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TDGL Simulation on Dynamics of Helical Vortices in Thin Superconducting Wires in the Force-Free Configuration

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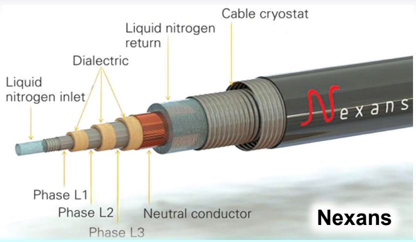
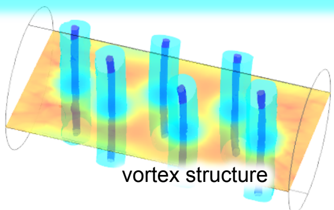
1. Time-Dependent Ginzburg-Landau (TDGL) Equations
2. 2D Simulation by TDGL equations
3. Vortex Dynamics and Electromagnetic Response
4. Longitudinal Field Effect in the Force-Free Configuration
5. 3D Simulation: Transverse Configuration
6. 3D Simulation: Longitudinal (Force-Free) Configuration
7. Summary

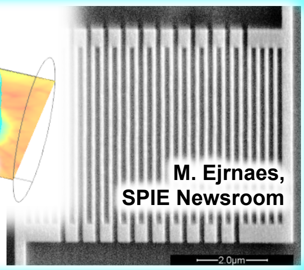
Modelling of Superconductors

- **Macroscopic** Modelling (> mm)
  - modelling of wires, conductors, and power devices
  - critical state model, power-law model $E \sim J^n$

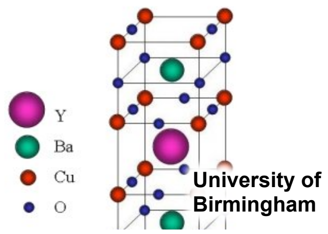


- **Mesoscopic** Modelling (nm ~ μm)
  - modelling of nanoscale phenomena and electronics devices
  - **Ginzburg-Landau** equation, London model, Josephson equation
  - length scales: coherence length $\xi \sim$ 1nm, penetration depth $\lambda \sim 0.1\mu$m





- **Microscopic** Theory (< nm)
  - theory for electronic state and superconducting mechanism
  - BCS theory, first-principle calculation



Time-Dependent Ginzburg-Landau Equation

A. Schmid, Phys. Kondens. Materie **5**, 302 (1966).

- Time-dependent Ginzburg-Landau (**TDGL**) equation:
  - order parameter $\psi = |\psi| \exp(i\varphi)$; superfluid density $|\psi|^2$, phase φ
  - ψ normalized ($|\psi| = 1$ for the equilibrium state at zero field)

$$\tau_0\left(\frac{\partial}{\partial t}+i\frac{2\pi}{\phi_0}\boldsymbol{\Phi}\right)\psi=\xi_0^2\left(\nabla-i\frac{2\pi}{\phi_0}\boldsymbol{A}\right)^2\psi+\left(1-|\psi|^2\right)\psi$$

- τ_0 : GL relaxation time
- ξ_0 : coherence length
- λ_0 : penetration depth
- Φ : scalar potential
- $\boldsymbol{A}$: vector potential
- ϕ_0 : flux quantum

- Current density, $\mathrm{rot\,rot}\boldsymbol{A}/\mu_0 = \boldsymbol{j}_s + \boldsymbol{j}_n$

$$\begin{cases}\boldsymbol{j}_s = \dfrac{|\psi|^2}{\mu_0\lambda_0^2}\left(\dfrac{\phi_0}{2\pi}\nabla\varphi-\boldsymbol{A}\right), \\ \boldsymbol{j}_n = \sigma_n\boldsymbol{E} = -\sigma_n\left(\dfrac{\partial\boldsymbol{A}}{\partial t}+\nabla\Phi\right)\end{cases}$$

➔ fluxoid quantization

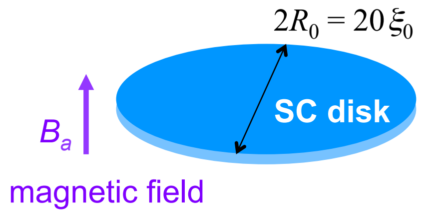
  - upper critical field $B_{c2} = \dfrac{\phi_0}{2\pi\xi^2}$
  - depairing current density

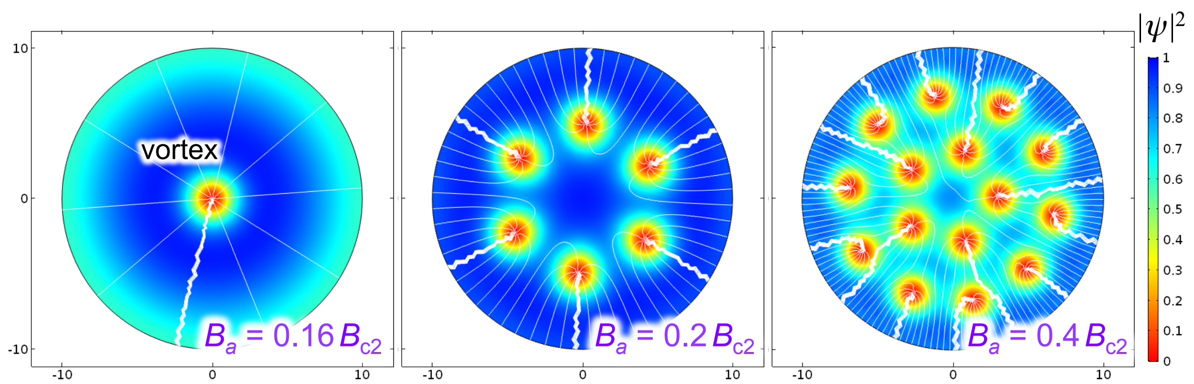
$$J_d = \frac{2\sqrt{2}}{3\sqrt{3}}\frac{B_c}{\mu_0\lambda} = \frac{\phi_0}{3\sqrt{3}\pi\mu_0\lambda^2\xi}$$

	NbTi	MgB_2	$YBa_2Cu_3O_y$
T_c	9.5K	39K	95K
τ_0	0.3ps	0.08ps	0.03ps
ξ_0	4nm	5nm	0.4–3nm
λ_0	300nm	140nm	30–200nm

2D TDGL Simulation for SC Disks

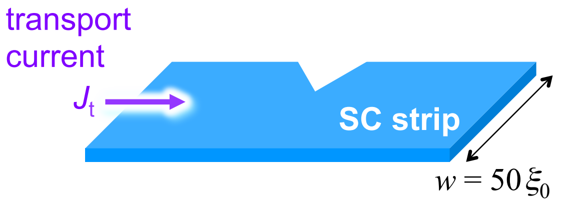
- SC disk in a perpendicular magnetic field (2D)
  - vortex penetration



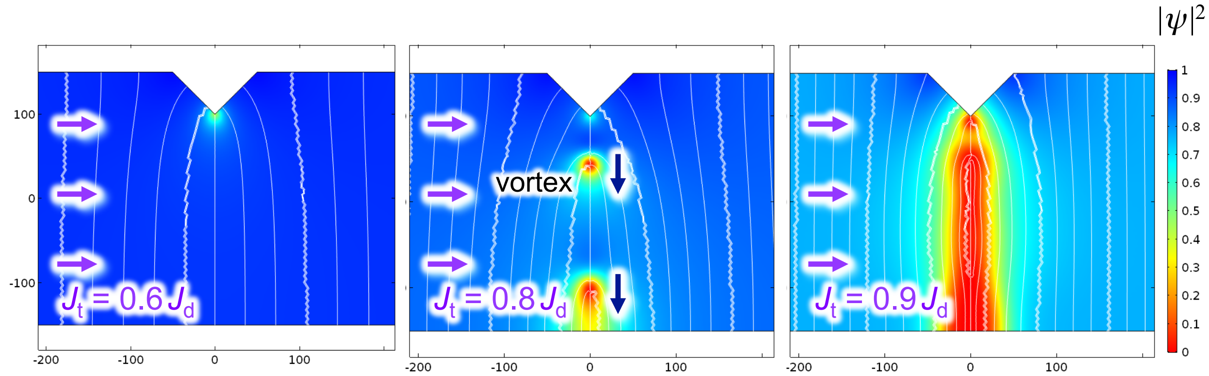


2D TDGL Simulation for Current Carrying SC Strips

- current-carrying SC strip
  with an edge defect (2D)



- vortex nucleation at the edge defect
- vortex flow

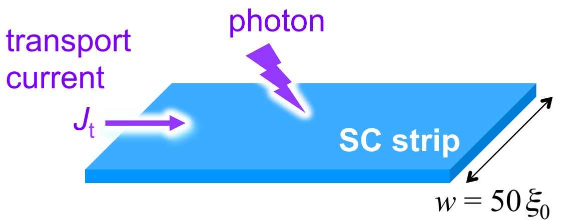


similar to the weak link (Josephson junction)

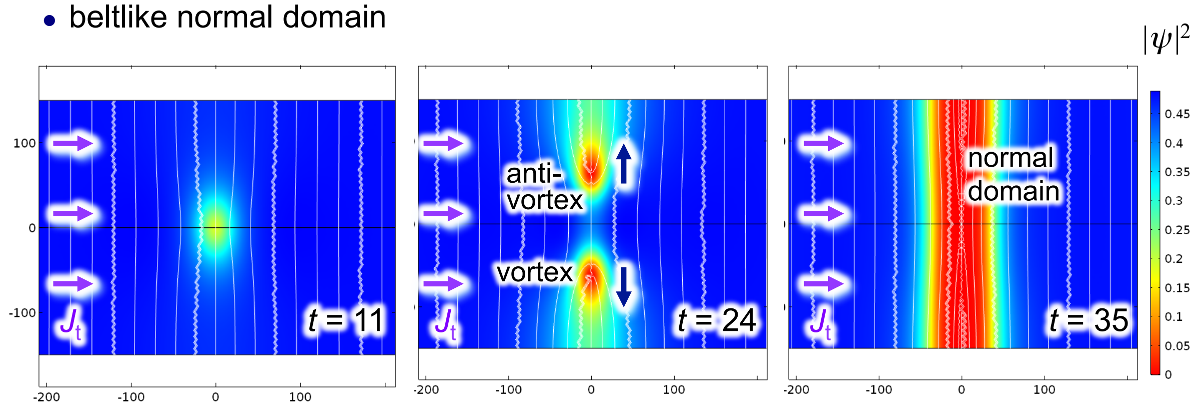
2D TDGL Simulation for SC Strip Photon Detectors

- SC strip photon detector (2D)

  simulation based on the TDGL
  and heat diffusion equations

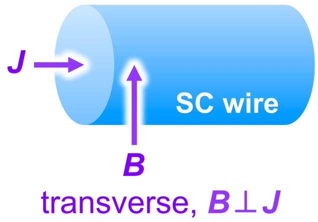


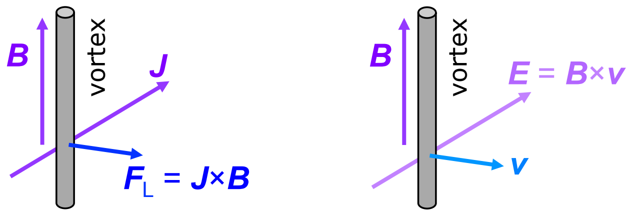
- hot spot due to the photon incidence
- vortex and antivortex nucleation
- vortex flow
- beltlike normal domain



Vortex Dynamics and Electromagnetic Response

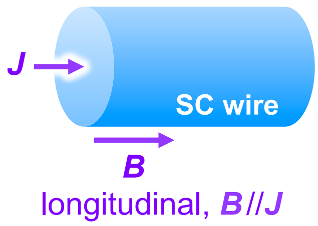
- Vortex Dynamics in the **Transverse** ($\boldsymbol{B}\perp\boldsymbol{J}$) Configuration:
  - driving (Lorentz) force on vortices, $\boldsymbol{F}_L = \boldsymbol{J}\times\boldsymbol{B}$
    ~ current density, $\boldsymbol{J}$
  - vortex velocity, $\boldsymbol{v}$
    ~ electric field, $\boldsymbol{E} = \boldsymbol{B}\times\boldsymbol{v}$
  - force balance equation for vortices, $\boldsymbol{F}_L + \boldsymbol{F}_p + \boldsymbol{F}_v + ... = \boldsymbol{0}$
    ($\boldsymbol{F}_p$: pinning force and $\boldsymbol{F}_v$: viscous drag force)
    ~ ***E-J*** characteristics, critical state model
    ~ electromagnetic response of superconductors

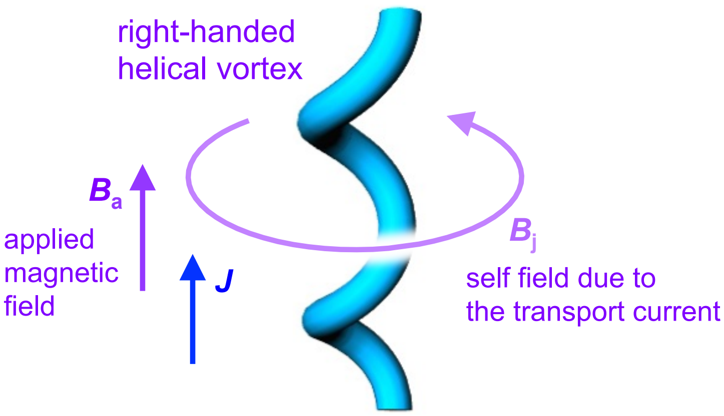




Longitudinal Field Effect

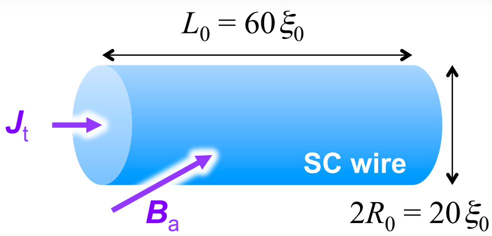
- Vortex Dynamics in the **Longitudinal** ($\boldsymbol{B}/\!/\boldsymbol{J}$) Configuration:
  - $\boldsymbol{F}_L = \boldsymbol{J}\times\boldsymbol{B} \sim 0$ (**force free ?**)
  - $\boldsymbol{v}$? ~ $\boldsymbol{E} \neq \boldsymbol{B}\times\boldsymbol{v}$?
  - force balance equation ?
    ~ ***E-J*** characteristics ??
    (double critical state model,...)
    ~ electromagnetic response ???
    (e.g., large I_c, paramagnetic effect,...)





Lehninger et al.
"Principles of Biochemistry"

TDGL Simulation on Thin Superconducting Wires



- Superconducting cylinder:
  - radius $R_0 = 10\,\xi_0$, length $L_0 = 60\,\xi_0$
  - applied magnetic field $B_a = 0.2$–$0.4\,B_{c2}$
  - transport current density $J_t = 0.3$–$0.7\,J_d$
- TDGL equation for thin superconducting wires:
  - self field neglected, $\boldsymbol{A} = \boldsymbol{B}_a \times \boldsymbol{r}/2$

$$\begin{cases} \tau_0 \left(\dfrac{\partial}{\partial t} + i\dfrac{2\pi}{\phi_0}\Phi \right)\psi = \xi_0^2 \left(\nabla - i\dfrac{2\pi}{\phi_0}\boldsymbol{A} \right)^2 \psi + \left(1 - |\psi|^2 \right)\psi , \\ \nabla \cdot \left(\boldsymbol{j}_s + \boldsymbol{j}_n \right) = 0 \end{cases}$$

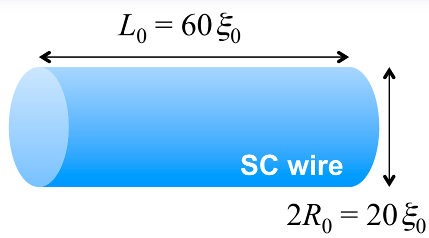
Parameters for $YBa_2Cu_3O_7$ at T = 0K
- τ_0 : GL relaxation time ~0.03ps
- ξ_0 : coherence length ~1nm
- λ_0 : penetration depth ~100nm
- B_{c2} : upper critical field ~100T
- J_d : depairing current density ~10^{12} A/m^2

  - boundary conditions

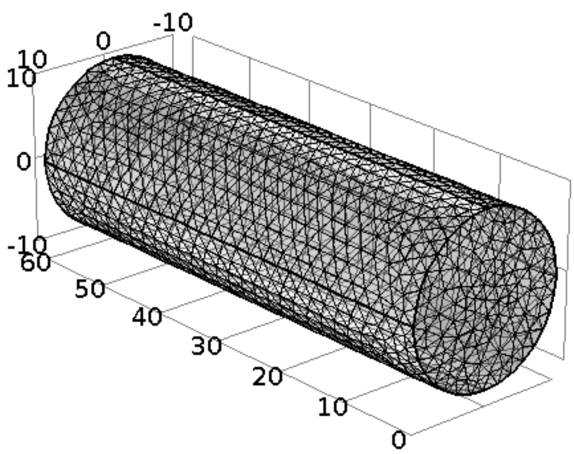
$$\begin{cases} \boldsymbol{n} \cdot \left(\nabla - (2\pi i/\phi_0)\boldsymbol{A} \right)\psi = 0 & \text{at the surface,} \\ \boldsymbol{n} \cdot \nabla\Phi = 0 & \text{at the side surface,} \\ z \cdot \nabla\Phi = -J_t/\sigma_n & \text{at the ends (normal current injected)} \end{cases}$$

FEM Calculation by COMSOL Multiphysics®

Numerical TDGL-FEM simulation has been done by using the commercial software, **COMSOL Multiphysics®** (www.comsol.com).



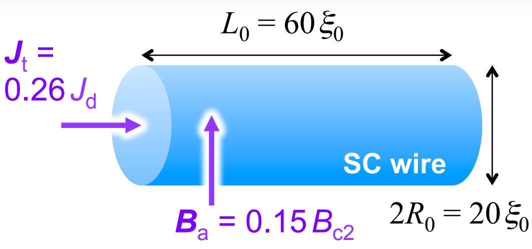
- Coefficient Form PDE
  - order parameter $\psi(x,y,z,t)$
  - scalar potential $\Phi(x,y,z,t)$
- Mesh:
  - Element size parameter (in units of ξ_0): max. element size 2.1, min. element size 0.09
  - Complete mesh consists of: 33,317 domain elements, 2,576 boundary elements, and 180 edge elements

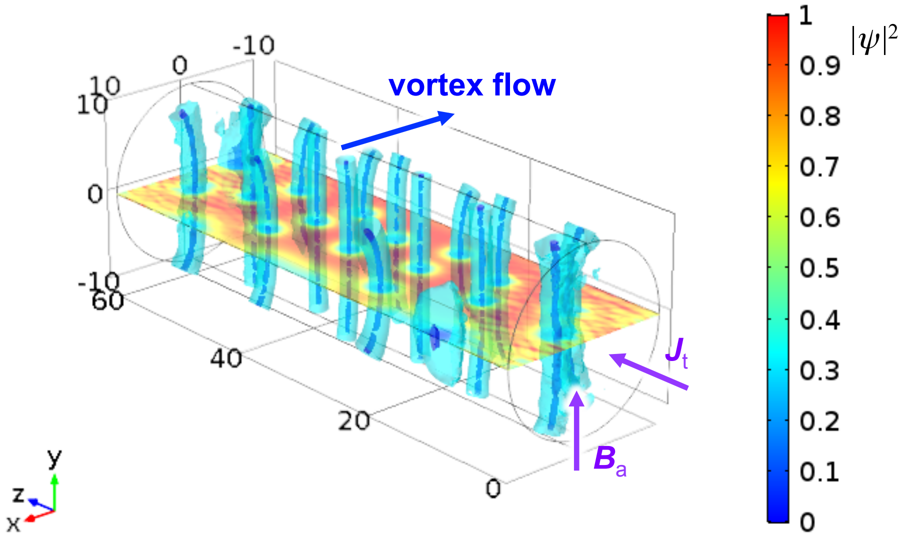


- Calculation:
  - Number of degrees of freedom solved for 188,608 (plus 22,072 internal DOFs)
  - Calculation time ~15 hours for 0 < t < 1,000

Simulation Results (I): Transverse Field

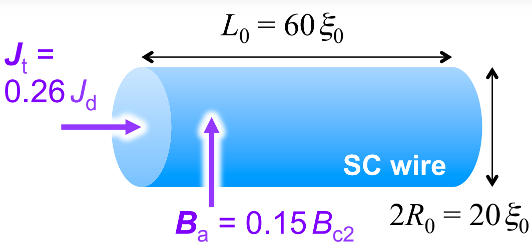
- Superconducting cylinder:
  - **transverse** magnetic field $B_a = 0.15 B_{c2}$
  - transport current density $J_t = 0.26 J_d$



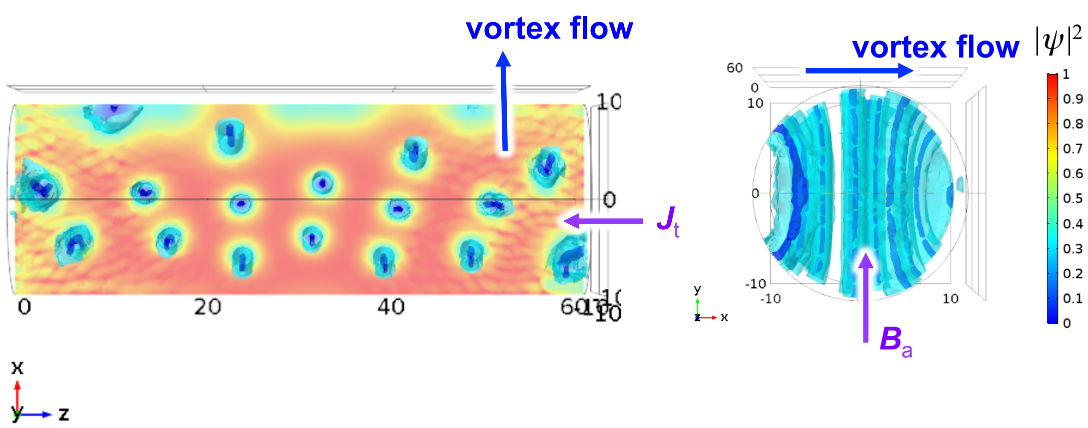


Simulation Results (II): Transverse Field

- Superconducting cylinder:
  - **transverse** magnetic field $B_a = 0.15 B_{c2}$
  - transport current density $J_t = 0.26 J_d$

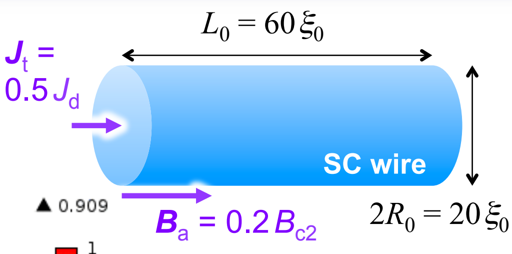


Time=170 s , Ba/Bc2 = 0.15, Jt/Jd = 0.26

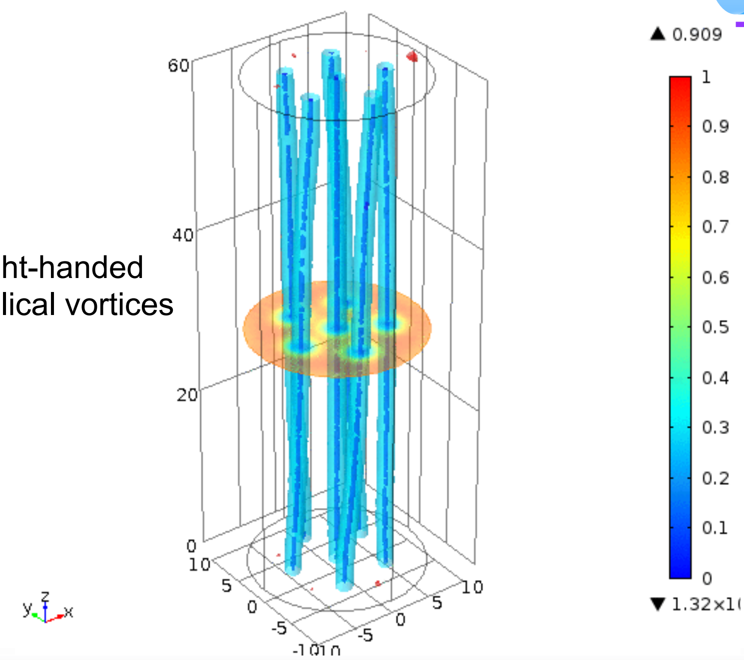


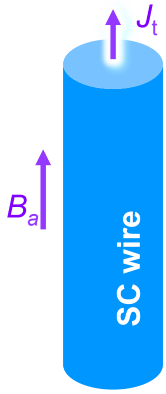
Simulation Results (III): Longitudinal Field

- Superconducting cylinder:
  - **longitudinal** magnetic field $B_a = 0.2\,B_{c2}$
  - transport current density $J_t = 0.5\,J_d$



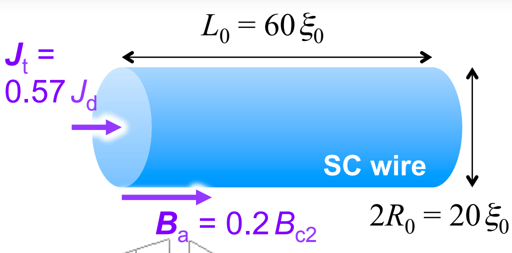
- right-handed helical vortices

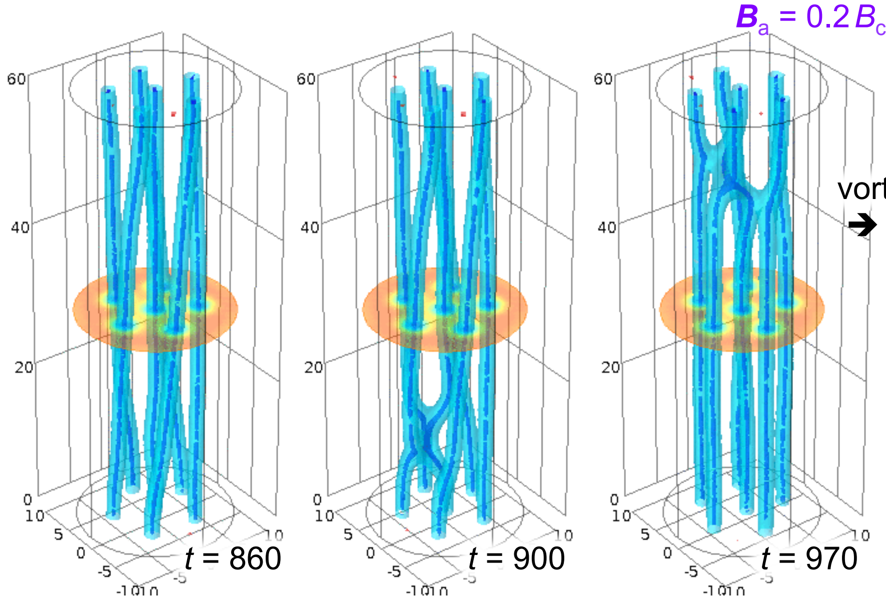




Simulation Results (IV): Longitudinal Field

- Superconducting cylinder:
  - **longitudinal** magnetic field $B_a = 0.2\,B_{c2}$
  - transport current density $J_t = 0.57\,J_d$



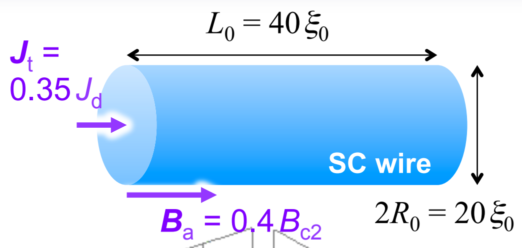


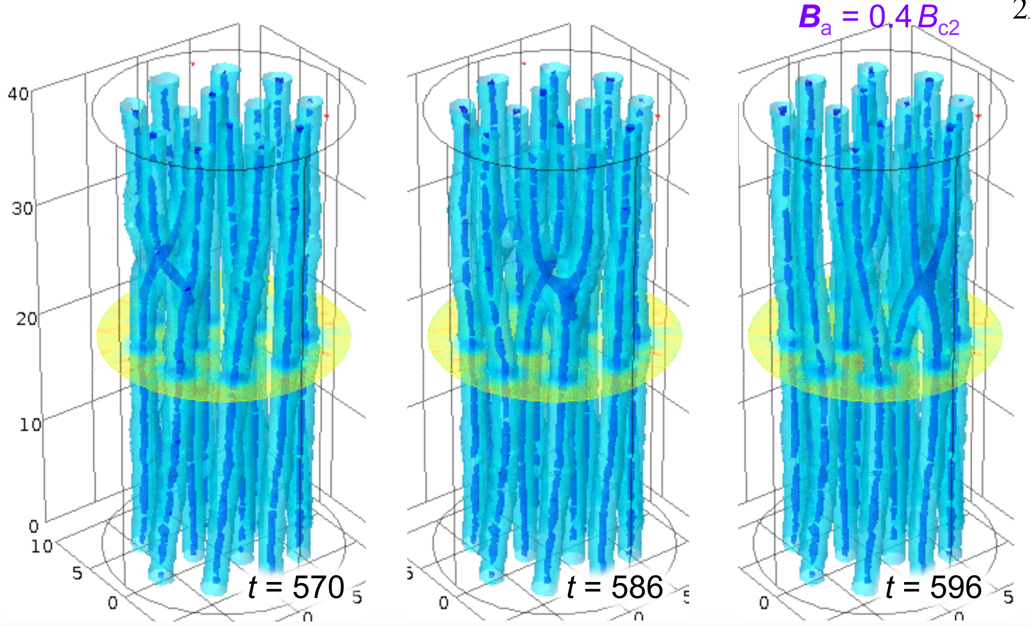
vortex cutting
➔ voltage pulse

Simulation Results (V): Longitudinal Field

- Superconducting cylinder:
  - **longitudinal** magnetic field $B_a = 0.4 B_{c2}$
  - transport current density $J_t = 0.35 J_d$

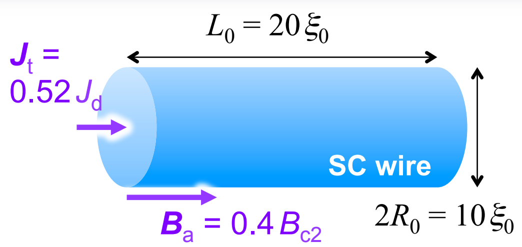
  - propagation of vortex cutting

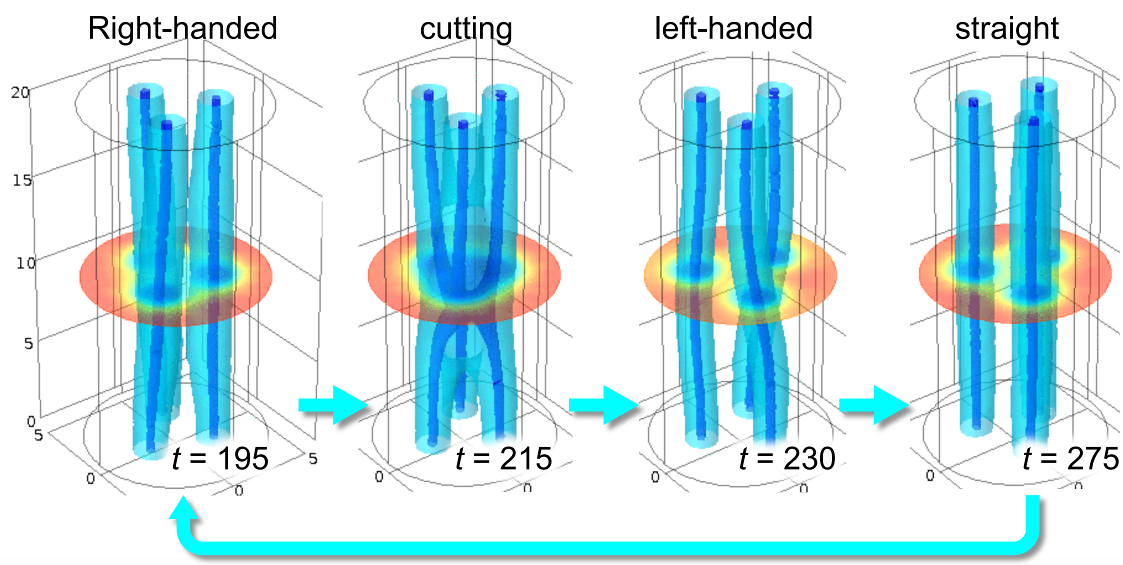




Simulation Results (VI): Longitudinal Field

- Superconducting cylinder:
  - **longitudinal** magnetic field $B_a = 0.4 B_{c2}$
  - transport current density $J_t = 0.52 J_d$





Summary

We have numerically investigated three-dimensional dynamics of vortices in thin superconducting (SC) cylinders exposed to transport currents and longitudinal magnetic fields.

- The time-dependent Ginzburg-Landau (TDGL) equations are useful for modelling of nanoscale (nm–μm) SC phenomena (e.g., vortex dynamics, SC electronics devices,…).
- Helical vortex structure in current carrying SC wires in longitudinal magnetic fields
- Complicated vortex dynamics with cutting for large current:
  - propagation of vortex cutting in many vortices
  - right-handed helices ➔ cutting ➔ left-handed helices ➔…

Future issues:

- Further investigation (by changing applied field, current, and sample size,…) to clarify the complicated vortex dynamics
- Large scale simulation for bulk behavior
- Pinning, anisotropy, thermal fluctuation,…