Discontinuity lines in Nb thin films with artificial border micro-indentations

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Ref: J. Brisbois, O.-A. Adami, J. I. Avila, M. Motta, W. A. Ortiz, N. D. Nguyen, P. Vanderbemden, B. Vanderheyden, R. B. G. Kramer, A. V. Silhanek, PRB **93**, 054521 (2016)

Defects in superconductor films

- Defects play a key role in type II superconductors acting as pinning sites for vortices, allowing dissipation-less current transport in the mixed state.
- Defects in the borders generate long range regions where the current changes direction abruptly: the discontinuity or *d-lines*.

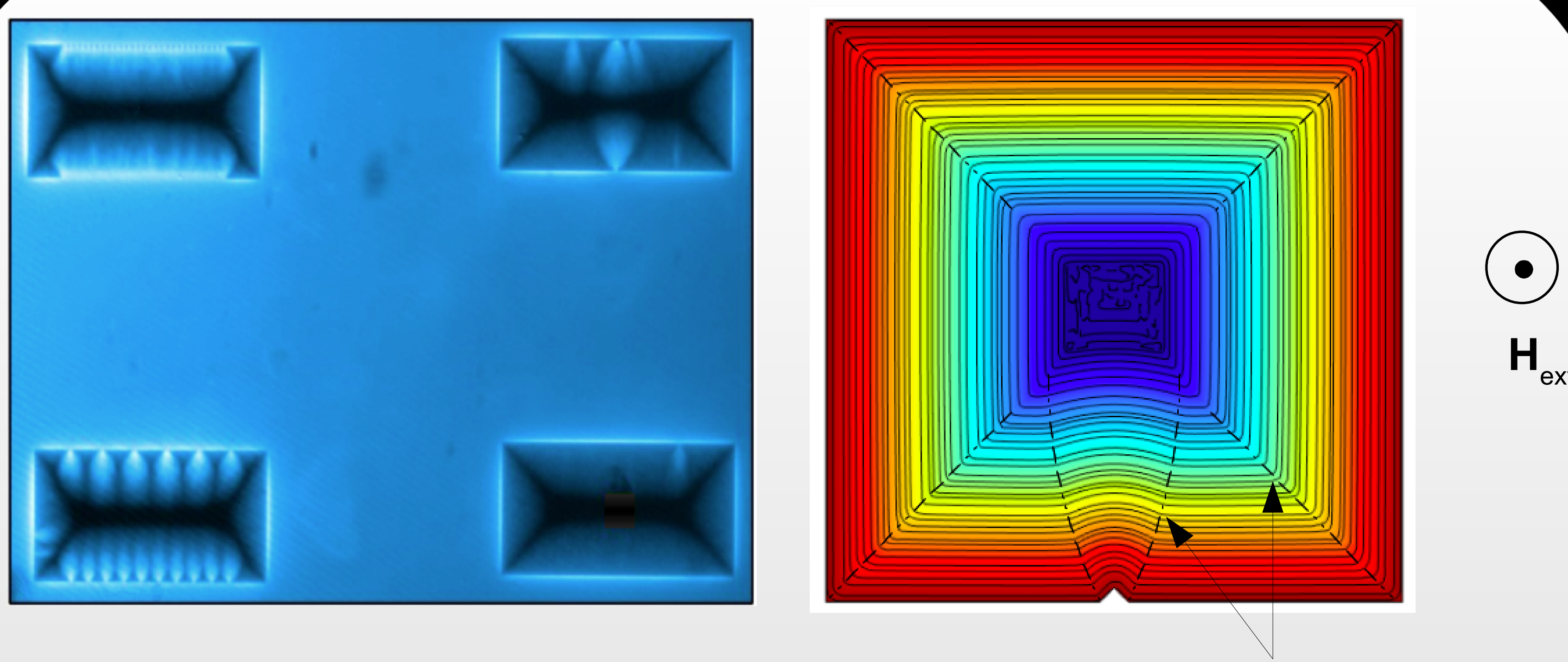


FIG.I1: Left, Magneto-optical image of thin film samples with defects,
Right: representation of d-lines

Simple current model: current has uniform magnitude j_c

Simple d-lines model: d-lines are equidistant from 2 nearest straight edges. For semi-circular and triangular defect of height **R**, its associated d-line will be (BB model):

$$y \approx \frac{x^2}{2R} - \frac{R}{2} \qquad \text{(I1)}$$

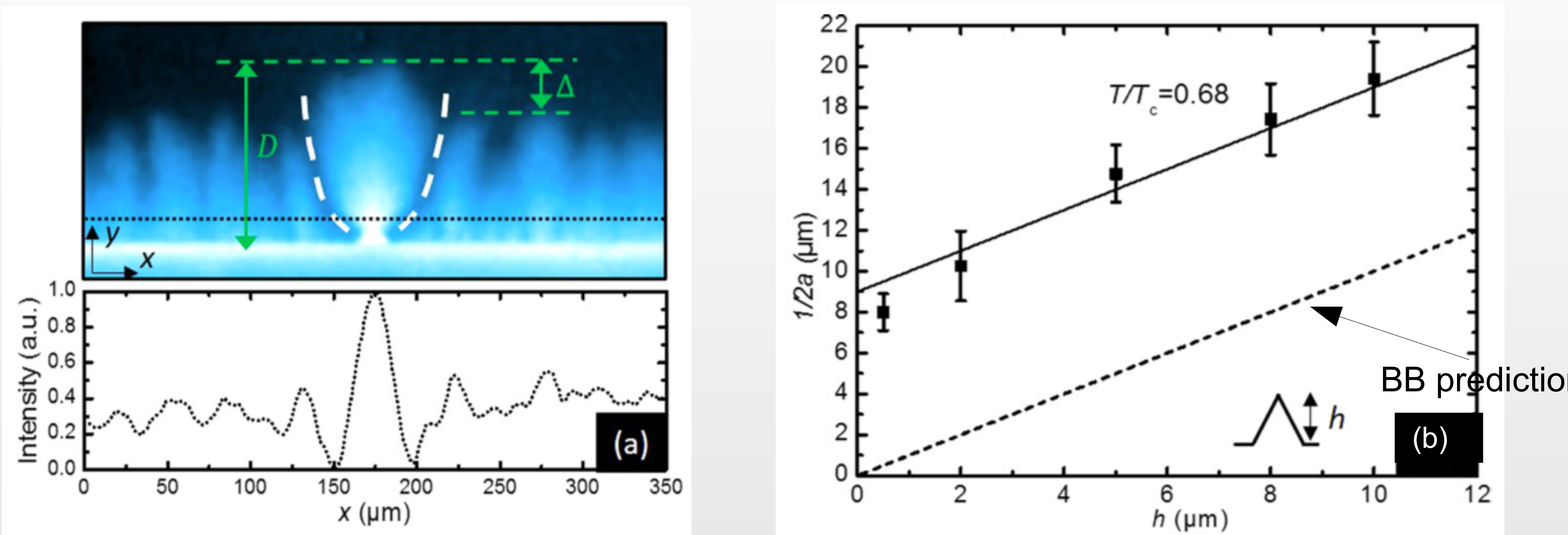


FIG.I2: (a) upper flux penetration in triangular indentation of 10μm height and lower intensity profile along dotted line. (b) 1/2*a* vs triangular defect height, where parameter '*a*' is obtained from d-line shape fit $y=ax^2+b$

From Eqn (I1)

$$\frac{1}{2a}=R=h \qquad \text{(I2)}$$

Experimental values much higher than predicted!

-Calculating the magnetic field distribution of a 2D superconductor in the mixed state. Two cases: Local and Non-Local

Local or longitudinal case:

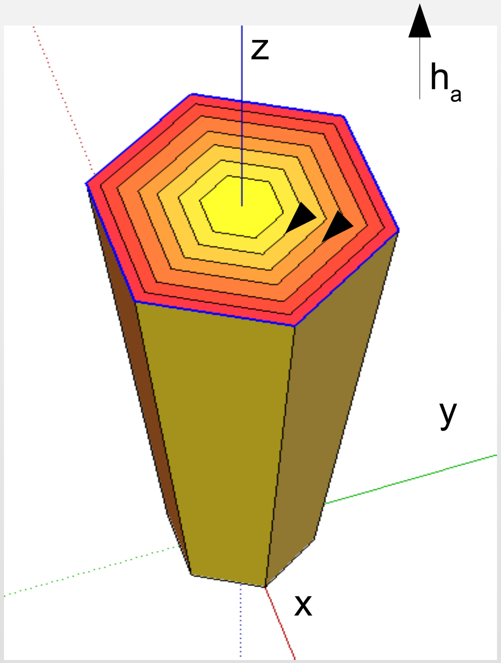


FIG.1

Neglecting displacement currents, Maxwell eqns are:

$$\boldsymbol{j} = \nabla \times \boldsymbol{H} \qquad (1)$$
$$\mu_0 \frac{\partial \boldsymbol{H}}{\partial t} = -\nabla \times \boldsymbol{E} \qquad (2)$$

Replacing (1) in the constitutive relationship, and replacing in (2):

$$\boldsymbol{E} = \rho\, \boldsymbol{j} = \rho \nabla \times \boldsymbol{H} \qquad (2)$$
$$\mu_0 \frac{\partial \boldsymbol{H}}{\partial t} = -\nabla \times (\rho \nabla \times \boldsymbol{H}) \qquad (3)$$

Because of the symmetry, the currents must be planar, and therefore $\boldsymbol{H}$ must point in the *z* direction and be independent of *z*:

$$\boldsymbol{H} = \left(h(x, y, t) + h_a(t)\right)\hat{z} \qquad (4)$$

$$\rho = \rho(x, y) \qquad (5)$$

$$\frac{\partial h}{\partial t} = \nabla \cdot (\rho \nabla h)/\mu_0 - \frac{\partial h_a}{\partial t} \qquad (6)$$

Boundary condition h=0

Integrated in COMSOL Multyphysics

It follows that h is also a current stream-function, since:

$$\boldsymbol{j} = \nabla \times (h\hat{z}) \qquad (7)$$

And then $$j = |\boldsymbol{j}| = |\nabla \times (h\hat{z})| = |\nabla h| \qquad (8)$$

Ref: E. H. Brandt, PRB **52**, 21 (1995), 15442

Magnetic field in thin film: non-local or transversal case

Introducing the sheet current:

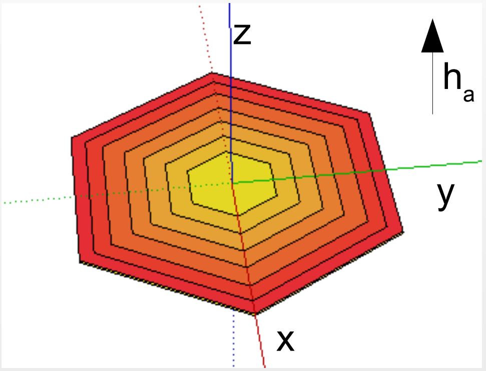


FIG.3

$$\boldsymbol{J} = d\,\boldsymbol{j} \qquad (12)$$

Where d is the thickness of the film, and introducing the stream function g:

$$\boldsymbol{J} = \nabla \times (g\,\hat{z}) \qquad (13)$$

The current induced field H_d time derivative is now given by:

$$\frac{\partial H_d}{\partial t} = \iint Q(r, r') \frac{\partial g(r')}{\partial t} dS', \qquad \text{with} \qquad (14)$$

$$Q(r, r') = \lim_{z \to 0} \frac{1}{4\pi} \frac{2z^2 - R^2}{(z^2 + R^2)^{5/2}}, \qquad R = |\boldsymbol{r} - \boldsymbol{r}'| \qquad (15)$$

And the equation for the time evolution of g is given by:

$$\nabla\cdot(\rho\nabla g)/d\mu_0=\frac{\partial H_a}{\partial t}+\frac{\partial H_d}{\partial t} \qquad (17)$$

$$\nabla\cdot(\rho\nabla g)/d\mu_0=\frac{\partial H_a}{\partial t}+\iint Q\frac{\partial g}{\partial t} \qquad (18)$$

Implemented in Matlab

Integration of eqn (18) is not trivial as $\partial g/\partial t$ is inside the integral.
Here an algorithm based on space Fourier transforms (F) was used.

$$\frac{\partial H_d}{\partial t}=\iint Q(\mathbf{r}-\mathbf{r}')\frac{\partial g}{\partial t},\ \text{apply}\ F() \qquad (19)$$

$$F(\frac{\partial H_d}{\partial t})=F(\iint Q(\mathbf{r}-\mathbf{r}')\frac{\partial g}{\partial t})=\frac{k}{2}F(\frac{\partial g}{\partial t}) \qquad (20)$$

$$\frac{\partial g}{\partial t}=F^{-1}\left(\frac{2}{k}F\left(\frac{\partial H_d}{\partial t}\right)\right) \qquad (22)$$

$$\frac{\partial g}{\partial t}=F^{-1}\left(\frac{2}{k}F\left(\nabla\cdot(\rho\nabla g)/d-\frac{\partial H_a}{\partial t}\right)\right) \qquad (23)$$

Where it was used that the Fourier transform of $Q(\mathbf{r}-\mathbf{r}')$ is $k/2$

Ref: J I Vestgården, P Mikheenko, Y M Galperin, T H Johansen, *New Journal of Physics* 15 (2013) 093001

-What about the resistivity?

In a type II superconductor, the resistivity above H_{c1} can be modeled by:

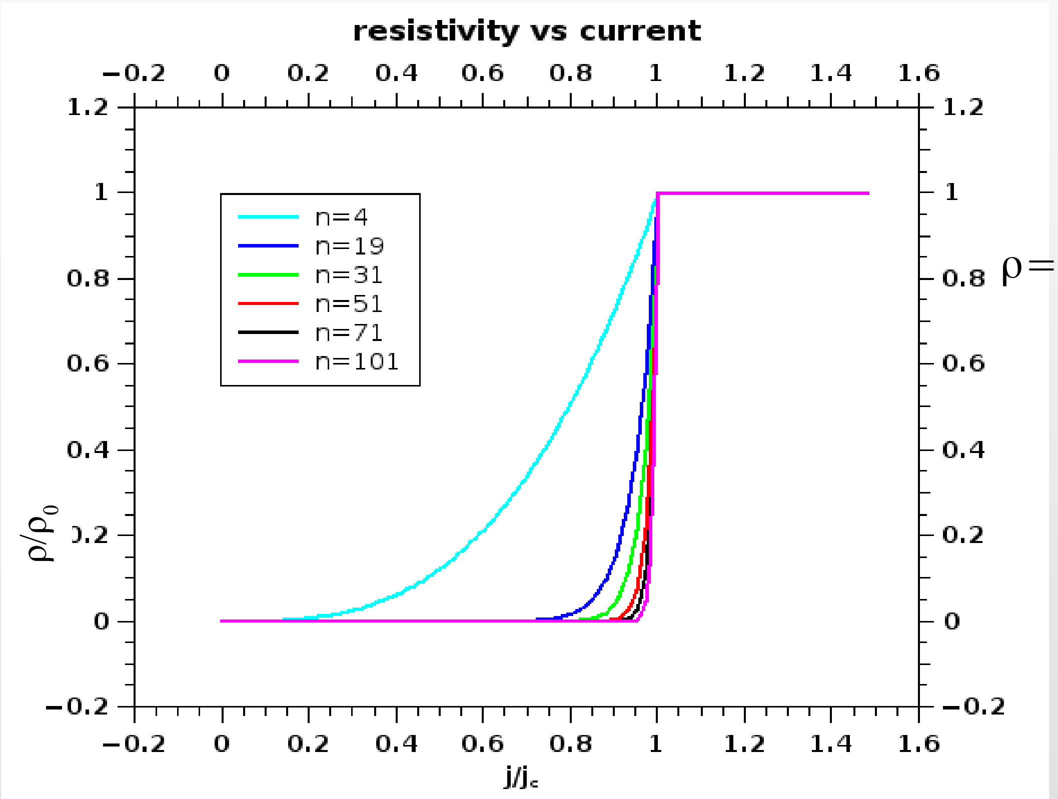


FIG.2: Resistivity vs current for different creep exponents for T<T_c

MODEL 1: constant j_c

$$\rho = \begin{cases} \rho_0 \left| \nabla h / j_c \right|^{n-1}, & \text{for } \left| \nabla h \right| < j_c, T < T_c \\ \rho_{0,} \quad \text{for } \left| \nabla h \right| > j_c, T < T_c \\ \rho_n, \quad \text{for } T > T_c \end{cases} \tag{9}$$

Where $\boldsymbol{j_c}$ is the critical current, and n is the creep exponent.

Taking a magnetic field dependent critical current:

$$j_c = j_{c0}\left(\frac{H}{H_{c2}}\right)^{-\gamma} \qquad (10)$$

Wide range of 0<γ<1.9 in literature, here used 0.37 from experimental fit

MODEL 2
$j_c = j_c(H)$:

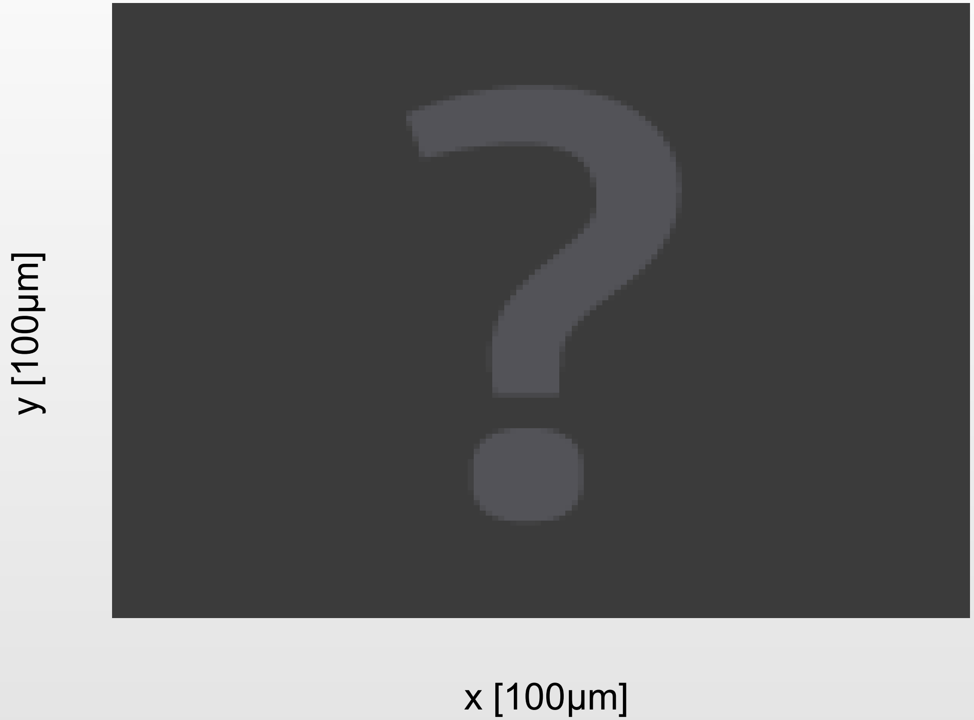
$$\rho = \begin{cases} \rho_0\left(\frac{H}{H_{c2}}\right)^{\gamma(n-1)} |\nabla h / j_{c0}|^{n-1}, & \text{for } T < T_c \\ \rho_n, \text{ for } T > T_c \end{cases} \qquad (11)$$

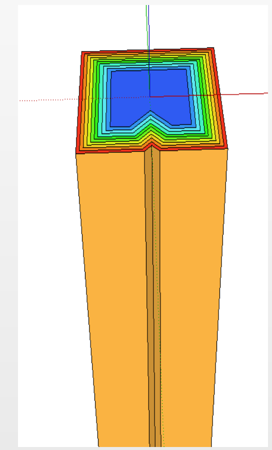
Calculations summary:

400μm x 400μm, 10 μm height, 20 μm base triangular defect
Constant field rate 10^{-3} T/s

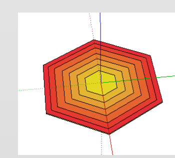
1)Local or longitudinal case considering constant j_c
2)Non-local or transversal case considering constant j_c
3)Non-local or transversal case considering $j_c = j_c(H)$

Results

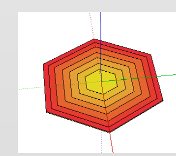




ANIM.1: Local case: d-line under external field ramp of 0.001 T/s. N=51, $\rho_0=10^{-14}$ Ωm, $j_c=10^{10}$ A/m^2 . Triangular defect of 20µm base and 10µm height.



ANIM.2:Non-Local MODEL 1: d-line under external field ramp of 0.001 T/s. N=51, $\rho_0=10^{-14}$ Ωm, $j_c=10^{10}$ A/m^2, d=100nm. Triangular defect of 20μm base and 10μm height.



ANIM.3: Non-Local case MODEL 2: d-line under external field ramp of 0.001 T/s. N=51, ρ_0=1.57x10^{-7} Ωm, j_{c0}=1.36x10^{9} A/m^2, H_{c2}=1.625T, , d=100nm, triangular defect of 20µm base and 10µm height.

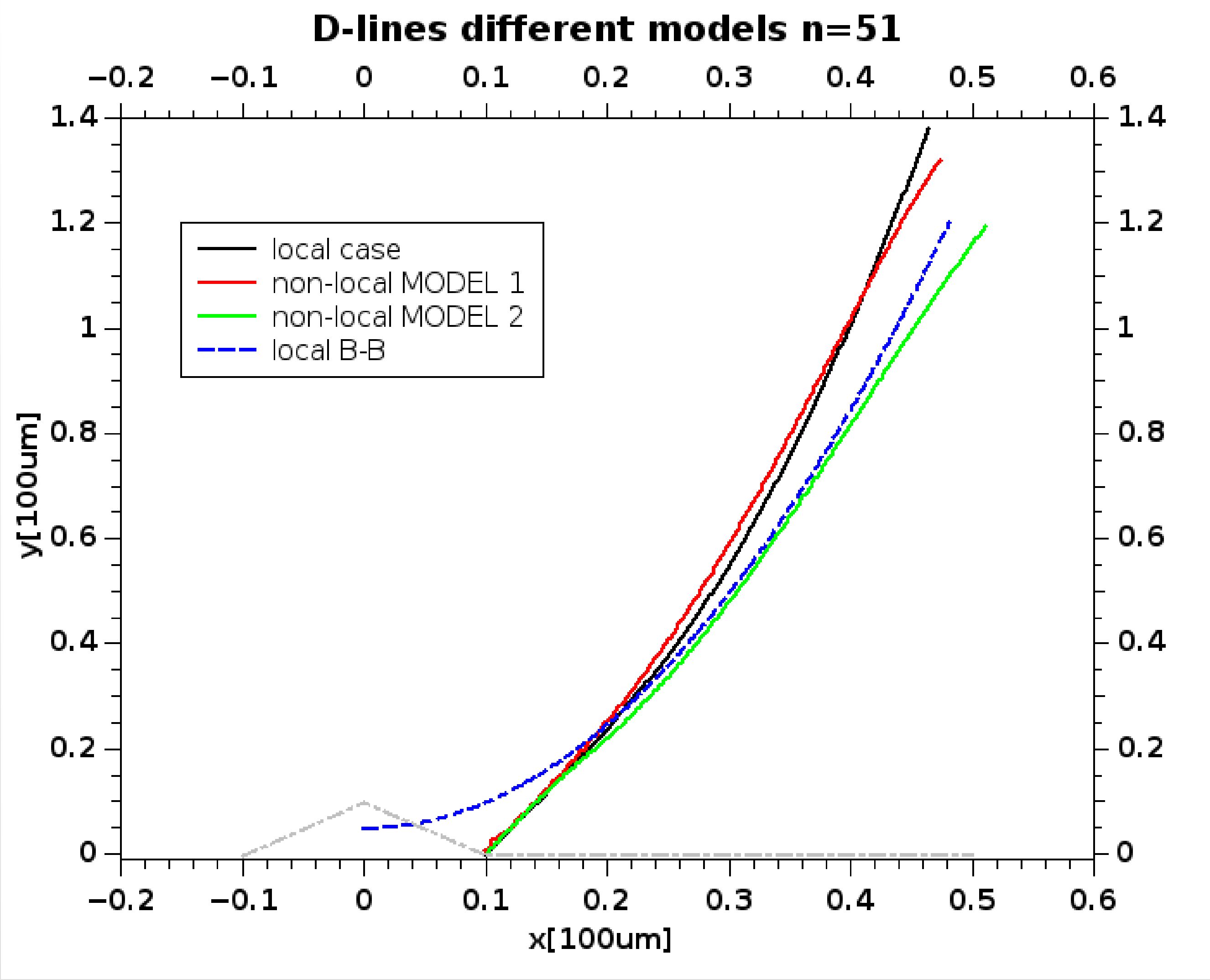


FIG.5: D-lines for previous configurations + bean model prediction

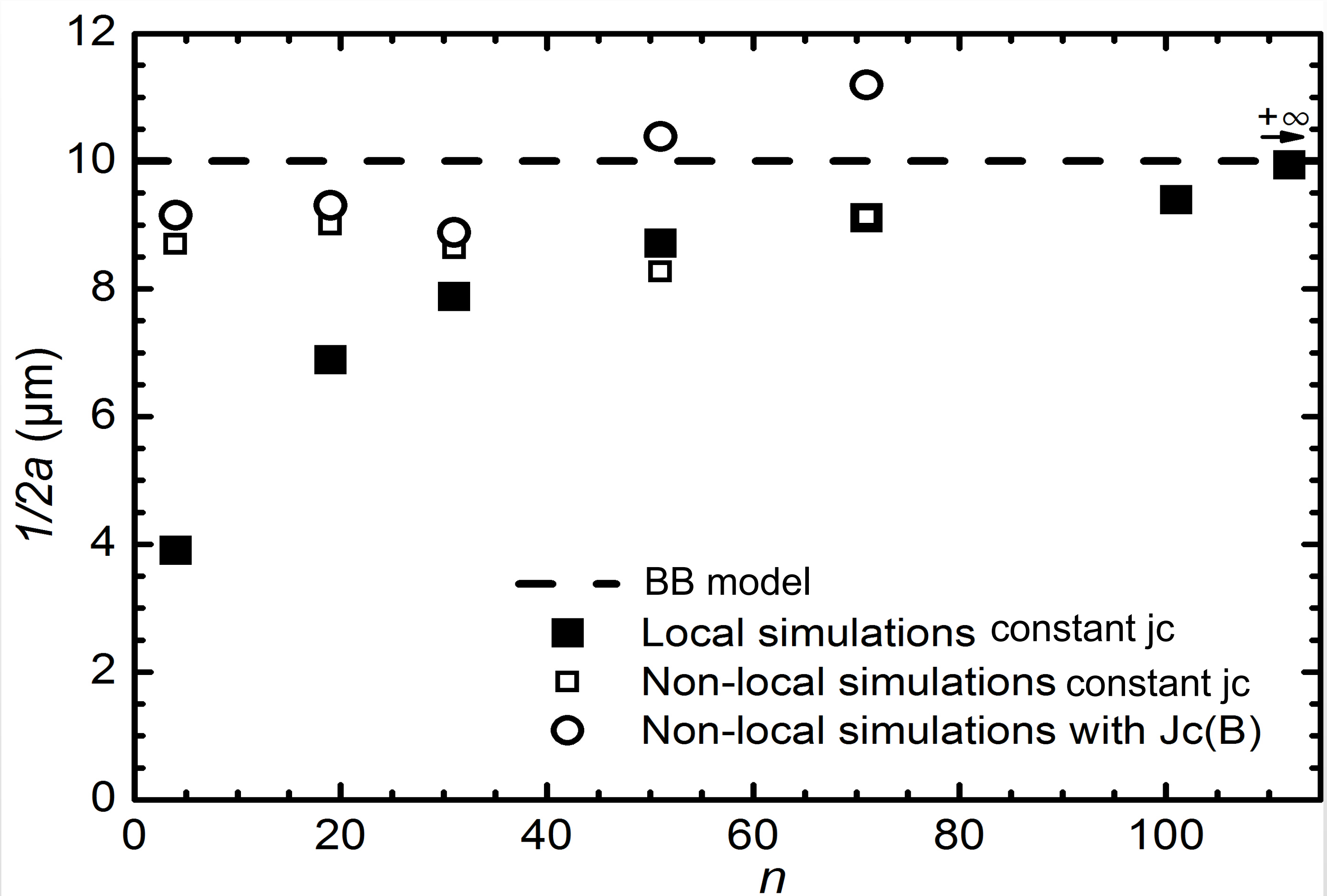


FIG.6: Graph 1/2a vs n for different models, obtained from second order fit of d-lines $y=ax^2+const$.

Conclusions

Resistivity model is relevant as it modifies the dynamics of magnetization and therefore the shape of d-lines.

jc=jc(H) provide 1/2a values higher than BB model, in better accordance with experimental measurements.

Necessary to include other parameters like temperature variations

Acknowledgements

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