Practical considerations on the use of $J_c(B,\theta)$ in numerical models of the electromagnetic behavior of HTS

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Motivation

Given a tape with known $I_c(B, \theta)$, how can we calculate the effective critical current of devices (cables, coils) made of that tape?

Picture sources:
Univ. Houston
Daibo et al. 10.1109/TASC.2011.2179691
Example: Roebel cable

Tape, \(I_c=340\) A

77 K, self-field

Strand, \(I_c=150\) A

10-strand cable, \(I_c=?\)

\[10 \times 150 = 1500\] A?

No, 1000 A!

33 % self-field reduction

We need a tool to predict this value!
Let’s start from the model for calculating \( I_c \).

- The model solves Ampere’s law in terms of \( \mathbf{A} \)

\[
\nabla \times \frac{1}{\mu} \nabla \times \mathbf{A} = \mathbf{J}
\]

- In the asymptotic limit \( t \to \infty \) from Faraday’s equation

\[
\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} - \nabla V \quad \Rightarrow \quad \mathbf{E} = -\nabla V
\]

- In the 2-D approximation, the scalar variable \( \mathbf{E} \)
  - represents the voltage drop (per unit length)
  - must be constant in each conductor

- Superconductor simulated with power-law resistivity

\[
E = \frac{\mathbf{J}}{J_c(B)} \left| \frac{J}{J_c(B)} \right|^{n-1}
\]

Reference: Zermeno et al. 2015 SuST 28 085004
How does the model work?

- Inversion of the $E$-$J$ relationship

\[
E = E_c \left| \frac{J}{J_c(B)} \right|^{n-1}
\]

\[
J = J_c(B) P
\]

\[
P = \frac{E}{E_c} \left| \frac{E}{E_c} \right|^{1-n^{-1}}
\]

- If $I_a$ is the transport current flowing in the i-th conductor, one has

\[
I_a = \int_{\Omega_i} P_i J_c(B) d\Omega_i
\]

\[
P_i = I_a \int_{\Omega_i} J_c(B) d\Omega_i
\]

- And the voltage drop per unit length $E_i$ in the i-th conductor

\[
E_i = E_c P_i \left| P \right|^{n-1}
\]
Test of the model against experimental data
Main features of the Roebel cables assembled at KIT

- 3 designs: 10, 17, 31 strands, transposition length 126, 226, 426 mm
- 12 mm tapes from two manufacturers: SuperOx and SuperPower
- 3 sizes x 2 manufacturers = 6 cables in total
- Length: 2.5 x transposition length
How to define the critical current of a Roebel cable?

2-D calculation

Two possible criteria:

1. Current at which $E=E_c$ in at least one conductor (MAX criterion)

2. Current at which $E_{AVG}=E_c$ (AVG criterion)
The starting tapes have very different $I_c(B, \theta)$. 
The in-field behavior determines the cable’s $I_c$.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Measured $I_c$</th>
<th>(# of strands) x ($I_c$ of the strands)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SuperOx</td>
<td>2747 A</td>
<td>3999 A</td>
</tr>
<tr>
<td>SuperPower</td>
<td>2264 A</td>
<td>4247 A</td>
</tr>
</tbody>
</table>
Measured and computed $I_c$ values agree within 9 %

Statistics on $I_c$ of 20 strands
SuperOx: mean=140 A, $\sigma$=10 A
SuperPower: mean=147 A, $\sigma$=7 A

$J_c(B,\theta)$ measured on a tape.
The calculated $I_c$ of the strand is:
SuperOx: 125.1 A
SuperPower: 146.0 A

For SuperOx, the sample used for $J_c(B,\theta)$ was a below-average one.

For SuperPower, it was very close to average.
With a correction factor 1.12 (dashed lines) the agreement for SOx is much better than before.
Considerations on cable design
What is the influence of the spacing between the superconducting layers?

Question #1:
Does a loose packing of the strands lead to higher $I_c$ due to the reduction of self-field?

Example: SuperOx cable (31 strands)

Standard spacing: 125 µm $\rightarrow$ $I_c = 2509$ A

Increased spacing: 350 µm $\rightarrow$ $I_c = 2700$ A $\quad +7.6\%$
The dependence of AC loss characteristics on the spacing between strands in YBCO Roebel cables

Zhenan Jiang¹, K P Thakur¹, Mike Staines¹, R A Badcock¹, N J Long¹, R G Buckley¹, A D Caplin² and Naoyuki Amemiya³

4. Conclusion

Transport AC loss in a nine strand YBCO Roebel cable with 0.25 mm spacers between the strands was measured and compared with that in a nine strand YBCO Roebel cable without spacers. Critical current was increased by 6.8% by spacing, due to a reduced self-field effect. AC loss in a Roebel cable with spacers may be improved.
What is the influence of the distance between the superconducting layers?

Question #2:
Can we then increase $J_e$ by pushing the superconducting layers closer to each other?

HTS coated conductors with 30 µm will be available soon

Example: SuperOx cable (31 strands)

Standard spacing: 125 µm $\rightarrow I_c = 2509$ A

Reduced spacing: 75 µm $\rightarrow I_c = 2446$ A

$I_c$ down by 2.5 %
$J_e$ up by 60 %
How does $I_c$ increase with increasing number of strands?

- More strands $\rightarrow$ more self-field
- Important role of $J_c(B,\theta)$
What is the influence of a background magnetic field?

50 strands
self-field

50 strands
background field 200 mT
What is the influence of a background magnetic field?

\[ \eta = 1 - \frac{l_{cc}}{ns \times l_{cs}} \]
Conclusion (1)

- A DC model was used to evaluate critical current of Roebel cables for low-field applications.
- In-field performance of composing strands plays a major role on the effective $I_c$ of the cable.
- Distance between superconducting layers has little influence $\rightarrow$ great potential for new tapes with thin substrate.
- With moderate fields (hundreds of mT), $I_c$ can be simply calculated from the $I_c$ of the strands.
How does $J_c(B, \theta)$ vary along the length of a tape?

- For modeling devices made of (hundreds of) meters of tape, we use a $J_c(B, \theta)$ model derived from data of a 15 cm long sample.
- We know that the self-field $I_c$ varies along the length.

How does $J_c(B, \theta)$ vary along the length? Simply a multiplicative factor? (e.g. 1.12 factor we used here)

Recent work says “no”.

Source: SuperPower, Inc.
Sample and length-dependent variability of 77 and 4.2K properties in nominally identical RE123 coated conductors

L Rossi\(^1\), X Hu, F Kametani, D Abraimov, A Polyanskii, J Jaroszynski and D C Larbalestier

![Graph 1](image1.png)

**Figure 3.** \(I_c\) as a function of position at \(B||c, 0.5\ T\) and at \(B||ab, 0.6\ T\) at 77 K in conductor S4 as a function of position. A tendency for \(I_c\) to drop for \(H||c\) that correlates to \(I_c\) rising for \(H||ab\) is evident.

![Graph 2](image2.png)

**Figure 4.** \(J_c\) angular dependence for tapes S1, S2, and S4.
Extracting an analytic expression for $J_c(B,\theta)$ is a time consuming process:

1. Find an analytic expression reproducing the angular dependence
2. Find the correct parameters that reproduce the data → calculation of effective $I_c$ necessary for a large number of field/angle combinations!

In the example on the left:

1. the $J_c(B,\theta)$ has 11 parameters → brute force approach time-consuming → manual tweak
2. Still, the agreement is far from perfect.
With the parameter-free approach (see Victor Zermeno’s poster), we reach an excellent agreement with experimental data in just six steps.

• No need of thinking about an analytic formula for $J_c(B, \theta)$.

• No need of manual or automatic tweaking of parameters.

• The interpolated $J_c(B, \theta)$ is ready to be used in successive simulations (e.g. calculation of $I_c$ or AC losses in a device).
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From experimental data to a ready-to-go model in 5 minutes!
$I_c$ calculated with the parameter-free method and with analytic expressions agree well.

maximum difference <3 %
Conclusion (2)

- It is important to check how the short sample on which $I_c(B_{\text{EXT}}, \theta)$ is measured is representative of the whole tape.
  - Recent work suggests variations of pinning center quality along the length.

- Parameter-free method allows going from experimental $I_c(B_{\text{EXT}}, \theta)$ data to a ready-to-use local $J_c(B_{\text{LOCAL}}, \theta)$ model in a few minutes.
  - No complex analytic expressions
  - No parameter tweaking
The codes for $I_c$ calculation are available for free. The one for extracting $J_c(B, \theta)$ will be soon.

Open-Source Codes for Computing the Critical Current of Superconducting Devices

Víctor M. R. Zermeño, Salman Quaiyum, and Francesco Grilli

www.htsmodelling.com
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