Two ways of evaluating the loss per AC cycle in a superconducting coil

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1) Motivation

2) Macroscopic and microscopic dissipation

3) Example – 2 FEM techniques:
   - A-formulation FEM, \( J = J_c \tanh(E/E_c) \)
   - H-formulation FEM, \( J = J_c (E/E_c)^{1/n} \)

4) Conclusions

CC (12 mm wide) pancake coil with 10 turns
Motivation

how to test a numerical model?

• experiment

  real materials rather complex

• comparing with the results of other models

  how to distinguish the correct one?
Macroscopic and microscopic dissipation

AC loss – particular advantage: two ways for evaluation

\[ \int_{t_0}^{t} I_{coil}(t)U_{coil}(t)dt = \int_{0}^{T} \int_{V} j(r,z,\theta)E(r,z,\theta)dV \]

- **Power supply**
  - \( U_{ps} \)
  - \( I_{ps} \)

- **Load**
  - \( U_{coil} \)
  - \( I_{coil} \)

**Macroscopic point of view:**
\[ P_{ps}(t) = I_{ps}(t)U_{ps}(t) \quad \cong \quad P_{coil}(t) = I_{coil}(t)U_{coil}(t) \]

**Microscopic point of view**
\[ P_L(r,z,\theta) = j(r,z,\theta)E(r,z,\theta) \]

Local values:
- \( j(r,z,\theta) \)
- \( E(r,z,\theta) \)

F. Grilli et al. 2014 IEEE Trans Appl Supercond 24 8200433
Evaluation of macroscopic quantities: A-FEM method

\[ I_{i(t)} = \int_{S} j(r, z, t) dS \]

constraint: \( I_{i(t)} = I_{\text{coil}}(t) \)

fulfilled by choosing the set of \( \nabla \varphi_i \)

Acknowledgements:
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\[ E(r, z) |_i = -\frac{\partial A(r, z)}{\partial t} - \nabla \varphi_i \]

Acknowledgement:
Victor Zermeno
neutral zone (in each turn)

\[ E(r_{0i}, z_{0i}) = 0 \]

then from

\[ E_i(r, z) = -\frac{\partial A(r, z)}{\partial t} - \nabla \varphi_i \]

follows

\[ \nabla \varphi_i = -\frac{\partial A(r_{0i}, z_{0i})}{\partial t} = -\frac{\partial A_{0i}}{\partial t} \]

E. Pardo 2008
Evaluation of macroscopic quantities: A-FEM method

**Turn i:**

\[ L_i = 2\pi R_i \]
\[ S_i = \pi R_i^2 \]
\[ \nabla \varphi_i = -\frac{\partial A_{0i}}{\partial t} \]

\[
U_i(t) = \int_{L_i} \tilde{E}_i(t) \cdot d\tilde{l} = \int_{L_i} \left( -\frac{\partial \tilde{A}}{\partial t} - \nabla \varphi_i(t) \right) \cdot d\tilde{l} =
\]
\[
= \int_{L_i} \left( -\frac{\partial \tilde{A}}{\partial t} + \frac{\partial A_{0i}}{\partial t} \right) \cdot d\tilde{l} = U_{\Phi,i}(t) + U_{\text{turn},i}(t)
\]

\[
U_{\Phi,i}(t) = -\int_{L_i} \frac{\partial \tilde{A}}{\partial t} \cdot d\tilde{l} = -\oint_{\partial S_i} \nabla \times \frac{\partial \tilde{A}}{\partial t} \cdot dS = -\frac{\partial}{\partial t} \oint_{\partial S_i} \nabla \times \tilde{A} \cdot dS = -\frac{\partial}{\partial t} \oint_{S_i} \tilde{B} \cdot dS = -\frac{\partial \Phi_i}{\partial t}
\]

\[
U_{\text{turn},i}(t) = \int_{L_i} \frac{\partial A_{0i}}{\partial t} \cdot d\tilde{l} = -\int_{L_i} \nabla \varphi_i \cdot d\tilde{l} \approx -2\pi R_i \nabla \varphi_i
\]

\[
U_{\text{coil}}(t) = \sum_{i=1}^{N_{\text{turns}}} U_{\text{turn},i}(t) = -2\pi \sum_{i=1}^{N_{\text{turns}}} R_i \nabla \varphi_i
\]
Evaluation of macroscopic quantities: $H$-FEM method

Formulation:

$$
\begin{align*}
1 \frac{1}{r} \frac{\partial E_\varphi}{\partial z} &= -\mu_0 \mu_r \frac{\partial H_r}{\partial t} \\
1 \frac{1}{r} \frac{\partial r E_\varphi}{\partial r} &= -\mu_0 \mu_r \frac{\partial H_z}{\partial t} \\
J_\varphi &= \frac{\partial H_r}{\partial z} - \frac{\partial H_z}{\partial r} \\
E_\varphi &= E_c \left( \frac{J_\varphi}{J_{c0}} \right)^n
\end{align*}
$$

Constraint:

$$
I_{i(t)} = \int_{S} j(r, z, t) dS \\
I_{i(t)} = I_{coil}(t)
$$
Evaluation of macroscopic quantities: \textit{H-FEM} method

Current distribution (\(J\)) is setting as external current density for \(mf\) model. \(Mf\) model is used just for calculating \(A_J\).

\[
\sum_{i=1}^{N_{\text{turns}}} \int_{S_{i}} r d\vec{s} \cdot \left[ \vec{E}_i(t) + \frac{\partial \vec{A}_J}{\partial t} + \frac{\partial \vec{A}_a}{\partial t} \right]
\]

\[
\nabla^2 \vec{A}_J = -\mu_0 \vec{J} \quad ; \quad \vec{A}_a = 0
\]

\[
U_{\text{coil}}(t) = \sum_{i=1}^{N_{\text{turns}}} U_{\text{turn},i}(t) = -2\pi \sum_{i=1}^{N_{\text{turns}}} R_i \nabla \phi_i
\]

\(E(J)\) is calculated by PDE model.

\textbf{Settings:}

\[
\begin{align*}
\vec{J}_e &= \vec{J} \\
\sigma_{\text{HTS}} &= 1 \text{S} / \text{m}; \\
\sigma_{\text{Air}} &= 0 \text{S} / \text{m};
\end{align*}
\]

computation details: A-FEM method

$$j(r, z, t) = j_c \tanh \left( \frac{E(r, z, t)}{E_c} \right) = j_c \tanh \left( - \frac{1}{E_c} \frac{\Delta [A(r, z) - A_{0,i}]}{\Delta t} \right)$$

not power law!

$$E_c = 10^{-4} \text{ V/m}$$

$$N_{steps} = 40$$

variable

$$\Delta t = \frac{T}{N_{steps}} = \frac{1}{fN_{steps}}$$

grid for saving $A$ data

points per thickness

$$d_{max}$$

points per tape width:

$$N_w$$

numerical tolerance

$$h_{SC} = 10 \mu m$$

points per thickness

$$N_h$$

width:

$$N_w$$

not power law!
computation details: \( H\)-FEM method

E-J power law: \( E = E_c (J / J_{c0})^n \)

\[
dt = \frac{T}{N_{\text{steps}}} = \frac{1}{fN_{\text{steps}}}
\]

\( E_c = 10^{-4} \text{ V/m} \)

\( n = 23 \)

\( N_{\text{step}} = 400 \)

**Applied current:**

\( N_w \): Points along the width during meshing;

\( N_t \): Points along the thickness during meshing;

\( n_t \): Numerical error tolerance during computing

**Meshing:** Mapping in HTS domain, Finer triangle in other domain

Lagrange or Curl
checking of calculation correctness : A-FEM method

\[
f = 2500 \text{ Hz}
\]
\[
d_{\text{max}} = 18.5 \text{ m}, N_w = 101, N_h = 3, n_{\text{tol}} = 10^{-8}
\]

\[
\Gamma = \frac{2\pi Q l}{\mu_0 I_a^2}
\]

\[
F = \frac{I}{I_c}
\]

\[
j_c = \text{const.}
\]

1 data point – 4 hours

Could the experimental data help?
checking of calculation correctness: A-FEM method

\[ f = 2500 \text{ Hz} \]
\[ d_{\text{max}} = 18.5 \mu m, \quad N_w = 101, \quad N_h = 3, \quad n_{\text{tol}} = 10^{-8} \]

\[ j_c = \text{const.} \]
1 data point – 4 hours

Acknowledgement:
Jano Šouc

probably not!
checking of calculation correctness: A-FEM method

\[ f = 2500 \text{ Hz} \]
\[ d_{\text{max}} = 18.5 \mu\text{m}, N_w = 221, N_h = 7, n_{\text{tol}} = 10^{-12} \]

**Graphs:**

- Left graph: Plot of \( Q \) vs. \( I_a \)
- Right graph: Plot of \( I' = 2\pi Q/\mu J^2 \) vs. \( F = I/I_c \)

\( j_c = \text{const.} \)
1 data point – 4 hours
checking of calculation correctness : A-FEM method

\[ f = 2500 \text{ Hz} \]
\[ d_{\text{max}} = 18.5 \, \mu\text{m}, \quad N_w = 221, \quad N_h = 11, \quad n_{\text{tol}} = 10^{-12} \]

\[ j_c = \text{const.} \]

1 data point – 5 hours
checking of calculation correctness: A-FEM method

\[ f = 2500 \text{ Hz} \]
\[ d_{\text{max}} = 18.5 \mu \text{m}, \ N_w = 451, \ N_h = 11, \ n_{\text{tol}} = 10^{-12} \]

\[ j_c = \text{const.} \]

1 data point – 7 hours

-> good resolution along the tape width is essential
checking of calculation correctness : A-FEM method

$f = 3.85 \text{ Hz}$

$d_{max} = 18.5 \text{ µm}$, $N_w = 451$, $N_h = 11$, $n_{tol} = 10^{-12}$

$\Gamma = \frac{2\pi Q}{\mu_0 I_a^2}$

$F = \frac{I_d}{I_c}$

$\dot{j}_c = \text{const.}$

1 data point – 4.5 hours

consequence of the used $E(j)$ relation
checking of calculation correctness: A-FEM method

\[ j_c = \text{const.} \]

consequence of the used \( E(j) \) relation
checking of calculation correctness: A-FEM method

\[ f = 36 \text{ Hz} \]
\[ d_{\text{max}} = 18.5 \mu\text{m}, N_w = 451, N_h = 11, n_{\text{tol}} = 10^{-12} \]

\[ j_c = \text{const.} \]

1 data point – 3 hours
checking of calculation correctness: \textit{H-FEM} method

\[ f = 36 \text{ Hz} \]

\[ N_w = 100, \quad N_h = 1, \quad n_{tol} = 10^{-7} \]

Shape function (Curl linear)

\[ j_c = \text{const.} \]

2 h46 min (each point)

\[ t_{step} = 1e^{-6} \text{ s} \]

Good agreement between two evaluation methods
checking of calculation correctness: \textit{H-FEM} method

\[ f = 36 \text{ Hz} \]
\[ N_w = 25, \ N_h = 1, \ n_{tol} = 10^{-7} \] Shape function (Curl linear)

\[ j_c = \text{const.} \]

25 min (each point)
\[ t_{\text{step}} = 1 \times 10^{-6} \text{ s} \]

very rough meshing was used in HTS zone, results wrong but still agree with each other
checking of calculation correctness: \( H \)-FEM method

\[ f = 72 \text{ Hz} \]
\[ N_w = 100, \quad N_h = 1, \quad n_{tol} = 10^{-7} \]
Shape function (Curl linear)

\[ j_c = \text{const.} \]

1 h 32 min (each point)
\[ t_{\text{step}} = 1 \times 10^{-6} \text{ s} \]
checking of calculation correctness: \( H\text{-FEM method} \)

\[ f = 1 \text{ Hz} \]
\[ N_w = 100, \quad N_h = 1, \quad n_{tol} = 10^{-7} \quad \text{Shape function (Curl linear)} \]

\[ j_c = \text{const.} \]

3 h 16 min (each point)
\[ t_{\text{step}} = 5 \times 10^{-5} \text{ s} \]

agreement between two methods does not change with frequency

Do these two methods always agree with each other in H-FEM method?
checking of calculation correctness: \( H \)-FEM method

\[
f = 36 \text{ Hz} \\
N_w = 25, \ N_h = 1, \ n_{tol} = 10^{-7} \ (\text{Lagrange linear})
\]

\[
j_c = \text{const.}
\]

20 min (each point)
\( t_{\text{step}} = 1 \times 10^{-5} \text{ s} \)
checking of calculation correctness: $H$-FEM method

$f = 36 \text{ Hz}$

$N_w = 100$, $N_h = 1$, $n_{tol} = 10^{-7}$ (Lagrange linear)

$j_c = \text{const.}$

$t_{\text{step}} = 5 \times 10^{-6} \text{ s}$

$1 \text{ h 5 min (each point)}$
checking of calculation correctness: *H*-FEM method

\[ f = 36 \text{ Hz} \]
\[ N_w = 400, \; N_h = 1, \; n_{tol} = 10^{-7} \text{ (Lagrange linear)} \]

\[ j_c = \text{const.} \]

1 h 2 min (each point)
\[ t_{step} = 1 \times 10^{-5} \text{ s} \]

better agreement can be achieved by finer meshing!
calculation correctness: comparison of two methods

\( f = 36 \text{ Hz} \)

<table>
<thead>
<tr>
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\( j_c = \text{const.} \)

comparison of calculated voltage waveforms:

apparently only inductive signal
calculation correctness: comparison of two methods

\( f = 36 \text{ Hz} \)

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\( j_c = \text{const.} \)

comparison of calculated voltage waveforms after compensating the inductive component
Conclusions

- Comparing the macroscopic and microscopic approach of AC loss evaluation is a useful tool in checking the numerical model for AC problem.
- In the methods using the macroscopic current as a constraint, main task is the determination of macroscopic voltage.
- In $A$-formulation, macroscopic voltage is calculated as an independent variable, and such comparison is straightforward; calculation parameters e.g. necessary mesh density can be found.
- In $H$-formulation, macroscopic voltage is derived from the solution, then it is not an independent variable, but still some preliminary checks are possible; e.g. shape function Curl (linear) much better than Lagrange (linear).
Comparing the macroscopic and microscopic approach of AC loss evaluation is a useful tool in checking the numerical model for AC problem.

In the methods using the macroscopic current as a constraint, main task is the determination of macroscopic voltage.

In A-formulation, macroscopic voltage is calculated as an independent variable, and such comparison is straightforward; calculation parameters e.g. necessary mesh density can be found.

In H-formulation, macroscopic voltage is derived from the solution, then it is not an independent variable, but still some preliminary checks are possible; e.g. shape function Curl (linear) much better than Lagrange (linear).

Voltage on coil is a measurable quantity – allows more detailed comparison with experiments than the check of AC loss value.