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(June 17, Friday)

Modelling pancake coils and solenoid coils wound with coated conductors for electromagnetic field analyses: using exact coil geometry or using axisymmetric approximation

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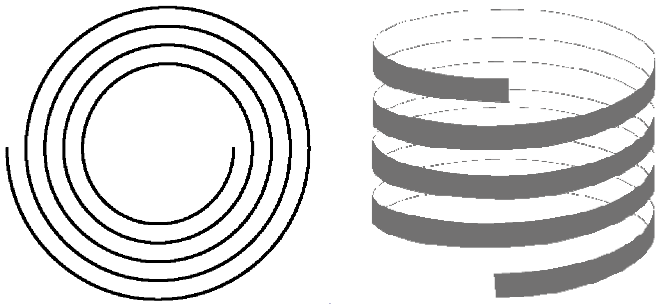
Motivation

- Non-uniform current distributions in HTS tapes affect the field qualities of magnets wound with HTS tapes: shielding (screening) current issue.
- Electromagnetic field analyses are the popular approach to this issue.

Motivation (continued)

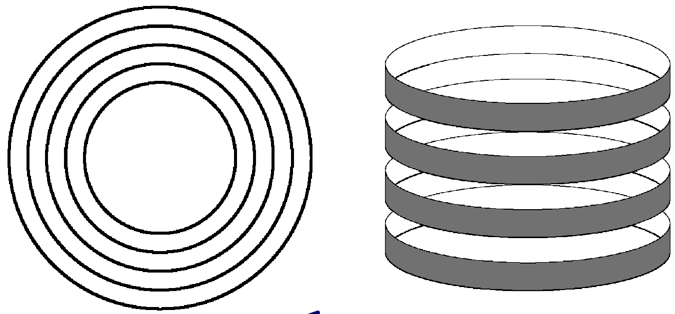
Exact geometry



Pancake coil: plane spiral
Solenoid coil: helix

A concern has been raised over the axisymmetric approximation for shielding current calculation (by Ueda et al.).

Axisymmetric approximation

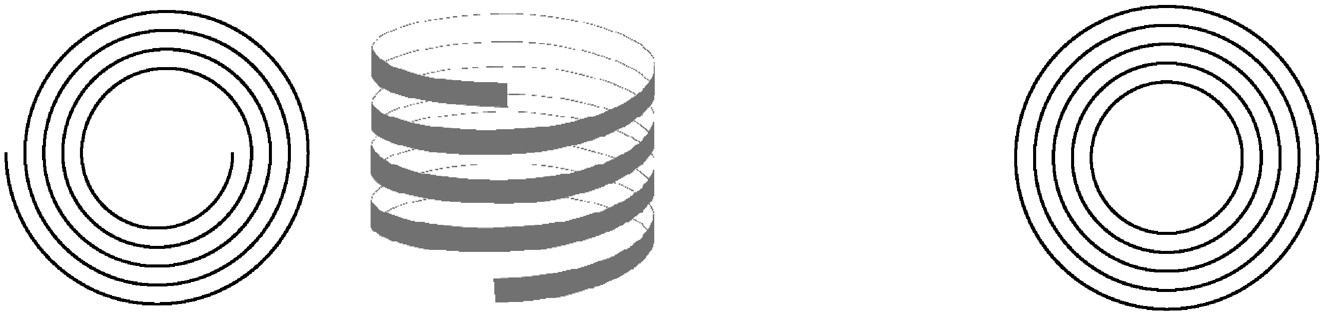


Pancake coil: nested one-turn loops of coated conductors
Solenoid coil: stacked one-turn loops of coated conductors

- Cross-sectional analyses
- Reduction of CPU time and memory

Objective

Comparison between the model based on the exact coil geometry and the model based on the axisymmetric approximation



- ✓ Focus on shielding-current-induced field (SCIF)
- ✓ Detailed results on stacked-pancake coils
- ✓ Some results on a multilayered solenoid coil: the difference from the pancake coils

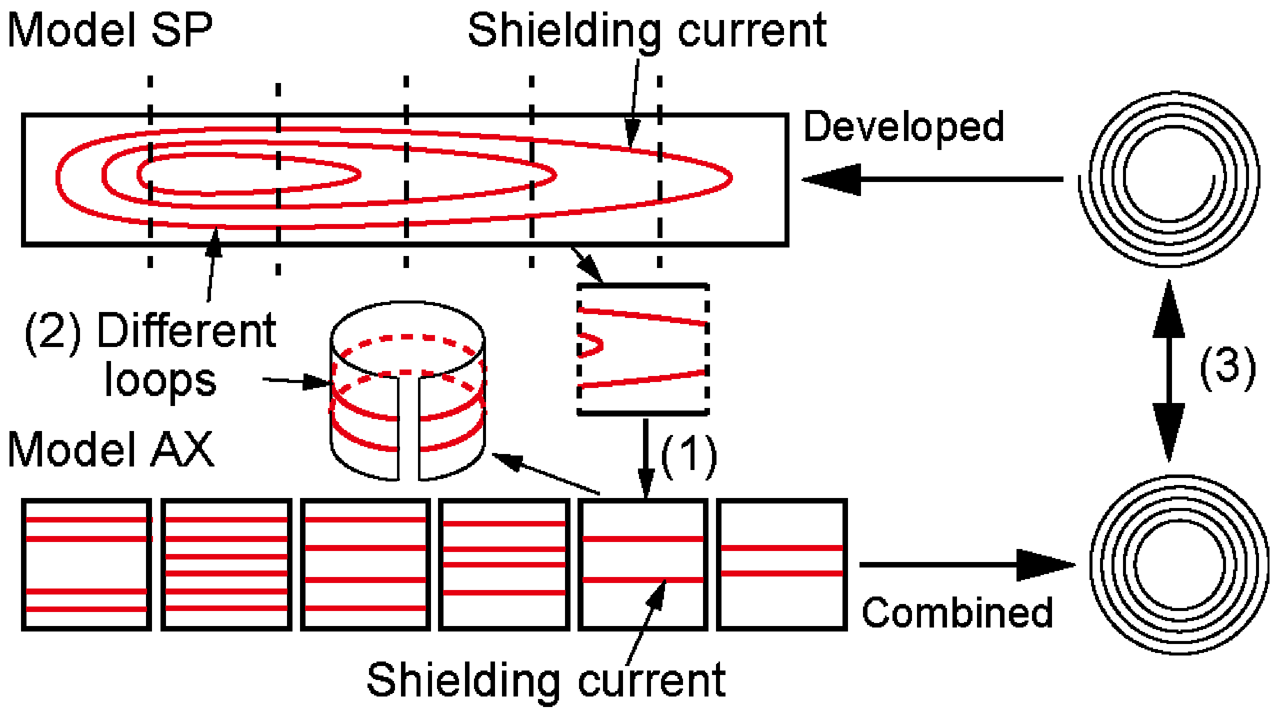
Outline

- Stacked pancake coils
  - Key issues of the concern on the axisymmetric approximation
  - Models for analyses
  - Results
- Multilayered solenoid coil
  - Model for analyses
  - Results
- Conclusion

Pancake coils

Key issues of the concern on the axisymmetric approximation

Key issues of concern on axisymmetric approximation



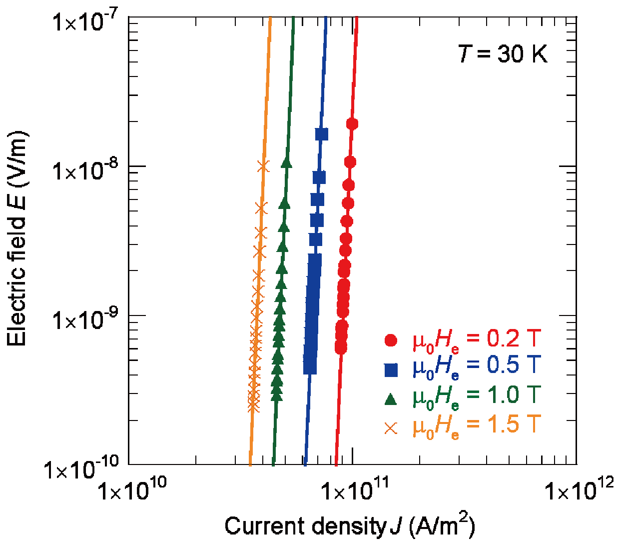
1. Different current path: neglecting transverse *J* -> SCIF
2. Different inductance -> decay time constant of shielding current
3. Different tape geometry (even if assuming uniform current distribution)

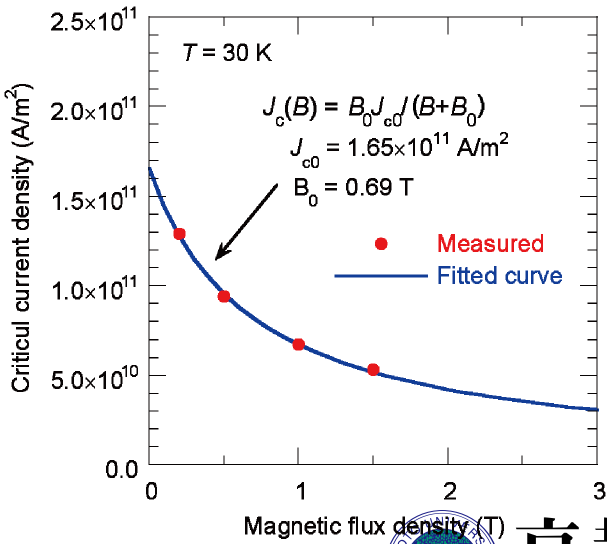
Models for analyses

E–J and J_c–B_n characteristics

$$E = E_0\left(\frac{J}{J_c}\right)^n \qquad J_c = J_{c0}\frac{B_0}{B_0 + |B_n|}$$

Parameters	
E_0	1×10^{-4} V/m
n	32.6
J_{c0}	1.65×10^{11} A/m^2
B_0	0.69 T



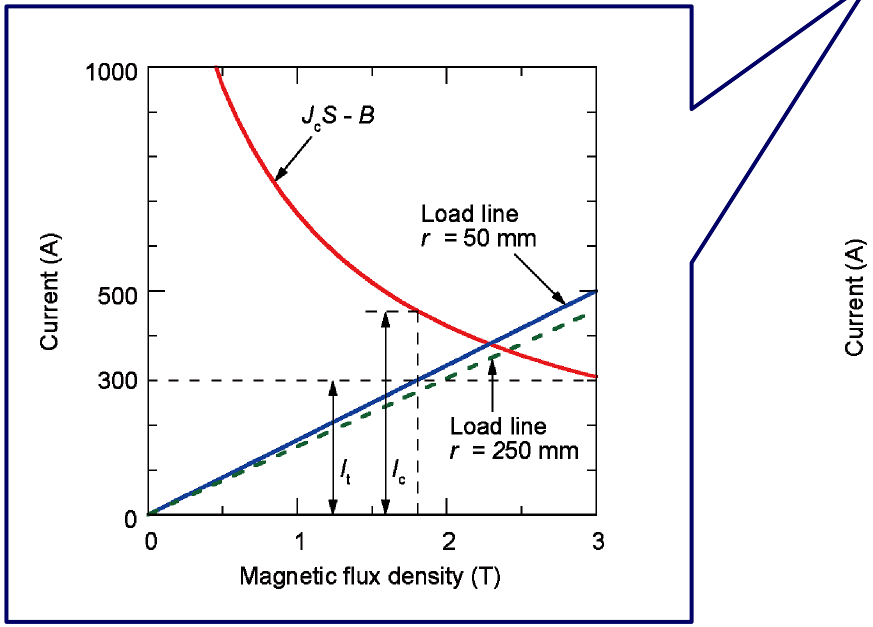


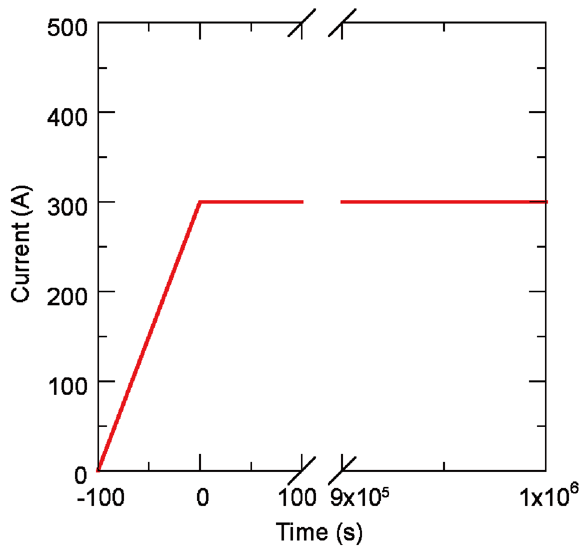
Stacked pancake coils for analyses

	PC-S	PC-L
Conductor width	5 mm	
Conductor / SC layer thickness	0.2 mm / 2 μm	
Inner / outer radius	50 mm / 62.5 mm	250 mm / 262.5 mm
Number of turn per PC	50	50
Turn separation	0.05 mm	0.05 mm
Conductor length per PC	18 m	80.4 m
Number of PCs	30	30
Separation between PCs	0.5 mm	
Magnetic field at coil center	2.83 T @ 300 A	1.05 T @ 300 A
Maximum normal field component	1.8 T @ 300 A	1.97 T @ 300 A
Load ratio of coil @ 300 A	0.66	0.70

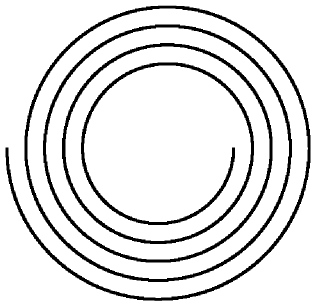
Operating point on *I* – *B* plots and excitation pattern

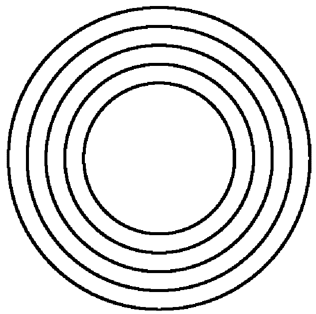
	PC-S	PC-L
Magnetic field at coil center	2.83 T @ 300 A	1.05 T @ 300 A
Maximum normal field component	1.8 T @ 300 A	1.97 T @ 300 A
Load ratio of coil @ 300 A	0.66	0.70





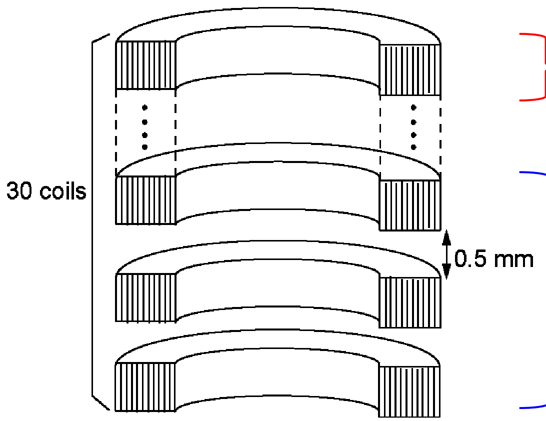
Two models for analyses





Model SP
(Exact spiral geometry)

Model AX
(Nested axisymmetric one-turn loops)



Analyzing one PC

Other PCs are represented by line currents in order to apply the field to the analyzed PC

Pancake by pancake analysis

Repeated for all PCs

Governing equation and constitutive equation

〔Faraday's law〕

$$\nabla \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{0}$$

〔Biot-Savart's law〕

$$\boldsymbol{B} = \frac{\mu_0}{4\pi} \int_V \frac{\boldsymbol{J} \times \boldsymbol{r}}{r^3} dV$$

〔Definition of current vector potential〕

$$\boldsymbol{J} = \nabla \times \boldsymbol{T}$$

〔Extended Ohm's law〕

$$\boldsymbol{E} = \boldsymbol{J} / \sigma(J)$$

Equivalent conductivity derived from *E-J* characteristic: Ex. $E = E_0 \left(J / J_c \right)^n$

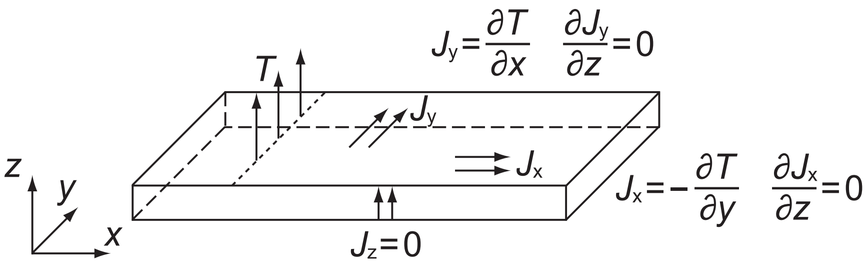
$$\nabla \times \left(\frac{1}{\sigma} \nabla \times \boldsymbol{T}_{\mathrm{f}} \right) + \frac{\mu_0}{4\pi} \frac{\partial}{\partial t} \int_V \frac{(\nabla \times \boldsymbol{T}_{\mathrm{s}}) \times \boldsymbol{r}}{r^3} dV = \boldsymbol{0}$$

Integrate along thickness of coated-conductor

$$\nabla \times \left(\frac{1}{\sigma} \nabla \times \boldsymbol{T}_{\mathrm{f}} \right) + \frac{\mu_0 t_{\mathrm{s}}}{4\pi} \frac{\partial}{\partial t} \int_S \frac{(\nabla \times \boldsymbol{T}_{\mathrm{s}}) \times \boldsymbol{r}}{r^3} dS = \boldsymbol{0}$$

Thin strip approximation

High cross-sectional aspect ratio of coated-conductor allows its use.



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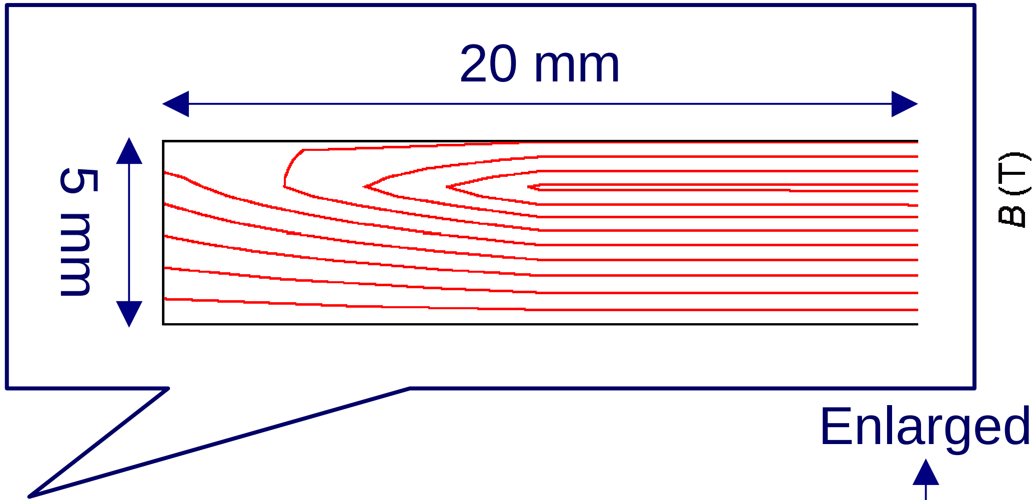
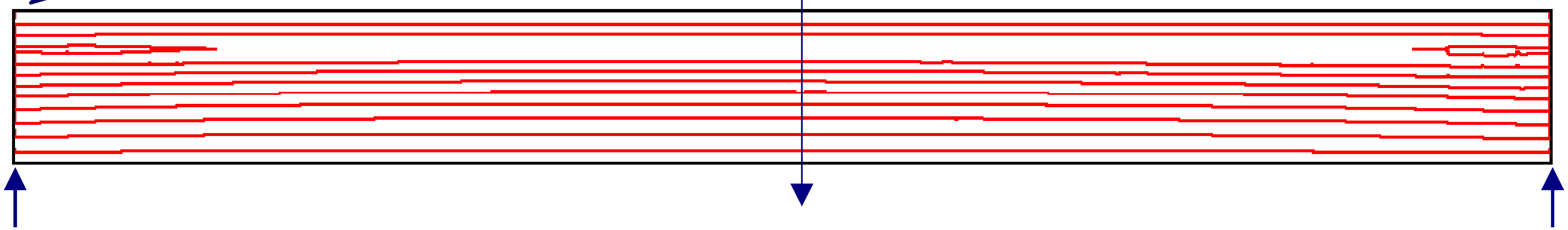
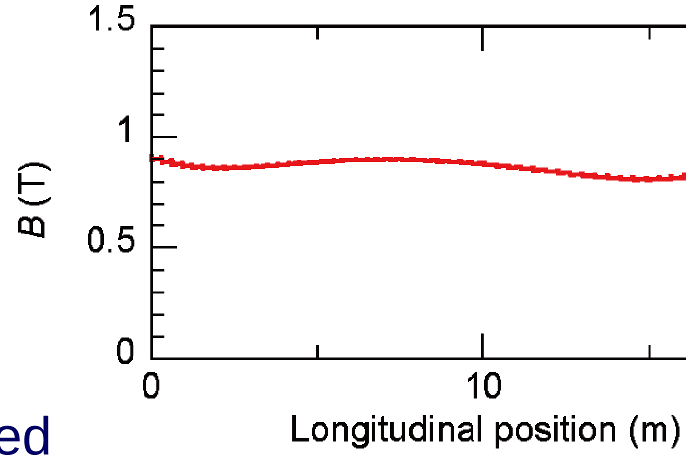
Results

Model SP: normal field component and current profile with induced shielding current







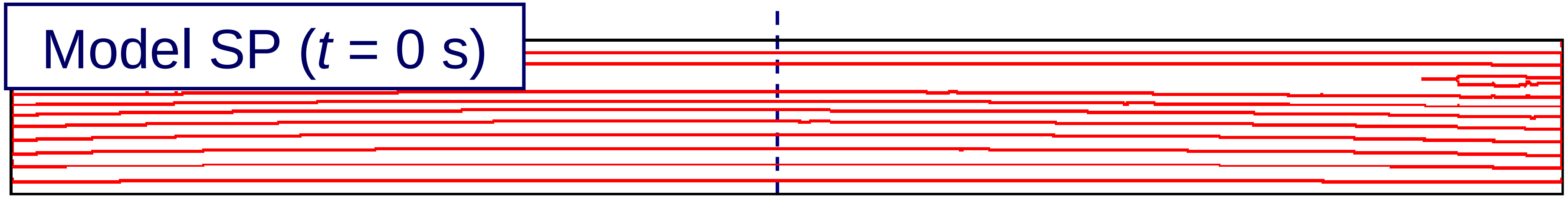
Current lines on the entire length of coated conductor of the top pancake coil
(t = 0 s)

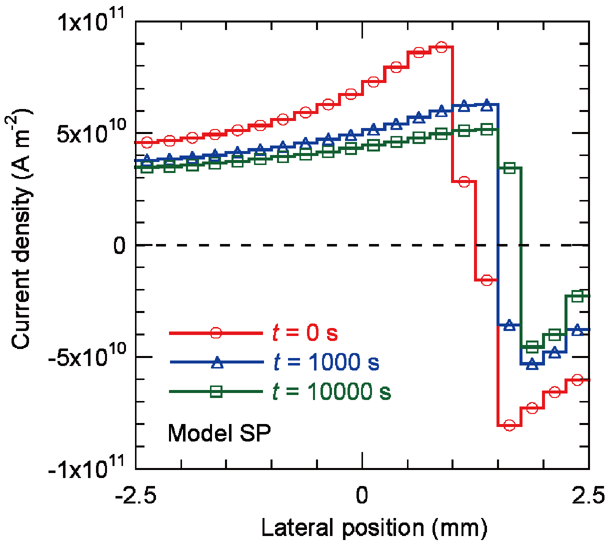
60 A / line

PC-S

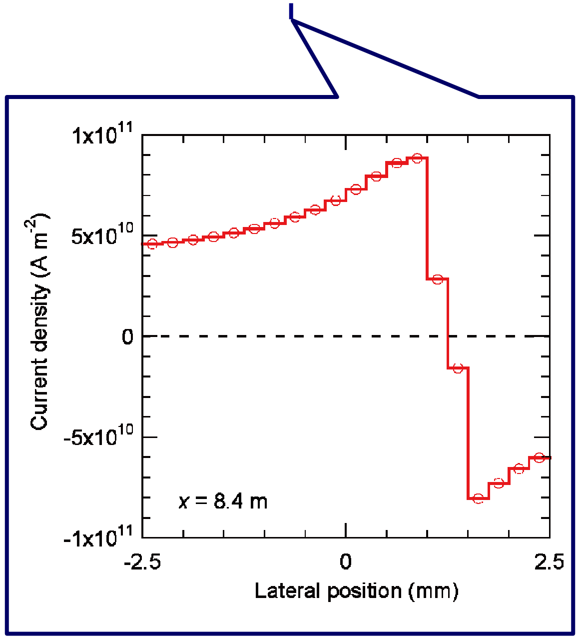
Lateral current distributions (top pancake)

Model SP (t = 0 s)

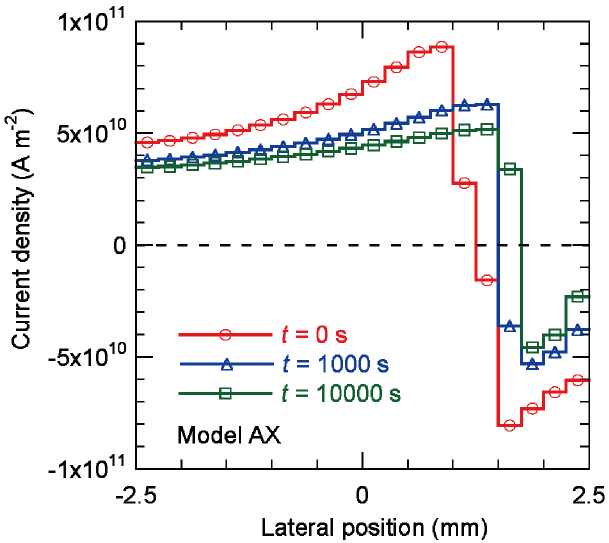




Model SP



Center of the 25th turn of top pancake



Model AX

$\Delta J/J \sim 10^{-4}$

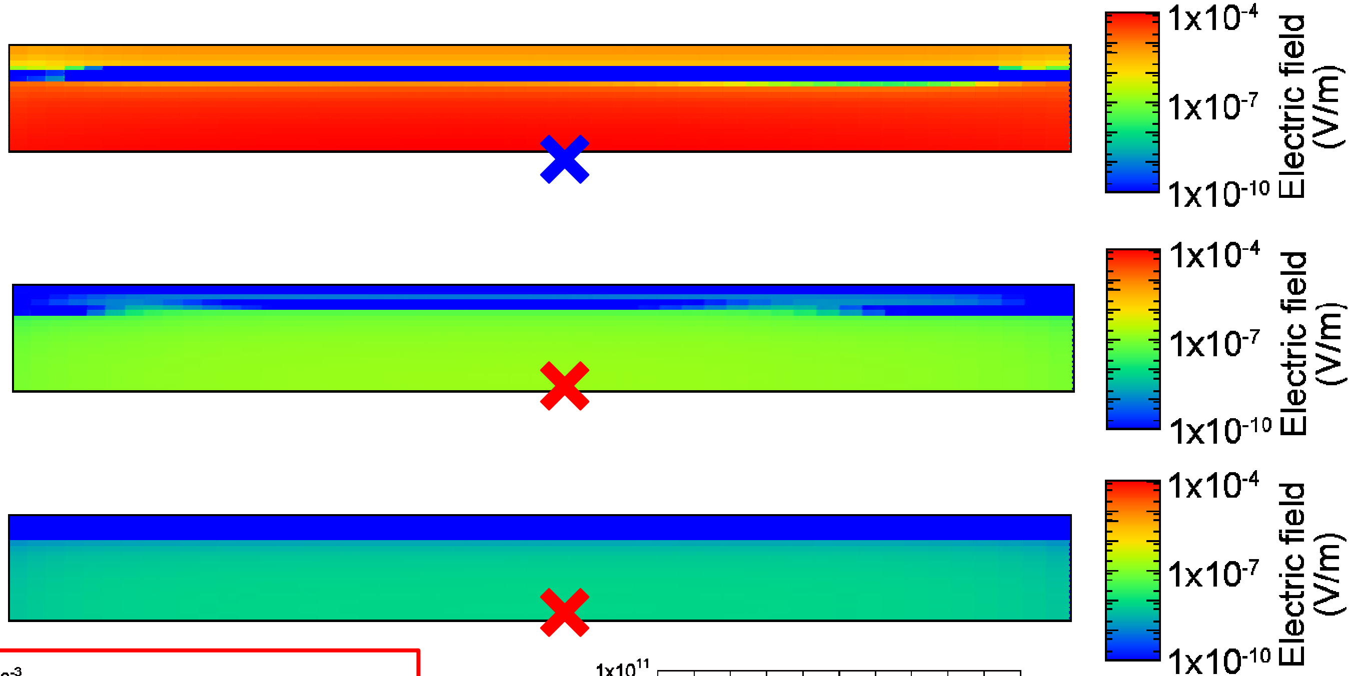
PC-S

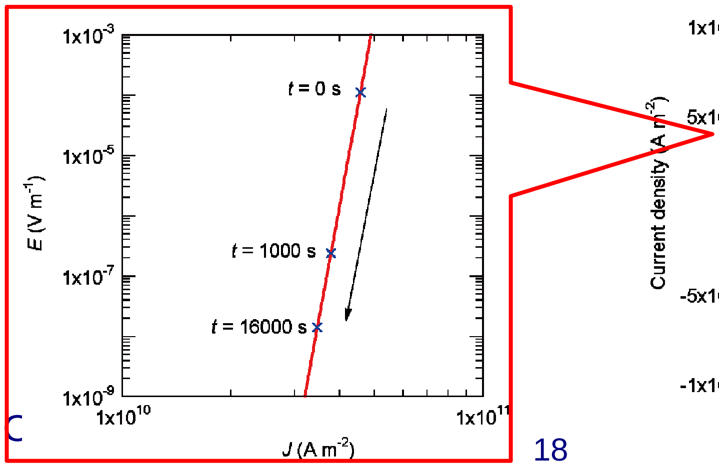
Electric field and change in current distribution

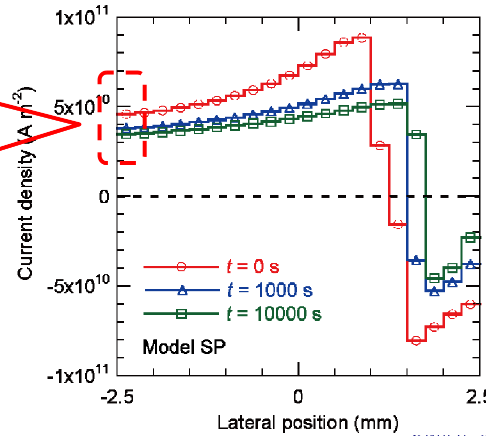
t = 0 s

t = 1000 s

t = 16000 s







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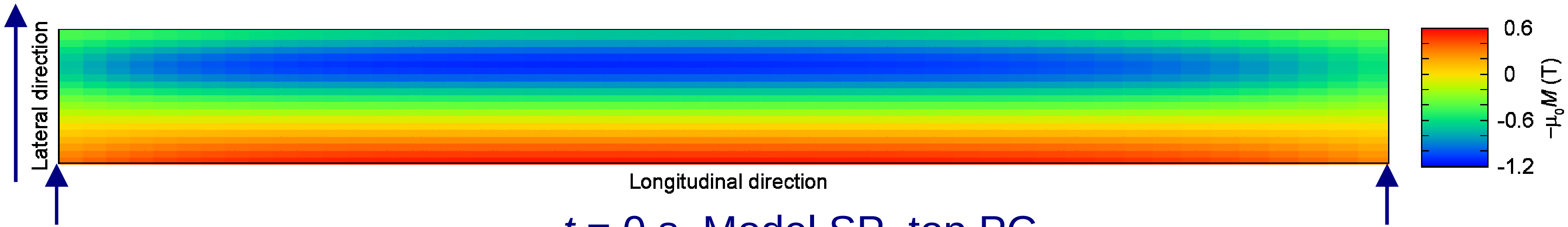
Magnetization distribution on coated conductor

$$-\mu_0 M = B - \mu_0 H$$

-$\mu_0 M$: magnetization
B: Magnetic flux density $\mu_0 H$: Field generated by other PCs

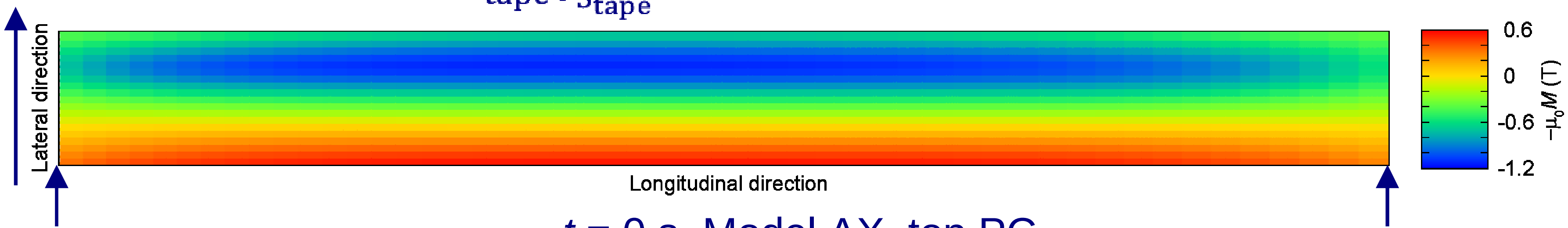


Inner end

t = 0 s, Model SP, top PC

Outer end

$$-\mu_0 M_{\mathrm{tape}} = \frac{1}{S_{\mathrm{tape}}} \int_{S_{\mathrm{tape}}} -\mu_0 M \mathrm{d}S = 345.9\ \mathrm{mT}$$



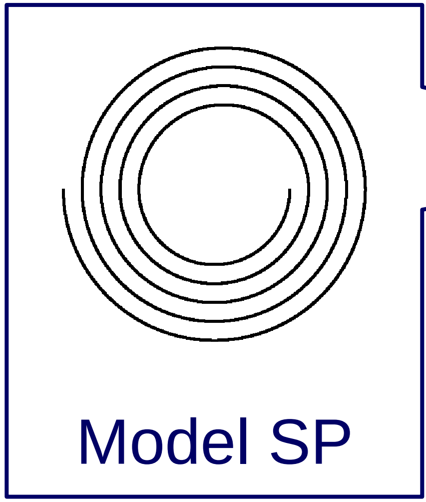
Inner end

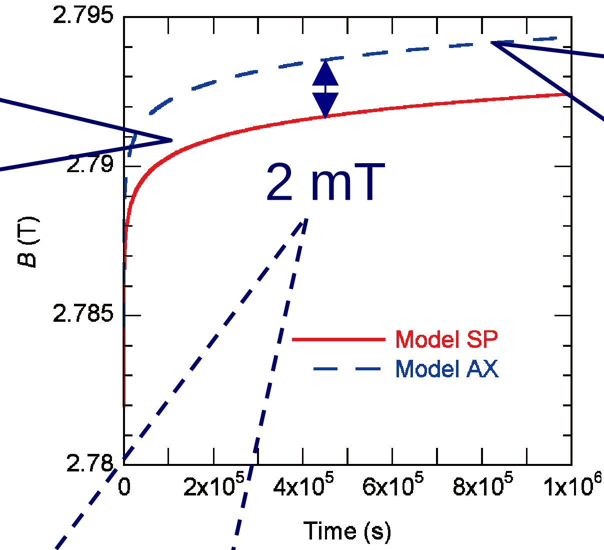
t = 0 s, Model AX, top PC

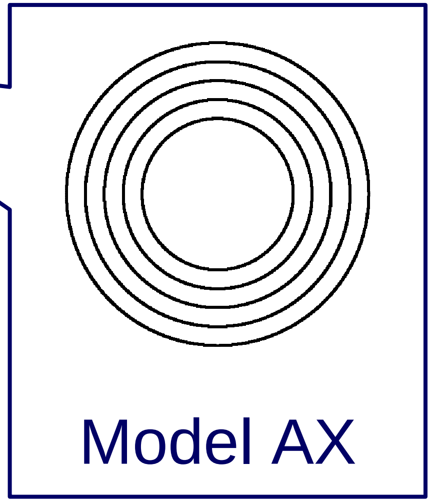
Outer end

$$-\mu_0 M_{\mathrm{tape}} = \frac{1}{S_{\mathrm{tape}}} \int_{S_{\mathrm{tape}}} -\mu_0 M \mathrm{d}S = 346.3\ \mathrm{mT}$$

Temporal evolution of magnetic field at coil center

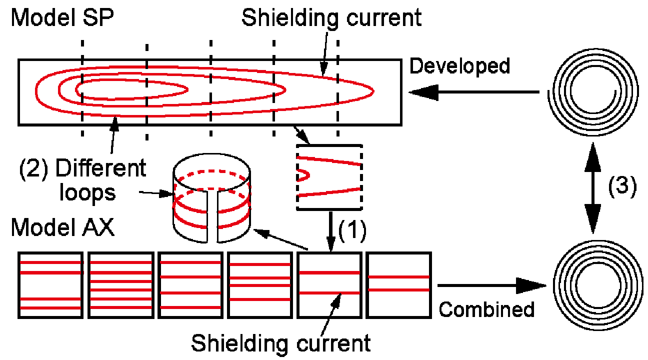






Is this difference caused by

- different current path: neglecting transverse *J*?
- different geometry?



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PC-S

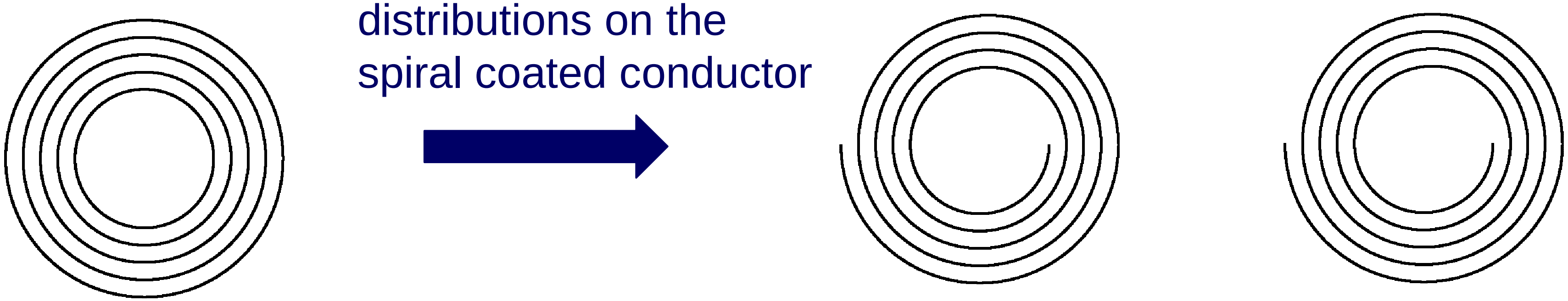
Compensation of influence of different geometry

Model AX

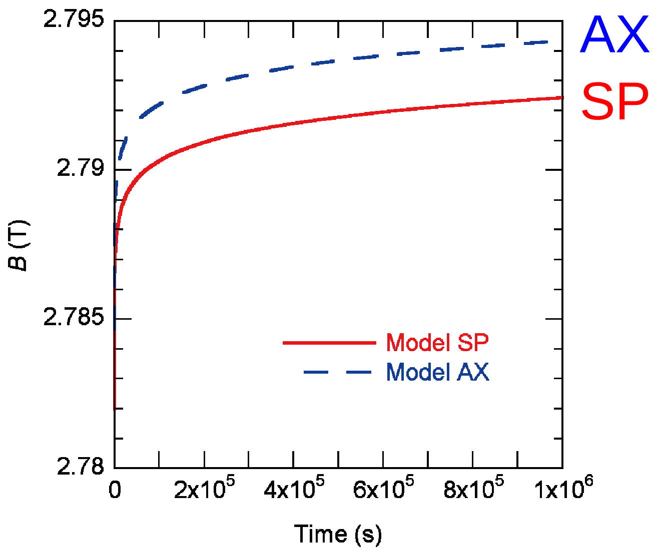
2. Projecting the current distributions on the spiral coated conductor

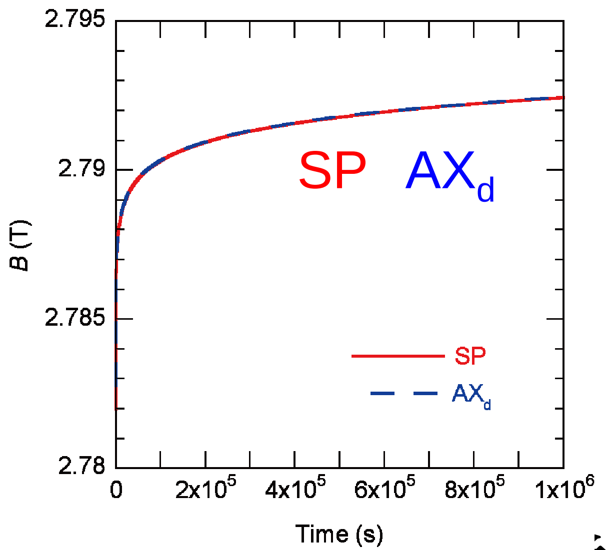
AX_d

Model SP

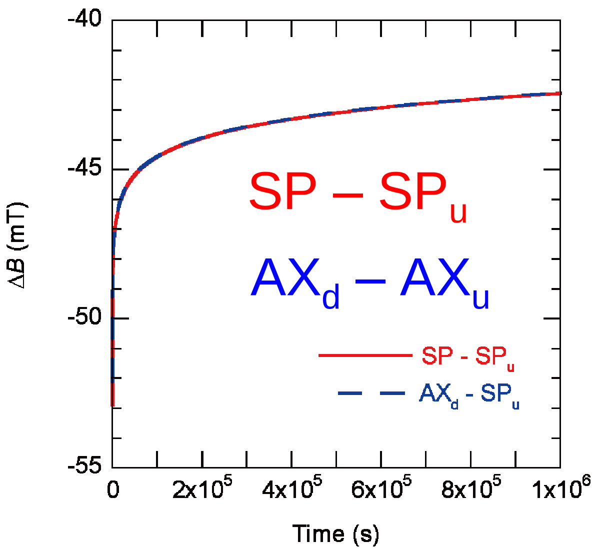


1. Calculating current distributions by using model AX

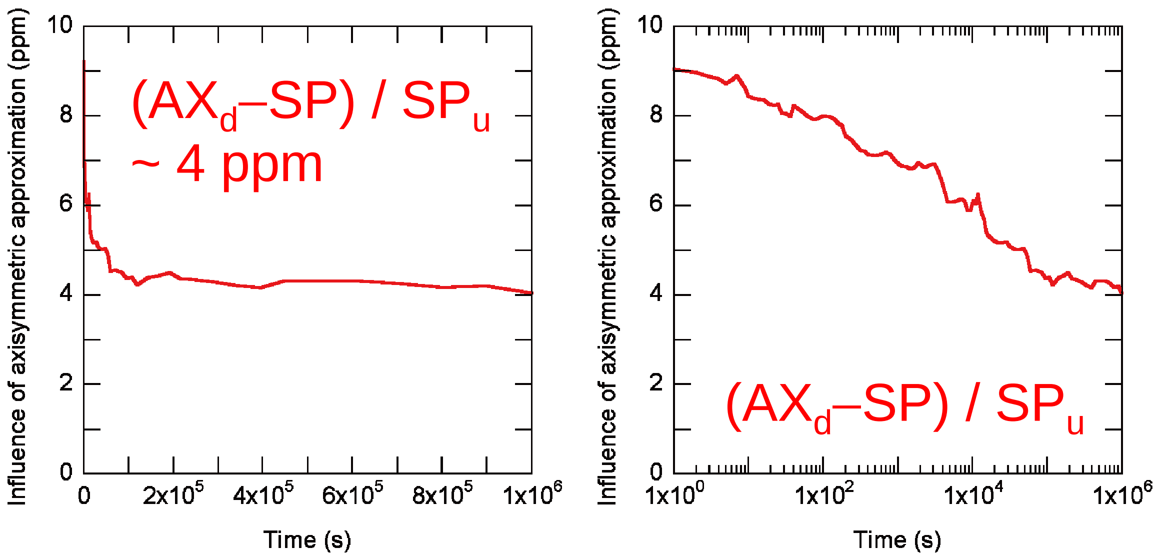




Influence of SCIF



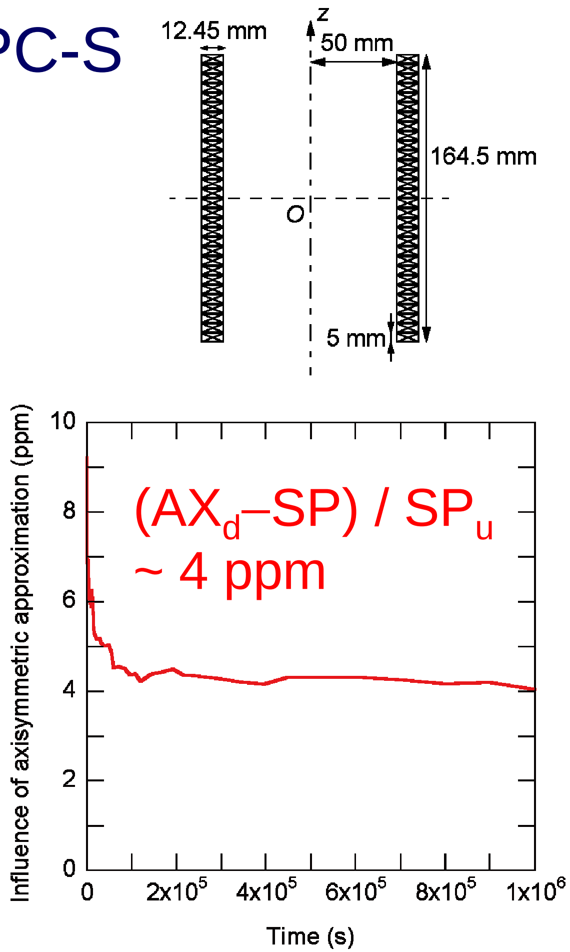
Influence of SCIF



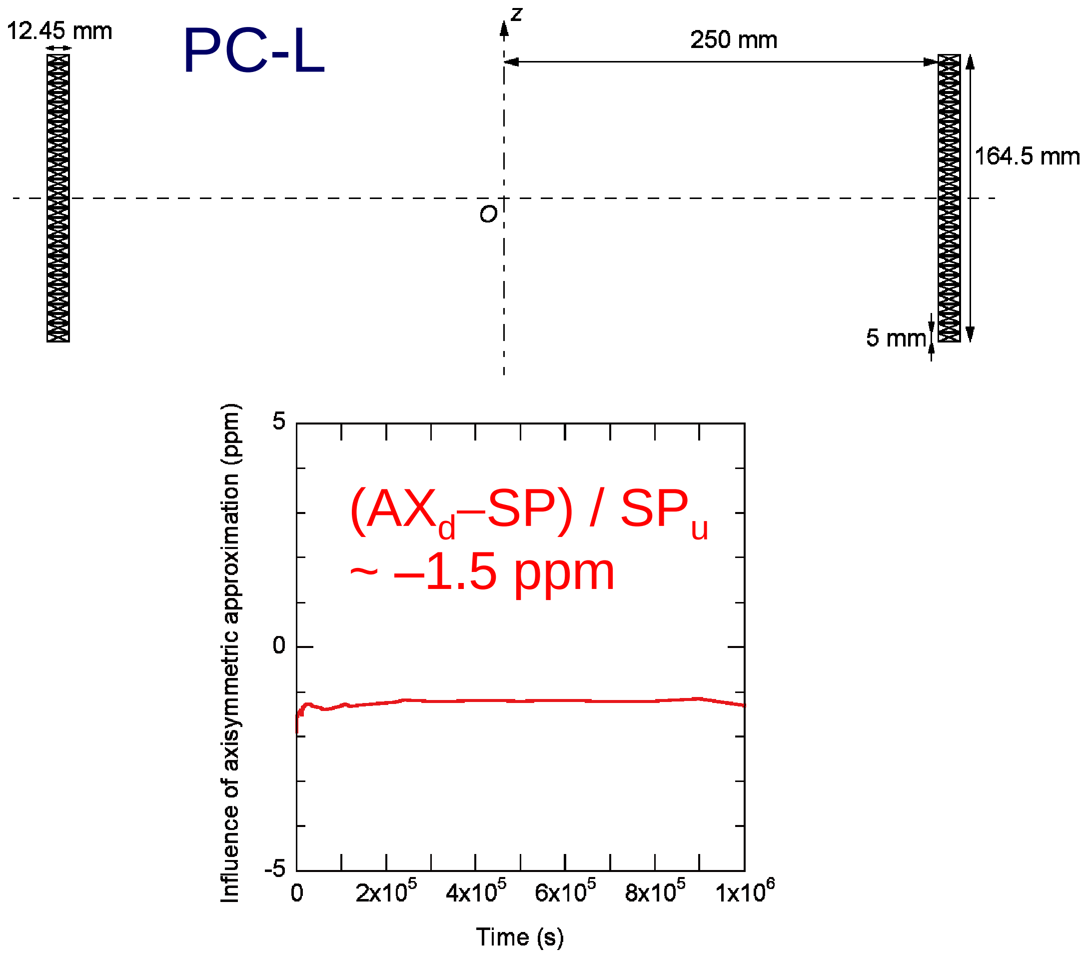
Difference between AX$_d$ and SP normalized by the field calculated with uniform current distribution

Comparison between PC-S and PC-L

PC-S



PC-L

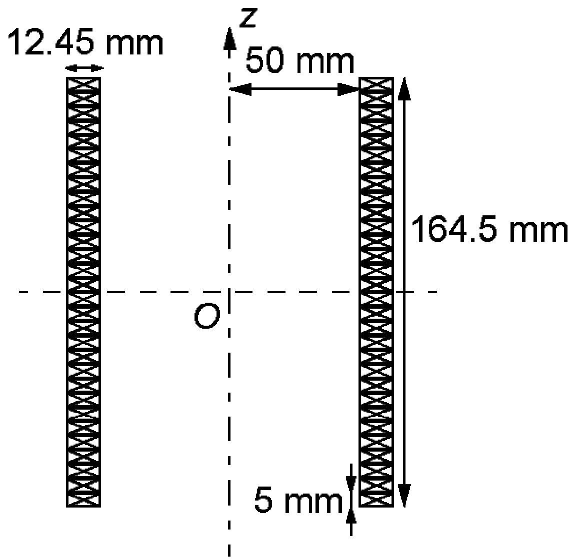


Solenoid coil

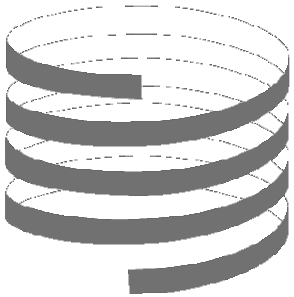
Models for analyses

Multilayered solenoid coil

	SL
Conductor width	5 mm
Conductor / SC layer thickness	0.2 mm / 2 μm
Inner / outer radius	50 mm / 62.5 mm
Number of turn per layer	30
Turn separation	0.5 mm
Conductor length per layer	12 m
Number of layers	50
Separation between layer	0.05 mm



Models for analyses



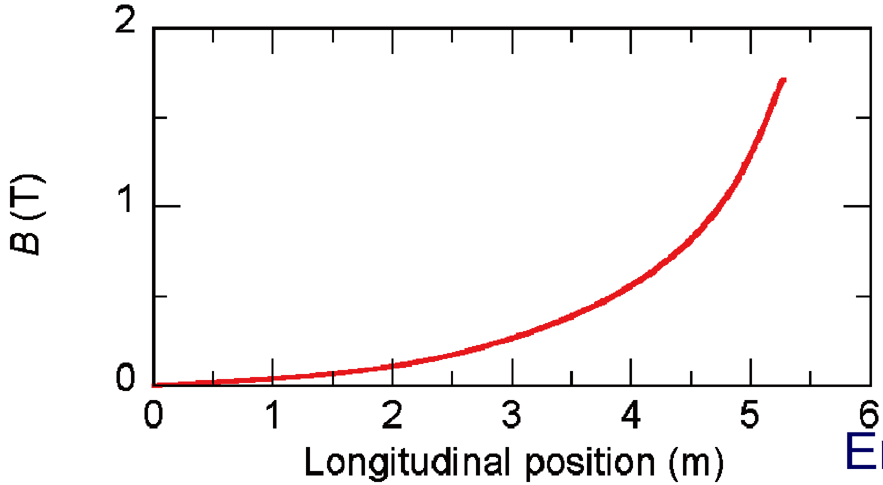
Model HX
(Exact spiral geometry)

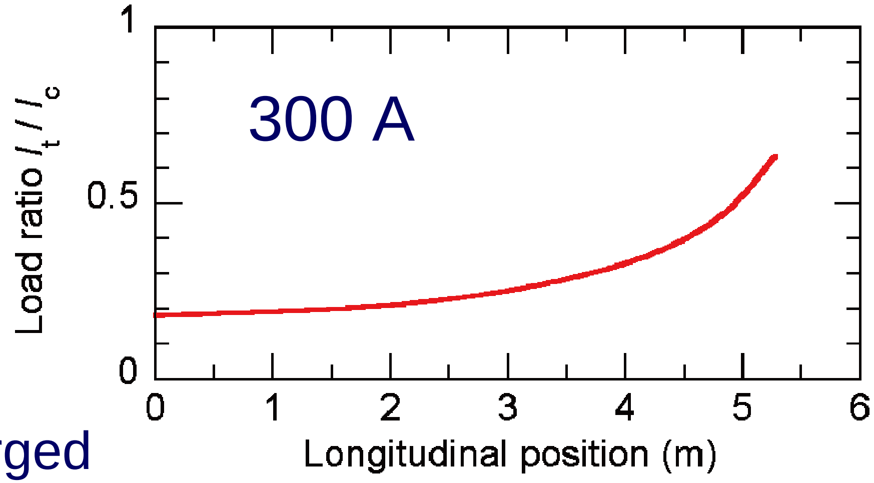
- ✓ The layer-by-layer analysis applied: not analyzed layers represented by sets of line currents in order to apply the field to the analyzed layer

Results

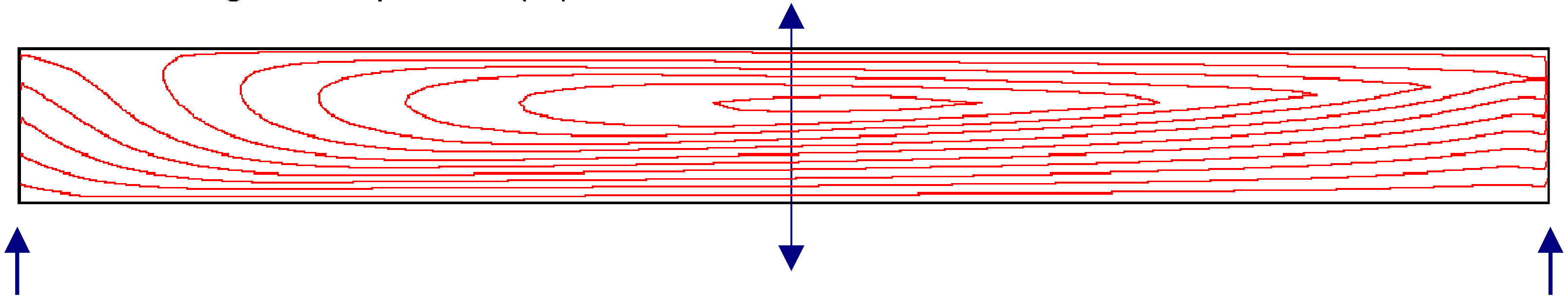
Model HX: normal field component, load ratio and current profile with induced shielding current

SL





Enlarged



Inner end

Current lines on the entire length of coated conductor of the 25th layer (t = 0 s)

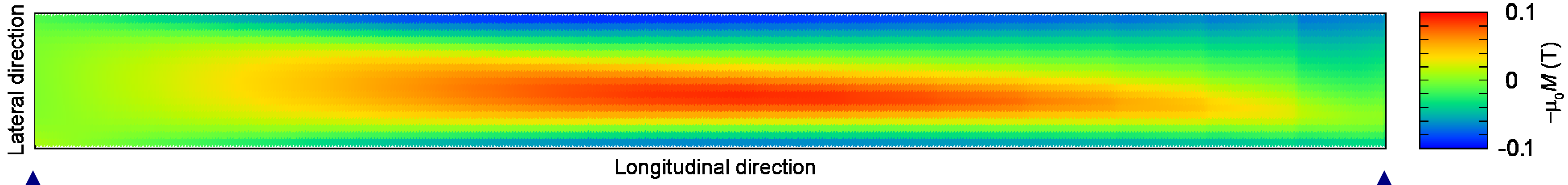
Outer end

60 A / line

Magnetization distribution on coated conductor

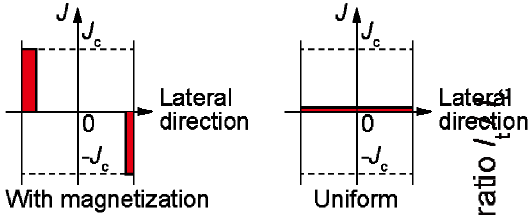
$$-\mu_0 M = B - \mu_0 H$$

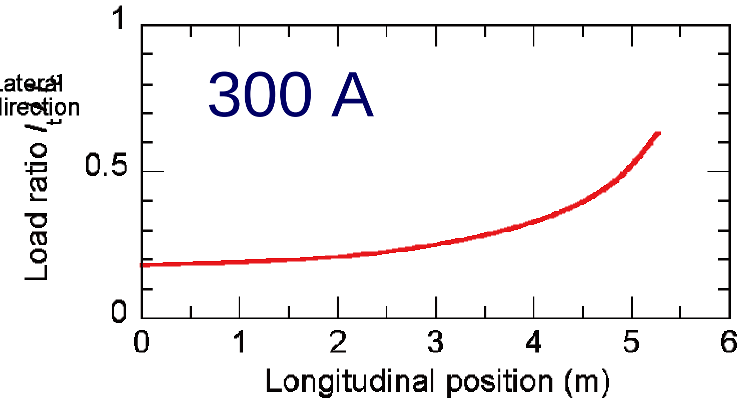
$-\mu_0 M$: magnetization
B: Magnetic flux density $\mu_0 H$: Field generated by other PCs

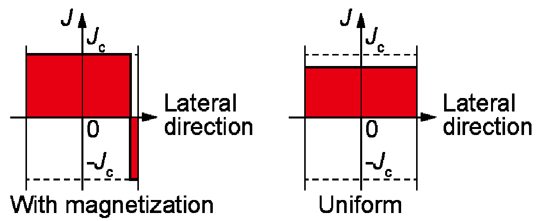


Center of layer End of layer

t = 0 s, Model HX, the 25th layer



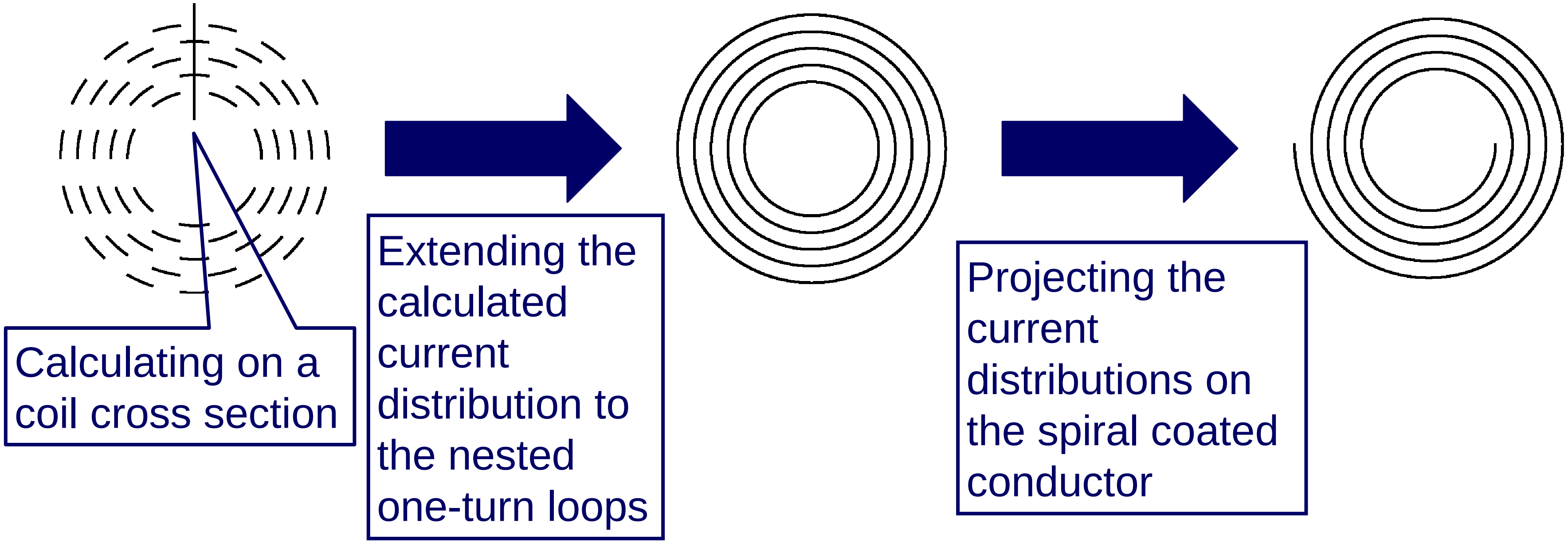




Low load ratio High load ratio

Conclusion

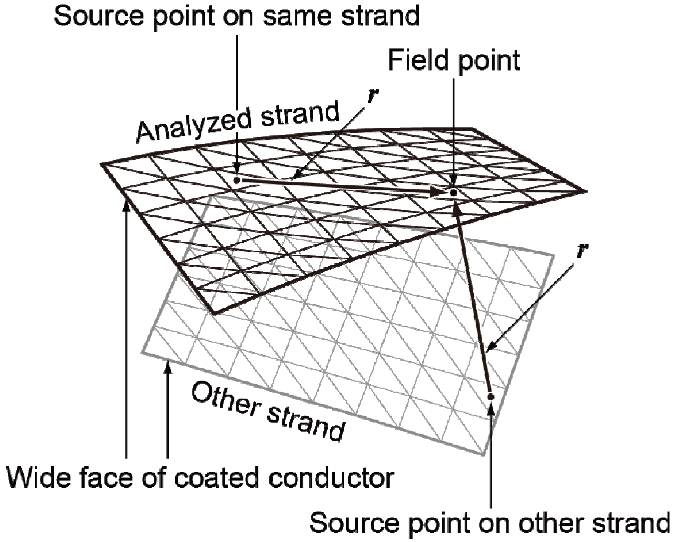
- The following scheme enables us to calculate the magnetic field of a stacked pancake coils accurately and with less CPU time and memory.



- The approximation of nested one-turn loops is also useful for non-axisymmetric coils such as saddle-shape coils.

Back-up slides

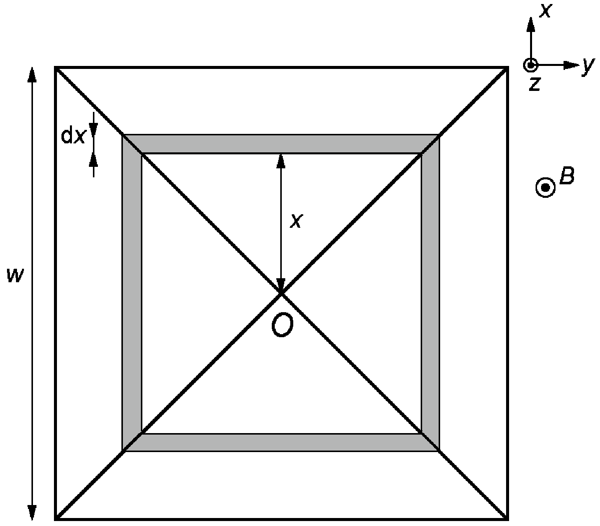
Consideration of three-dimensionally-curved coated conductors



$$\nabla \times \left(\frac{1}{\sigma} \nabla \times \boldsymbol{n} T \right) \cdot \boldsymbol{n} + \frac{\partial}{\partial t} \left(\frac{\mu_0 t_\mathrm{s}}{4\pi} \int_{S'} \frac{(\nabla \times \boldsymbol{n}' T') \times \boldsymbol{r} \cdot \boldsymbol{n}}{r^3} \mathrm{d}S' + \boldsymbol{B}_\mathrm{ext} \cdot \boldsymbol{n} \right) = 0$$

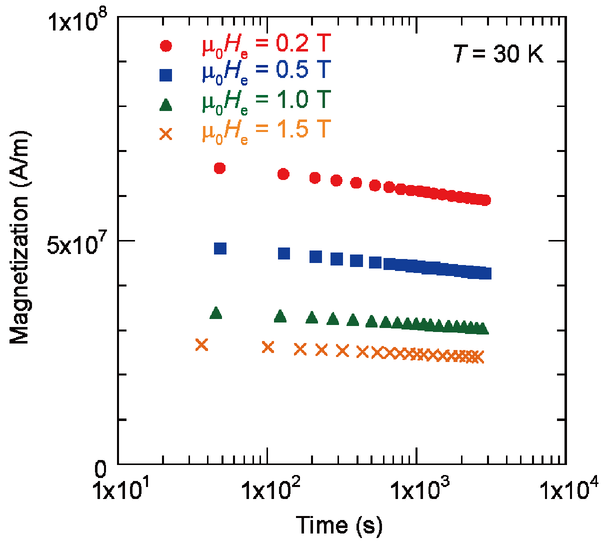
This term representing ***B*** in Faraday's law is calculated by Biot-Savart's law based on currents on arbitrary 3D-shaped conductors

Magnetization measurements to determine *E*–*J*



$$J = \frac{6m}{w^3 d} \qquad E = -\frac{\mu_0 G}{4wd} \cdot \frac{\mathrm{d}m}{\mathrm{d}t}$$

d: thickness of superconductor
G: geometrical factor 7.17×10^{-4}
m: magnetization of sample



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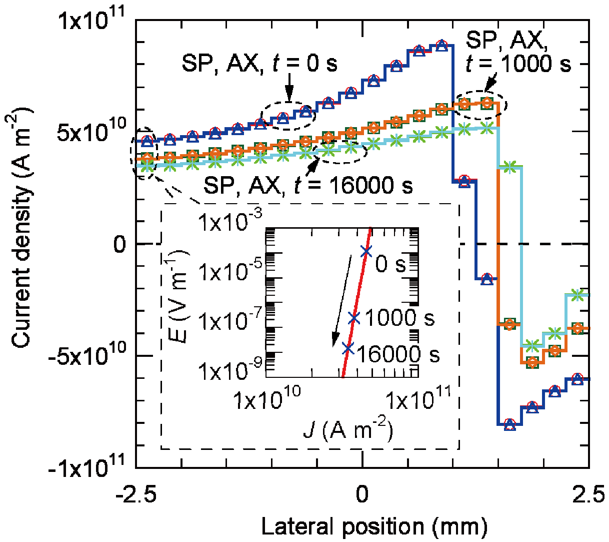
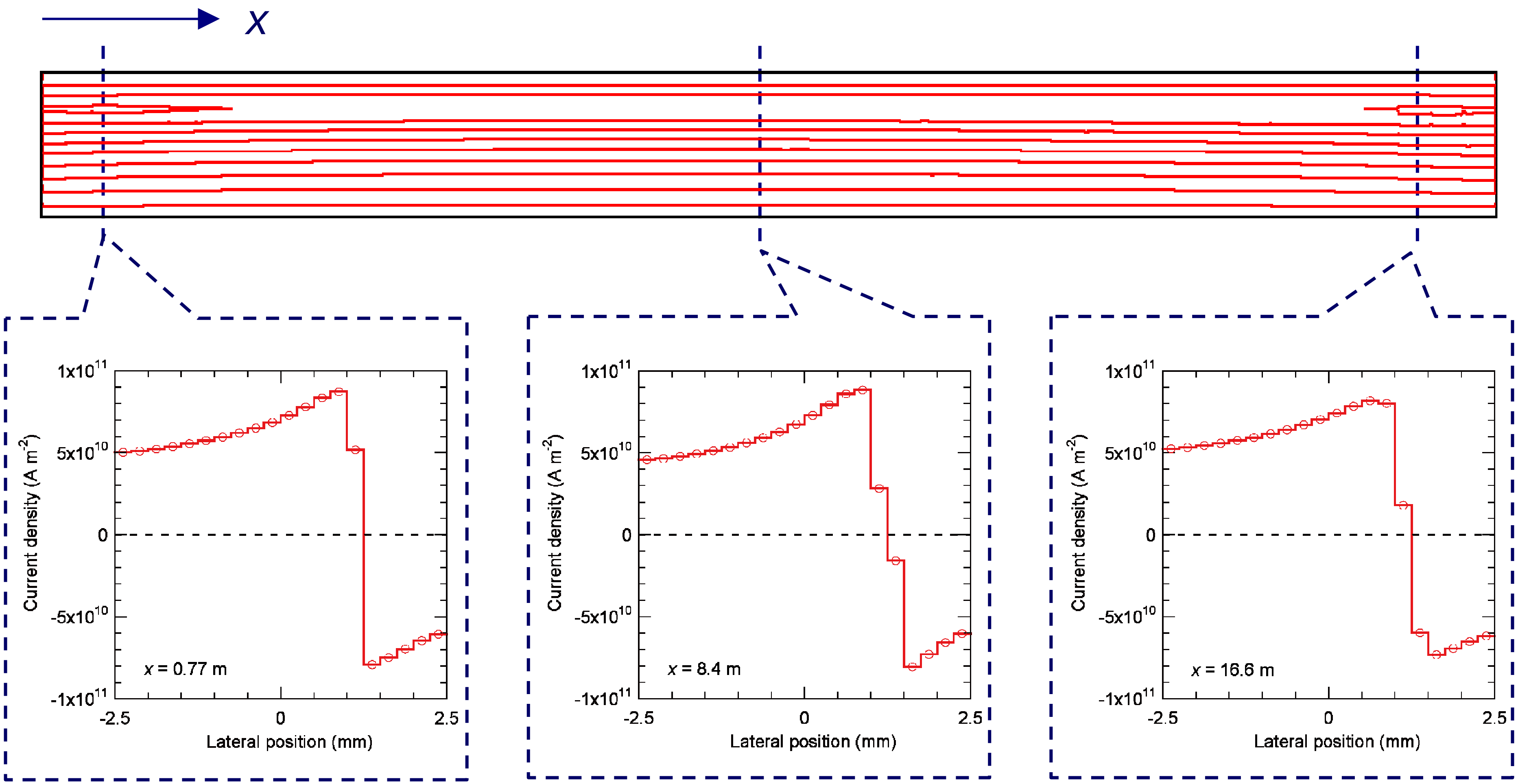
Figures not used

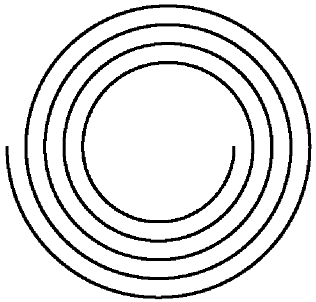


Lateral current distributions



PC-S

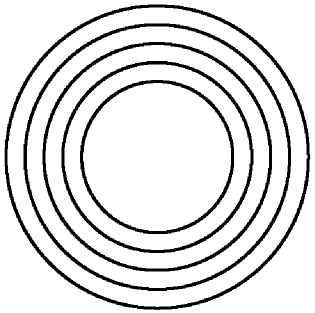
Influence of different geometry (uniform current)



Model SP

Magnetic field calculated with uniform current distribution	
Model SP (SP_u)	Model AX (AX_u)
2.83488 T	2.83686 T

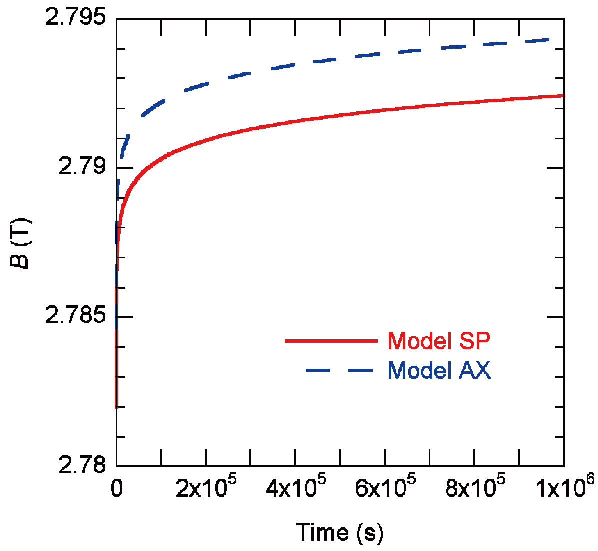
Difference ~ 2 mT

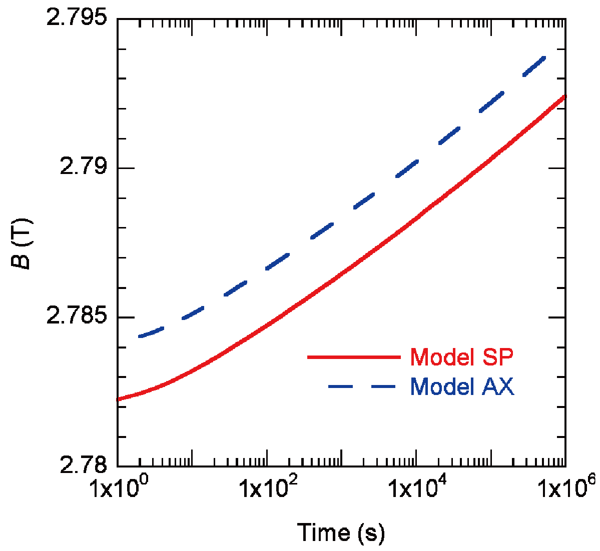


Model AX

PC-S

Temporal evolution of magnetic field at coil center

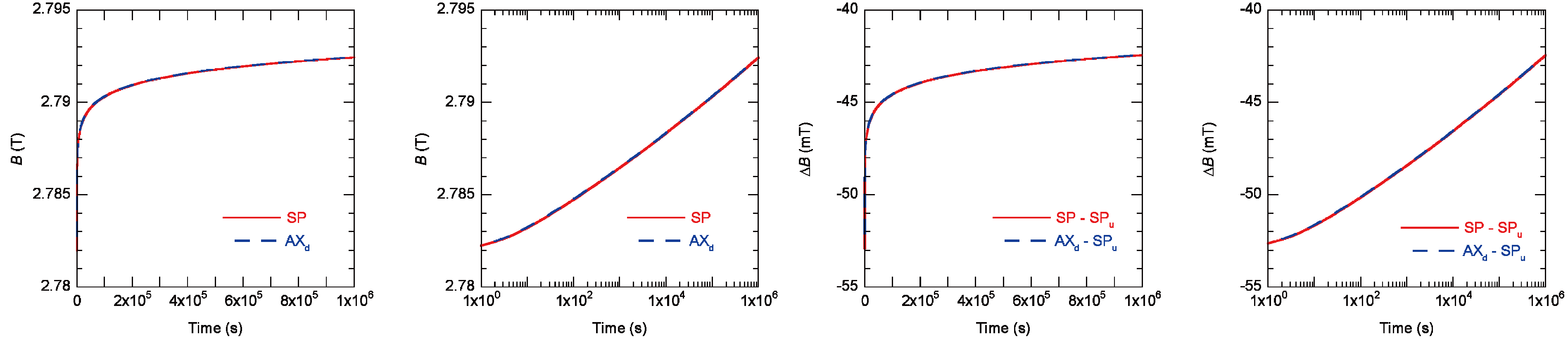




Log plot

PC-S

SP, AX, AX_d (linear, log, values)



遮蔽電流の影響を考慮した場合の発生磁場（電磁界解析結果）		
Model SP (SP)	Model AX (AX)	Model AX (AX_d)
2.78198 T (t = 0 s)	2.78389 T (t = 0 s)	2.78201 T (t = 0 s)

軸対称近似のコイル発生磁場に対する影響
$(AX_d–SP) / SP_u = 9.2$ ppm

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3. 解析結果

異なる時刻での軸対称近似が磁場に与える影響

遮蔽電流の影響を考慮した場合の発生磁場（電磁界解析結果）			
ランプアップ終了からの経過時間*t*	SP	AX_d	$(AX_d–SP) / SP_u$
0 s	2.7820 T	2.7820 T	9.2 ppm
100 s	2.7847 T	2.7848 T	8.0 ppm
10000 s	2.7883 T	2.7884 T	6.1 ppm
1000000 s	2.7924 T	2.7924 T	4.0 ppm

軸対称近似のコイル発生磁場に対する影響の時間変化は非常に小さい

↓

軸対称近似の適用による遮蔽電流の減衰時定数への影響は小さい

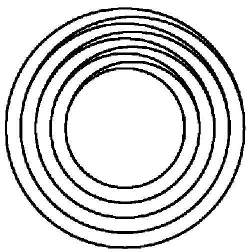
3. 解析結果

PC群中心での磁場のモデル間での差異に関する検討

線材内に電流が一様に分布していた場合の発生磁場	
Model SP (SP_u)	Model AX (AX_u)
1.05396 T	1.05442 T

コイル形状が異なることによって
0.5 mTの差が発生する。
（半径が大きくなった分、コイル形状の違いの影響も小さくなった）

遮蔽電流の影響を考慮した場合の発生磁場（電磁界解析結果）		
Model SP (SP)	Model AX (AX)	Model AX (AX_d)
1.05162 T (t = 0 s)	1.05208 T (t = 0 s)	1.05161 T (t = 0 s)

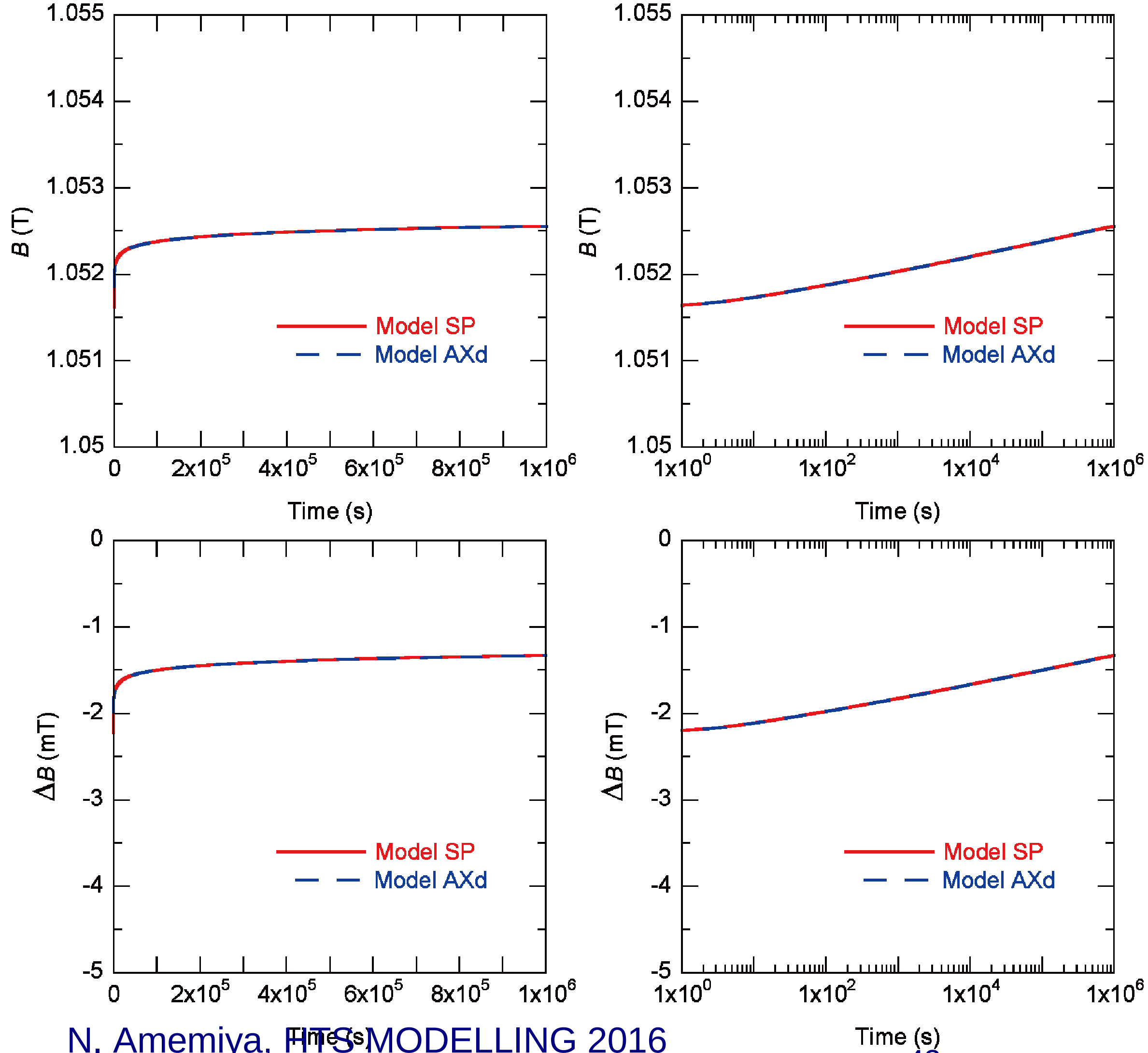


軸対称近似の厳密形状得基わた線材内電流写像を

軸対称近似のコイル発生磁場に対する影響
$(AX_d – SP) / SP_u$ = -1.9 ppm

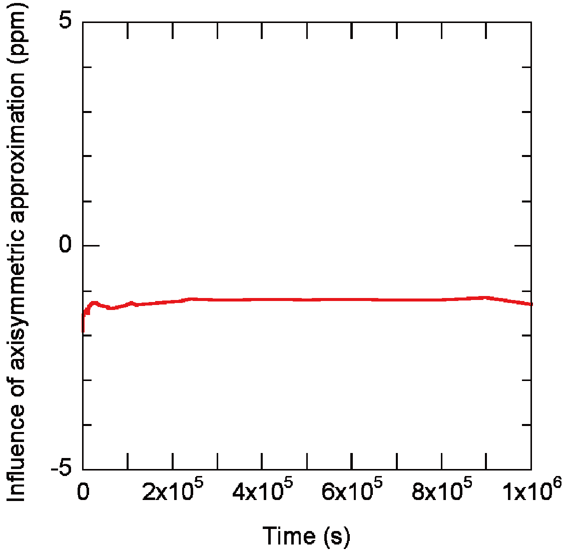
PC-L

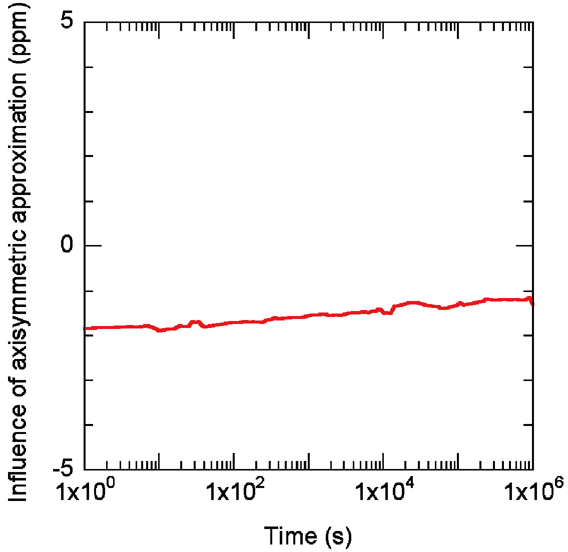
遮蔽電流の磁場への影響の時間変化



r = 50 mmの場合よりも
遮蔽電流磁場の減衰は小さい

軸対称近似の影響の変化

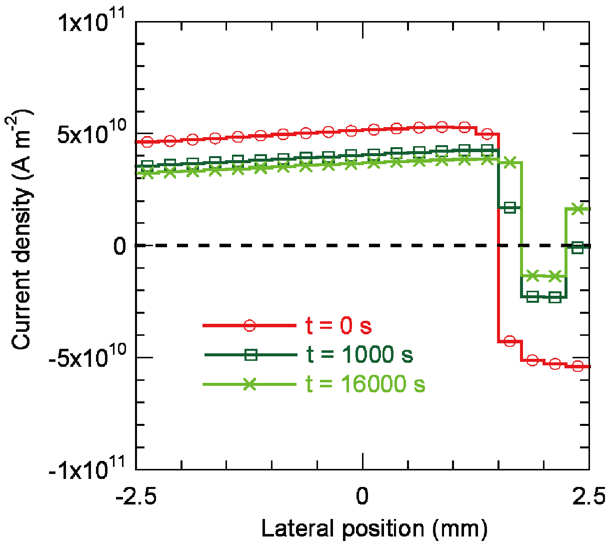


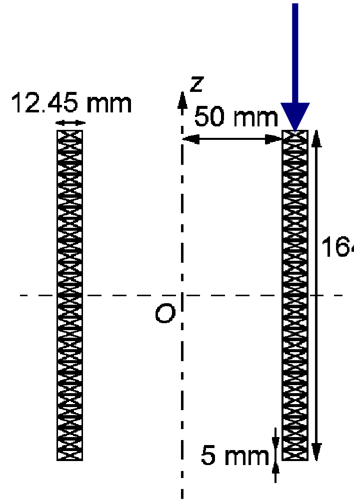


コイル半径が5倍になったとき、軸対称近似の影響は約1/4倍になった。
→コイル径が大きくなればなるほど、軸対称近似による解析で十分になる可能性が高い

SL

幅方向電流密度分布



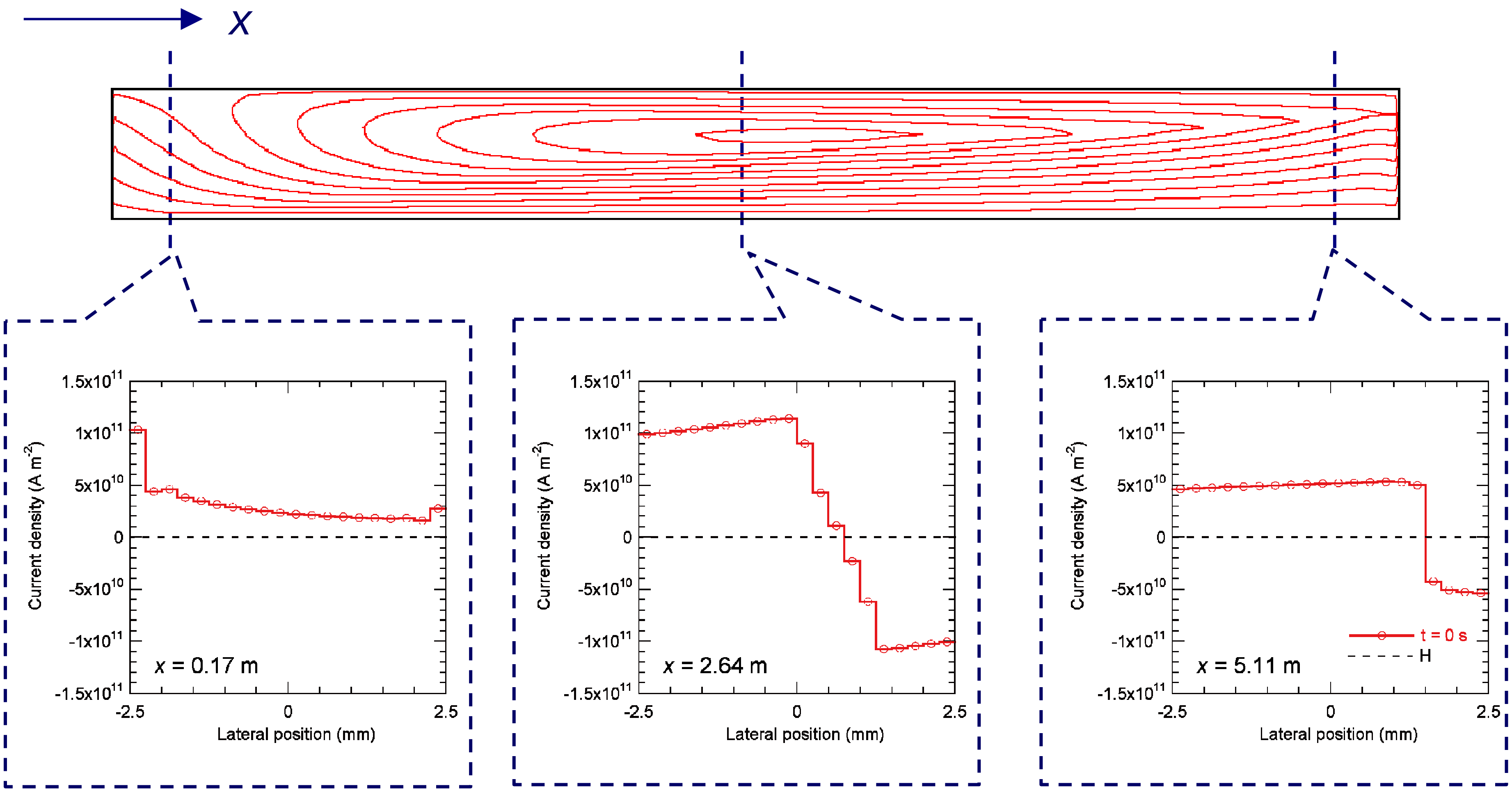


ソレノイドコイル25層目、30ターン目中央

外部磁場の与え方が異なっているため直接比較できないが、パンケーキコイルの場合とは分布が多少異なる。

時間変化の様子はパンケーキコイルと同様

Lateral current distributions



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