‘Charging’ of subharmonics in coupled Josephson Junctions

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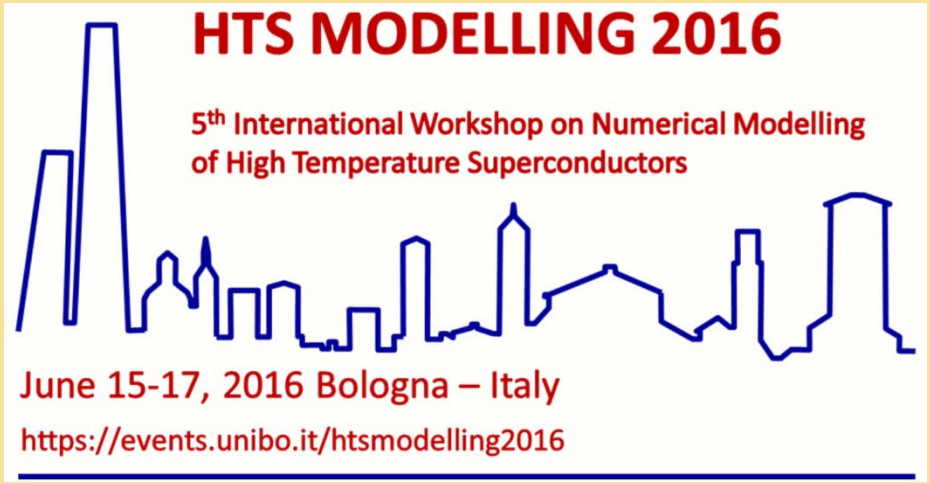
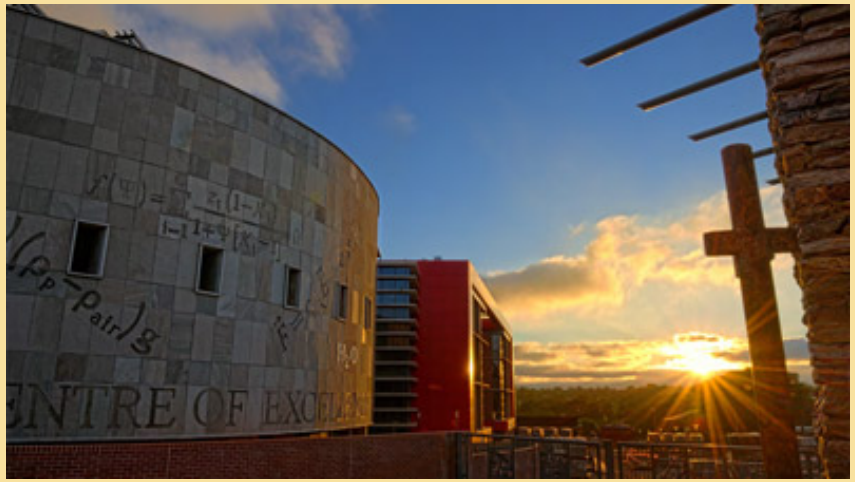




Motivation (Applications of JJs)

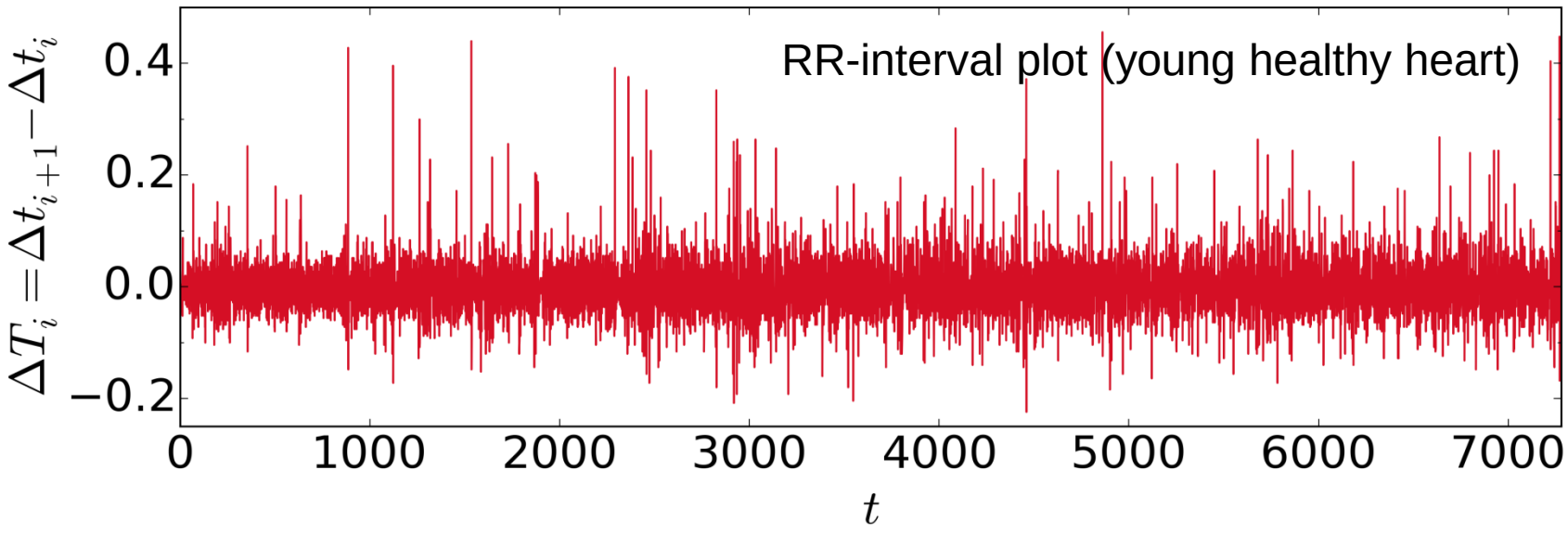
- Prospective objects for superconducting electronics (e.g. voltage standard, SQUID)
- Useful as models for HTSCs
- Sources of powerful coherent THz radiation (which can fill the 0.3-10 THz gap)

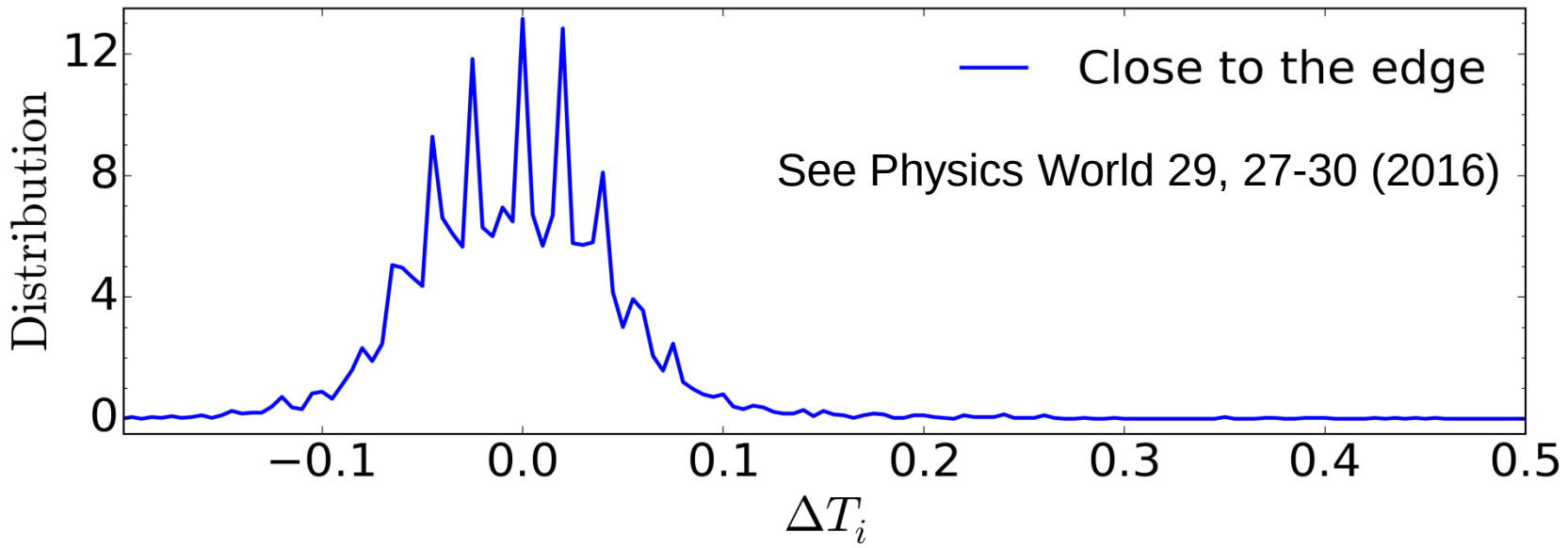
Motivation (Mathematical)

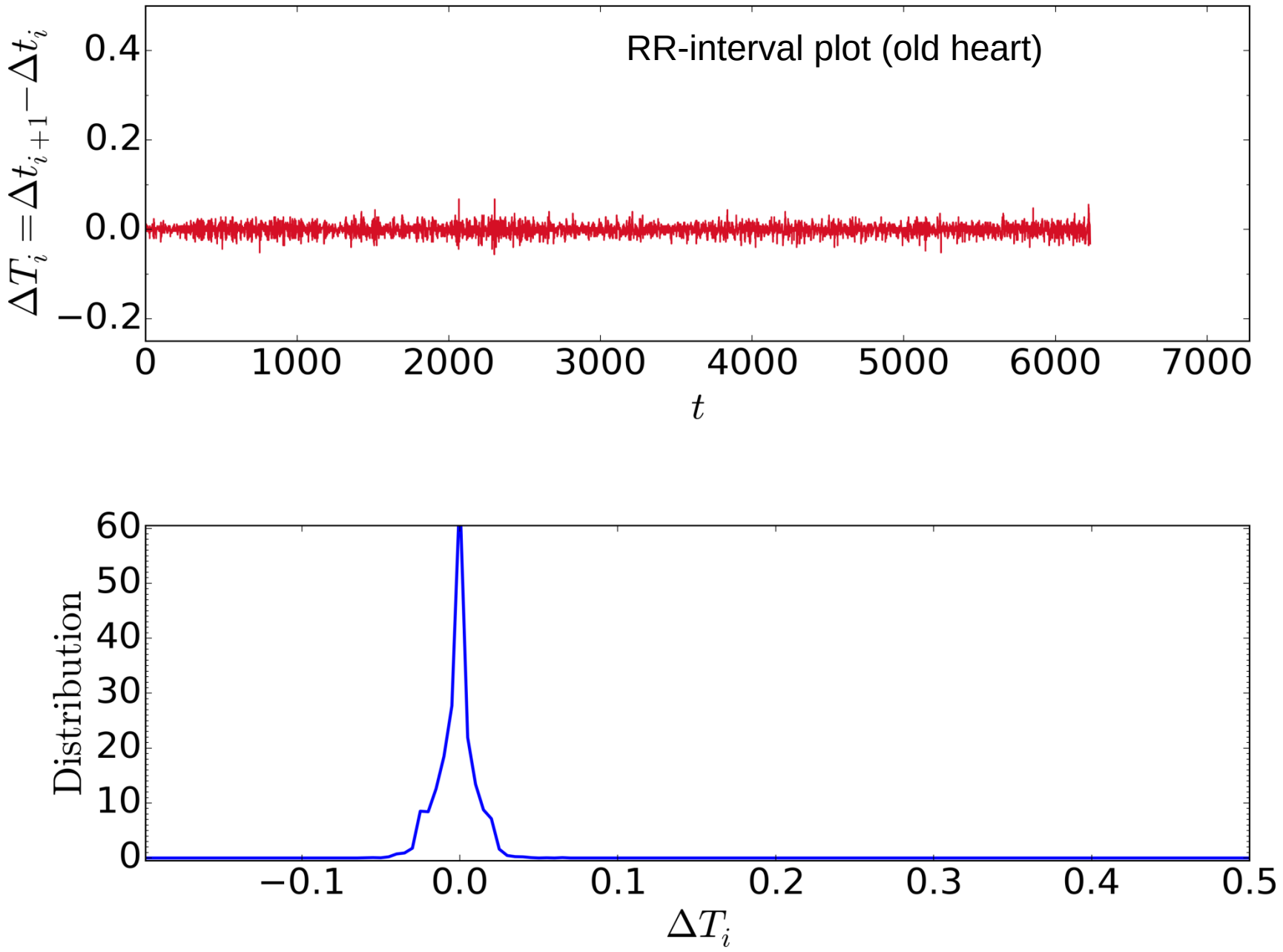
- Chaos synchronization and control
- Chimera states: Scientific Reports 6, 29213 (2016)
- Universality and Complexity Theory

“Instead of seeing chaotic behaviour as yet another tool to help us probe the microscopic world, we should think of this complexity as an essential part of the world around us, and science should attempt to understand it’’

Robert C. Hilborn

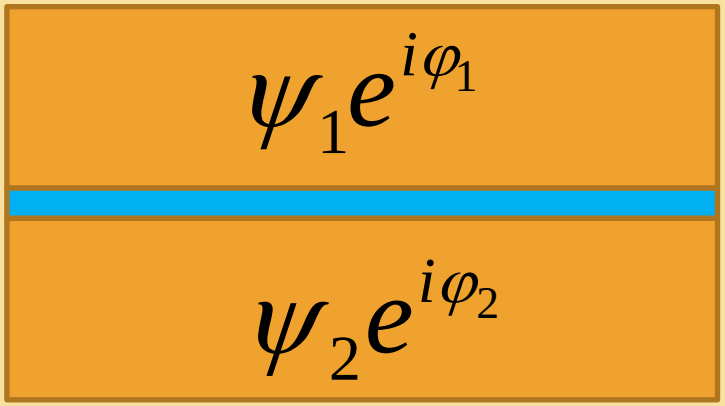

RR-interval plot (young healthy heart)
$\Delta T_i = \Delta t_{i+1} - \Delta t_i$
0.4
0.2
0.0
−0.2
0
1000
2000
3000
4000
5000
6000
7000
t


Close to the edge
See Physics World 29, 27-30 (2016)
Distribution
12
8
4
0
−0.1
0.0
0.1
0.2
0.3
0.4
0.5
ΔT_i


RR-interval plot (old heart)
$\Delta T_i = \Delta t_{i+1} - \Delta t_i$
0.4
0.2
0.0
−0.2
0
1000
2000
3000
4000
5000
6000
7000
t
Distribution
60
50
40
30
20
10
0
−0.1
0.0
0.1
0.2
0.3
0.4
0.5
ΔT_i

Introduction

- Josephson effect (1963)



Superconductor #1

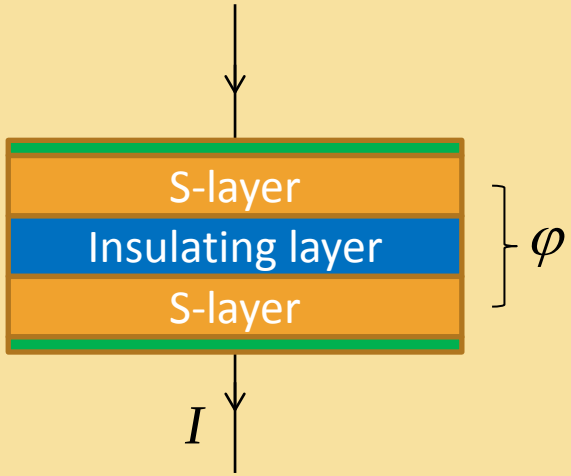
Weak coupling

Superconductor #2

Josephson current-phase relation: $I = I_c \sin\varphi\,, \quad \text{where } \varphi = \varphi_2 - \varphi_1$

Josephson voltage-phase relation: $V = \frac{\hbar}{2e}\frac{\mathrm{d}\varphi}{\mathrm{d}t}$

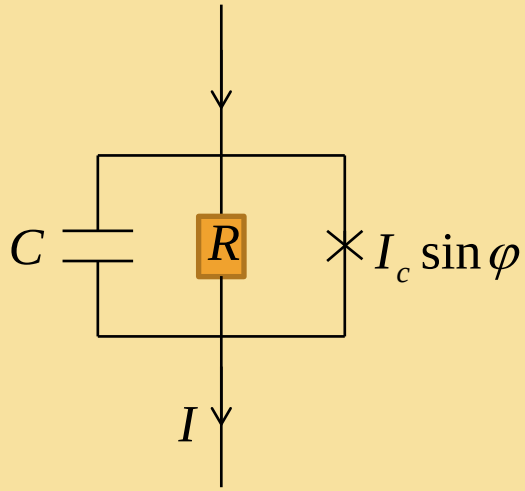
Introduction



$$V = \frac{\hbar}{2e}\frac{d\varphi}{dt}$$

$$I = C\frac{dV}{dt} + \frac{V}{R} + I_c \sin\varphi$$

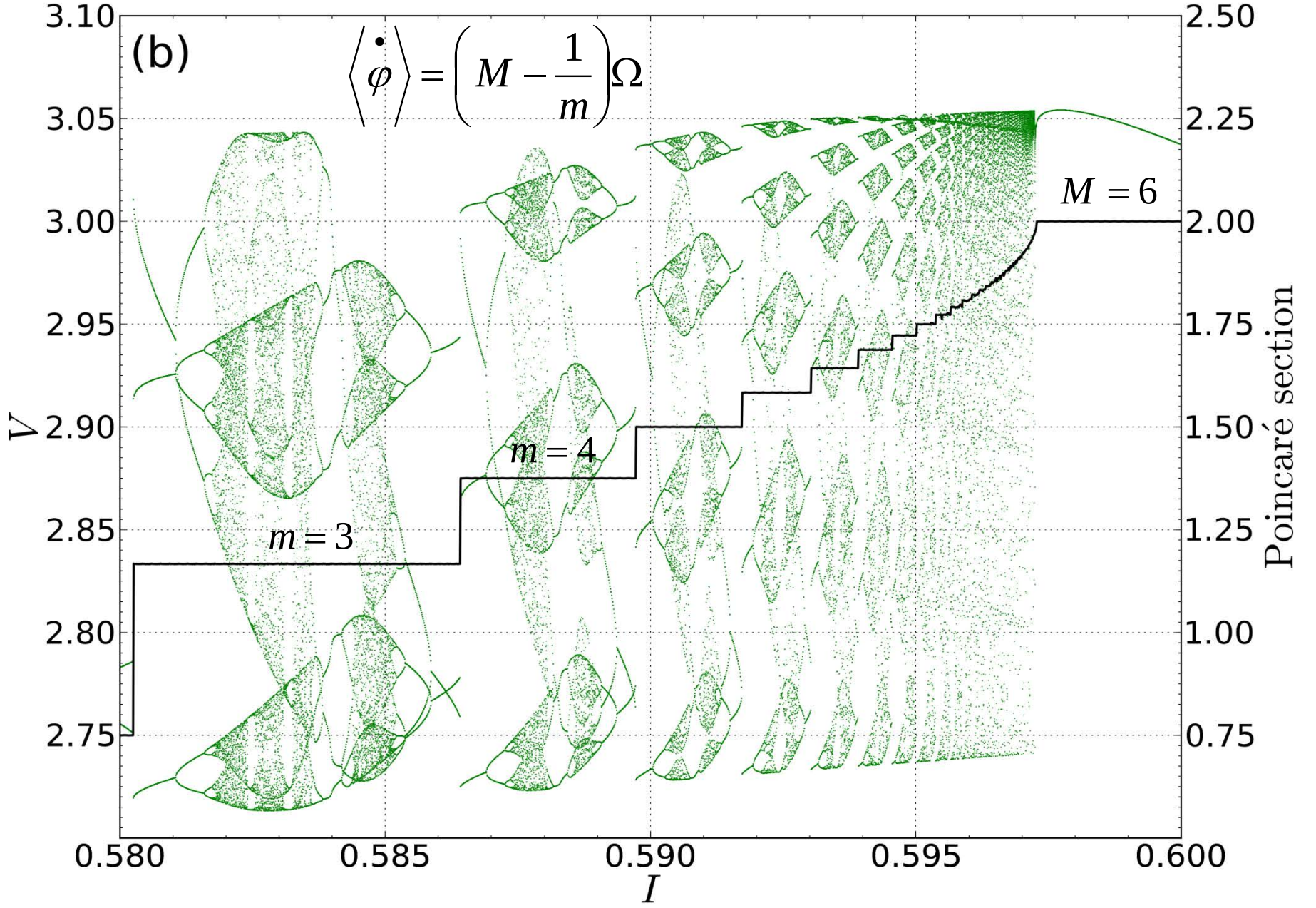
RCSJ model for single JJ



$$\frac{d\varphi}{d\tau} = V$$

$$\frac{dV}{d\tau} = I - \beta V - \sin\varphi + A\sin W$$

$$\frac{dW}{d\tau} = \omega$$


(b)
$\left\langle \dot{\varphi} \right\rangle = \left(M - \frac{1}{m} \right) \Omega$
$M = 6$
$m = 4$
$m = 3$
V
I
Poincaré section
3.10
3.05
3.00
2.95
2.90
2.85
2.80
2.75
2.50
2.25
2.00
1.75
1.50
1.25
1.00
0.75
0.580
0.585
0.590
0.595
0.600

Previous work

CHAOS **24** 033115 (2014)

Structured chaos in a devil's staircase of the Josephson junction

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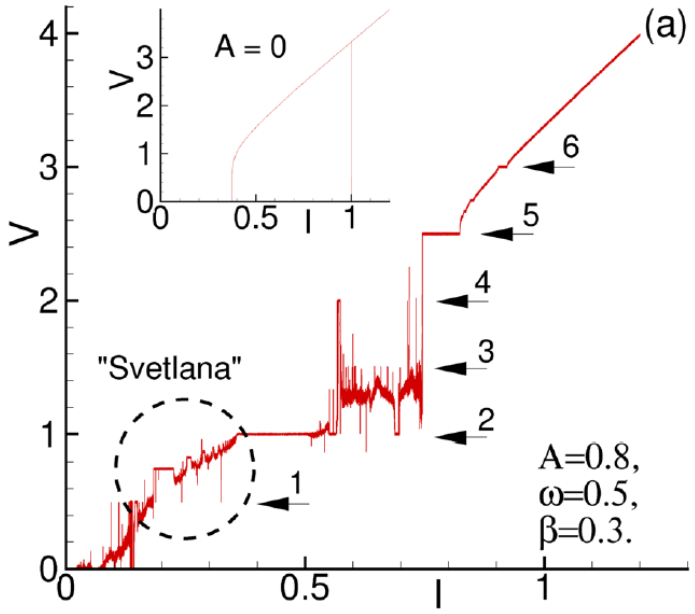
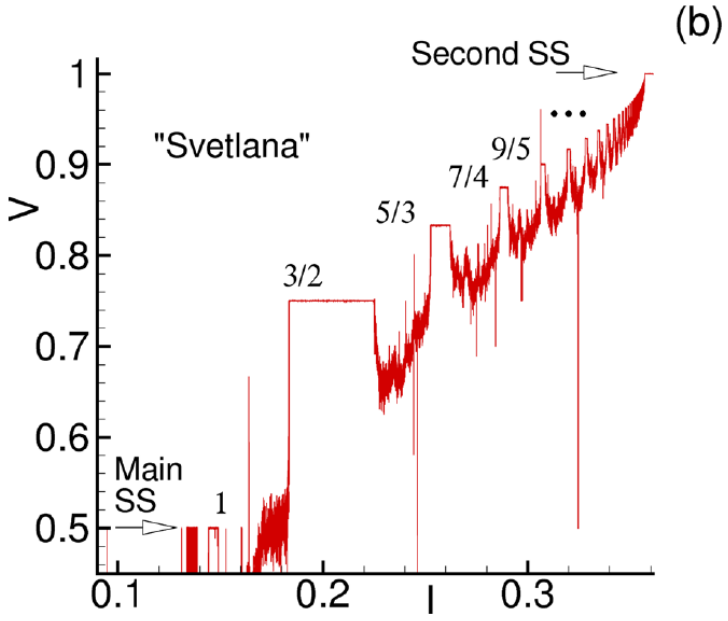
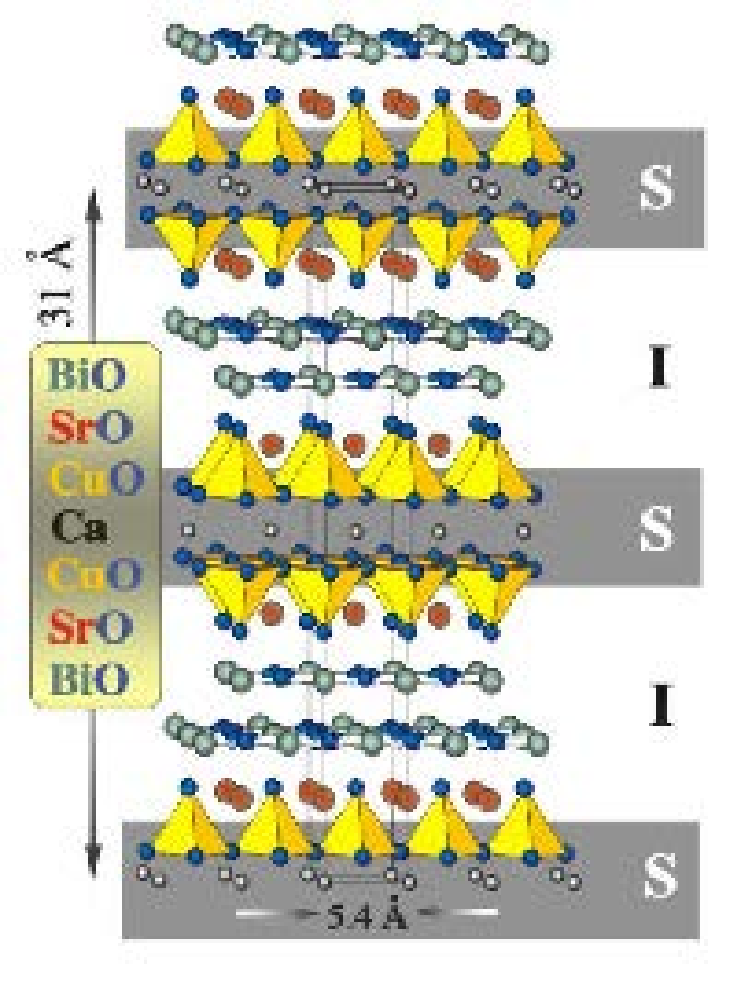
The phase dynamics of Josephson junctions (JJs) under external electromagnetic radiation is studied through numerical simulations. Current-voltage characteristics, Lyapunov exponents, and Poincaré sections are analyzed in detail. It is found that the subharmonic Shapiro steps at certain parameters are separated by structured chaotic windows. By performing a linear regression on the linear part of the data, a fractal dimension of D = 0.868 is obtained, with an uncertainty of ±0.012. The chaotic regions exhibit scaling similarity, and it is shown that the devil's staircase of the system can form a backbone that unifies and explains the highly correlated and structured chaotic

Chaos **24**, 033115 (2014)



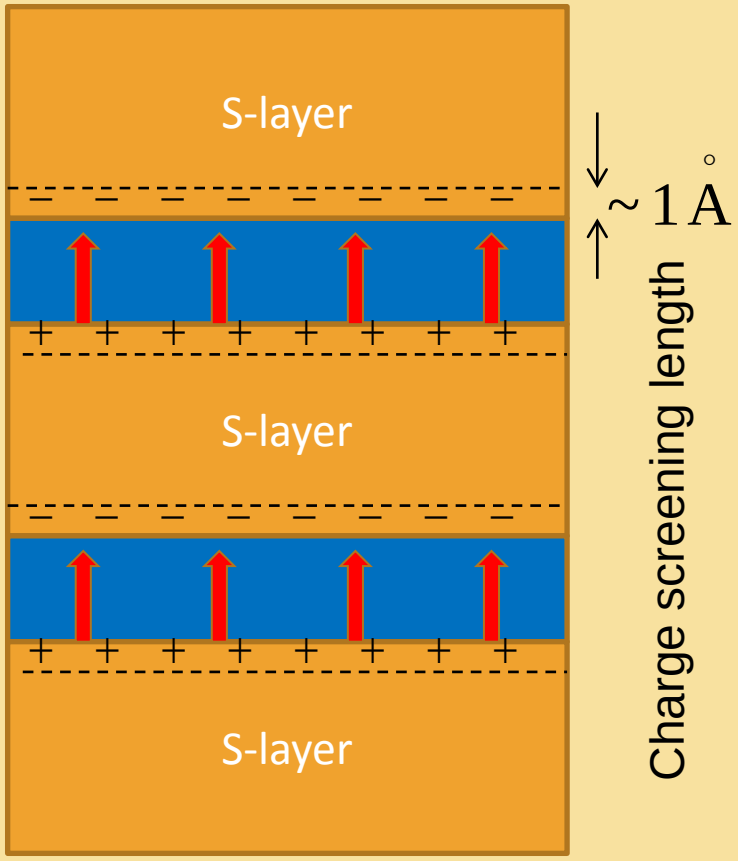


Intrinsic Josephson Junctions

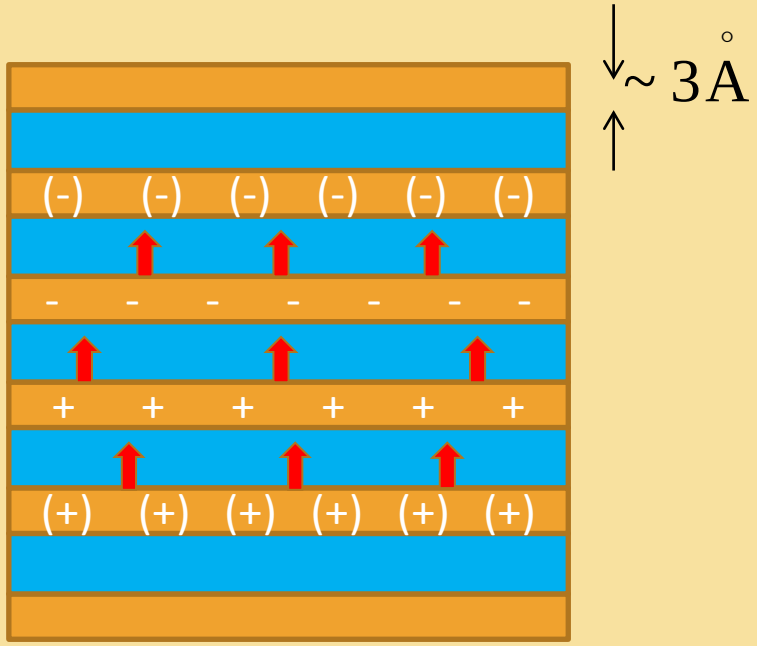


- High-temperature superconductors, like $Bi_2Sr_2CaCu_2O_{8+\delta}$ (Bi-2212), represent natural stacks of atomic scale *Intrinsic Josephson Junctions*.

Coupled Junctions



Conventional SIS array



High-T_c superconductor

Phenomenological models that including coupling

CCJJ

$$I = C\partial V/\partial t + \frac{V}{R} + I_c \sin\varphi$$

$$\frac{\hbar}{2e}\frac{\partial \varphi_{l,l+1}}{\partial t} = V_{l,l+1} + \frac{\varepsilon\mu^2}{d_s d_I}(V_{l+2,l+1} + V_{l-1,l} - 2V_{l,l+1})$$

CIB

$$J^l_{qp} = \frac{\hbar}{2eR}\dot{\varphi}_l + \frac{\Psi_{l-1} - \Psi_l}{R}$$

$$\frac{\hbar}{2e}\dot{\varphi}_l(t) = (1+2a)V_l - a(V_{l-1} + V_{l+1}) + \Psi_l - \Psi_{l-1}$$

CCJJ+DC

$$J^l_D = -\frac{\Phi_l - \Phi_{l+1}}{R}$$

$$J = C\frac{dV_l}{dt} + J^l_c \sin(\varphi_l) + \frac{\hbar}{2eR}\dot{\varphi}_l$$

$$\frac{\hbar}{2e}\frac{\partial \varphi_{l,l+1}}{\partial t} = V_{l,l+1} + \frac{\varepsilon\mu^2}{d_s d_I}(V_{l+2,l+1} + V_{l-1,l} - 2V_{l,l+1})$$

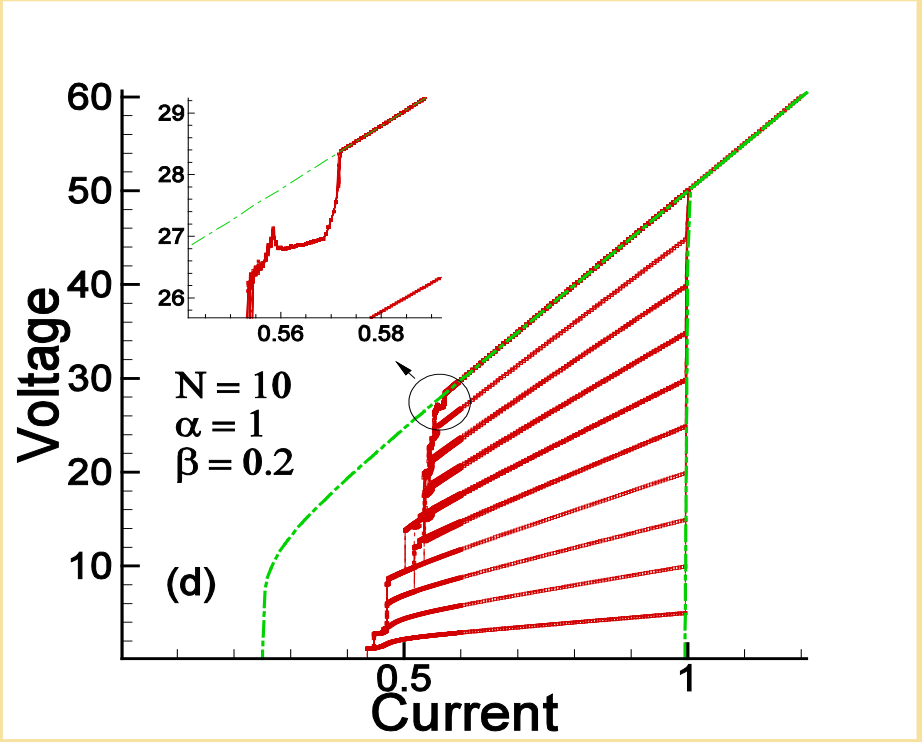
CCJJ+DC model

$$\frac{dV_\ell}{d\tau} = I - \sin\varphi_\ell - \beta\frac{d\varphi_\ell}{d\tau}$$

$$\frac{d\varphi_\ell}{d\tau} = V_\ell - \alpha\left(V_{\ell+1} + V_{\ell-1} - 2V_\ell\right)$$

where $\alpha = \dfrac{\varepsilon\mu^2}{d_S d_I}$ and

$\ell = 1, 2, ..., N.$



I-V characteristics

- large hysteresis
- equally spaced branch structure
- fine structure in BPR (chaos)

Previous work

J. Supercond. Nov. Mag. **28** 349 (2015)

ORIGINAL PAPER

Structured Chaos in 1-D Stacks of Intrinsic Josephson Junctions Irradiated by Electromagnetic Waves

A. E. Botha · Yu. M. Shukrinov · S. Yu. Medvedeva · M. R. Kolahchi

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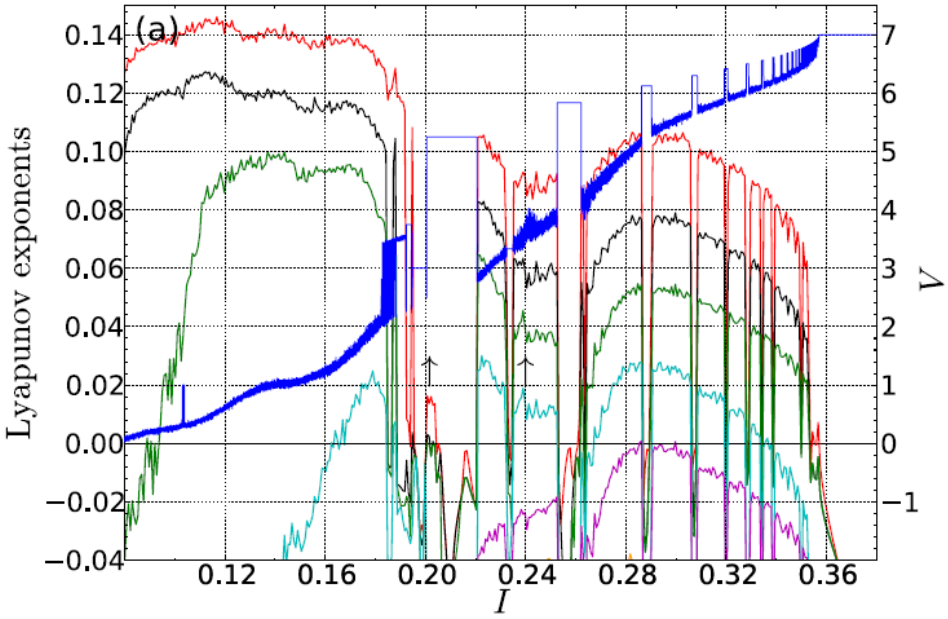
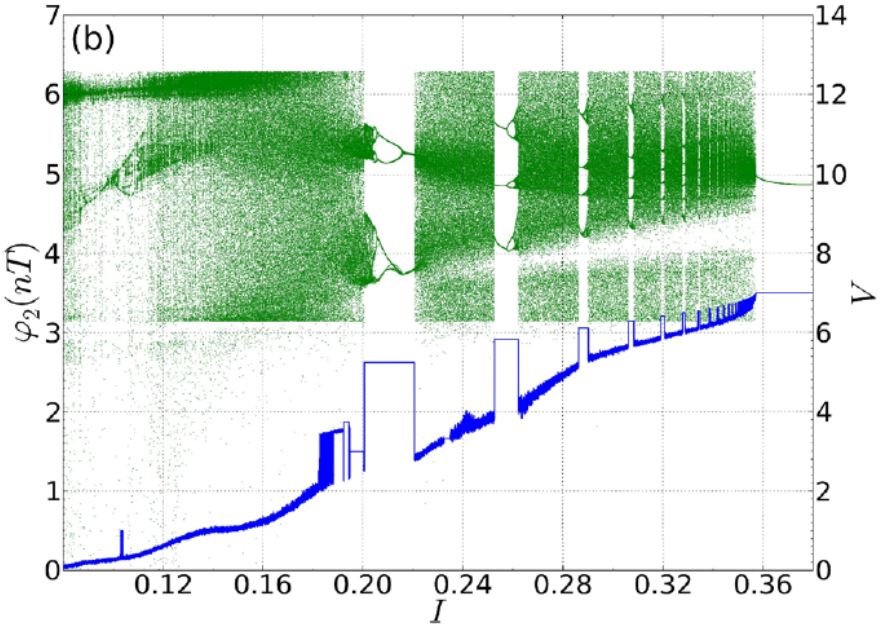
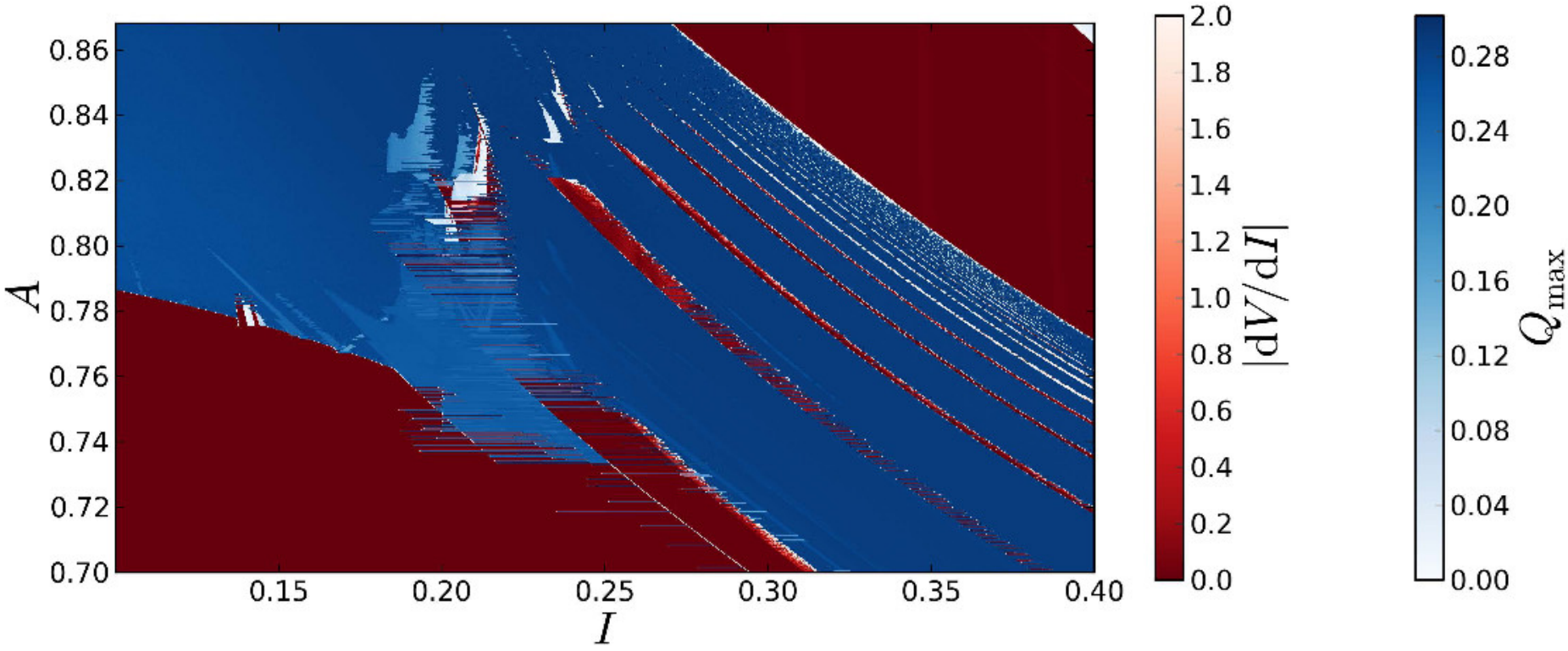
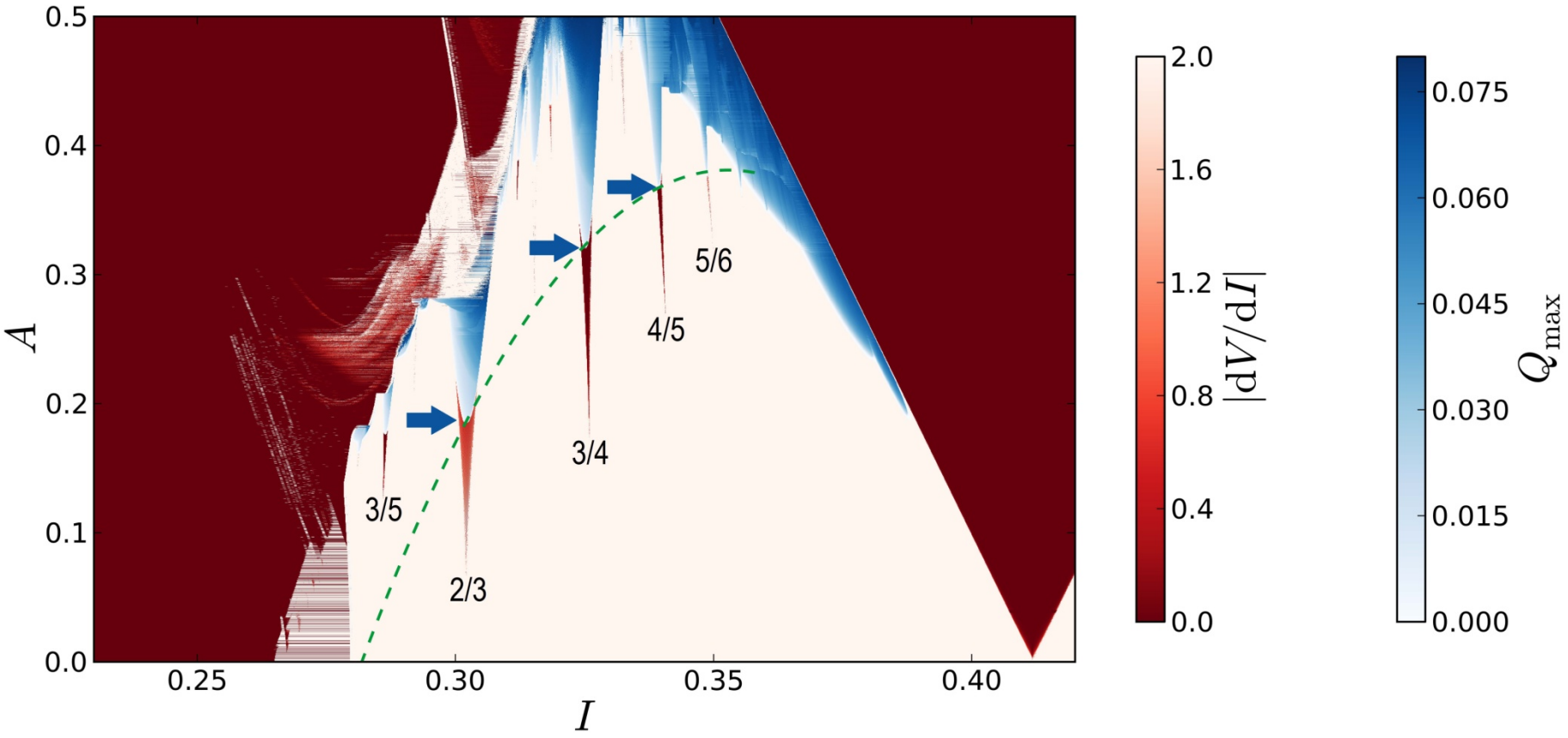




Chaos synchronization

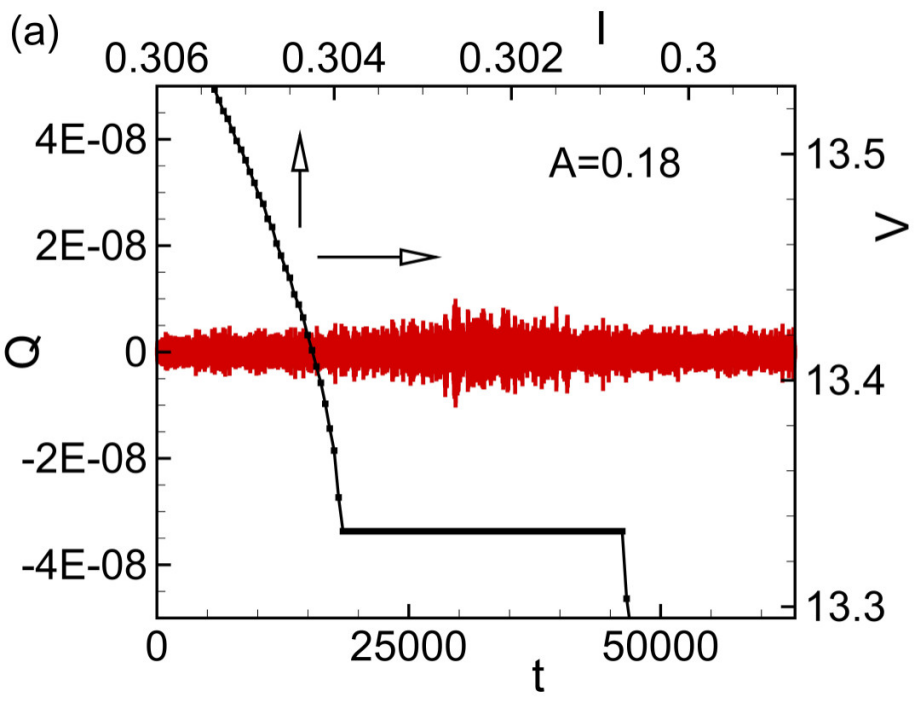


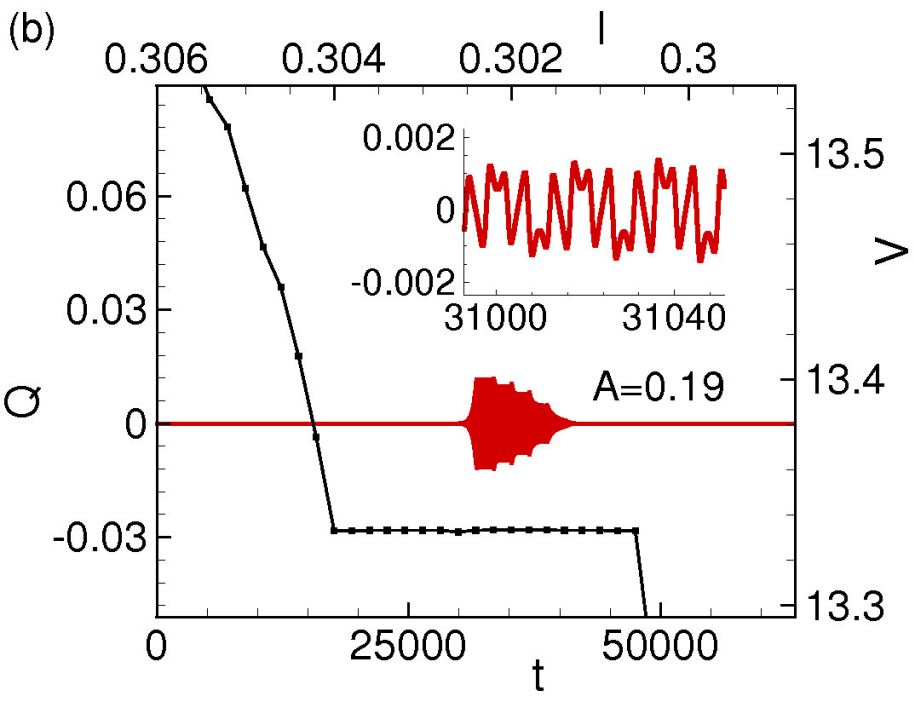
Charging of subharmonics



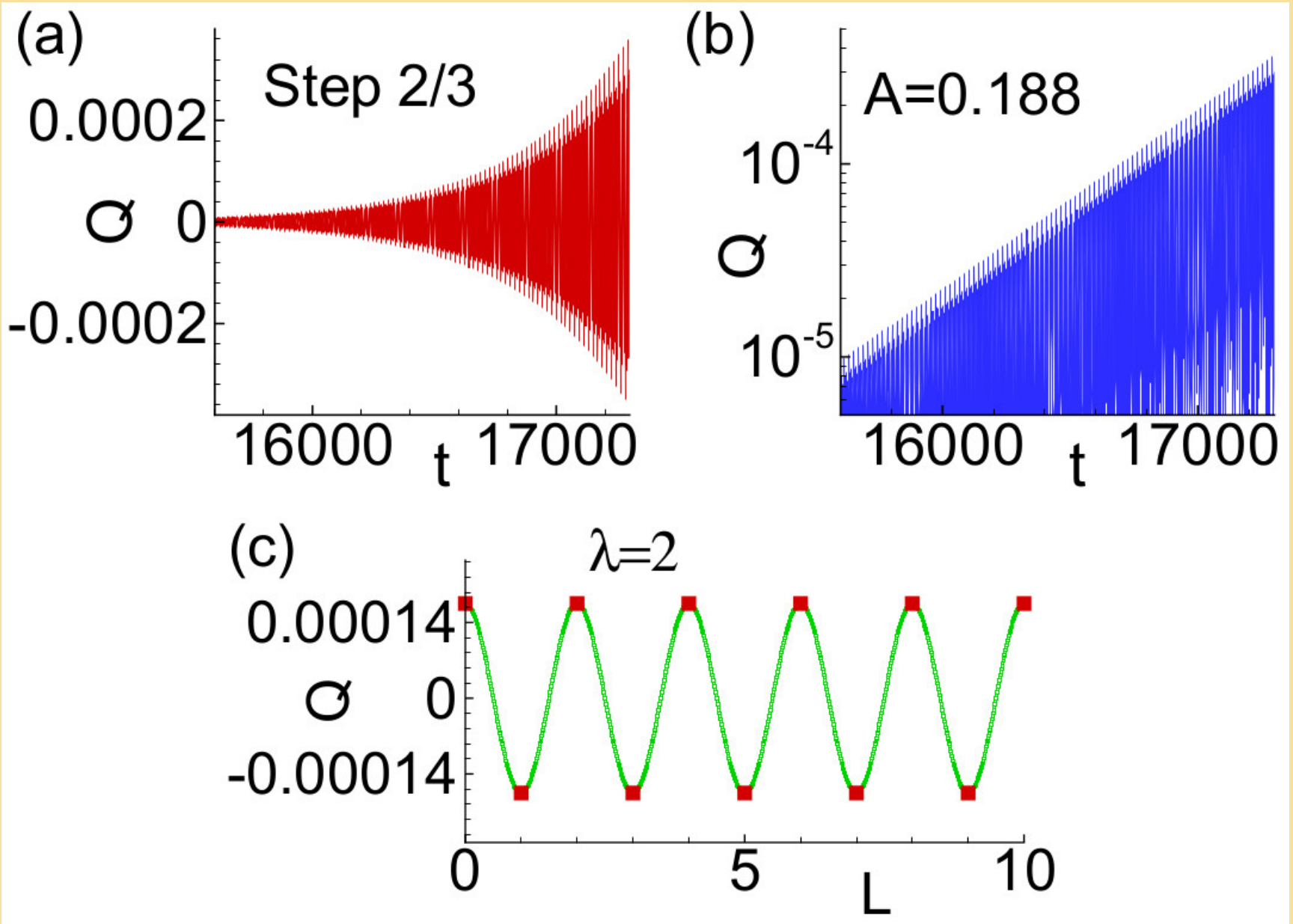
$$N = 10 (PBC), \quad \alpha = 0.05, \quad \beta = 0.2, \quad \omega = 2$$

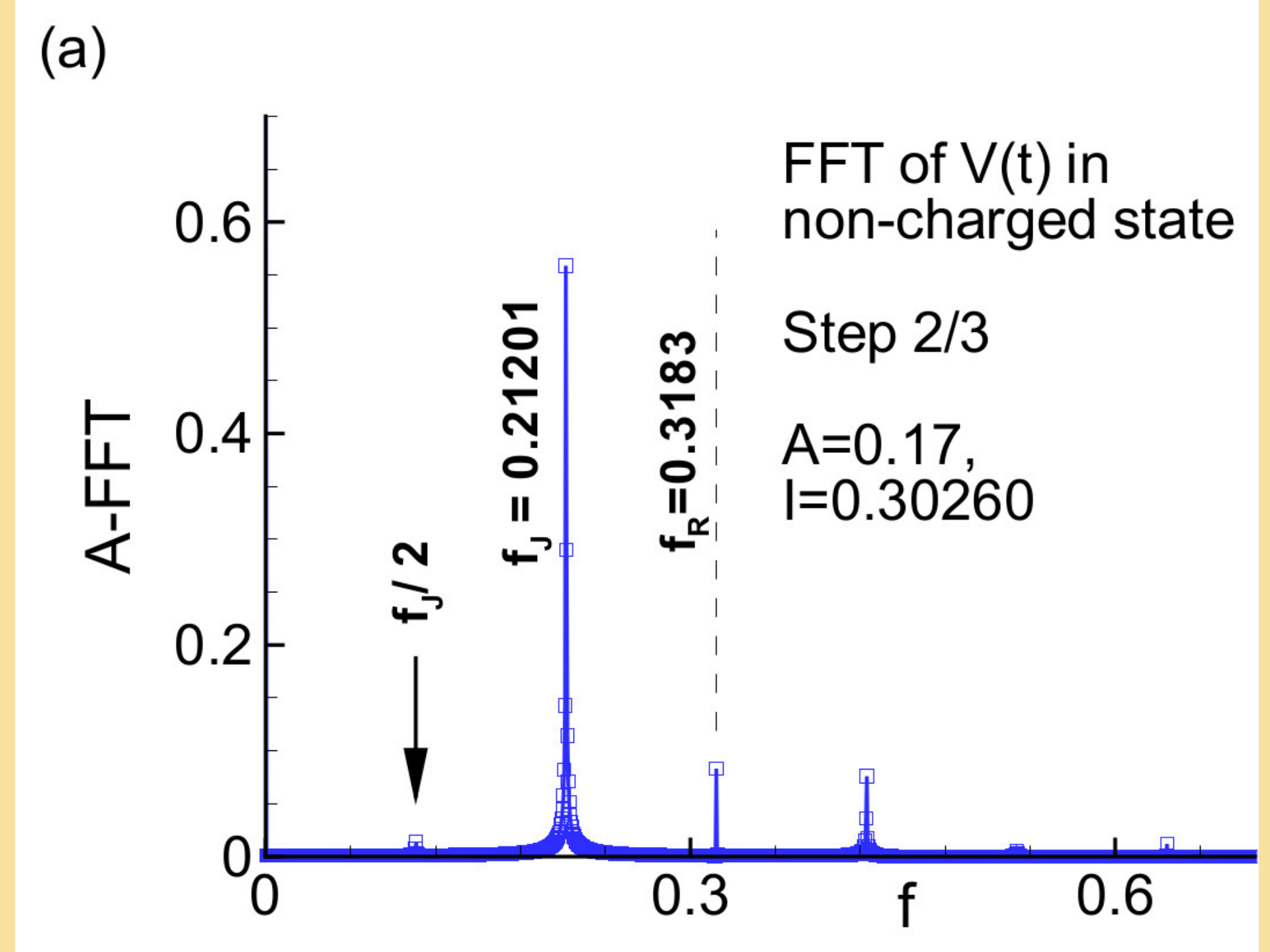
Onset of charging 2/3

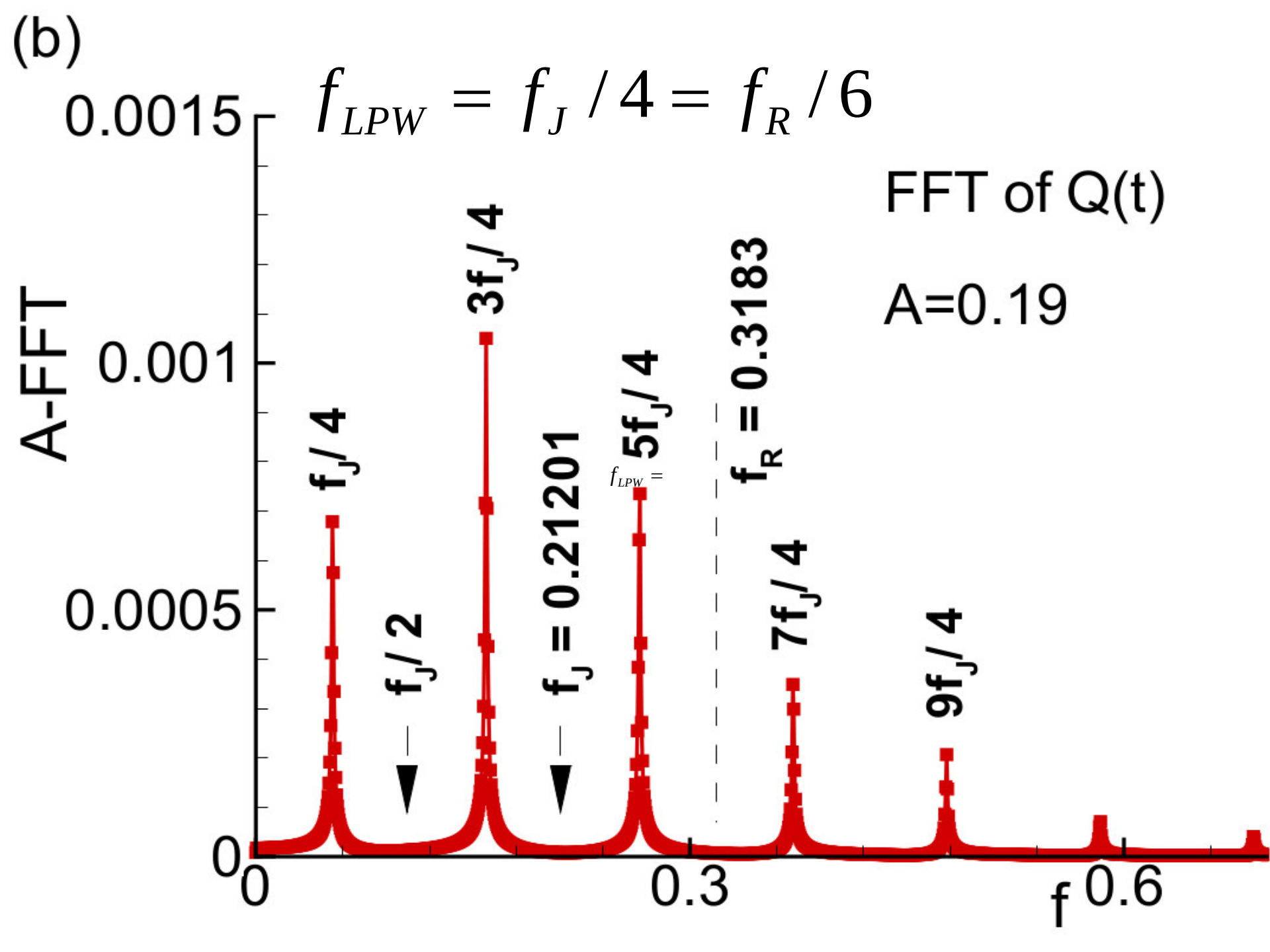




Parametric resonance




(a)
FFT of V(t) in
non-charged state
Step 2/3
A=0.17,
I=0.30260
A-FFT
0.6
0.4
0.2
0
0
0.3
0.6
f
f_J / 2
f_J = 0.21201
f_R=0.3183


(b)
$f_{LPW} = f_J / 4 = f_R / 6$
FFT of Q(t)
A=0.19
A-FFT
0.0015
0.001
0.0005
0
0
0.3
0.6
f
$f_J/4$
$f_J/2$
$3f_J/4$
$f_J = 0.21201$
$f_{LPW} = 5f_J/4$
$f_R = 0.3183$
$7f_J/4$
$9f_J/4$

Conclusion

- There is difference between the parametric instabilities that occur for subharmonics, in comparison to harmonics

- Remarkable ordering in the sequence of realizable subharmonic steps (continued fractions)

- Is there a simple rule that can predict the sequence of steps being charged as a function of N, A, I, etc?