Discontinuity lines in Nb thin films with artificial border micro-indentations

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Defects in superconductor films

- **Defects** play a key role in type II superconductors acting as **pinning sites** for vortices, allowing dissipation-less current transport in the mixed state.
- Defects in the borders generate long range regions where the current changes direction abruptly: the discontinuity or **d-lines**.
FIG.I1: Left, Magneto-optical image of thin film samples with defects, Right: representation of d-lines

**Simple current model**: current has uniform magnitude $j_c$

**Simple d-lines model**: d-lines are equidistant from 2 nearest straight edges. For semi-circular and triangular defect of height $R$, its associated d-line will be (BB model):

$$y \approx \frac{x^2}{2R} - \frac{R}{2} \quad (11)$$
FIG. I2: (a) upper flux penetration in triangular indentation of 10 \( \mu \)m height and lower intensity profile along dotted line. (b) \( 1/2a \) vs triangular defect height, where parameter 'a' is obtained from d-line shape fit \( y=ax^2+b \)

From Eqn (I1)

\[
\frac{1}{2a} = R = h \tag{I2}
\]

Experimental values much higher than predicted!
Calculating the magnetic field distribution of a 2D superconductor in the mixed state. Two cases: Local and Non-Local

**Local or longitudinal case:**

Neglecting displacement currents, Maxwell eqns are:

\[ j = \nabla \times H \quad (1) \]
\[ \mu_0 \frac{\partial H}{\partial t} = -\nabla \times E \quad (2) \]

Replacing (1) in the constitutive relationship, and replacing in (2):

\[ E = \rho \, j = \rho \, \nabla \times H \quad (2) \]
\[ \mu_0 \frac{\partial H}{\partial t} = -\nabla \times (\rho \, \nabla \times H) \quad (3) \]
Because of the symmetry, the currents must be planar, and therefore $H$ must point in the $z$ direction and be independent of $z$:

$$H = (h(x, y, t) + h_a(t)) \hat{z} \quad \text{(4)}$$

$$\rho = \rho(x, y) \quad \text{(5)}$$

$$\frac{\partial h}{\partial t} = \nabla \cdot (\rho \nabla h)/\mu_0 - \frac{\partial h_a}{\partial t} \quad \text{(6)}$$

Boundary condition $h=0$

It follows that $h$ is also a current stream-function, since:

$$j = \nabla \times (h \hat{z}) \quad \text{(7)}$$

And then

$$j = |j| = |\nabla \times (h \hat{z})| = |\nabla h| \quad \text{(8)}$$

Ref: E. H. Brandt, PRB 52, 21 (1995), 15442
Magnetic field in thin film: non-local or transversal case

Introducing the sheet current:

\[ J = d \ j \]  \hspace{1cm} (12)

Where \( d \) is the thickness of the film, and introducing the stream function \( g \):

\[ J = \nabla \times (g \ \hat{z}) \]  \hspace{1cm} (13)

The current induced field \( H_d \) time derivative is now given by:

\[
\frac{\partial H_d}{\partial t} = \iint Q(r, r') \frac{\partial g(r')}{\partial t} dS', \quad \text{with} \quad (14)
\]

\[ Q(r, r') = \lim_{z \to 0} \frac{1}{4\pi} \frac{2z^2 - R^2}{(z^2 + R^2)^{5/2}}, \quad R = |r - r'| \]  \hspace{1cm} (15)
And the equation for the time evolution of \( g \) is given by:

\[
\nabla \cdot (\rho \nabla g) / d \mu_0 = \frac{\partial H_a}{\partial t} + \frac{\partial H_d}{\partial t}
\]

\( (17) \)

\[
\nabla \cdot (\rho \nabla g) / d \mu_0 = \frac{\partial H_a}{\partial t} + \iint Q \frac{\partial g}{\partial t}
\]

\( (18) \) Implemented in Matlab

Integration of eqn (18) is not trivial as \( \frac{\partial g}{\partial t} \) is inside the integral. Here an algorithm based on space Fourier transforms \( (F) \) was used.

\[
\frac{\partial H_d}{\partial t} = \iint Q(r - r') \frac{\partial g}{\partial t}, \text{ apply } F()
\]

\( (19) \)

\[
F\left(\frac{\partial H_d}{\partial t}\right) = F\left(\iint Q(r - r') \frac{\partial g}{\partial t}\right) = \frac{k}{2} F\left(\frac{\partial g}{\partial t}\right)
\]

\( (20) \)

\[
\frac{\partial g}{\partial t} = F^{-1}\left(\frac{2}{k} F\left(\frac{\partial H_d}{\partial t}\right)\right)
\]

\( (22) \)

\[
\frac{\partial g}{\partial t} = F^{-1}\left(\frac{2}{k} F\left(\nabla \cdot (\rho \nabla g) / d - \frac{\partial H_a}{\partial t}\right)\right)
\]

\( (23) \)

Where it was used that the Fourier transform of \( Q(r-r') \) is \( k/2 \)

-What about the resistivity?
In a type II superconductor, the resistivity above $H_{c1}$ can be modeled by:

\[ \frac{\rho}{\rho_0} = \begin{cases} \rho_0 \left| \nabla h \right| j_c^{n-1}, & \text{for } \left| \nabla h \right| < j_c, \ T < T_c \\ \rho_0, & \text{for } \left| \nabla h \right| > j_c, \ T < T_c \\ \rho_n, & \text{for } T > T_c \end{cases} \]

(9)

Where $j_c$ is the critical current, and $n$ is the creep exponent.

FIG.2: Resistivity vs current for different creep exponents for $T<T_c$
Taking a magnetic field dependent critical current:

\[ j_c = j_{c0} \left( \frac{H}{H_{c2}} \right)^{-\gamma} \]  \hspace{1cm} (10)

Wide range of 0<\gamma<1.9 in literature, here used 0.37 from experimental fit

**MODEL 2**

\[ j_c = j_c(H) : \]

\[ \rho = \begin{cases} 
\rho_0 \left( \frac{H}{H_{c2}} \right)^{(n-1)} |\nabla h|^{n-1}, & \text{for } T<T_c \\
\rho_n, & \text{for } T>T_c 
\end{cases} \]  \hspace{1cm} (11)
Calculations summary:

400μm x 400μm, 10 μm height, 20 μm base triangular defect
Constant field rate $10^{-3}$ T/s

1) Local or longitudinal case considering constant $j_c$
2) Non-local or transversal case considering constant $j_c$
3) Non-local or transversal case considering $j_c = j_c(H)$
ANIM.1: Local case: d-line under external field ramp of 0.001 T/s. N=51, $\rho_0=10^{-14}$ Ωm, $j_c=10^{10}$ A/m². Triangular defect of 20μm base and 10μm height.
ANIM.2: Non-Local MODEL 1: d-line under external field ramp of 0.001 T/s. $N=51$, $\rho_0=10^{-14} \, \Omega \cdot m$, $j_c=10^{10} \, A/m^2$, $d=100\, nm$. Triangular defect of $20\, \mu m$ base and $10\, \mu m$ height.
ANIM.3: Non-Local case MODEL 2: d-line under external field ramp of 0.001 T/s.
N=51, $\rho_0=1.57\times10^{-7}$ $\Omega m$, $j_{c0}=1.36\times10^9$ A/m$^2$, $H_{c2}=1.625$T, $d=100$nm, triangular defect of 20$\mu$m base and 10$\mu$m height.
FIG. 5: D-lines for previous configurations + bean model prediction
FIG. 6: Graph 1/2a vs n for different models, obtained from second order fit of d-lines $y=ax^2+\text{const.}$
Conclusions

Resistivity model is relevant as it modifies the dynamics of magnetization and therefore the shape of d-lines.

\[ j_c = j_c(H) \] provide 1/2a values higher than BB model, in better accordance with experimental measurements.

Necessary to include other parameters like temperature variations

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