FEM-BEM approach for the numerical modeling of HTS-based conductors using A-V formulation for AC power application

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Agenda

- Introduction
- Objectives
- Motivation
- BEM-FEM (de-)Coupling
- Greens Functions over Finite Region
- Application to HTS A-V Formulation
- Illustrative Examples
- Conclusion and Further Work

Introduction

- This talk presents a development of the **Green's function** method into a **Boundary Element Method** (BEM) that enables efficient combination with the **Finite Element Method**
- The method developed also enables application to the **diffusion equation** which together with the **Laplace transform** method solves the High Temperature Superconductivity (HTS) modeling problem using the A-V Formulation

Objectives

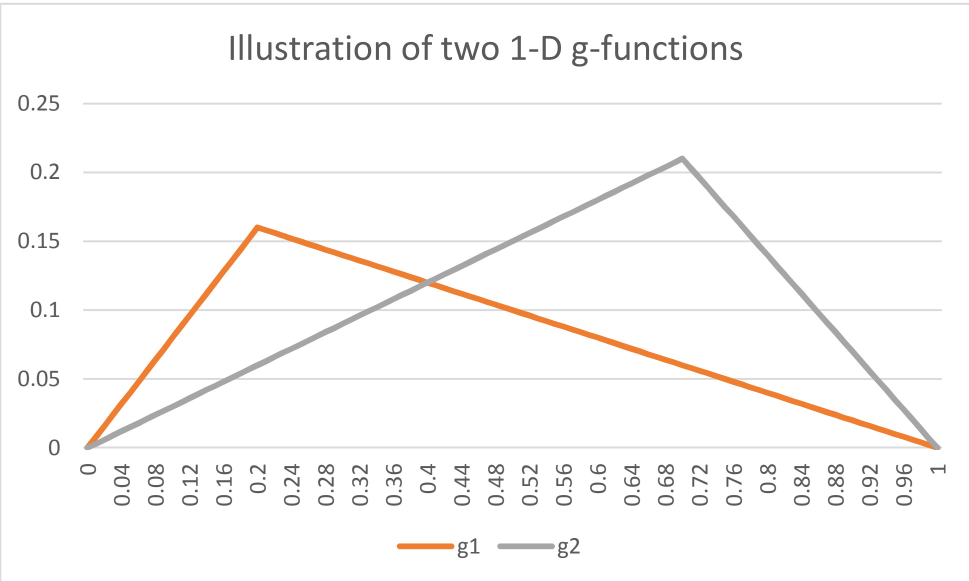
- Develop Green’s function over a uniform **finite region of arbitrary shape** and dimension for the diffusion equation into a BEM
- Develop Green’s function over a finite **multi-domain** region of arbitrary shape and dimension with different material properties for the diffusion equation into a BEM
- Use BEM over finite region with uniform properties and FEM over region with non-uniform properties in a manner that retains all the good aspects of both methods
- Apply a Laplace transform to enable coupling the BEM and FEM method for the diffusion equation

Motivation

- Construction of Green's function over a finite region enables **decoupling** of **FEM** and **BEM matrix forms**
- BEM enables collapsing bulk domains into boundary domains hence **reducing mesh requirement** whilst maintaining accuracy
- By transforming diffusion equation into the s-domain, the time aspects of the HTS model can be extracted from the domain onto the boundaries recovering the essential attractiveness of the BEM method

BEM-FEM (de)-Coupling

- Simple 1-dimensional illustration of Green's function for a uniform finite domain



$$\nabla^2 A = -\mu J$$

$$\nabla^2 g = -\delta(x)$$

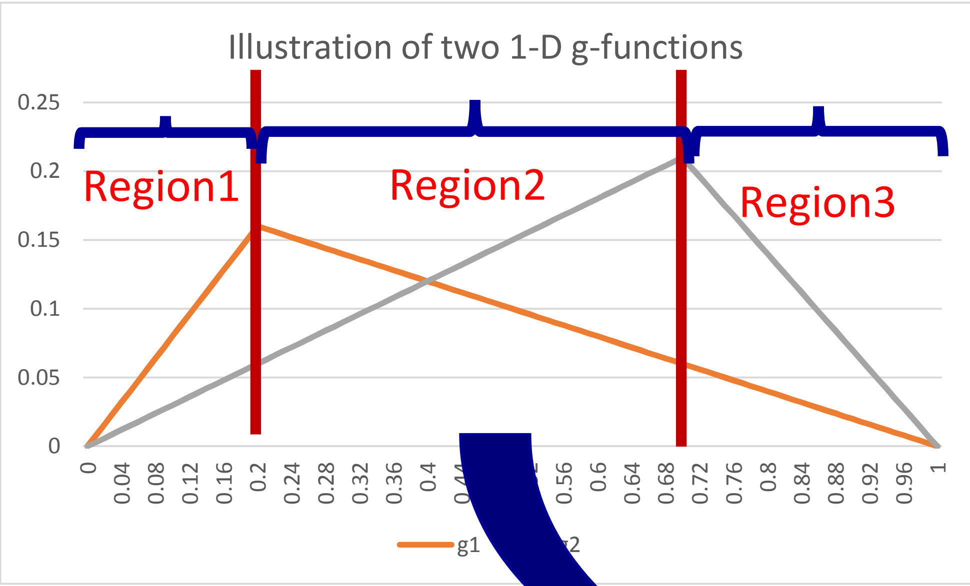
g is a **Piecewise affine function**

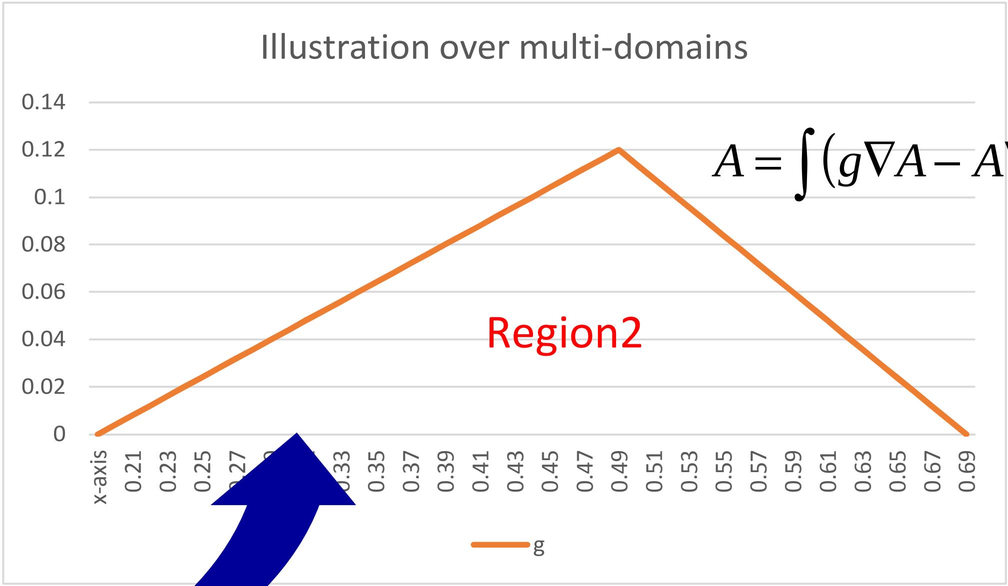
$$A_1 = \int (g_1 \nabla A - A \nabla g_1) d\Gamma + \int \mu g_1 J d\Omega$$

$$A_2 = \int (g_2 \nabla A - A \nabla g_2) d\Gamma + \int \mu g_2 J d\Omega$$

BEM-FEM (de)-Coupling

- Simple 1-dimensional illustration of Green's function for a multi-domain finite region



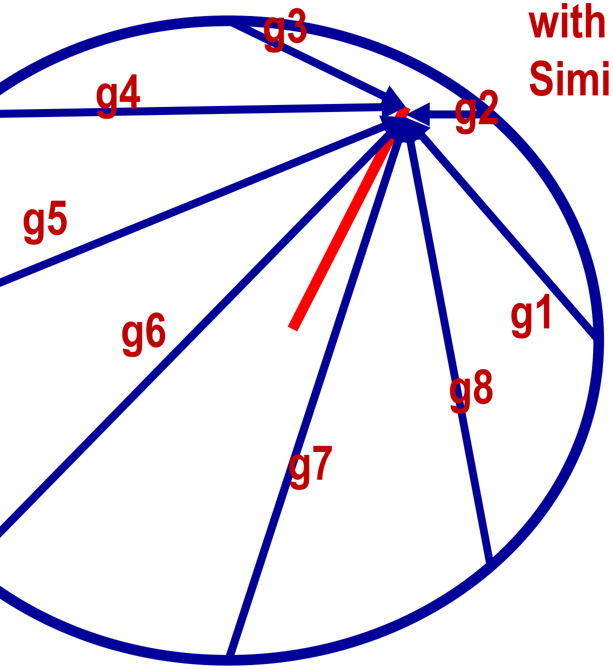


$$A = \int \left(g\nabla A - A\nabla g \right) d\Gamma_2 + \int$$

Boundary conditions for region 2 are obtained using the method of the earlier slide. If region has nonlinear properties, FEM can be used to solve within that region

BEM-FEM (de)-Coupling

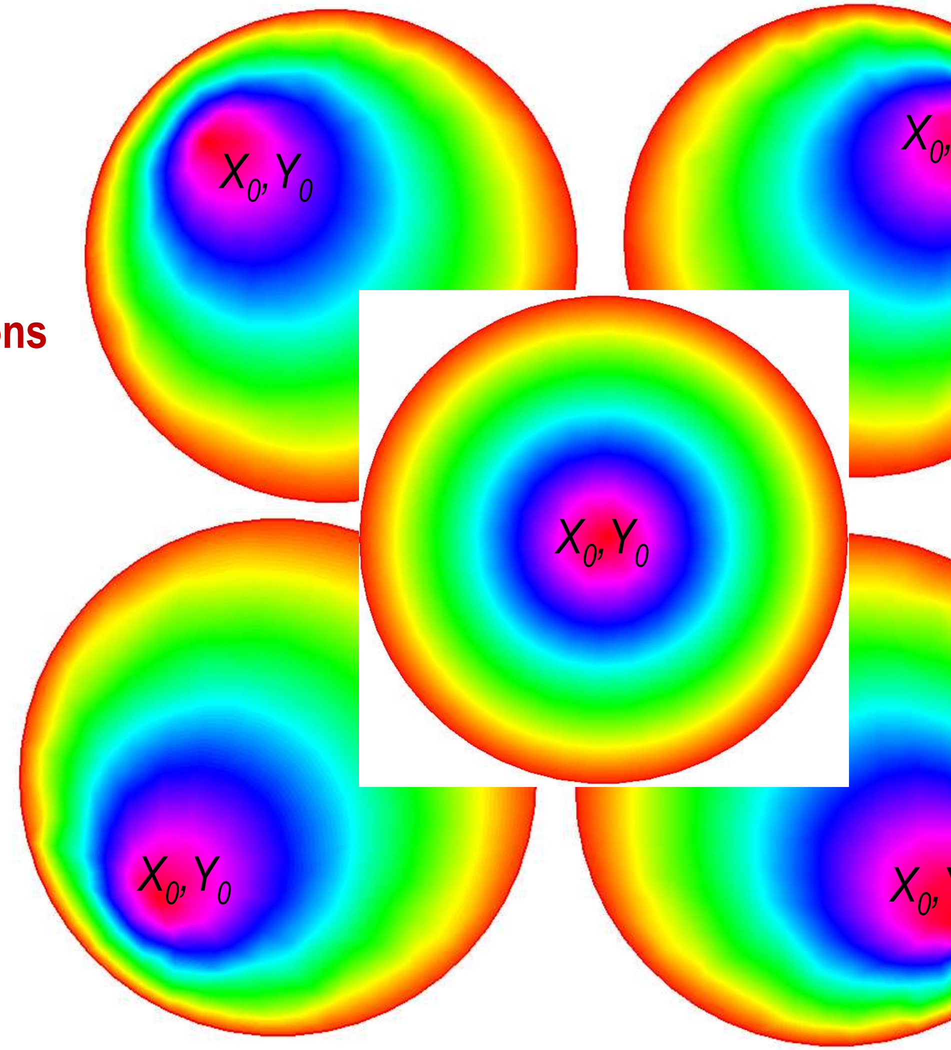
- Extension to 2 dimensions



Superposition of piecewise affine functions with radial and angular dependence
Similar extension to 3D

$$G = \sum_{i=1}^{N} g_i\left(\theta_i < \tan^{-1}(y/x) < \theta_{i+1}\right)$$

$$\nabla^2 G = -\delta(x)$$



Plot of G at different points, X_0, Y_0

BEM-FEM (de)-Coupling

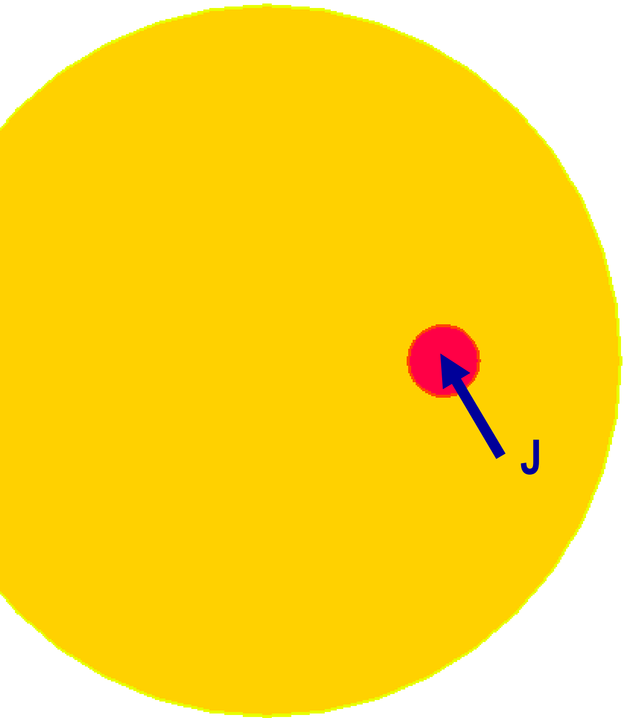
- Simple 1-dimensional illustration of Green's function for a multi-domain finite region with nonlinear material properties
  - **Conventionally** the FEM-BEM coupling is carried out for multi-domain systems. Where **BEM is used for the boundaries** of large bounding regions and **FEM for nonlinear sub-domains**
  - Using the fundamental solution for the **conventional BEM** method results in **coupled FEM-BEM matrix** forms that **may be difficult to solve** since resulting coupled matrices do not have the attractive features of conventional FEM matrix forms
  - The **conventional FEM-BEM coupled matrices** are also **full matrices** that can pose **memory storage problems** since they can not be stored sparse forms
  - Using the **green's functions** developed in this presentation for finite regions results in **decoupled matrix forms** that retains all the attractive features of FEM and BEM individually

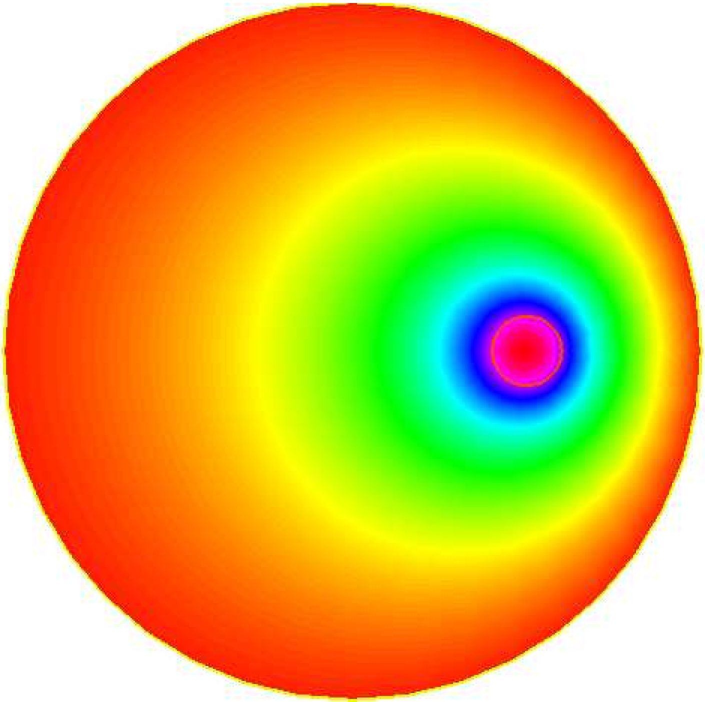
Implementation via Freefem++

- **Freefem++** does not explicitly implement any kind of BEM but the software package provides enough **computational capability** within its environment to enable the approach in this talk to be tested
    - **Boundary elements** and interior elements (nodes, edges, triangles, tetrahedra) for 2D and 3D can be accessed from created Freefem++ meshes
    - **Line integrals** and **surface integrals** can be easily calculated using modules within Freefem++
    - Computational and graphical **post-processing** is also very easily carried within the Freefem++ software environment
    - **Exporting data** for computational and graphical post-processing is also possible
    - Freefem++ also provides and links to a variety of **direct and iterative solvers** for sparse and full matrices

BEM-FEM (de)-Coupling

- Solving simple magneto-static problems





$$\nabla^2 A = -$$

$$\nabla^2 G = -$$

$$A = \int (G\nabla A - A\nabla G) d\Gamma + \int \mu G$$

BEM-FEM (de)-Coupling

- Application to the **Diffusion Equation**
  - **Green's functions** in this case become **exponentials of the affine functions** used for solving static Laplace or Poisson equations
  - Apply **Laplace transform** and **activation function** to convert bulk integrals into boundary integrals

Initial terms vanish because of activation function applied to both input signal and green's function

$$\nabla_{xyz}^{2} A(s) + skA(s) = 0$$

$$\nabla_{xyz}^{2} g(s) + skg(s) = -\delta(x, y, z, t)$$

$$A(s)e^{-Ct^{*}} = \int \left(g(s)\nabla_{xyz} A(s) - A(s)\nabla_{xyz} g(s) \right) d\Gamma$$

Boundary computation

Application to A-V Formulation for HTS modeling

ove from larger
nding domains to
ller sub-domains

Use BEM to obtain boundary conditions for bounding inner sub-domains

Greens functions are obtained for points on boundary of bounding sub-domain

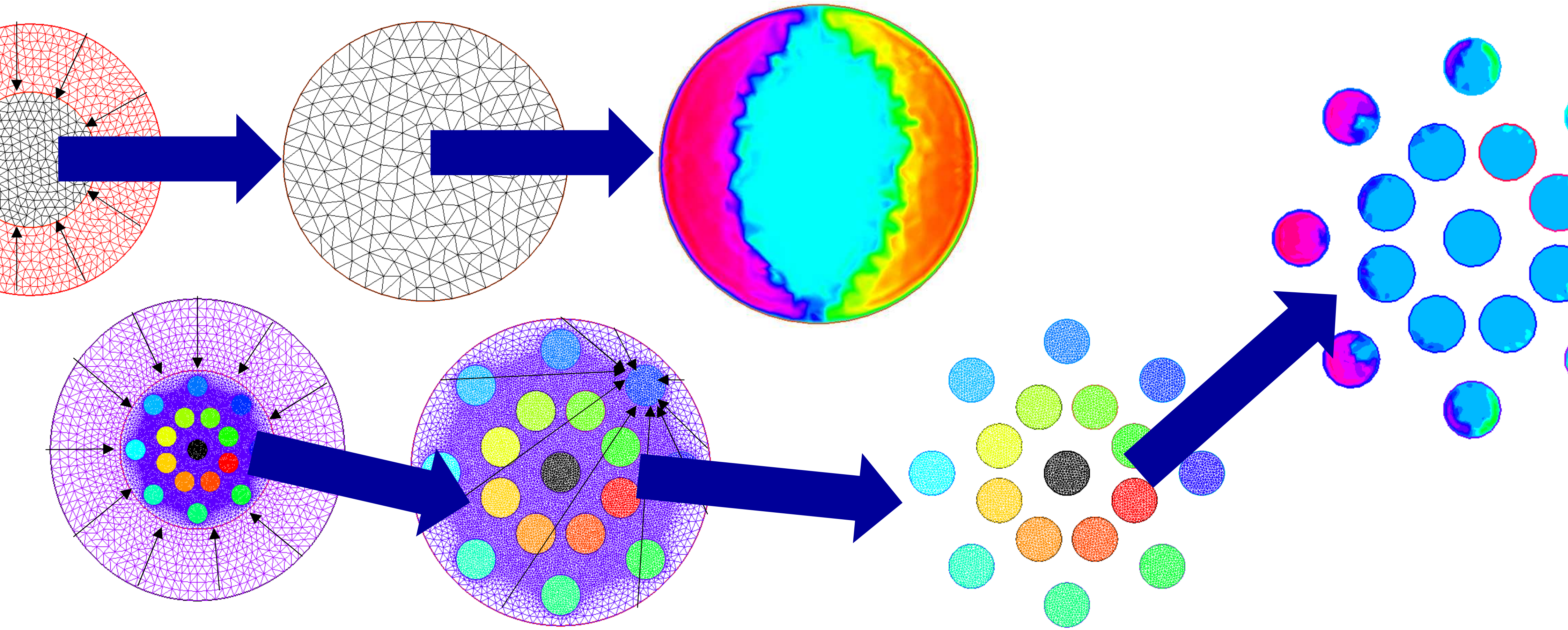
Obtain the Laplace transform of both the magnetic field input and greens functions

Obtain the inverse Laplace of the solution. Inversion is carried out once for the entire duration of the time simulation

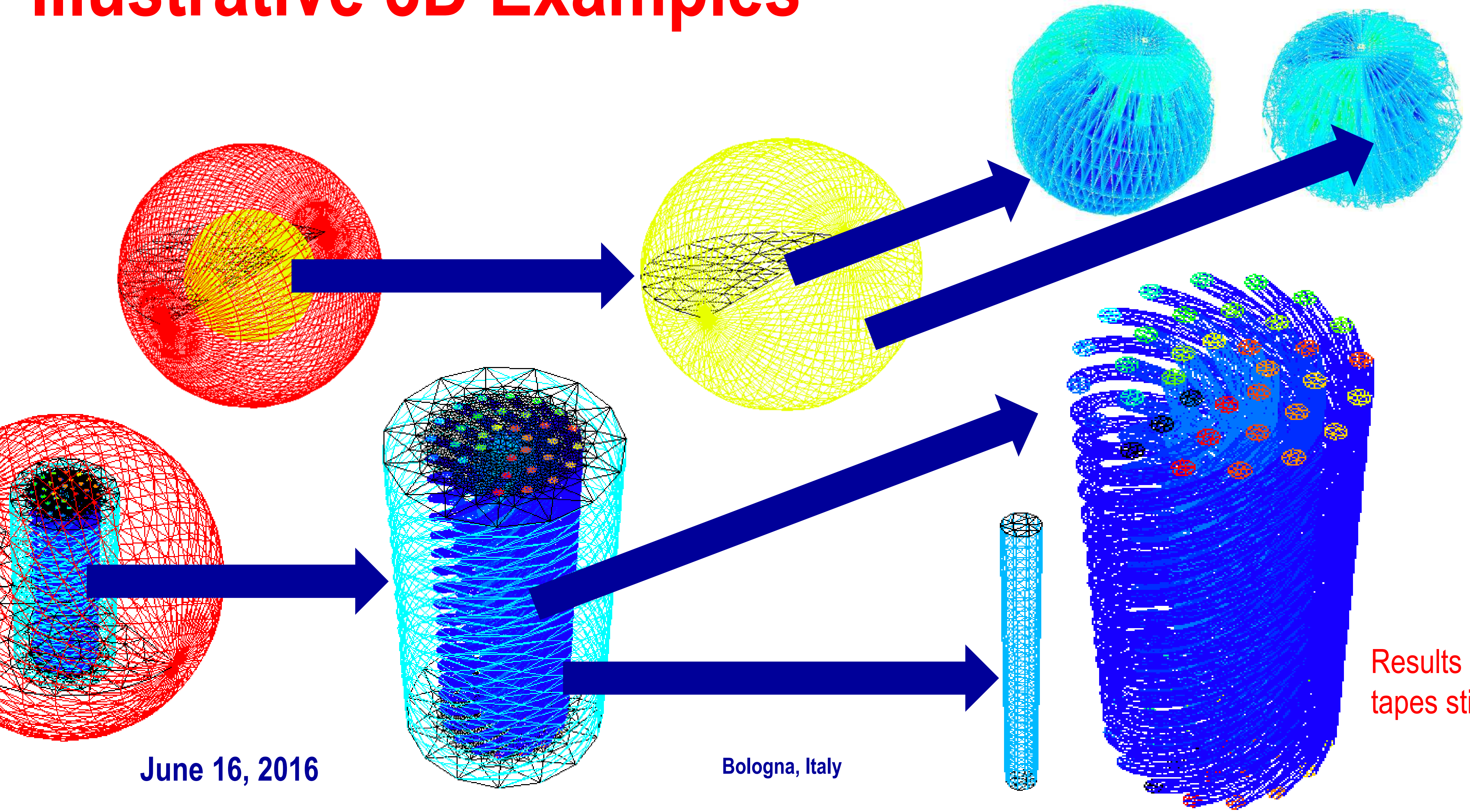
Use the Laplace transform of the diffusion equation to obtain solution of the diffusion at each bound on the bounding sub-domain

If the sub-domain has
nonlinear parameters, F
can be used to solve. The
electro-magnetic couplin
also easily incorporated i
FEM solution

Illustrative 2D Examples



Illustrative 3D Examples



Results for twist
tapes still in the

Conclusion and Further Work

- The BEM method outlined shows that simple piecewise functions can be easily constructed to satisfy the requirements of the Dirac function so it can be used for numerical computations involving the Poisson and Diffusion equations
- The Green's function obtained can be used to decompose sub-domains from larger domains that can be solved using the FEM method if nonlinear parameters are involved
- Using the Laplace transform, the diffusion equation can also be decomposed from the domain bulk onto the domain boundaries
- Verify outlined methodology experimentally and with standard numerical software computation tools