Two ways of evaluating the loss per AC cycle in a superconducting coil

F. Gömöry[1], J. Sheng[1,2]

[1]Institute of Electrical Engineering, Slovak Academy of Sciences, Bratislava, Slovakia
[2]Institute of Electrical Engineering. Shanghai Jiao Tong University, Shanghai, China

Outline

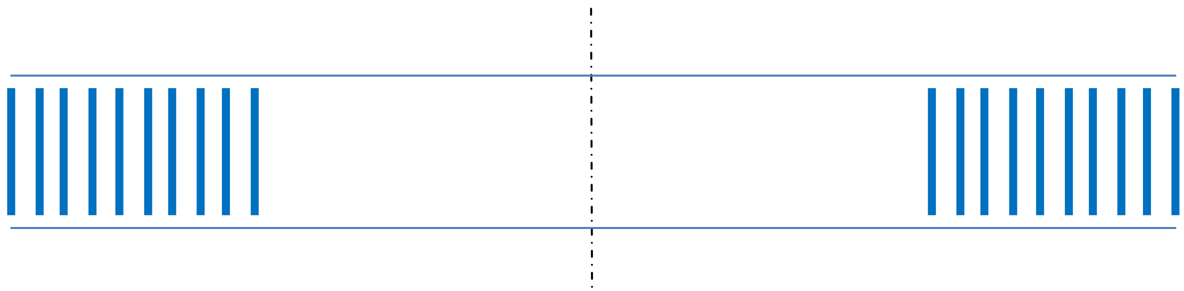
1) Motivation

2) Macroscopic and microscopic dissipation

3) Example – 2 FEM techniques:
   - *A*-formulation FEM, $J=J_c\ tanh(E/E_c)$
   - *H*-formulation FEM, $J=J_c\ (E/E_c)^{1/n}$

   Comsol Multiphysics

4) Conclusions



CC (12 mm wide) pancake coil with 10 turns

Motivation

how to test a numerical model ?

- experiment

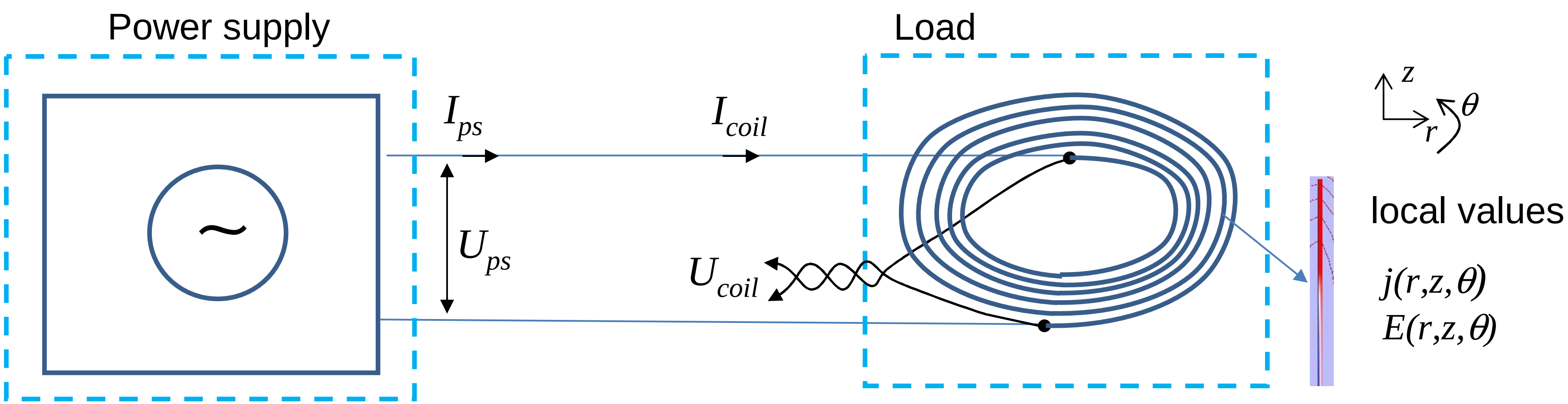
  real materials rather complex

- comparing with the results of other models

  how to distinguish the correct one ?

Macroscopic and microscopic dissipation

AC loss – particular advantage : two ways for evaluation



macroscopic point of view:

$$P_{ps}(t) = I_{ps}(t)U_{ps}(t) \quad \cong \quad P_{coil}(t) = I_{coil}(t)U_{coil}(t)$$

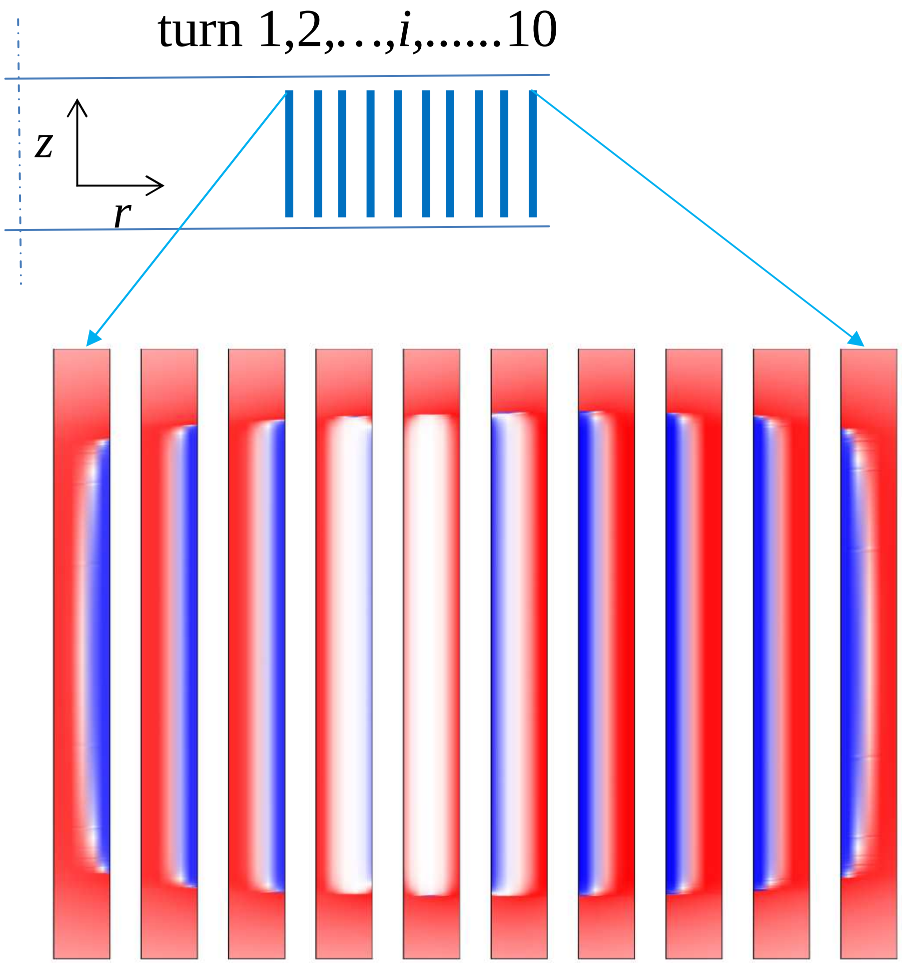
microscopic point of view

$$P_L(r,z,\theta) = j(r,z,\theta)E(r,z,\theta)$$

$$Q_{coil} = \int_0^T I_{coil}(t)U_{coil}(t)\mathrm{d}t \quad = \quad Q_L = \int_0^T \mathrm{d}t \int_V j(r,z,\theta)E(r,z,\theta)\mathrm{d}V$$

F. Grilli et al. 2014 IEEE Trans Appl Supercond 24 8200433

Evaluation of macroscopic quantities: *A*-FEM method



$$I_{i(t)} = \int_S j(r,z,t)\mathrm{d}S$$

constraint: $I_{i(t)} = I_{coil}(t)$

fulfilled by choosing the set of $\nabla\varphi_i$

Acknowledgements:

Ernst Helmut Brandt

Francesco Grilli

$$E(r,z)\big|_i = -\frac{\partial A(r,z)}{\partial t} - \nabla\varphi_i$$

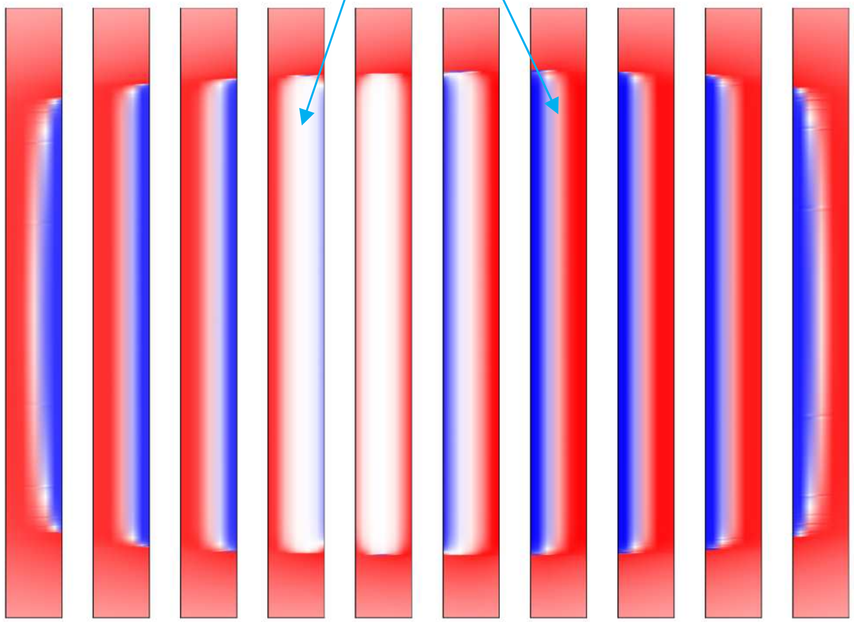
Acknowledgement:

Victor Zermeno

Evaluation of macroscopic quantities: *A*-FEM method

neutral zone (in each turn)

$$E(r_{0i}, z_{0i}) = 0$$



then from

$$E_i(r,z) = -\frac{\partial A(r,z)}{\partial t} - \nabla \varphi_i$$

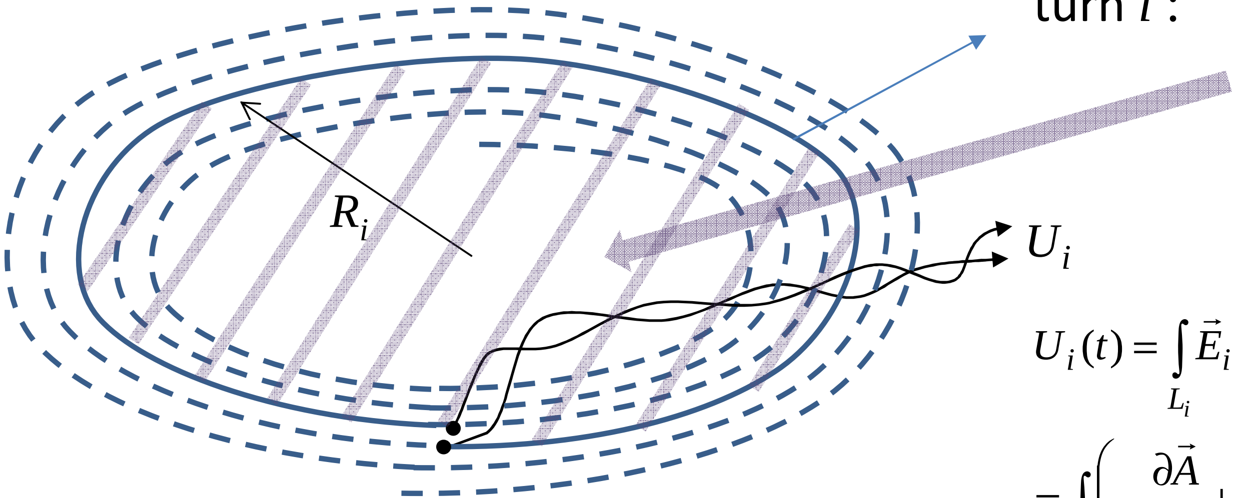
follows

$$\nabla \varphi_i = -\frac{\partial A(r_{0i}, z_{0i})}{\partial t} = -\frac{\partial A_{0i}}{\partial t}$$

E. Pardo 2008
Superconductor Sci Tech 21 (2008) 065014

Evaluation of macroscopic quantities: *A*-FEM method

turn *i* : $L_i \cong 2\pi R_i$ $\quad S_i \cong \pi R_i^2$ $\qquad \nabla\varphi_i = -\dfrac{\partial A_{0i}}{\partial t}$



$$U_i(t) = \int_{L_i} \vec{E}_i(t)\cdot \mathrm{d}\vec{l} = \int_{L_i}\left(-\frac{\partial \vec{A}}{\partial t} - \nabla\varphi_i(t)\right)\cdot \mathrm{d}\vec{l} =$$

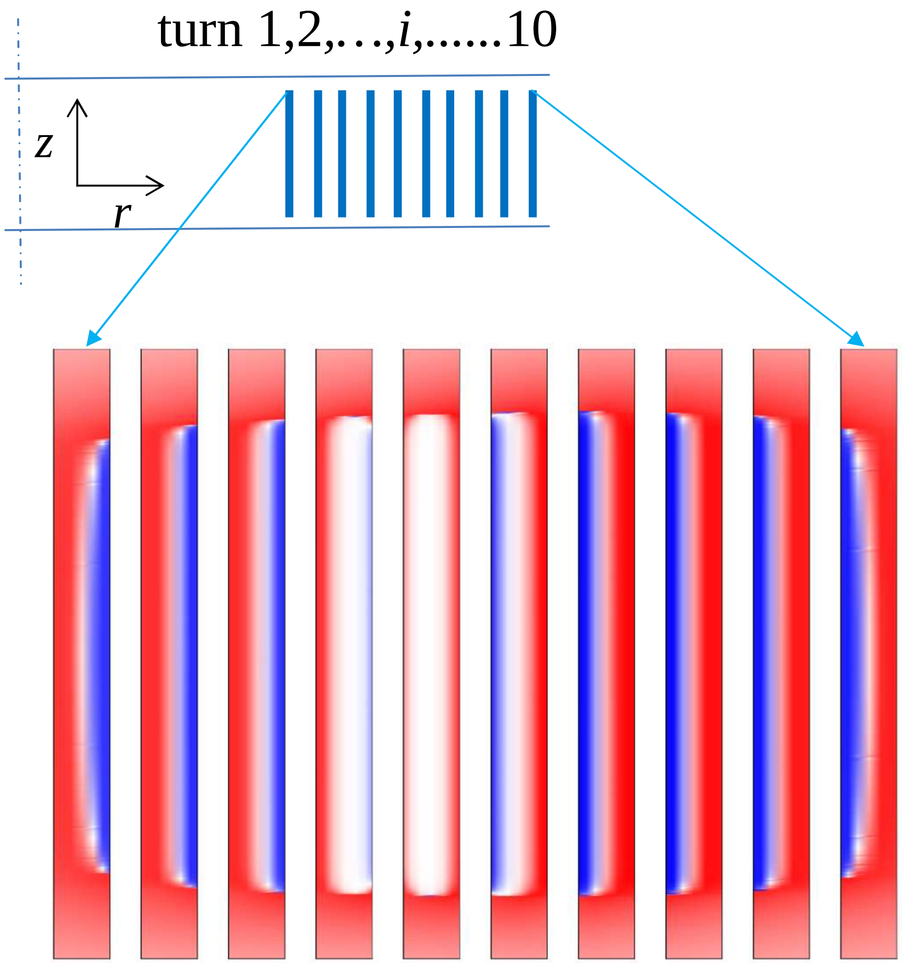
$$= \int_{L_i}\left(-\frac{\partial \vec{A}}{\partial t} + \frac{\partial \vec{A}_{0i}}{\partial t}\right)\cdot \mathrm{d}\vec{l} = U_{\Phi,i}(t) + U_{turn,i}(t)$$

$$U_{\Phi,i}(t) = -\int_{L_i}\frac{\partial \vec{A}}{\partial t}\cdot \mathrm{d}\vec{l} = -\oint_{S_i}\nabla\times\frac{\partial \vec{A}}{\partial t}\cdot \mathrm{d}\vec{S} = -\frac{\partial}{\partial t}\oint_{S_i}\nabla\times\vec{A}\cdot \mathrm{d}\vec{S} = -\frac{\partial}{\partial t}\oint_{S_i}\vec{B}\cdot \mathrm{d}\vec{S} = -\frac{\partial \Phi_i}{\partial t}$$

$$U_{turn,i}(t) = \int_{L_i}\frac{\partial \vec{A}_{0i}}{\partial t}\cdot \mathrm{d}\vec{l} = -\int_{L_i}\nabla\varphi_i\cdot \mathrm{d}\vec{l} \cong -2\pi R_i \nabla\varphi_i$$

$$U_{coil}(t) = \sum_{i=1}^{N_{turns}} U_{turn,i}(t) = -2\pi\sum_{i=1}^{N_{turns}} R_i \nabla\varphi_i$$

Evaluation of macroscopic quantities: *H*-FEM method



PDE model :

Formulation :

$$\begin{cases} -\dfrac{1}{r}\dfrac{\partial rE_\varphi}{\partial z} = -\mu_0\mu_r\dfrac{\partial H_r}{\partial t} \\ \dfrac{1}{r}\dfrac{\partial rE_\varphi}{\partial r} = -\mu_0\mu_r\dfrac{\partial H_z}{\partial t} \\ J_\varphi = \dfrac{\partial H_r}{\partial z} - \dfrac{\partial H_z}{\partial r} \end{cases}$$

$$E_\varphi = E_c\left(\frac{J_\varphi}{J_{c0}}\right)^n$$

Constraint:

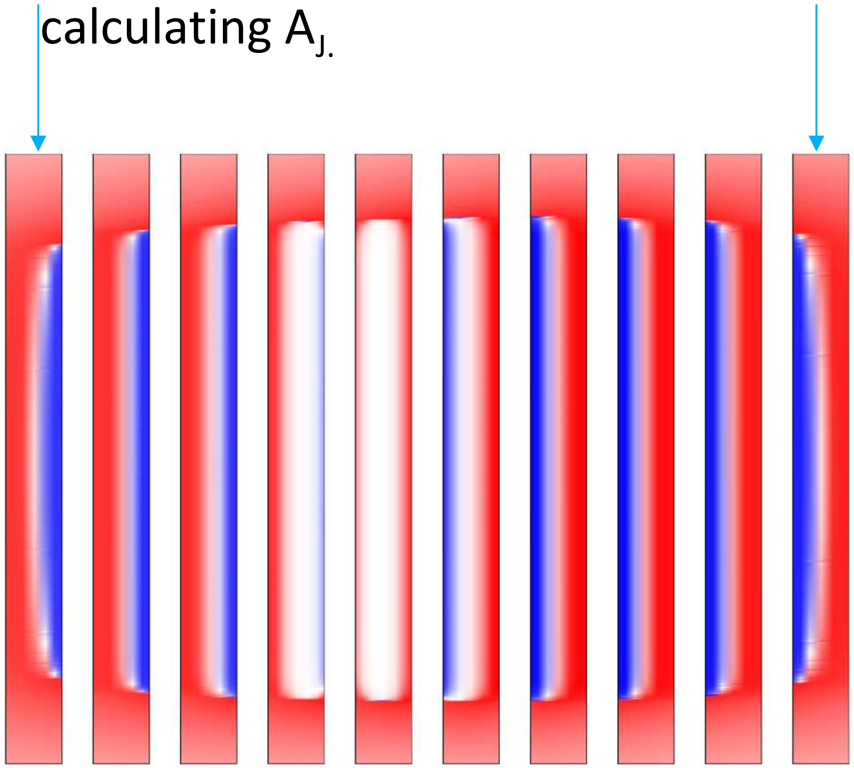
$$I_{i(t)} = \int_S j(r,z,t)\mathrm{d}S$$

$$I_{i(t)} = I_{coil}(t)$$

Evaluation of macroscopic quantities: *H*-FEM method

Magnetic Field (mf) model :

Current distribution (J) is setting as external current density for mf model. Mf model is used just for calculating A_J.



Formulation :

$$R_i \nabla \varphi_i = -\frac{1}{S_i}\int_{S_i} r d\vec{s} \cdot \left[\vec{E}_i(t) + \frac{\partial \vec{A}_J}{\partial t} + \frac{\partial \vec{A}_a}{\partial t} \right]$$ [1]

$$\nabla^2 \vec{A}_J = -\mu_0 \vec{J} \; ; \; \vec{A}_a = 0$$

$$U_{coil}(t) = \sum_{i=1}^{N_{turns}} U_{turn,i}(t) = -2\pi \sum_{i=1}^{N_{turns}} R_i \nabla \varphi_i$$

E(J) is calculated by PDE model.

Settings:

$$\vec{J}_e = \vec{J}$$
$$\sigma_{HTS} = 1 S/m;$$
$$\sigma_{Air} = 0 S/m;$$

[1] E Pardo Superconductor Science and Technology 28 (2015): 044003.

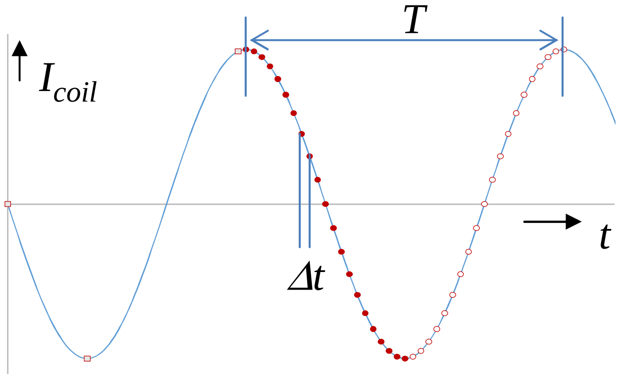
computation details : *A*-FEM method

$$j(r,z,t) = j_c \tanh\left(\frac{E(r,z,t)}{E_c}\right) = j_c \tanh\left(-\frac{1}{E_c}\frac{\Delta\left[A(r,z) - A_{0,i}\right]}{\Delta t}\right)$$

not power law !

$E_c = 10^{-4}\,\mathrm{V/m}$

time:



$$\Delta t = \frac{T}{N_{steps}} = \frac{1}{fN_{steps}}$$

$N_{steps} = 40$

variable

f

d_{max}

N_h

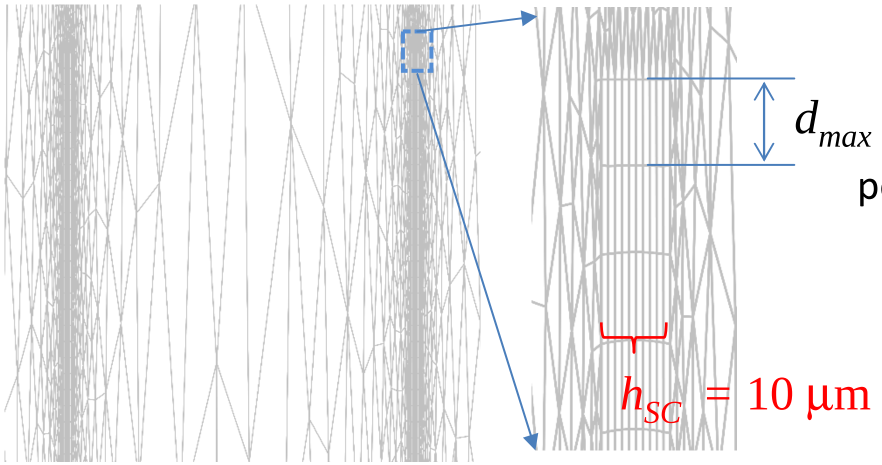
N_w

n_t

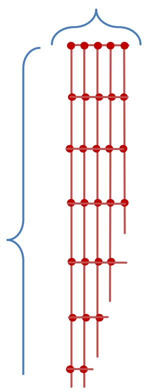
numerical tolerance

space:

asymmetric mesh (12x)



grid for saving A data



computation details : *H*-FEM method

E-J power law: $E = E_c (J / J_{c0})^n$

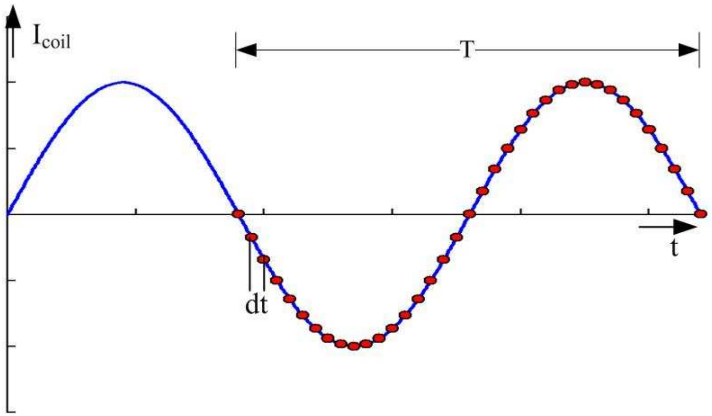
$E_c = 10^{-4}$ V/m

$n = 23$

$N_{step} = 400$

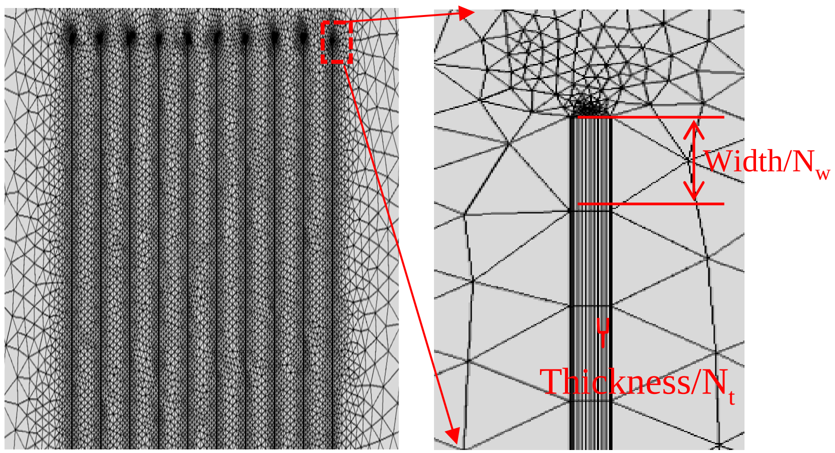
variable

Applied current:



$$dt = \frac{T}{N_{steps}} = \frac{1}{fN_{steps}}$$

Meshing: Mapping in HTS domain, Finer triangle in other domain



N_w: Points along the width during meshing;

N_t: Points along the thickness during meshing;

n_t: Numerical error tolerance during computing

f

n

N_w

N_t

n_t

Shape function

Lagrange or Curl

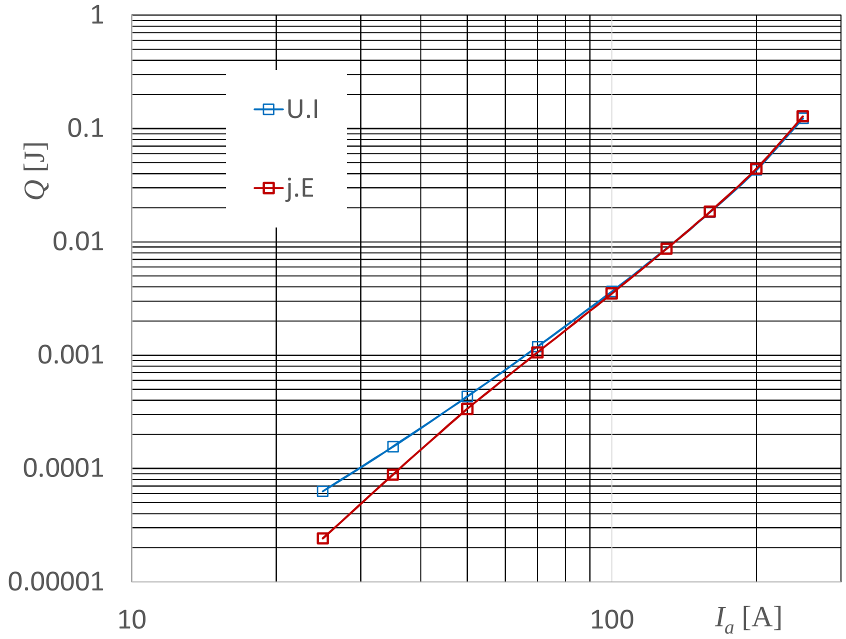
checking of calculation correctness : *A*-FEM method

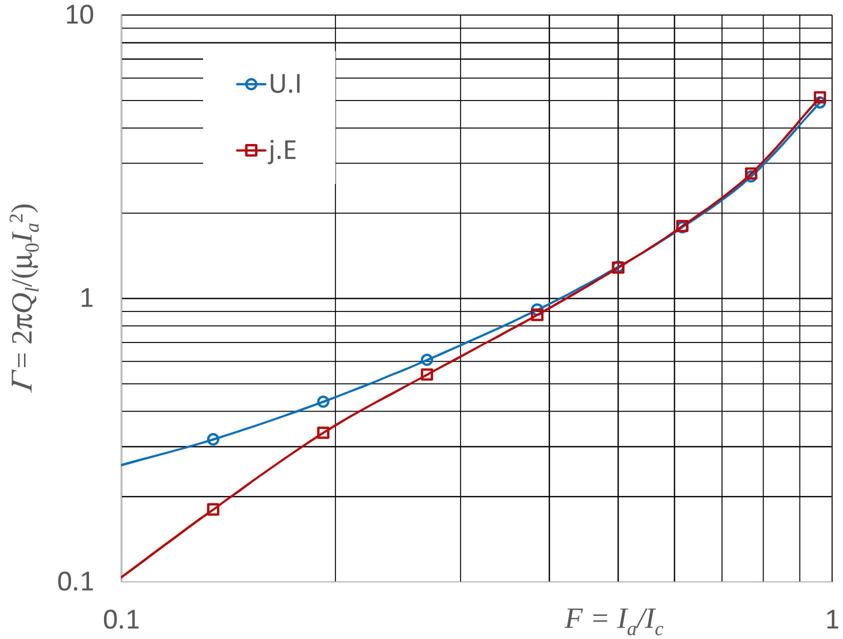
f = 2500 Hz

d_{max} = 18.5 μm, N_w = 101, N_h =3, n_{tol} = 10^{-8}

$j_c = const.$

1 data point – 4 hours





could the experimental data help ?

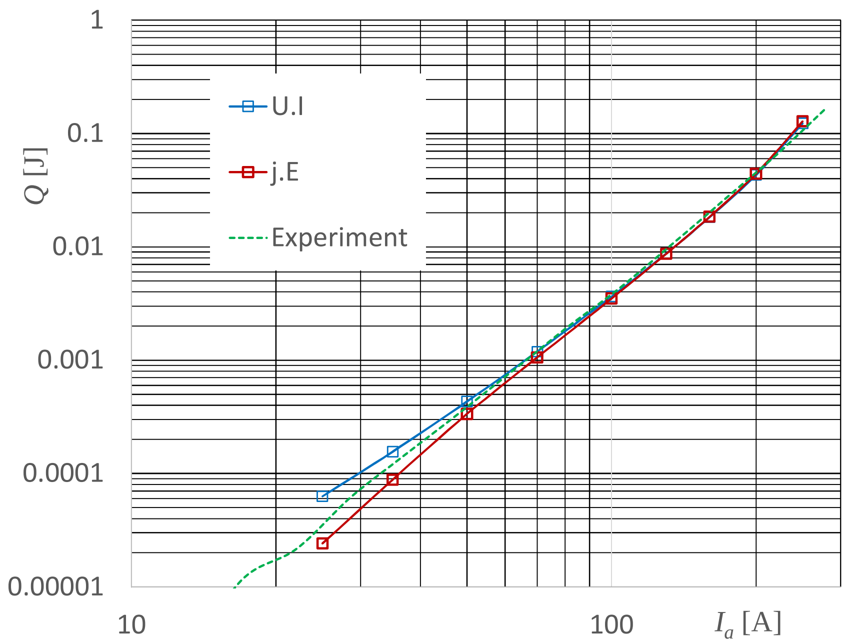
checking of calculation correctness : *A*-FEM method

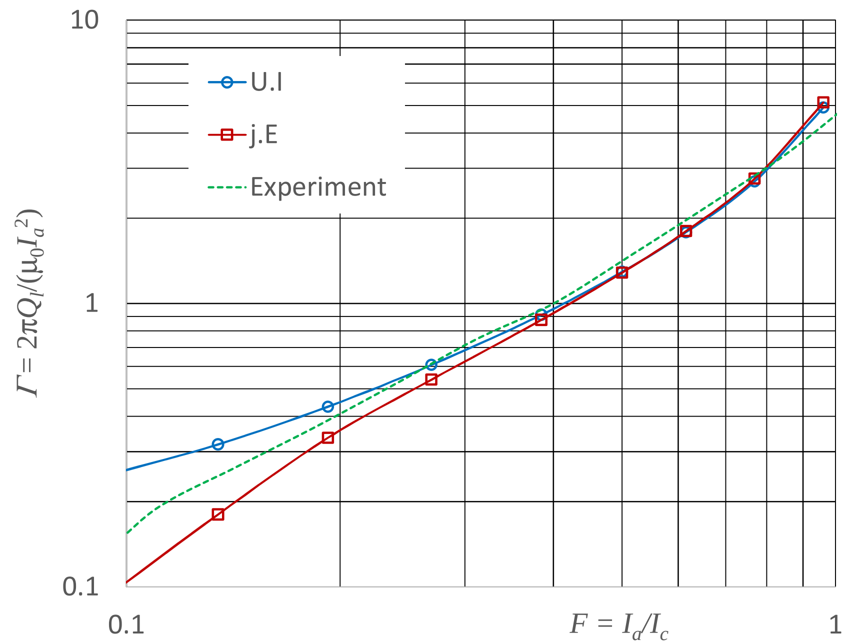
f = 2500 Hz

d_{max} = 18.5 μm, N_w = 101, N_h = 3, n_{tol} = 10^{-8}

$j_c = const.$

1 data point – 4 hours





Acknowledgement:
Jano Šouc

probably not!

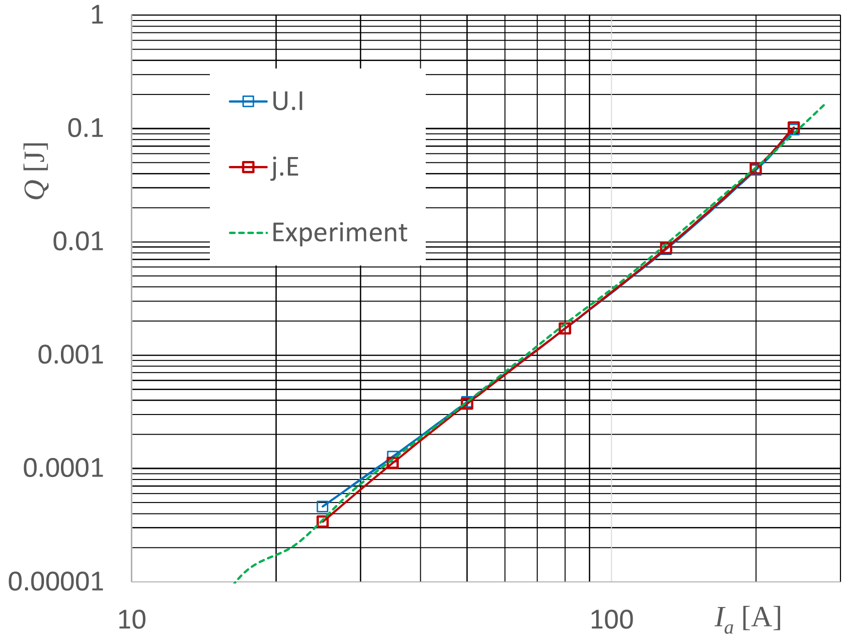
checking of calculation correctness : *A*-FEM method

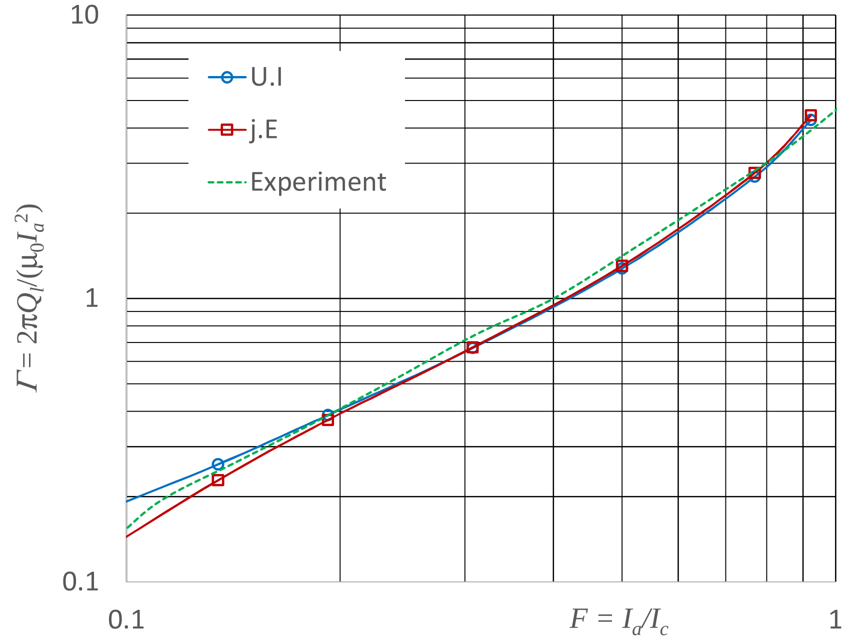
f = 2500 Hz

d_{max} = 18.5 μm, N_w = 221, N_h =7, n_{tol} = 10^{-12}

$j_c = const.$

1 data point – 4 hours





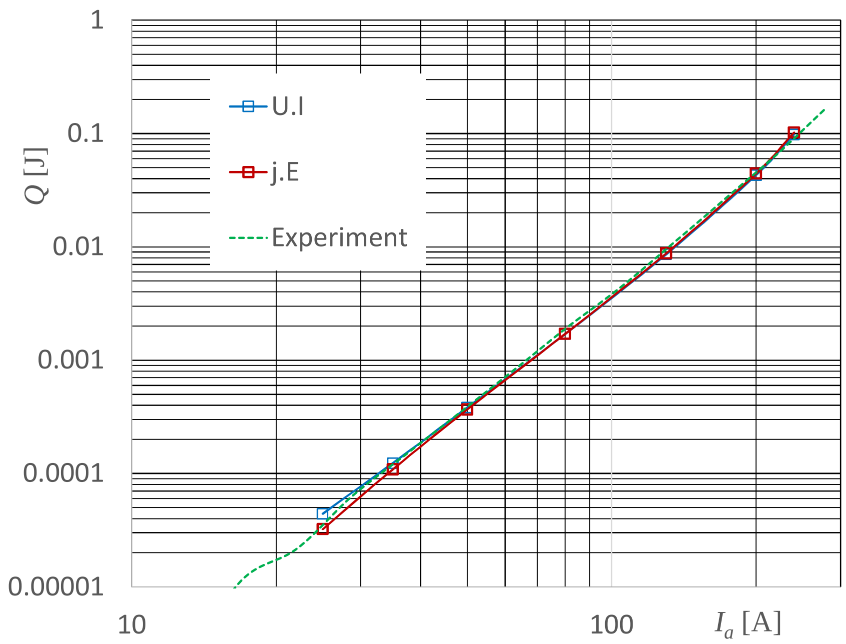
checking of calculation correctness : *A*-FEM method

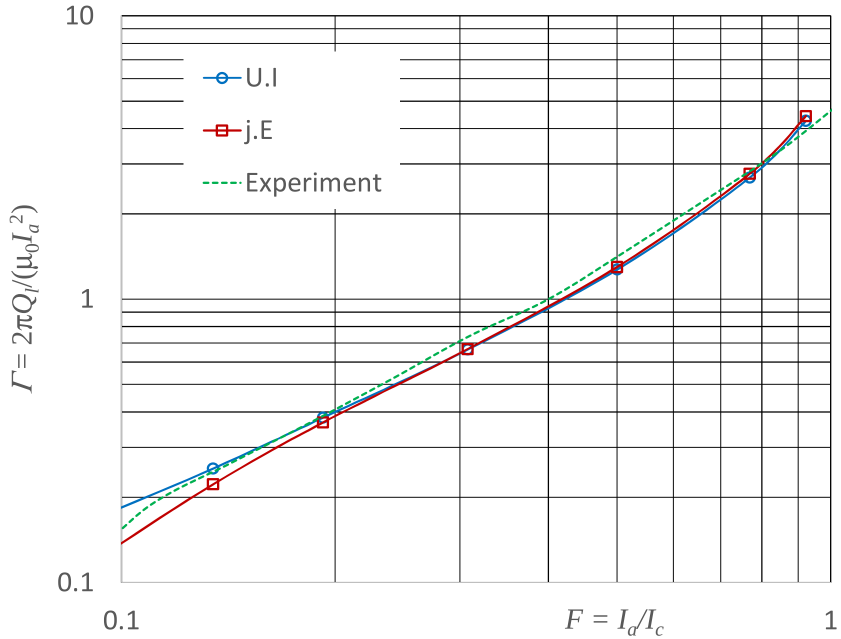
f = 2500 Hz

d_{max} = 18.5 μm, N_w = 221, N_h =11, n_{tol} = 10^{-12}

$j_c = const.$

1 data point – 5 hours





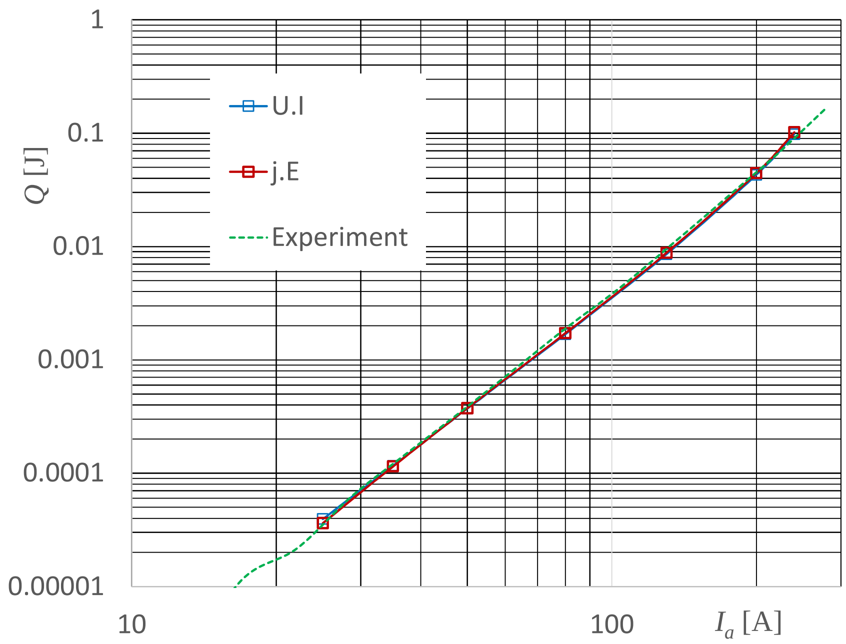
checking of calculation correctness : *A*-FEM method

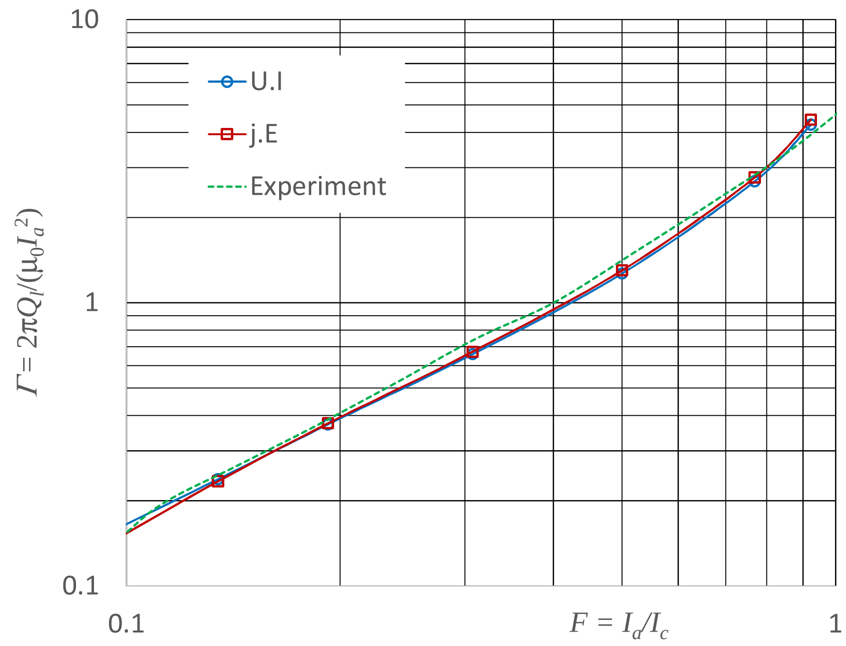
f = 2500 Hz

d_{max} = 18.5 μm, N_w = 451, N_h =11, n_{tol} = 10^{-12}

$j_c = const.$

1 data point – 7 hours





-> good resolution along the tape width is essential

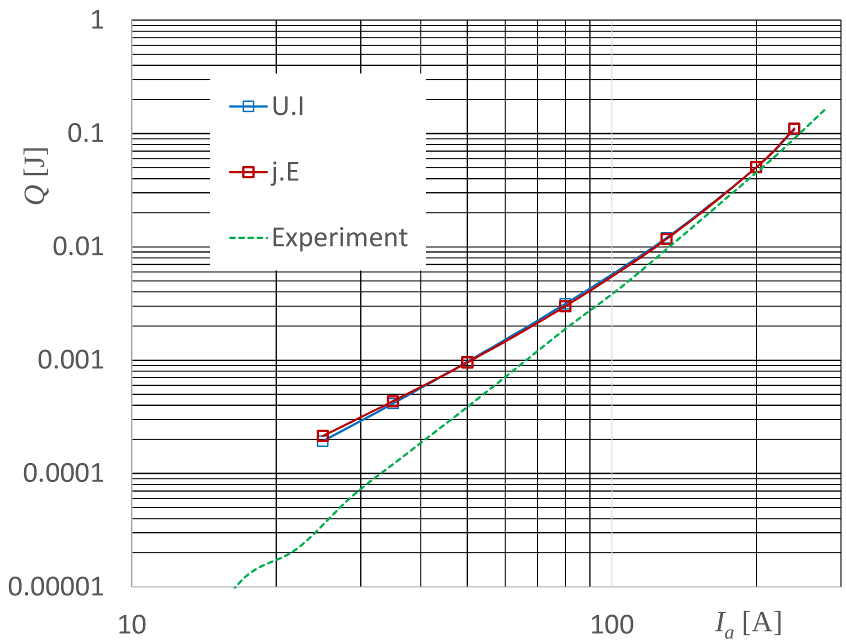
checking of calculation correctness : *A*-FEM method

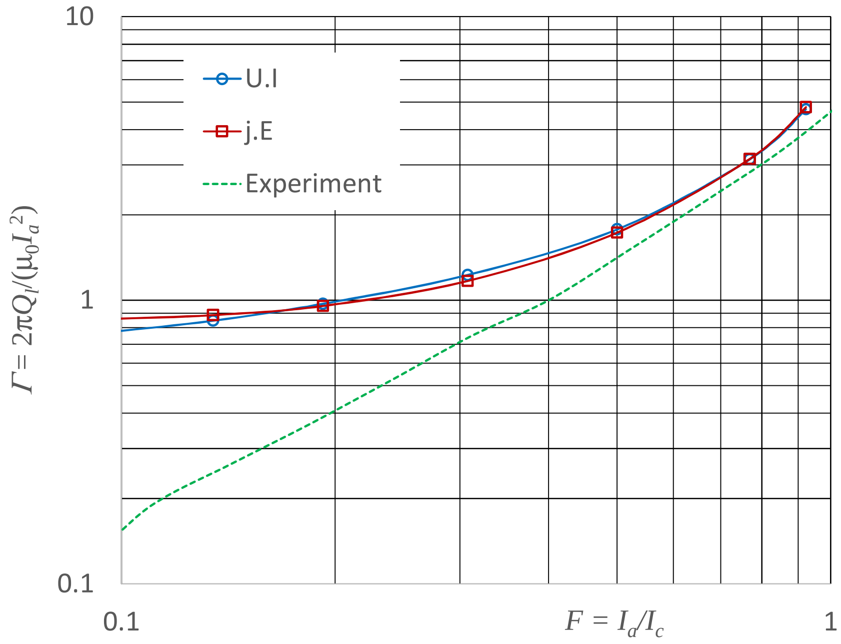
f = 3.85 Hz

$j_c = const.$

d_{max} = 18.5 μm, N_w = 451, N_h =11, $n_{tol} = 10^{-12}$

1 data point – 4.5 hours

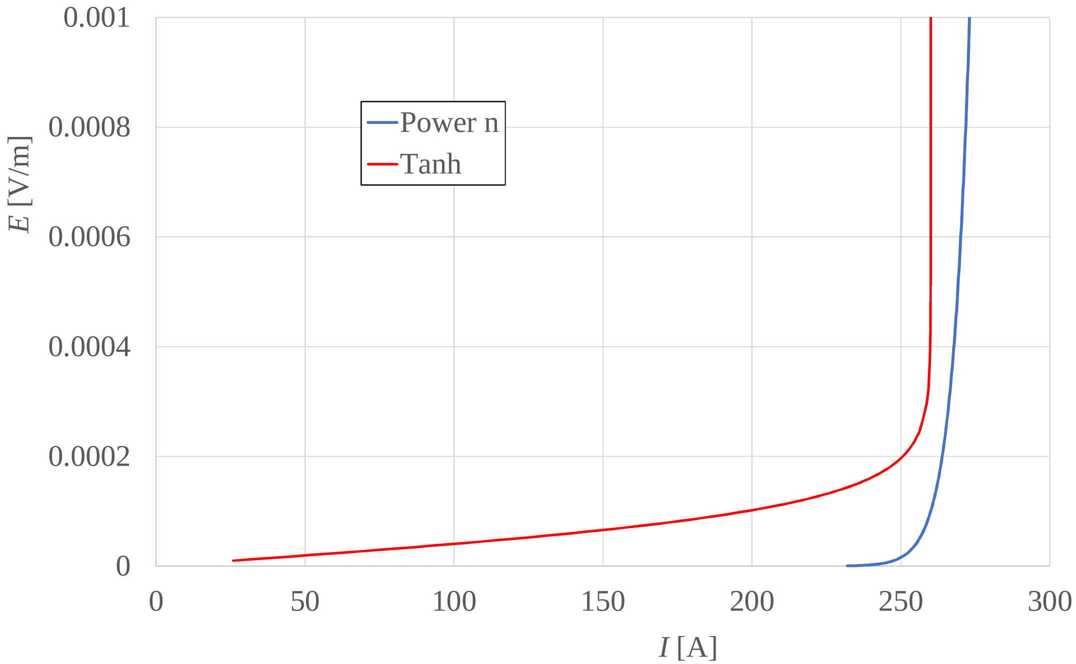




consequence of the used *E(j)* relation

checking of calculation correctness : *A*-FEM method

$j_c = const.$



consequence of the used *E(j)* relation

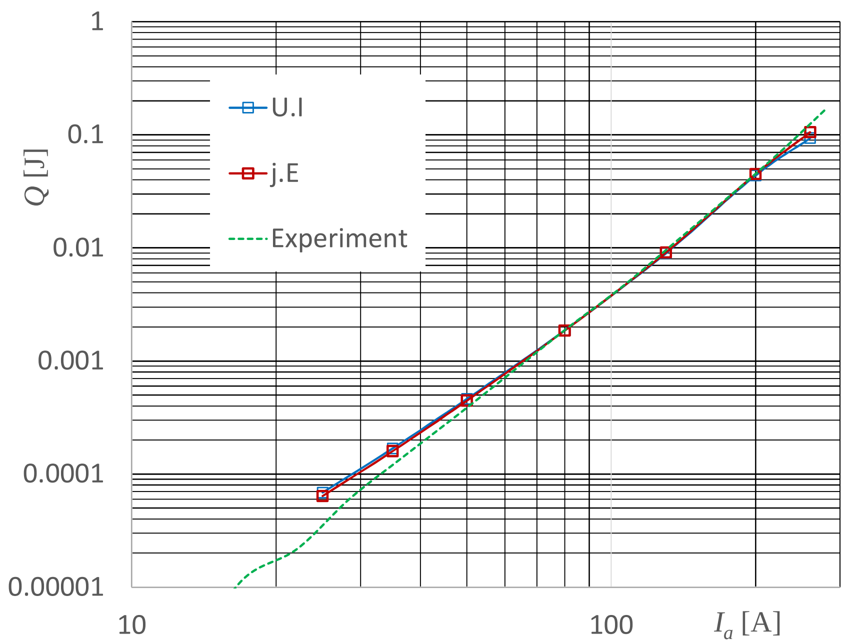
checking of calculation correctness : *A*-FEM method

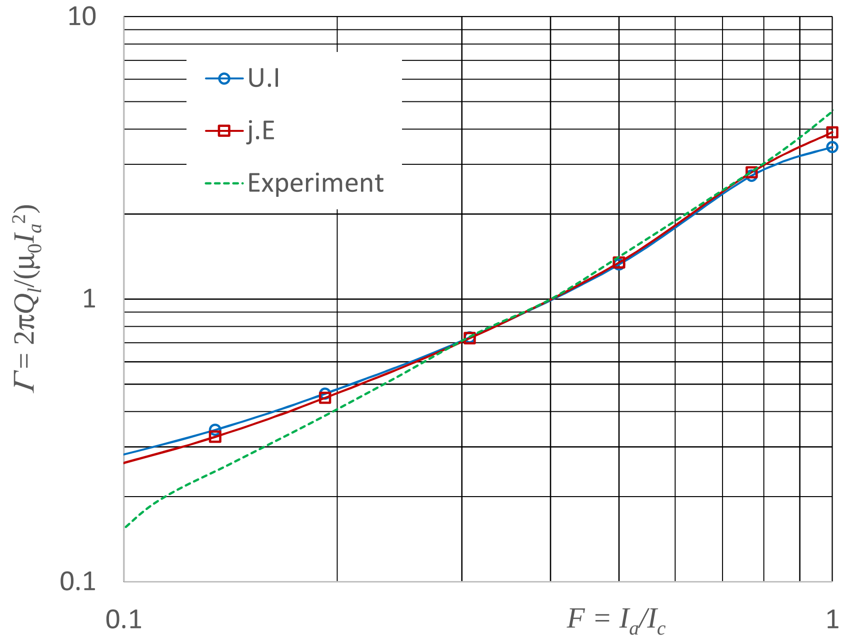
f = 36 Hz

d_{max} = 18.5 µm, N_w = 451, N_h = 11, $n_{tol} = 10^{-12}$

$j_c = const.$

1 data point – 3 hours





checking of calculation correctness : *H*-FEM method

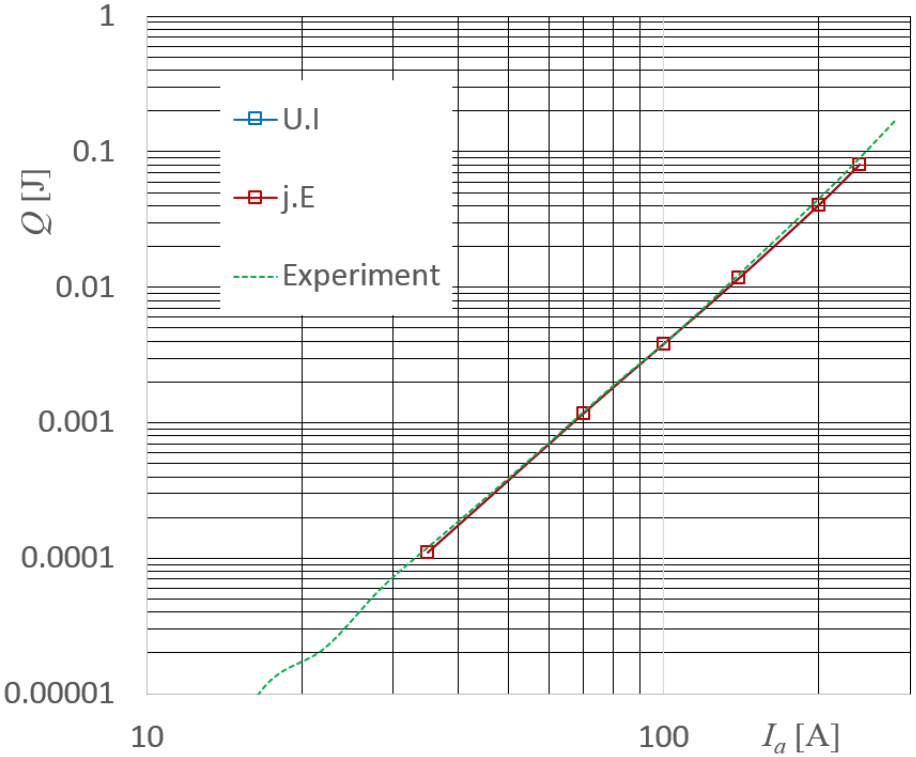
f = 36 Hz

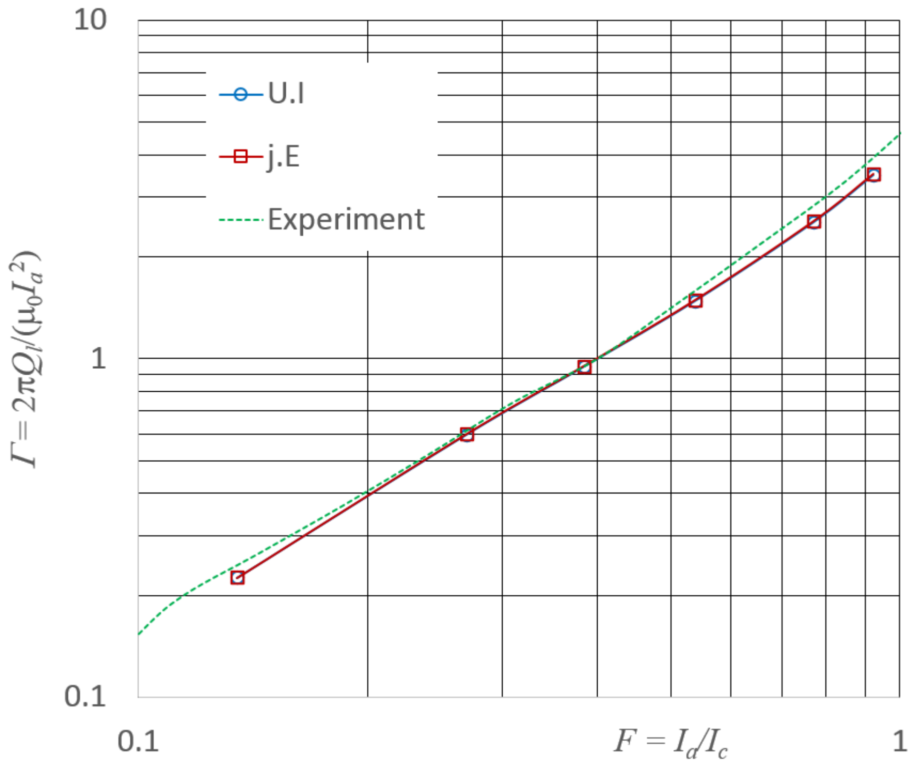
N_w = 100, N_h =1, n_{tol} = 10^{-7} , Shape function (Curl linear)

j_c = *const.*

2 h46 min (each point)

t_{step}=1e^{-6} s





good agreement between two evaluation methods

checking of calculation correctness : *H*-FEM method

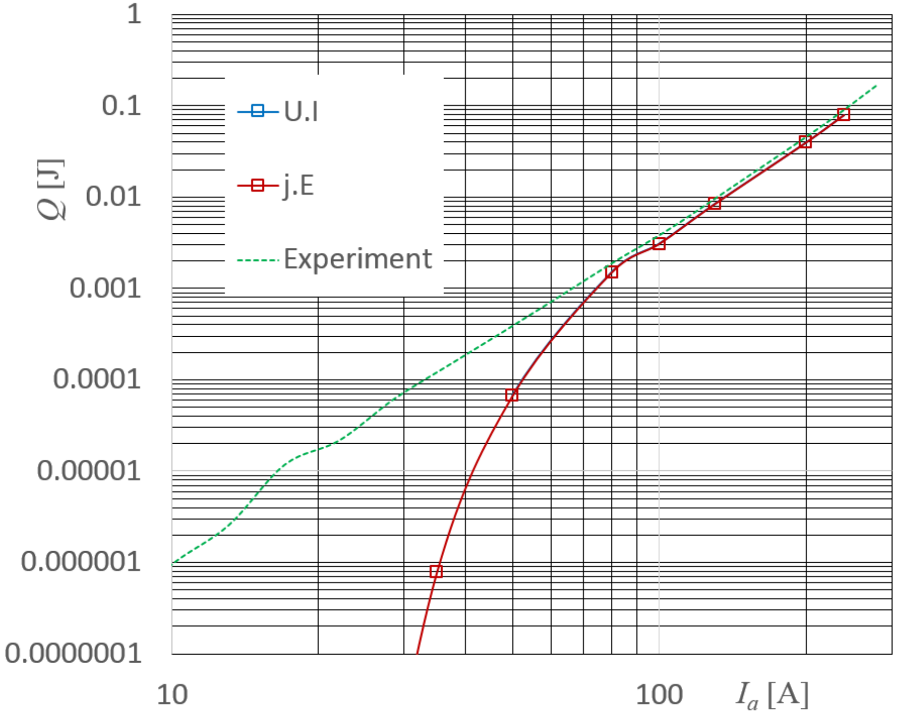
f = 36 Hz

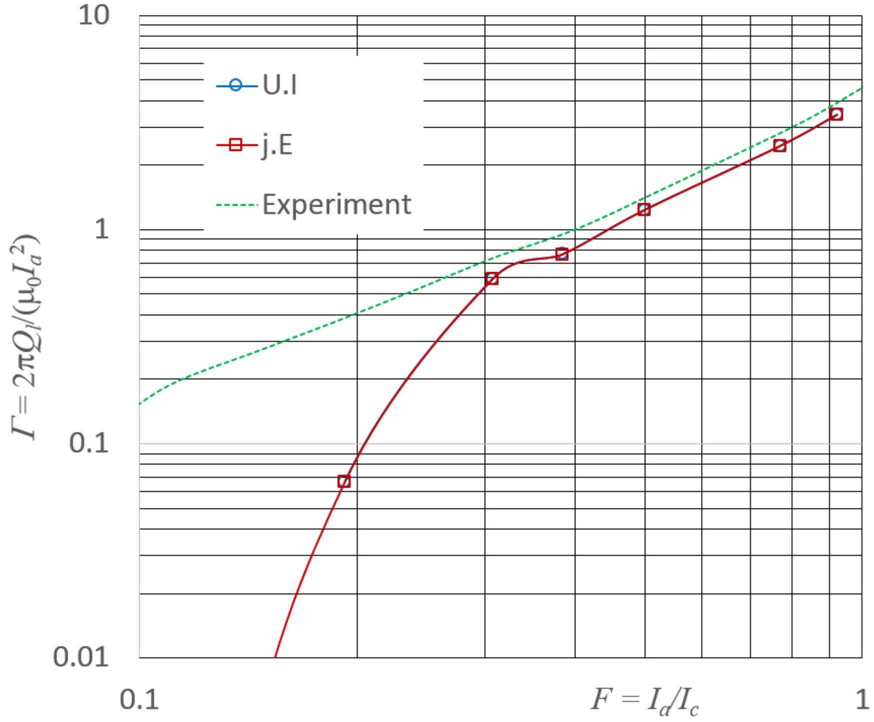
N_w = 25, N_h =1, n_{tol} = 10^{-7} Shape function (Curl linear)

j_c = *const.*

25 min (each point)

t_{step}=1e-6 s





very rough meshing was used in HTS zone, results wrong but still agree with each other

checking of calculation correctness : *H*-FEM method

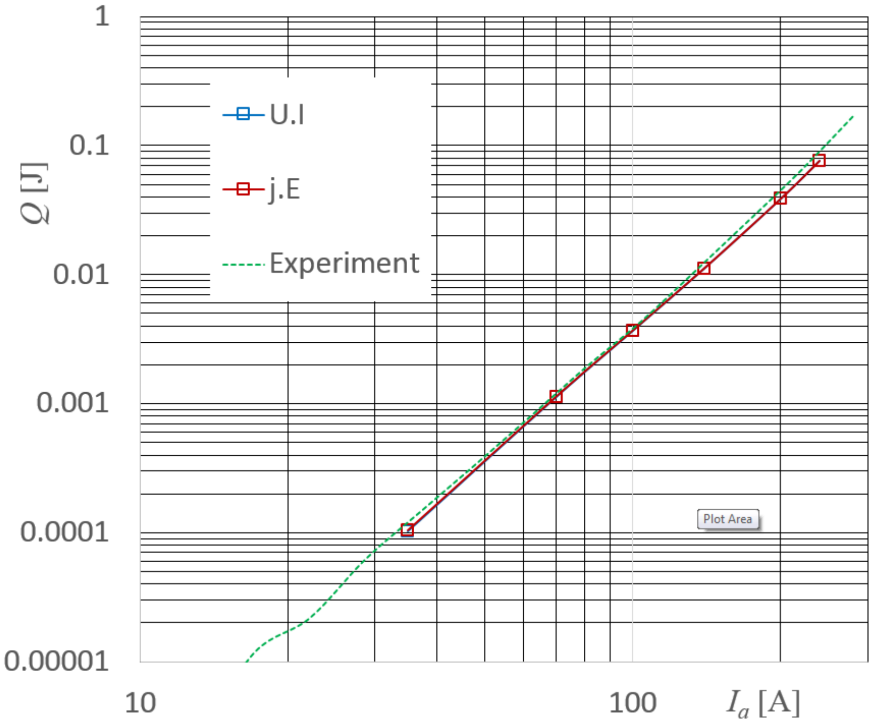
f = 72 Hz

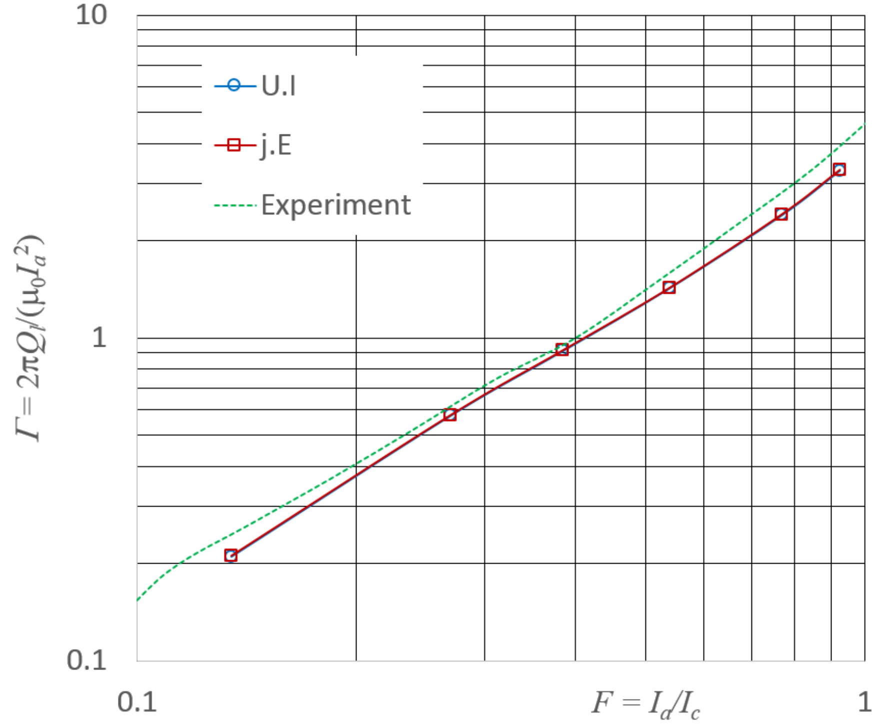
N_w = 100, N_h =1, n_{tol} = 10^{-7} Shape function (Curl linear)

j_c = *const.*

1 h 32 min (each point)

t_{step}=1e^{-6} s





checking of calculation correctness : *H*-FEM method

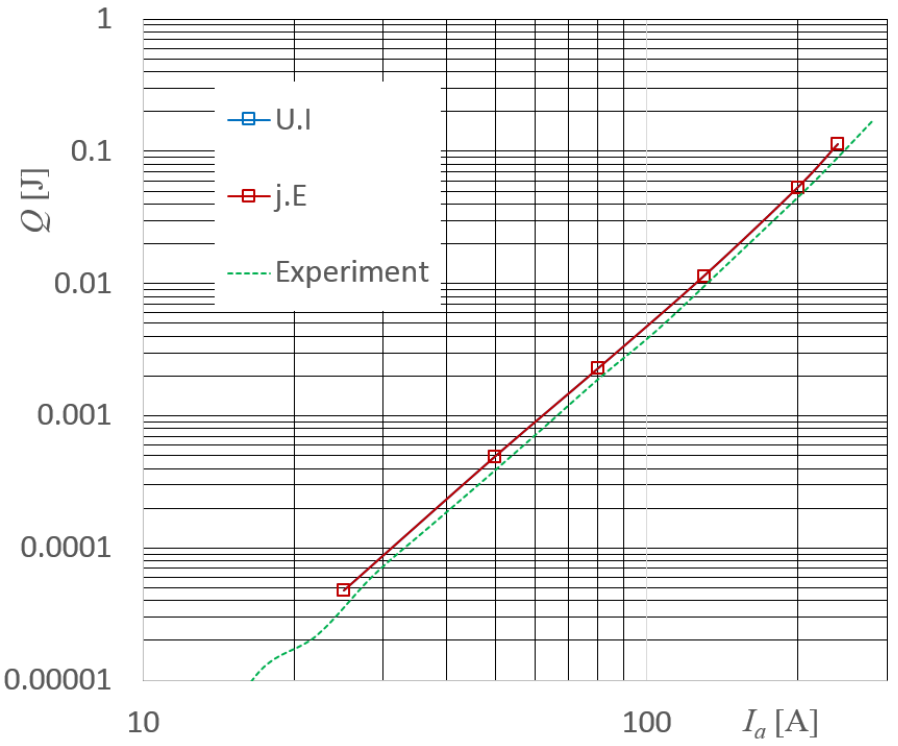
f = 1 Hz

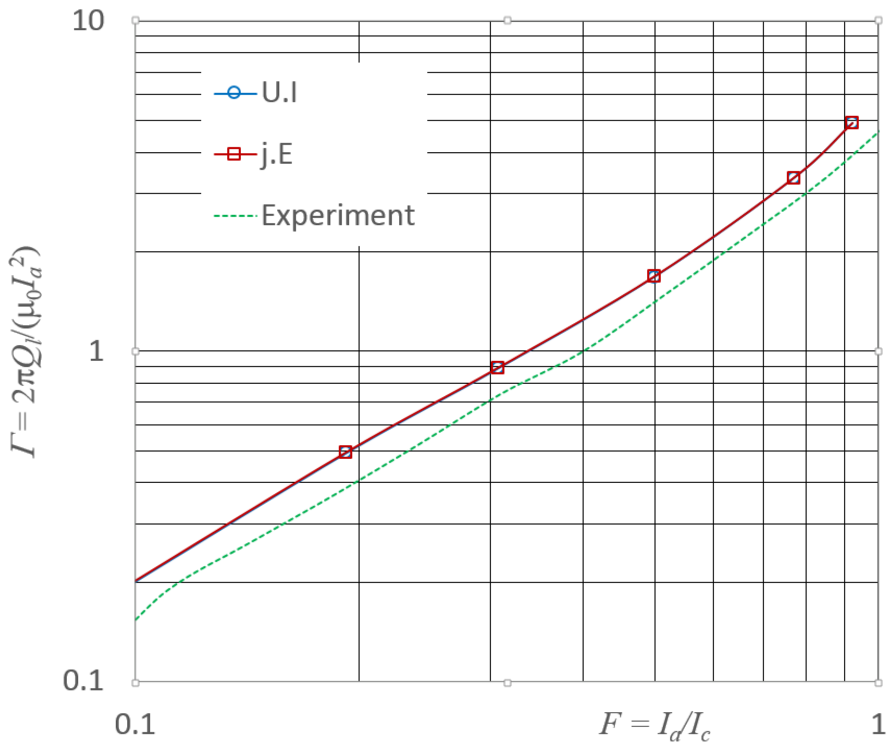
$N_w = 100$, $N_h = 1$, $n_{tol} = 10^{-7}$ Shape function (Curl linear)

$j_c = const.$

3 h 16 min (each point)

t_{step}=5e^{-5} s





agreement between two methods does not change with frequency

Do these two methods always agree with each other in H-FEM method?

checking of calculation correctness : *H*-FEM method

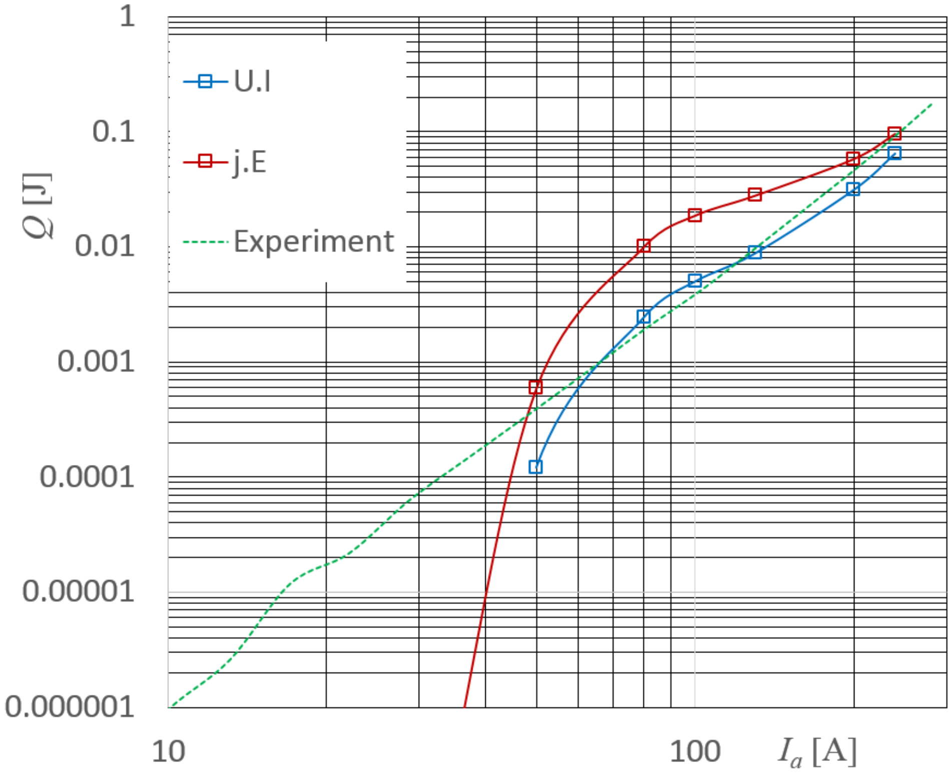
f = 36 Hz

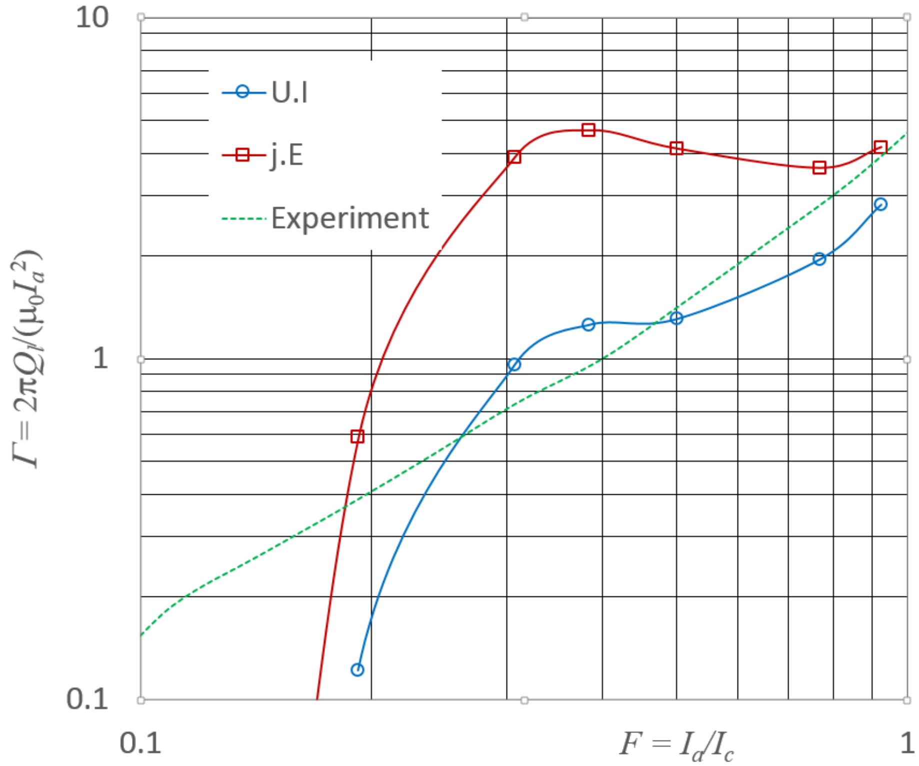
N_w = 25, N_h =1, n_{tol} = 10^{-7} (Lagrange linear)

$j_c = const.$

20 min (each point)

t_{step}=1e-5 s





NO!

checking of calculation correctness : *H*-FEM method

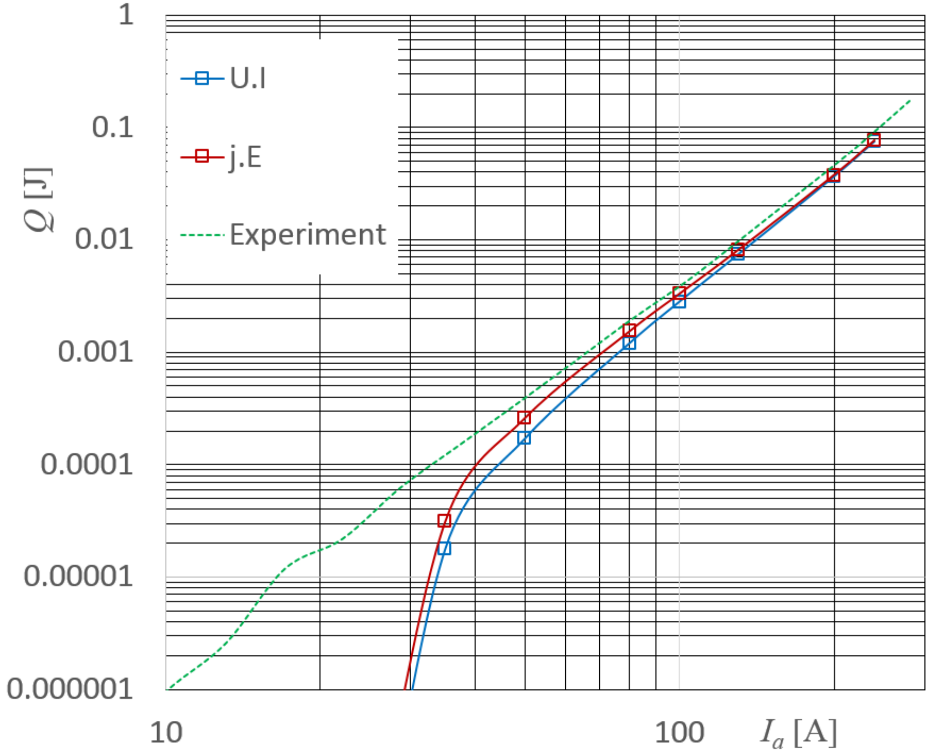
f = 36 Hz

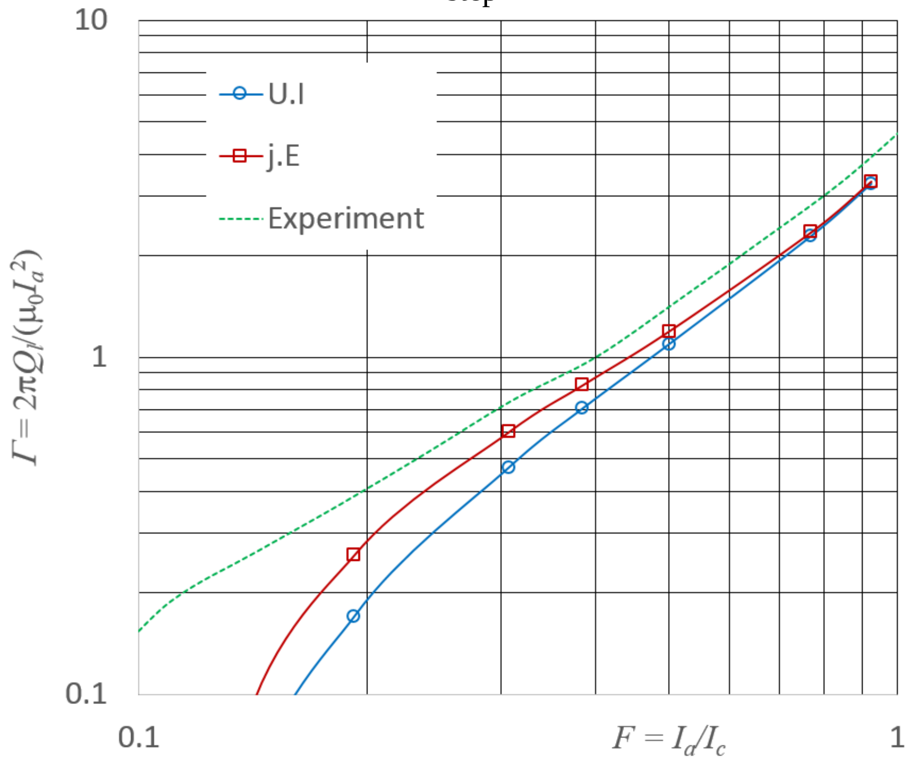
N_w = 100, N_h =1, n_{tol} = 10^{-7} (Lagrange linear)

j_c = *const.*

1 h 5 min (each point)

t_{step}=5e^{-6} s





checking of calculation correctness : *H*-FEM method

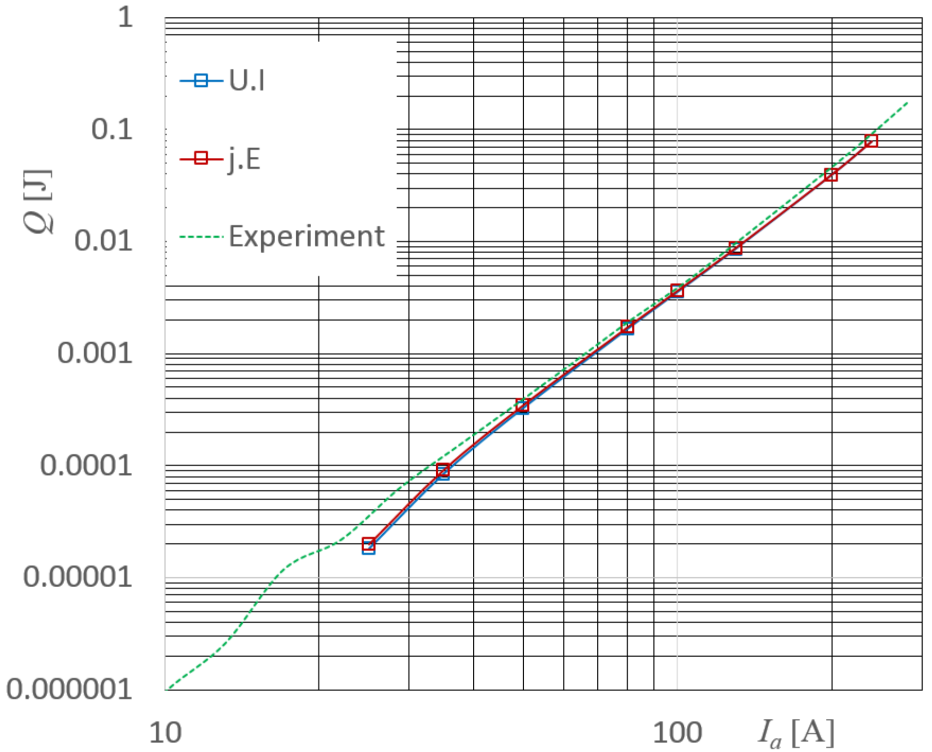
f = 36 Hz

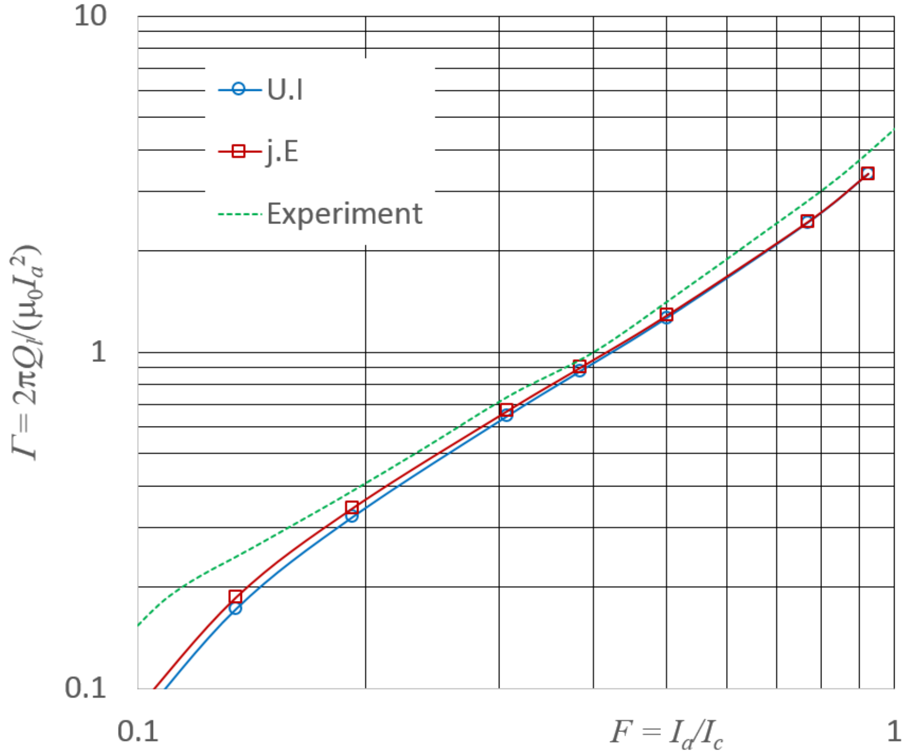
N_w = 400, N_h =1, $n_{tol} = 10^{-7}$ (Lagrange linear)

$j_c = const.$

1 h 2 min (each point)

t_{step}=1e^{-5} s





better agreement can be achieved by finer meshing!

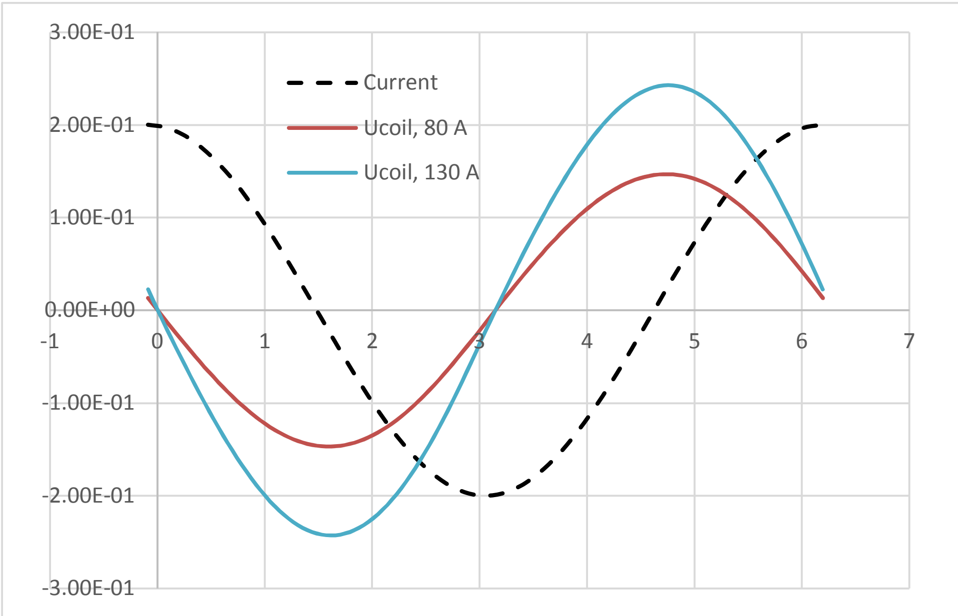
calculation corectness: comparison of two methods

f = 36 Hz

I_a [A]	Q [mJ] – A-FEM	Q [mJ] – H-FEM
80	1.87	1.82
130	9.06	9.10

$j_c = const.$

comparison of calculated voltage waveforms:



apparently only inductive signal

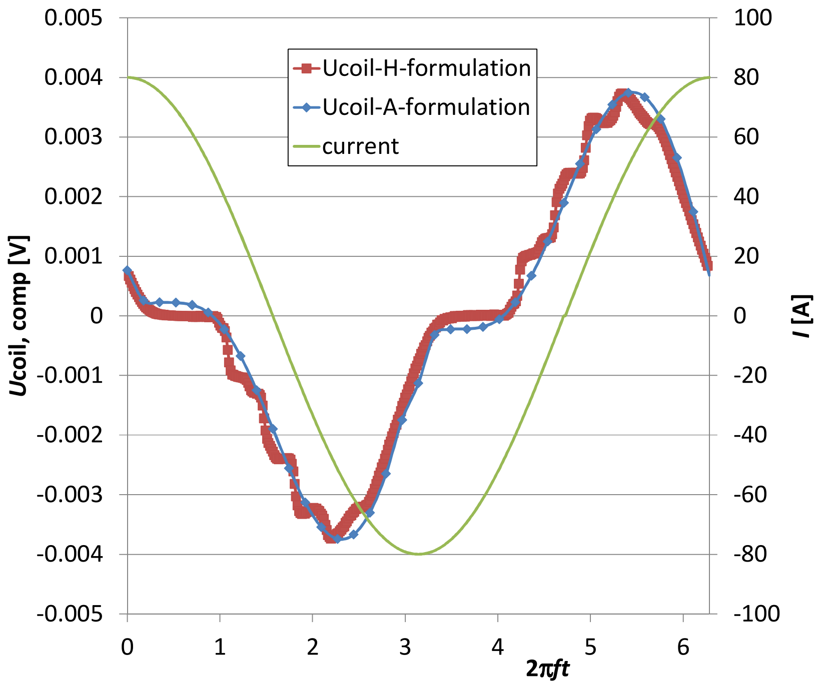
calculation corectness: comparison of two methods

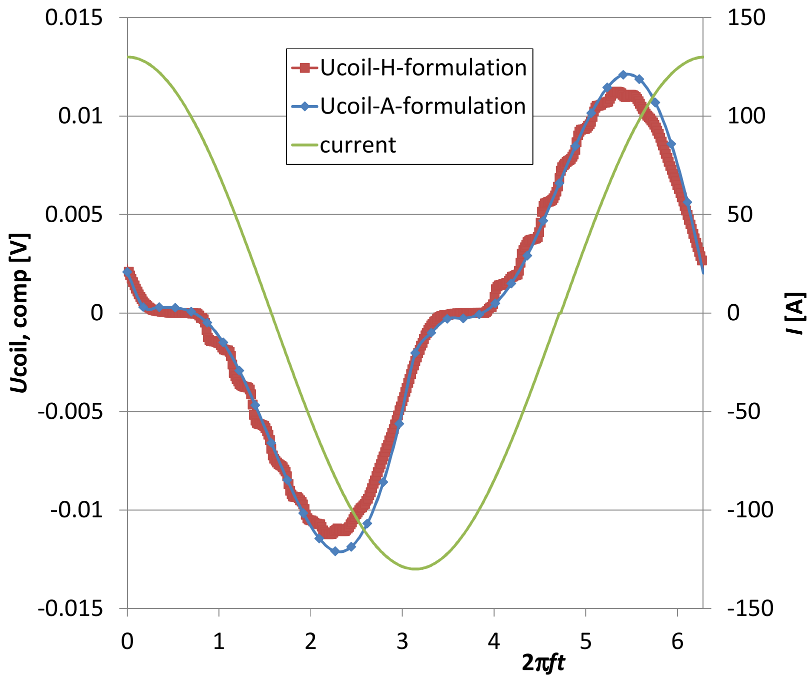
f = 36 Hz

I_a [A]	Q [mJ] – A-FEM	Q [mJ] – H-FEM
80	1.87	1.82
130	9.06	9.10

j_c = const.

comparison of calculated voltage waveforms after compensating the inductive component





Conclusions

- Comparing the macroscopic and microscopic approach of AC loss evaluation is a useful tool in checking the numerical model for AC problem
- In the methods using the macroscopic current as a constraint, main task is the determination of macroscopic voltage
- In *A*-formulation, macroscopic voltage is calculated as an independent variable, and such comparison is straightforward;

  calculation parameters e.g. necessary mesh density can be found
- In *H*-formulation, macroscopic voltage is derived from the solution, then it is not an independent variable, but still some preliminary checks are possible;

  e.g. shape function Curl (linear) much better than Lagrange (linear)

Conclusions

- Comparing the macroscopic and microscopic approach of AC loss evaluation is a useful tool in checking the numerical model for AC problem
- In the methods using the macroscopic current as a constraint, main task is the determination of macroscopic voltage
- In *A*-formulation, macroscopic voltage is calculated as an independent variable, and such comparison is straightforward;

  calculation parameters e.g. necessary mesh density can be found
- In *H*-formulation, macroscopic voltage is derived from the solution, then it is not an independent variable, but still some preliminary checks are possible;

  e.g. shape function Curl (linear) much better than Lagrange (linear)
- Voltage on coil is a measurable quantity – allows more detailed comparison with experiments than the check of AC loss value