnetization of sample
Figures not used
Lateral current distributions

$X$

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Influence of different geometry (uniform current)

Magnetic field calculated with uniform current distribution

<table>
<thead>
<tr>
<th>Model SP (SPₜₜ)</th>
<th>Model AX (AXₜₜ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.83488 T</td>
<td>2.83686 T</td>
</tr>
</tbody>
</table>

Difference ~ 2 mT

Model SP

Model AX

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Temporal evolution of magnetic field at coil center
遮蔽電流の影響を考慮した場合の発生磁場（電磁界解析結果）

<table>
<thead>
<tr>
<th>Model SP (SP)</th>
<th>Model AX (AX)</th>
<th>Model AX (AX_d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.78198 T (t = 0 s)</td>
<td>2.78389 T (t = 0 s)</td>
<td>2.78201 T (t = 0 s)</td>
</tr>
</tbody>
</table>

軸対称近似のコイル発生磁場に対する影響

\[(AX_d - SP) / SP_u = 9.2 \text{ ppm}\]
3. 解析結果
異なる時刻での軸対称近似が磁場に与える影響

<table>
<thead>
<tr>
<th>ランプアップ終了からの経過時間 $t$</th>
<th>SP</th>
<th>$AX_d$</th>
<th>$(AX_d-SP) / SP_u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 s</td>
<td>2.7820 T</td>
<td>2.7820 T</td>
<td>9.2 ppm</td>
</tr>
<tr>
<td>100 s</td>
<td>2.7847 T</td>
<td>2.7848 T</td>
<td>8.0 ppm</td>
</tr>
<tr>
<td>10000 s</td>
<td>2.7883 T</td>
<td>2.7884 T</td>
<td>6.1 ppm</td>
</tr>
<tr>
<td>1000000 s</td>
<td>2.7924 T</td>
<td>2.7924 T</td>
<td>4.0 ppm</td>
</tr>
</tbody>
</table>

軸対称近似のコイル発生磁場に対する影響の時間変化は非常に小さい

軸対称近似の適用による遮蔽電流の減衰時定数への影響は小さい

N. Amemiya, HTS MODELLING 2016
3. 解析結果
PC群中心での磁場のモデル間での差異に関する検討

線材内に電流が一様に分布していた場合の発生磁場

<table>
<thead>
<tr>
<th>Model SP (SP_u)</th>
<th>Model AX (AX_u)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.05396 T</td>
<td>1.05442 T</td>
</tr>
</tbody>
</table>

コイル形状が異なることによって0.5 mTの差が発生する。
(半径が大きくなった分、コイル形状の違いの影響も小さくなった)

遮蔽電流の影響を考慮した場合の発生磁場（電磁界解析結果）

<table>
<thead>
<tr>
<th>Model SP (SP)</th>
<th>Model AX (AX)</th>
<th>Model AX (AX_d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.05162 T (t = 0 s)</td>
<td>1.05208 T (t = 0 s)</td>
<td>1.05161 T (t = 0 s)</td>
</tr>
</tbody>
</table>

軸対称近似のコイル発生磁場に対する影響
\( (AX_d-SP) / SP_u = -1.9 \text{ ppm} \)

N. Amemiya, HTS MODELLING 2016
遮蔽電流の磁場への影響の時間変化

$r = 50 \text{ mm}$の場合よりも遮蔽電流磁場の減衰は小さい

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軸対称近似の影響の変化

コイル半径が5倍になったとき、軸対称近似の影響は約1/4倍になった。
→コイル径が大きくなればなるほど、軸対称近似による解析で十分になる可能性が高い

N. Amemiya, HTS MODELLING 2016
幅方向電流密度分布

ソレノイドコイル25層目、30ターン目中央

外部磁場の与え方が異なっているため直接比較できないが、パンケーキコイルの場合とは分布が多少異なる。
時間変化の様子はパンケーキコイルと同様
Lateral current distributions

N. Amemiya, HTS MODELLING 2016