Modeling of superconductors interacting with non-linear magnetic materials: 3D variational principles, force-free effects and applications

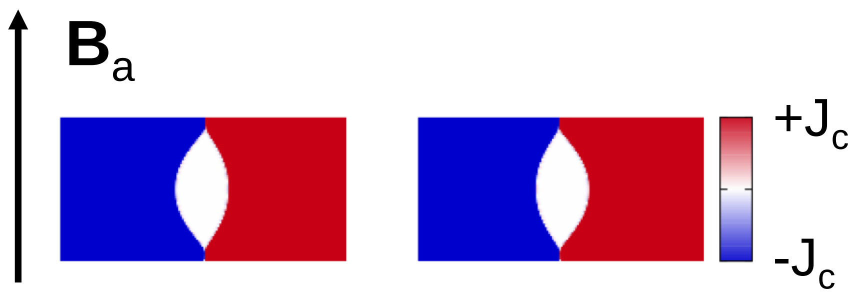
Enric Pardo, Milan Kapolka

Institute of Electrical Engineering
Slovak Academy of Sciences

What is the optimum enery loss in superconductors?

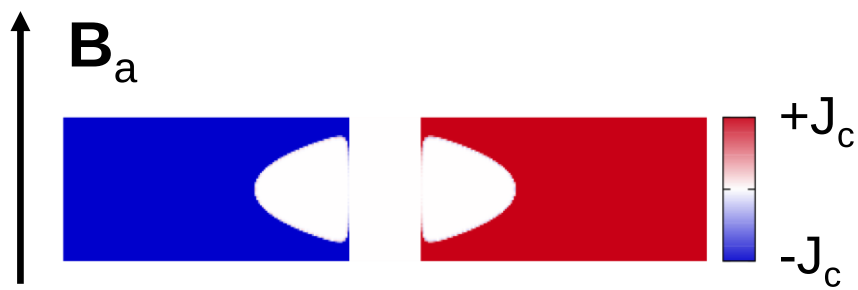
Two tapes connected at the ends

For you:



the lowest possible

For superconductors:



the highest possible!

Superconductors optimize the entropy production, not the loss!

Superconducors can be modelled as an optimization problem

Modeling of superconductors interacting with non-linear magnetic materials: 3D variational principles, force-free effects and applications

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eurotapes
NMP-LA-2012-280432

VEGA
2/0126/15

ERDF
ITMS 26240120011
ITMS 26230120002

Talk available at **zenodo.org**

Citable DOI

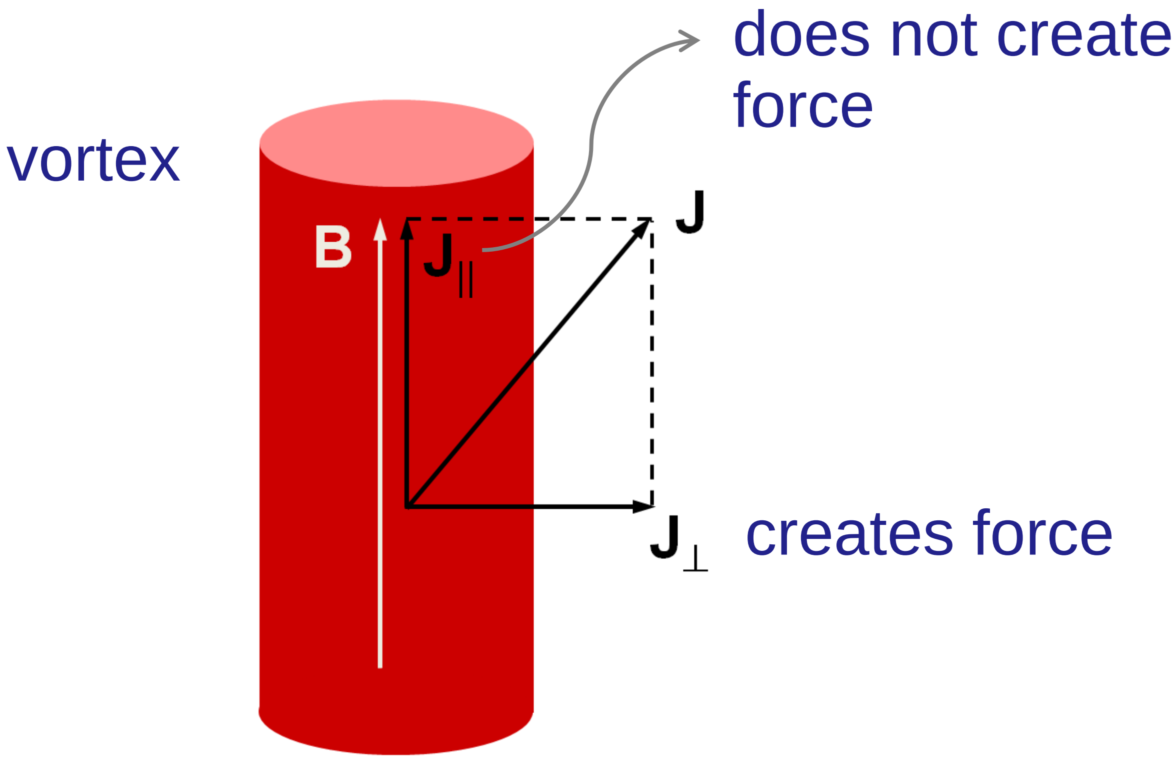
General variational principle

Power applications

3D modelling

Non-linear magnetic materials

Flux-free effects cause anisotropic E(J)



Two critical currents:

$\mathbf{J}_{c\perp}$

$\mathbf{J}_{c||}$

Anisotropic power law:

$$\mathbf{E}(\mathbf{J}) = E_c \left[\frac{J_{||}^2}{J_{c||}^2} + \frac{J_{\perp}^2}{J_{c\perp}^2} \right]^{\frac{n-1}{2}} \cdot \left(\frac{J_{||}}{J_{c||}} \frac{J_{\perp}}{J_{c||}} \mathbf{e}_{||} + \frac{J_{\perp}}{J_{c\perp}} \mathbf{e}_{\perp} \right)$$

A Badia, C Lopez DOI: 10.1088/0953-2048/28/2/024003

Minimum Magnetic Entropy Production (MEMEP)

Equations

$$\mathbf{E}(\mathbf{J}) = -\frac{\Delta \mathbf{A}}{\Delta t} - \nabla \phi$$

for given **E(J)** relation

$$\nabla \cdot \mathbf{J} = 0$$

are the Euler-Lagrange equations of

J change between two time instants

A from Δ**J**

A from applied field

scalar potential

$$L = \int_V \mathrm{d}V \left[\frac{1}{2} \Delta \mathbf{J} \cdot \frac{\Delta \mathbf{A}_J}{\Delta t} + \Delta \mathbf{J} \cdot \frac{\Delta \mathbf{A}_a}{\Delta t} + U(\mathbf{J}) + \nabla \phi \cdot \mathbf{J} \right]$$

$$U(\mathbf{J}) = \int_0^{\mathbf{J}} \mathrm{d}\mathbf{J}' \cdot \mathbf{E}(\mathbf{J})'$$

E Pardo et al. DOI: 10.1088/0953-2048/28/4/044003
A Bossavit DOI: 10.1109/20.312659
L Prigozin DOI: 10.1109/77.659440
A Badia, C Lopez DOI: 10.1088/0953-2048/28/2/024003

Minimum Magnetic Entropy Production (MEMEP)

You find **J** by minimizing the functional

scalar potential

$$L = \int_V \mathrm{d}V \left[\frac{1}{2} \Delta\mathbf{J} \cdot \frac{\Delta\mathbf{A}_J}{\Delta t} + \Delta\mathbf{J} \cdot \frac{\Delta\mathbf{A}_a}{\Delta t} + U(\mathbf{J}) + \cancel{\nabla\phi \cdot \mathbf{J}} \right]$$

$$U(\mathbf{J}) = \int_0^{\mathbf{J}} \mathrm{d}\mathbf{J}' \cdot \mathbf{E}(\mathbf{J})'$$

Cross-sectional models:

if you keep the current constrains,
you can ignore the scalar potential

E Pardo et al. DOI: 10.1088/0953-2048/28/4/044003

General variational principle

Power applications

Transformers

Magnets

3D modelling

Non-linear magnetic materials

General variational principle

Power applications

- Transformers
- Magnets

3D modelling

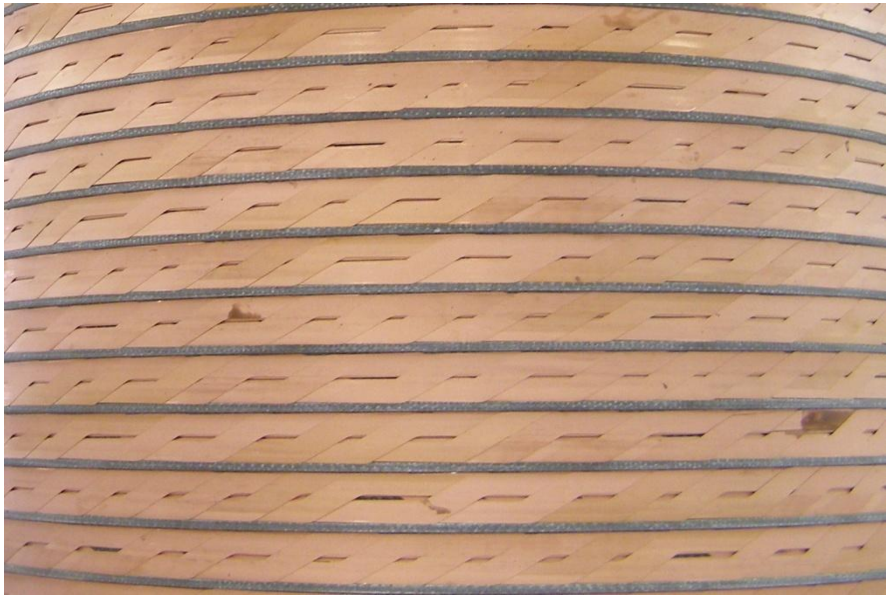
Non-linear magnetic materials

Transformer with Roebel cable in low-voltage winding

1 MVA 11 kV/415 V 3 phase transformer

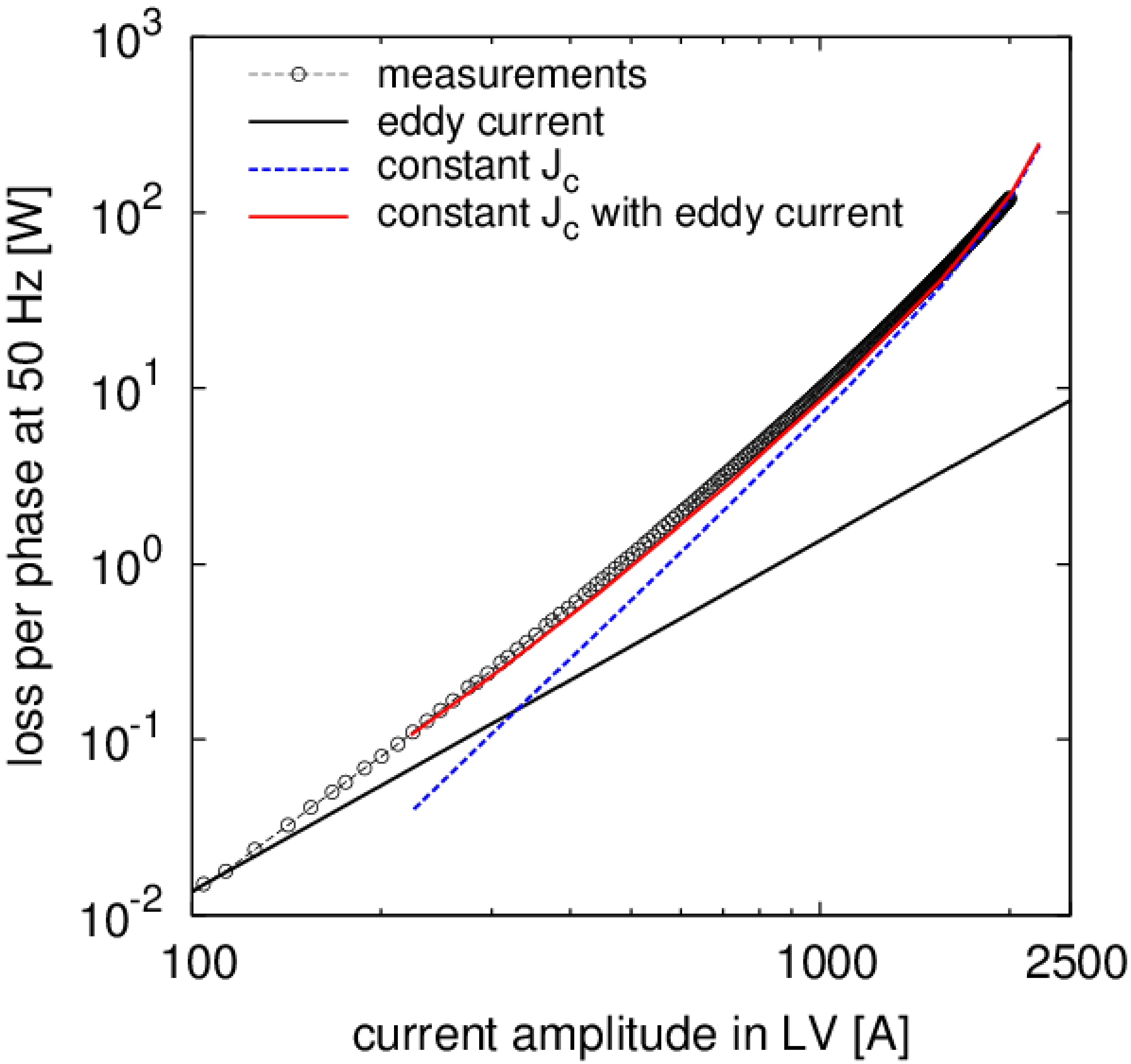
Robinson Research Institute in Wellington and industrial partners

Roebel cable solenoid



AC loss agrees with model

E Pardo et al. DOI: 10.1088/0953-2048/28/11/114008



Copper current leads cause eddy current loss

consistent with estimations

Real large scale application

~1200 turns or strands

General variational principle

Power applications

Transformers

Magnets

- Real geometry
- Continuous approximation
- Screening current induced field

3D modelling

Non-linear magnetic materials

General variational principle

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3D modelling

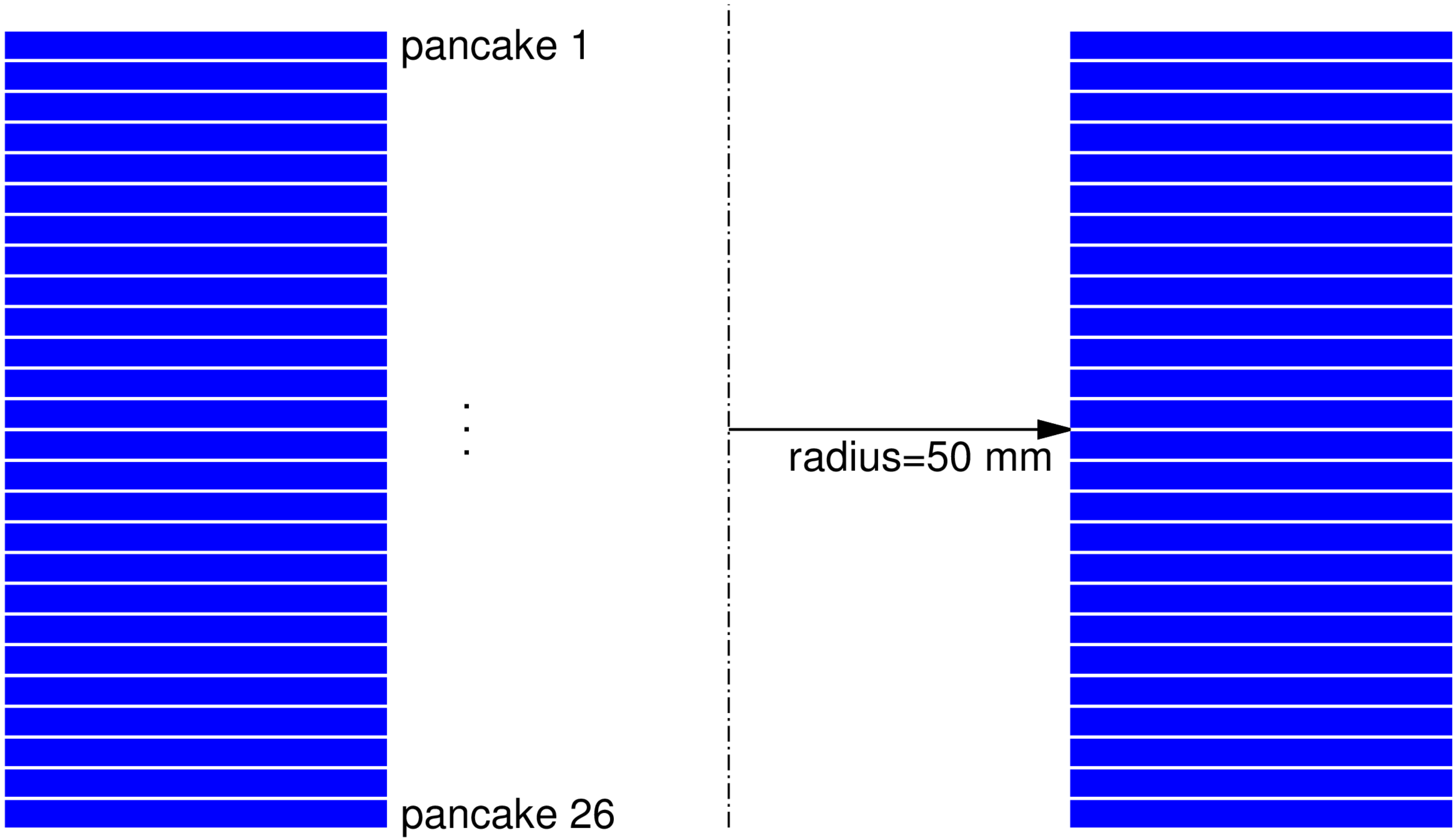
Non-linear magnetic materials

Example winding

26 pancakes

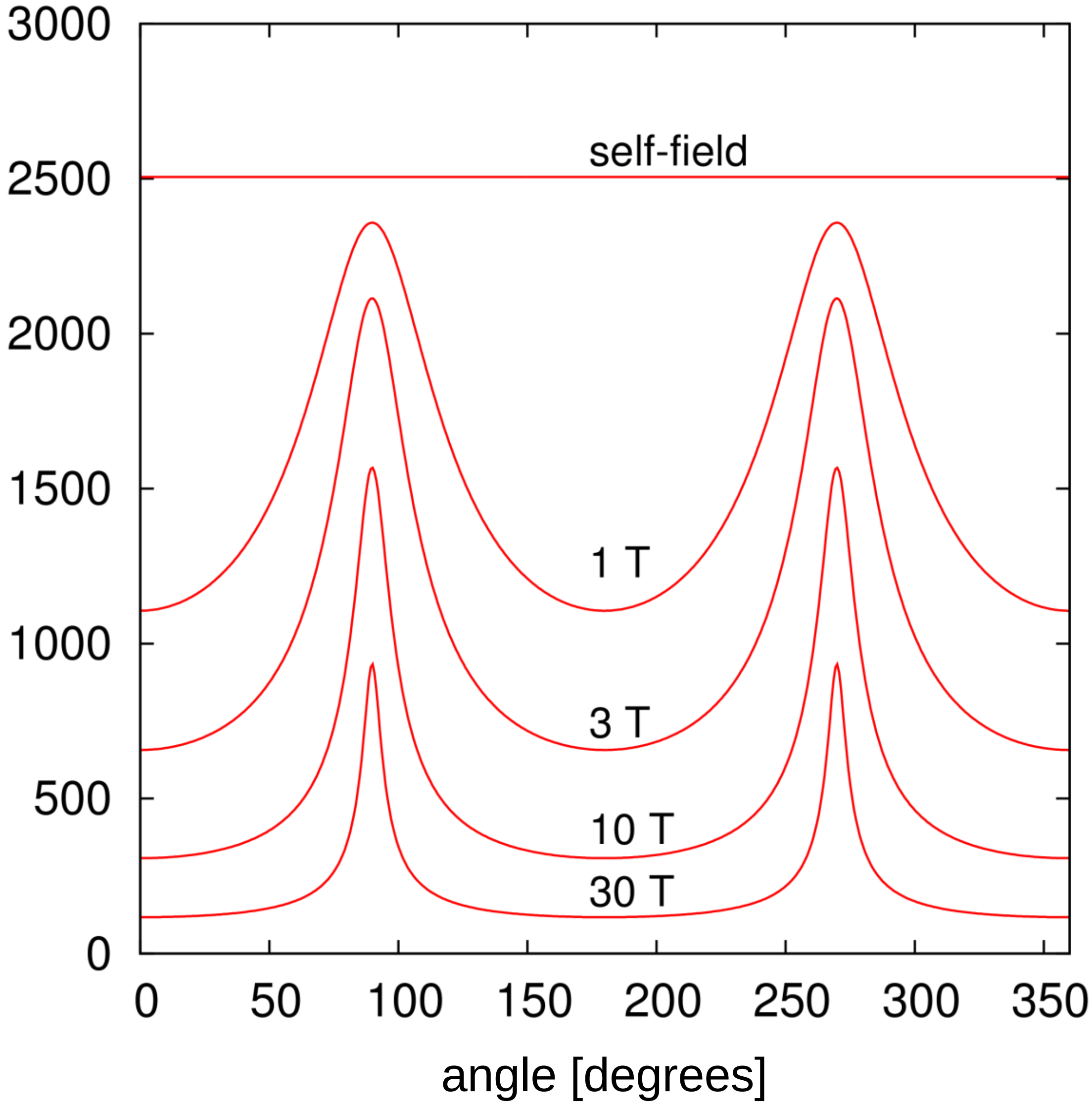
400 turns per pancake

more than 10000 turns



Anisotropic field dependent J_c

I_c of
4 mm tape
[A]

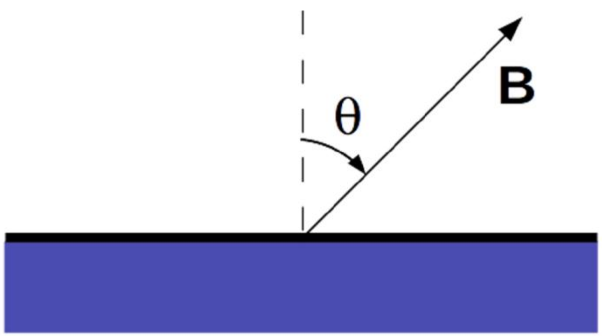


SuperPower tape

Fit of J_c from measurements at **4.2 K**

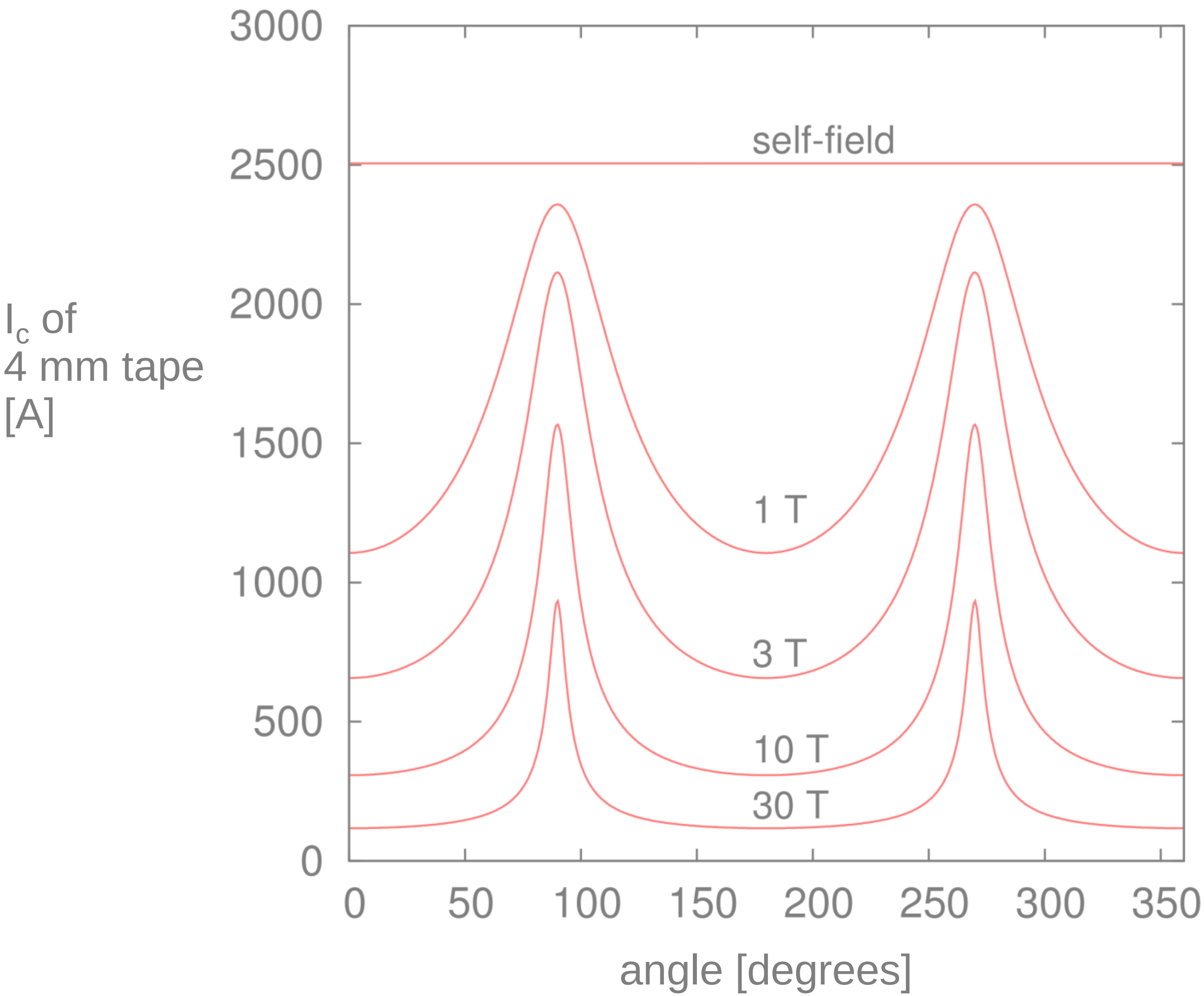
D K Hilton et al. 2015 SuST

Results useful for high-field magnets



Anisotropic field dependent J_c

Power-law exponent: **30**



SuperPower tape

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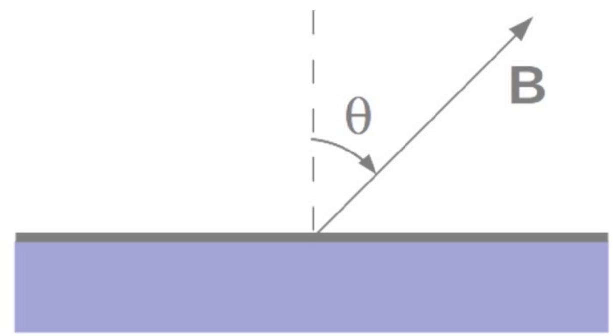
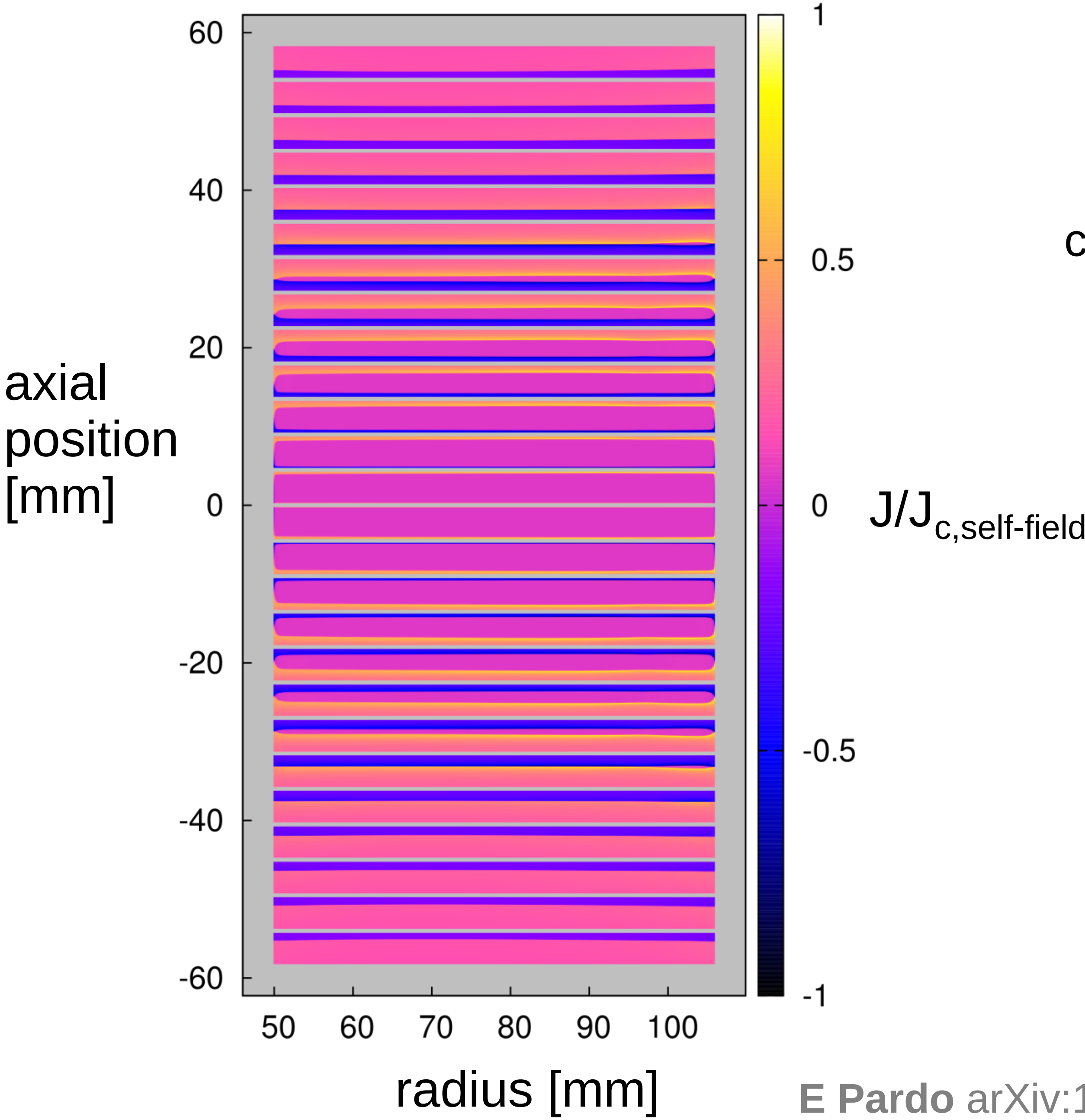
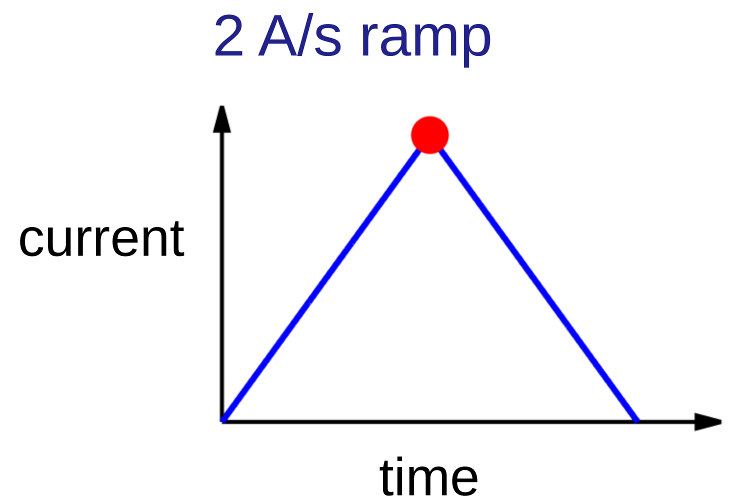


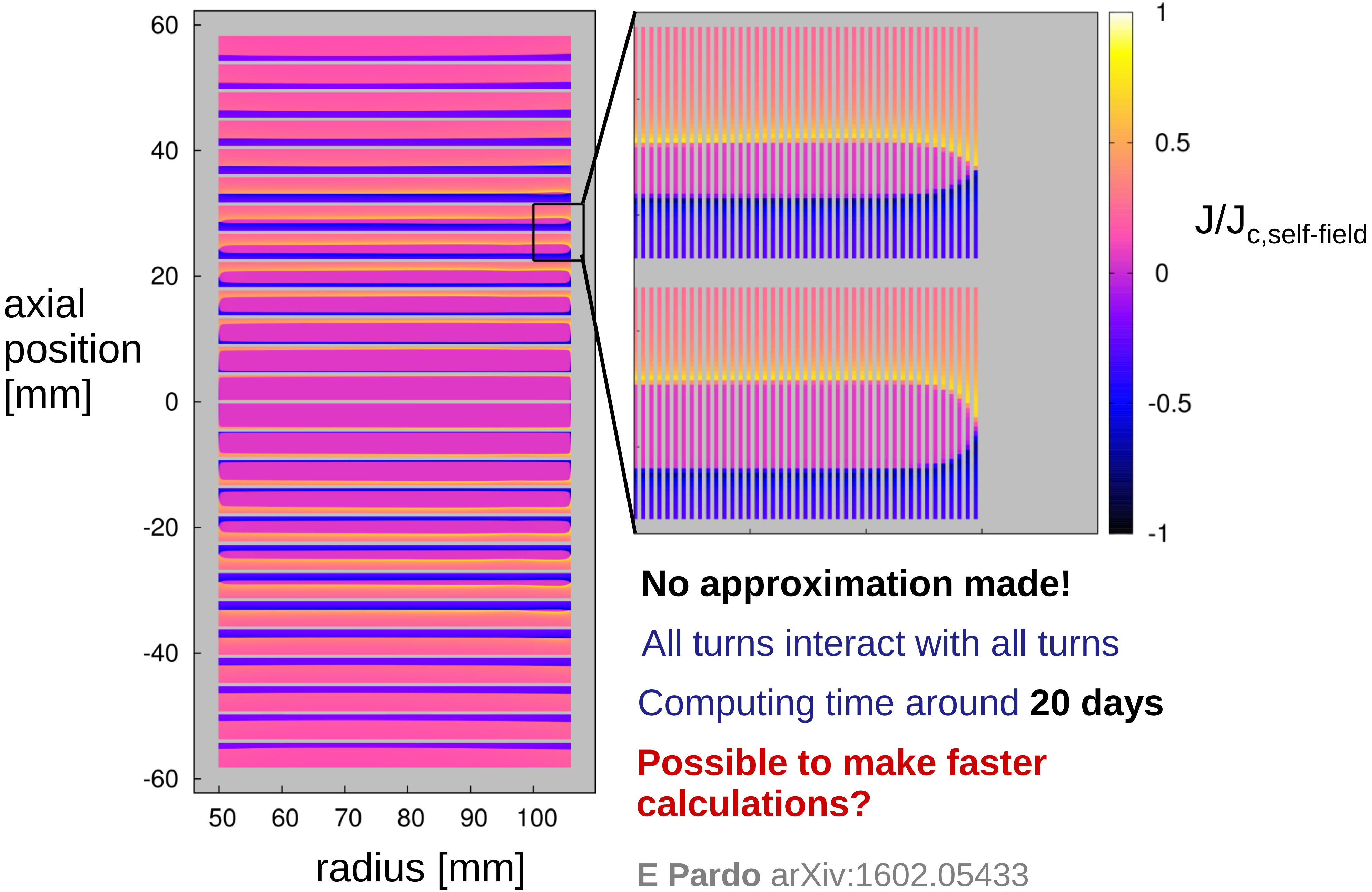
Important screening currents





E Pardo arXiv:1602.05433

Detailed current density at all turns



No approximation made!

All turns interact with all turns

Computing time around **20 days**

Possible to make faster calculations?

E Pardo arXiv:1602.05433

General variational principle

Power applications

Transformers

Magnets

- Real geometry
- Continuous approximation
- Screening current induced field

3D modelling

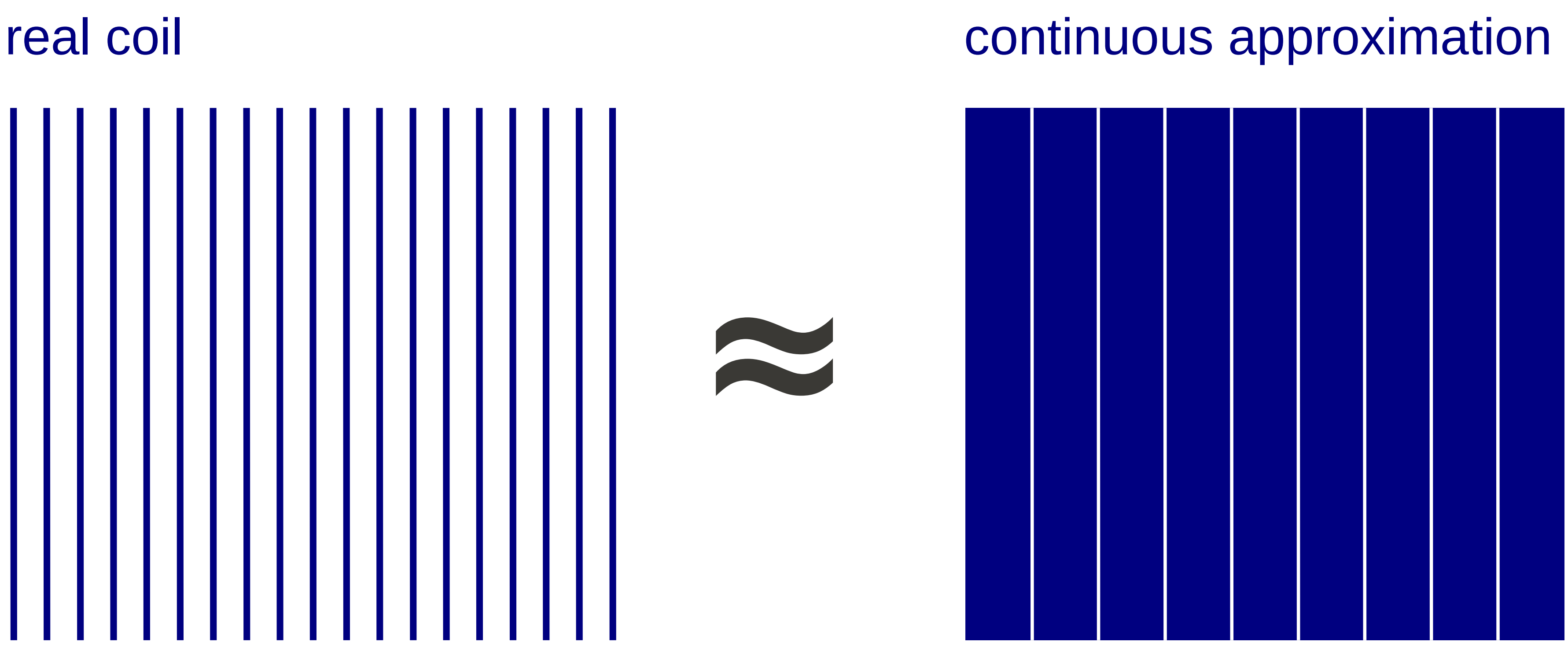
Non-linear magnetic materials

Continuous approximation

Pancake coil approximated by taking:

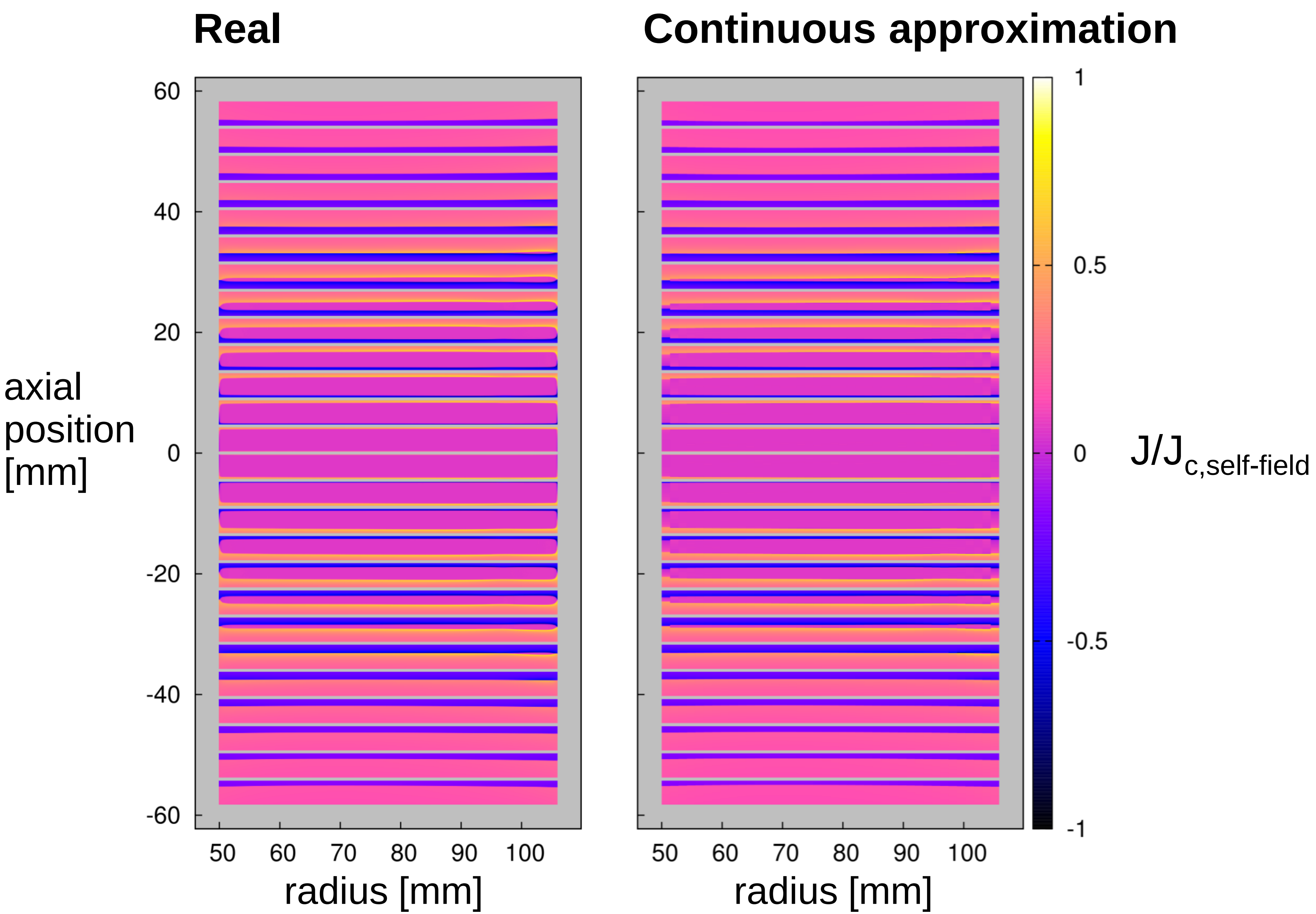
Less turns

No separation between turns



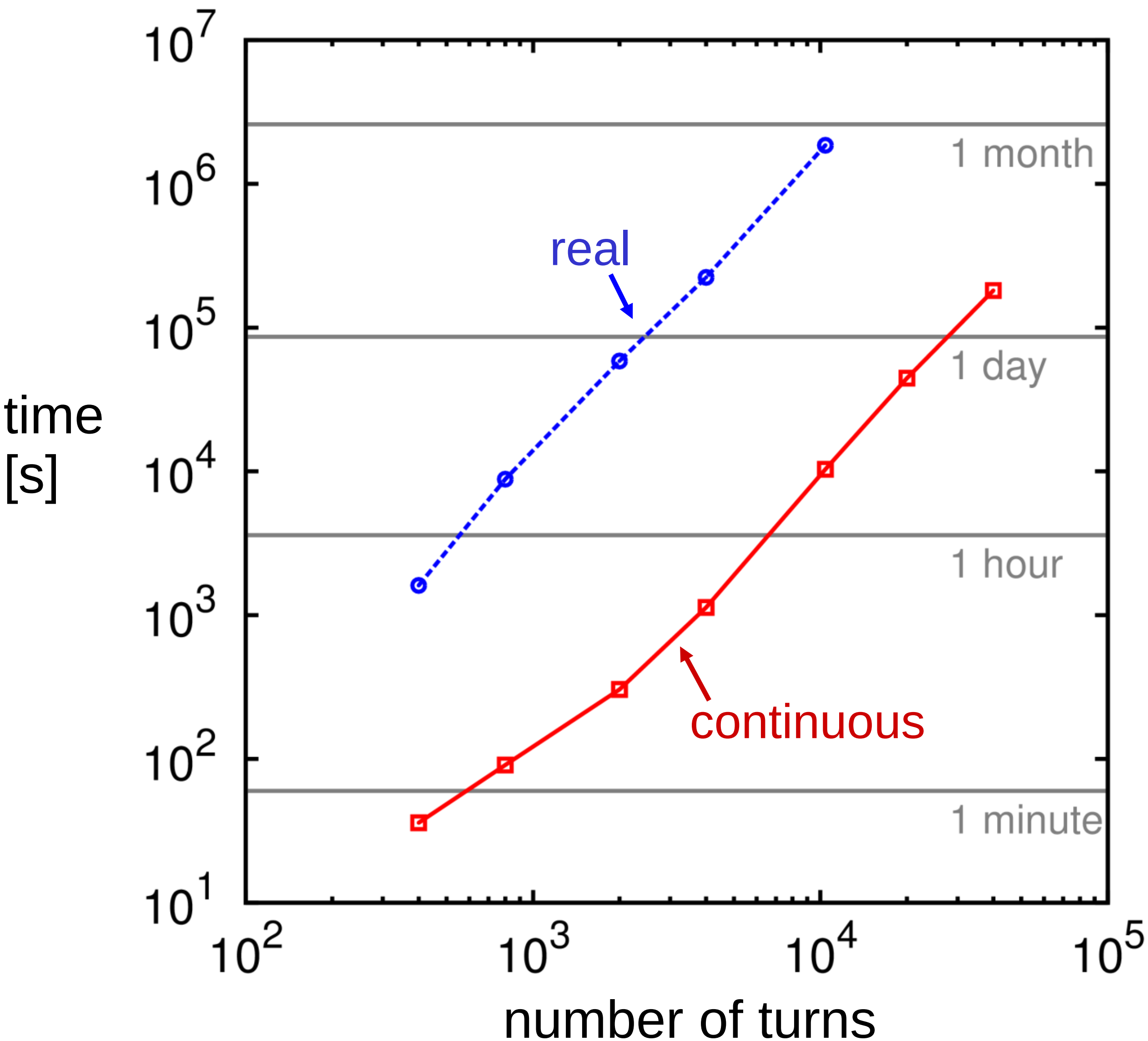
L Prigozhin, V Sokolovsky DOI: 10.1088/0953-2048/24/7/075012

Practically the same results but faster!



E Pardo and M Kapolka

We computed up to 40000 turns



10000 turns: **2.7 hours**

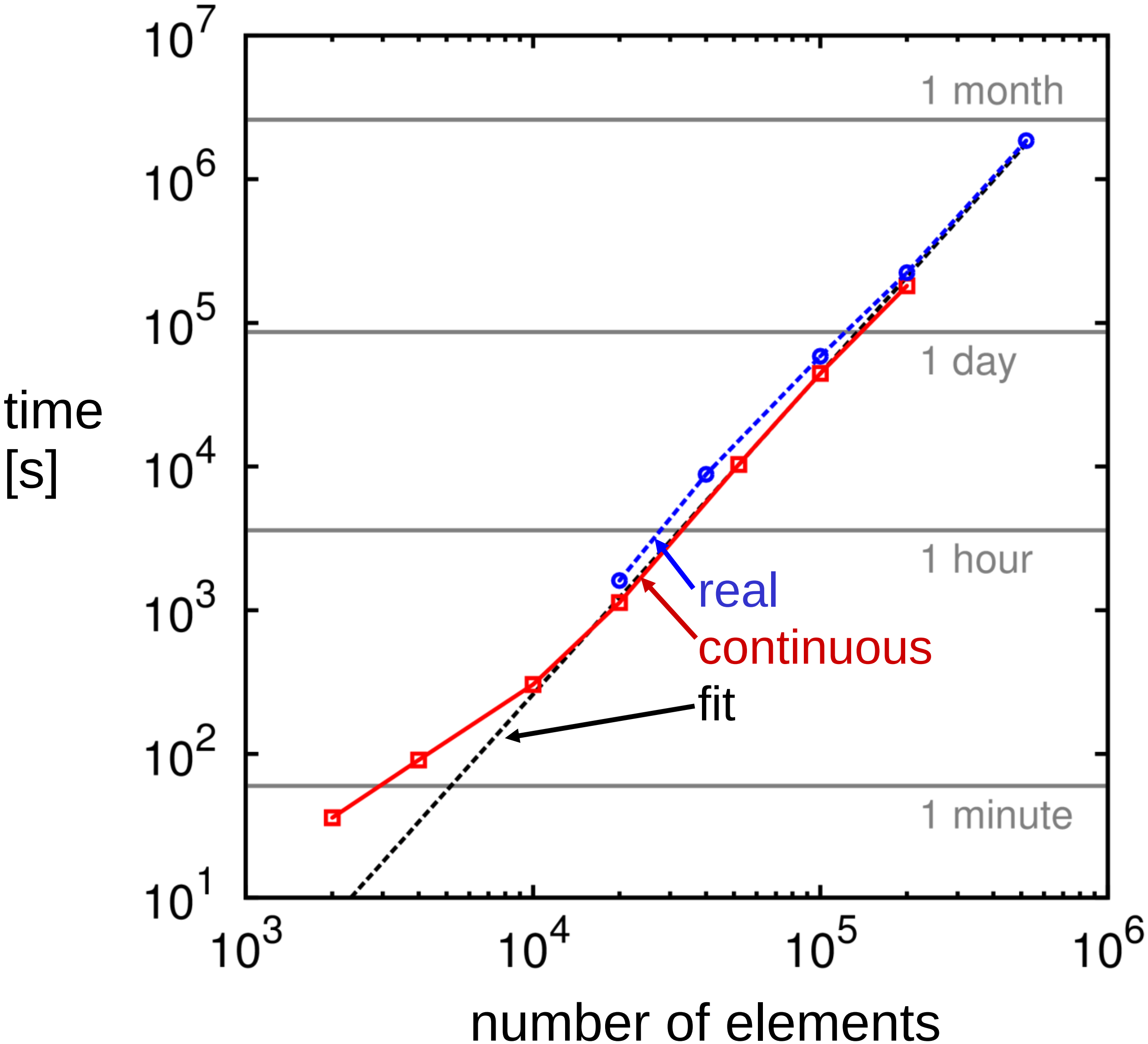
40000 turns: **2 days**

fulfills requirements for high-field magnets

H W Weijers et al. 2014 IEEE TAS

S Awaji et al. 2014 IEEE TAS

Up to 500 000 elements in the superconductor



Computing time scales as second power

General variational principle

Power applications

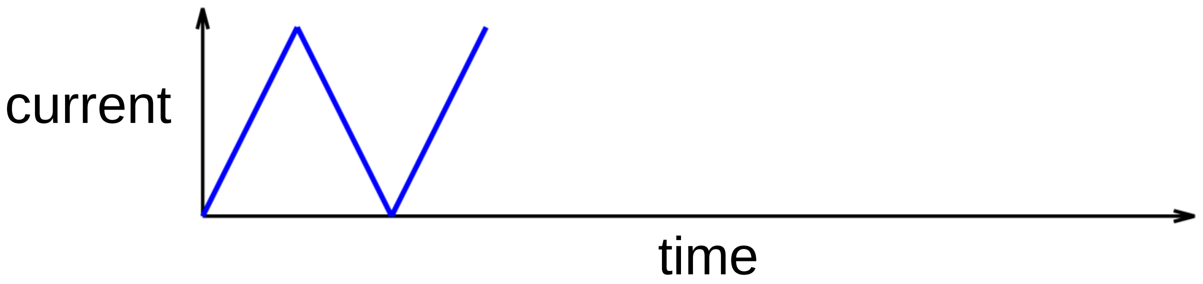
- Transformers
- Magnets
  - - Real geometry
  - - Continuous approximation
  - - Screening current induced field

3D modelling

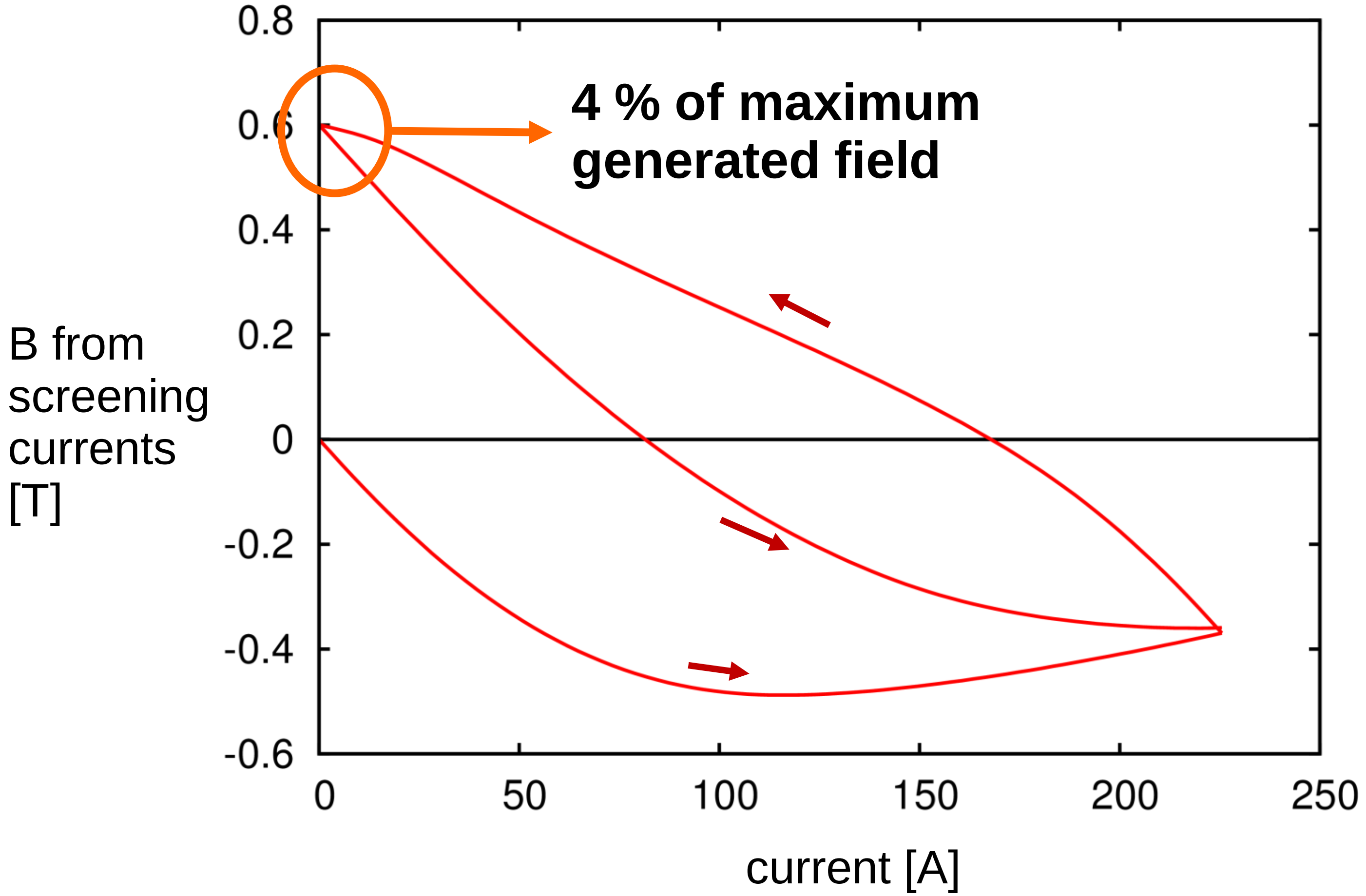
Non-linear magnetic materials

Screening currents are important

E Pardo arXiv:1602.05433

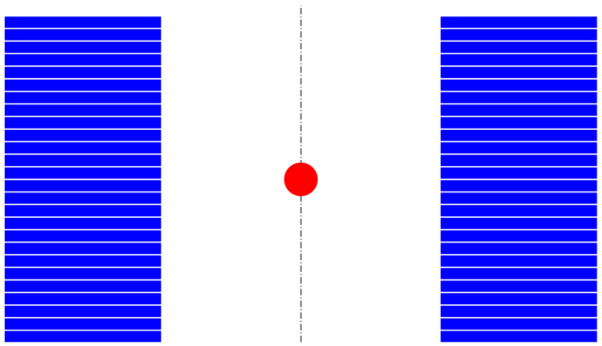


generated field
at maximum current **15 T**



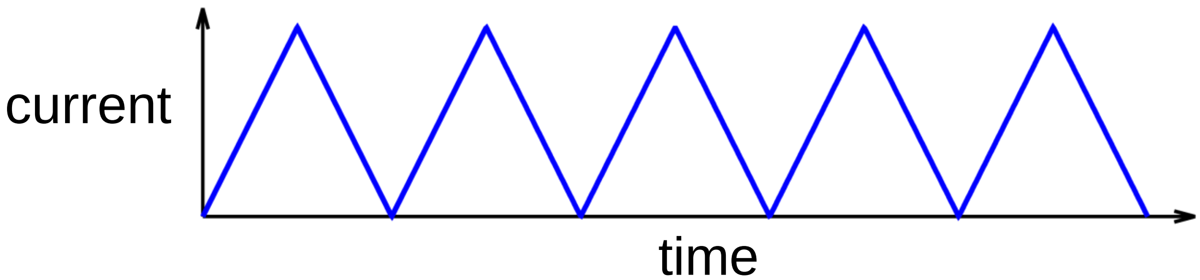
Important for MRI and NMR

magnetic field
at bore center

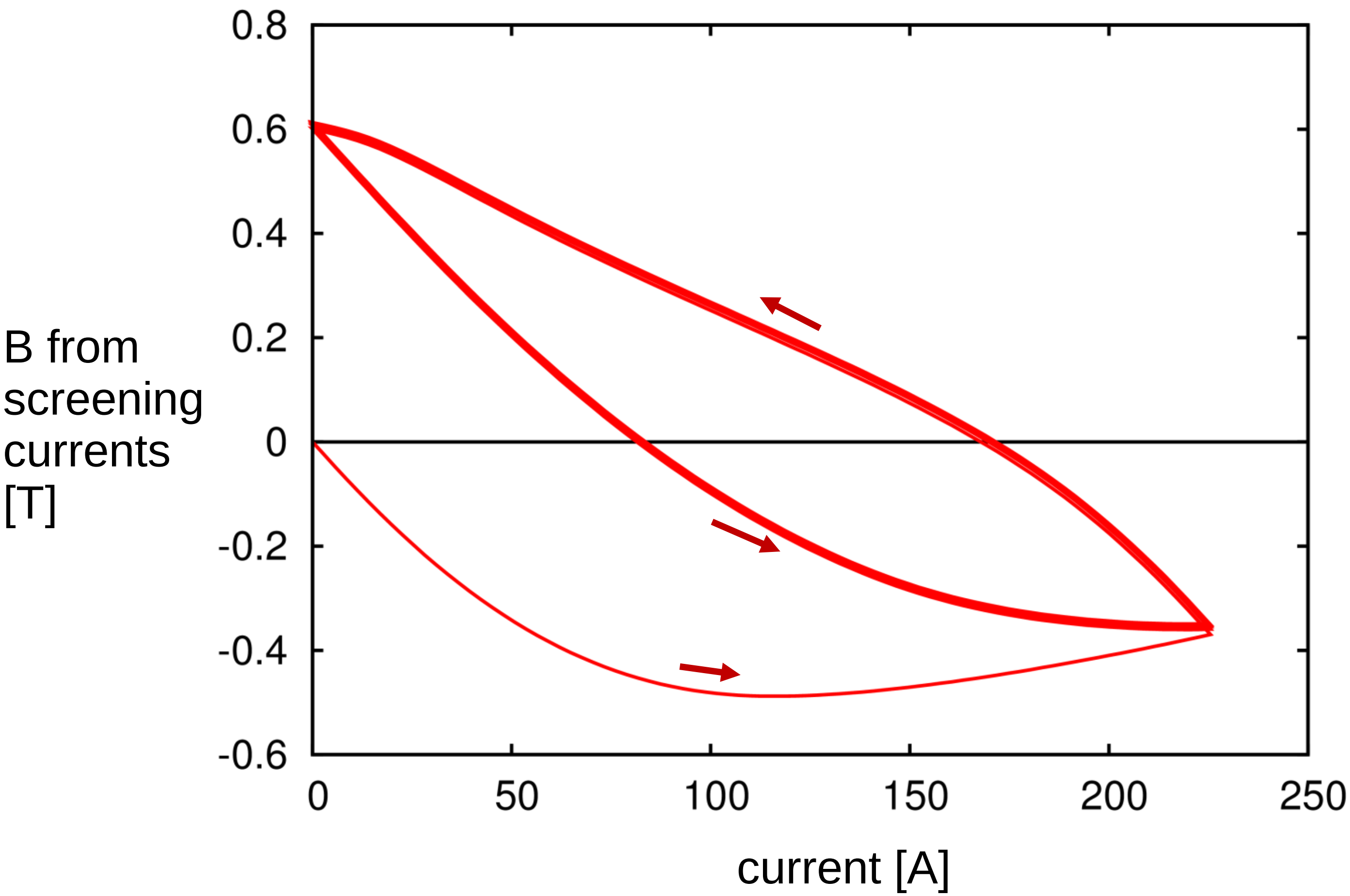


Screening currents are important

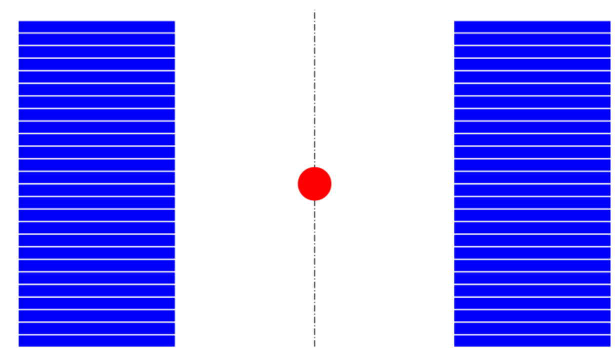
E Pardo arXiv:1602.05433



Stationary state after several cycles



magnetic field at bore center



The variational method is efficient for large number of elements

Promising for 3D modelling

General variational principle

Power applications

3D modelling

Non-linear magnetic materials

General variational principle

Power applications

3D modelling

- Novel variational principle
- Force-free effects in films
- 3D bulk

Non-linear magnetic materials

Novel 3D variacional principle

M Kapolka, E Pardo arXiv:1605.09610

$\mathbf{J} = \nabla \times \mathbf{T}$ current potential

T is the minimization variable

$$L = \int_V \mathrm{d}V \left[\frac{1}{2} \Delta\mathbf{J} \cdot \frac{\Delta\mathbf{A}_J}{\Delta t} + \Delta\mathbf{J} \cdot \frac{\Delta\mathbf{A}_a}{\Delta t} + U(\mathbf{J}) \right]$$

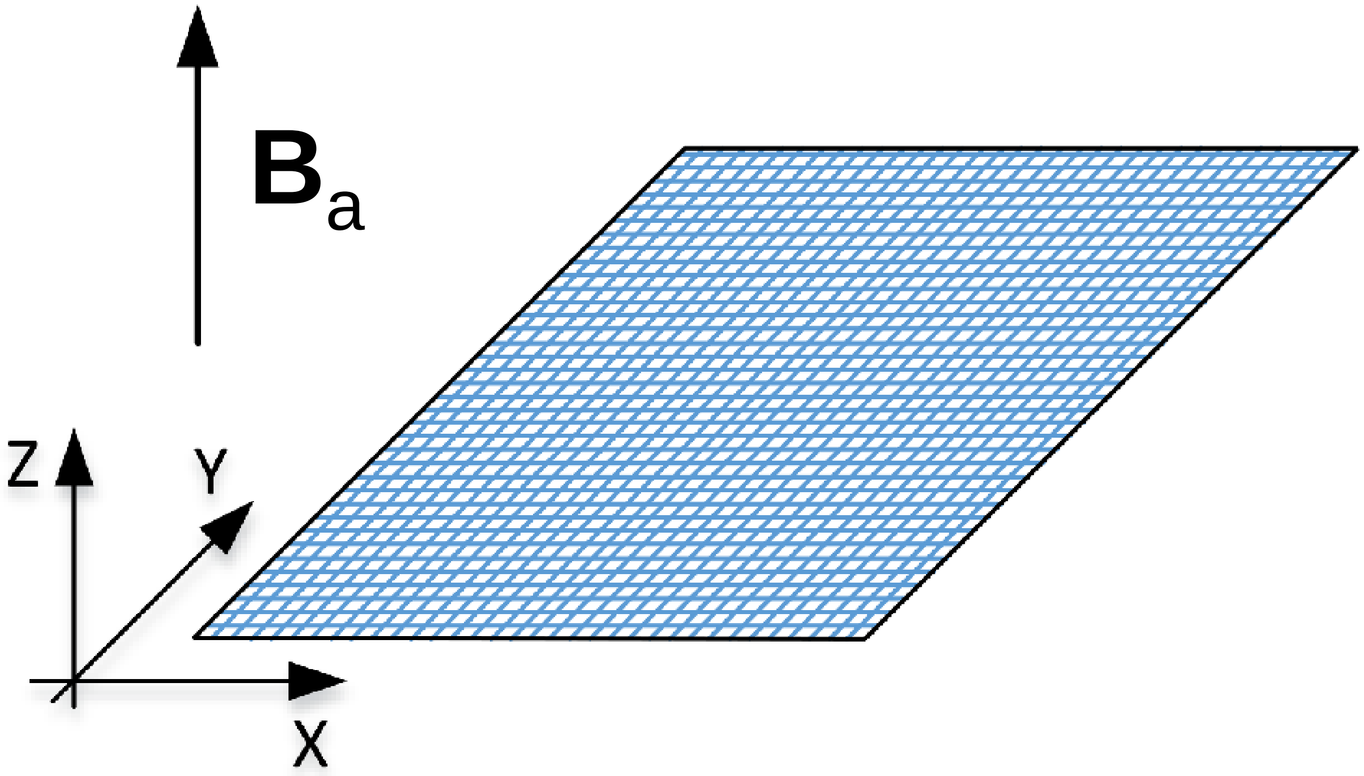
or

$$L = \int_V \mathrm{d}V \left[\frac{1}{2} \Delta\mathbf{T} \cdot \frac{\Delta\mathbf{B}_J}{\Delta t} + \Delta\mathbf{T} \cdot \frac{\Delta\mathbf{B}_a}{\Delta t} + U(\nabla \times \mathbf{T}) \right]$$

You can forget about scalar potential!

Still easy to take transport currents into account

Thin surface

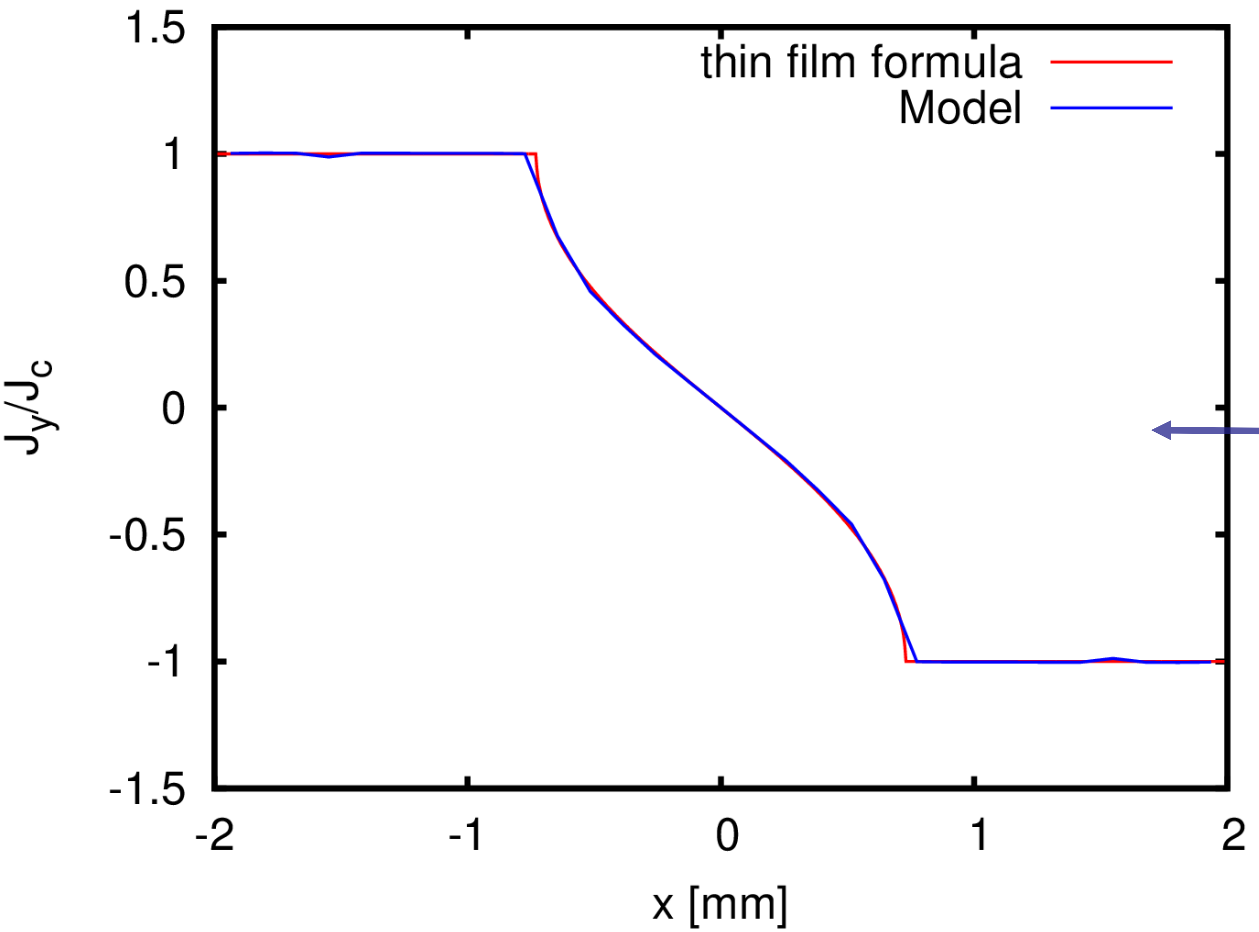


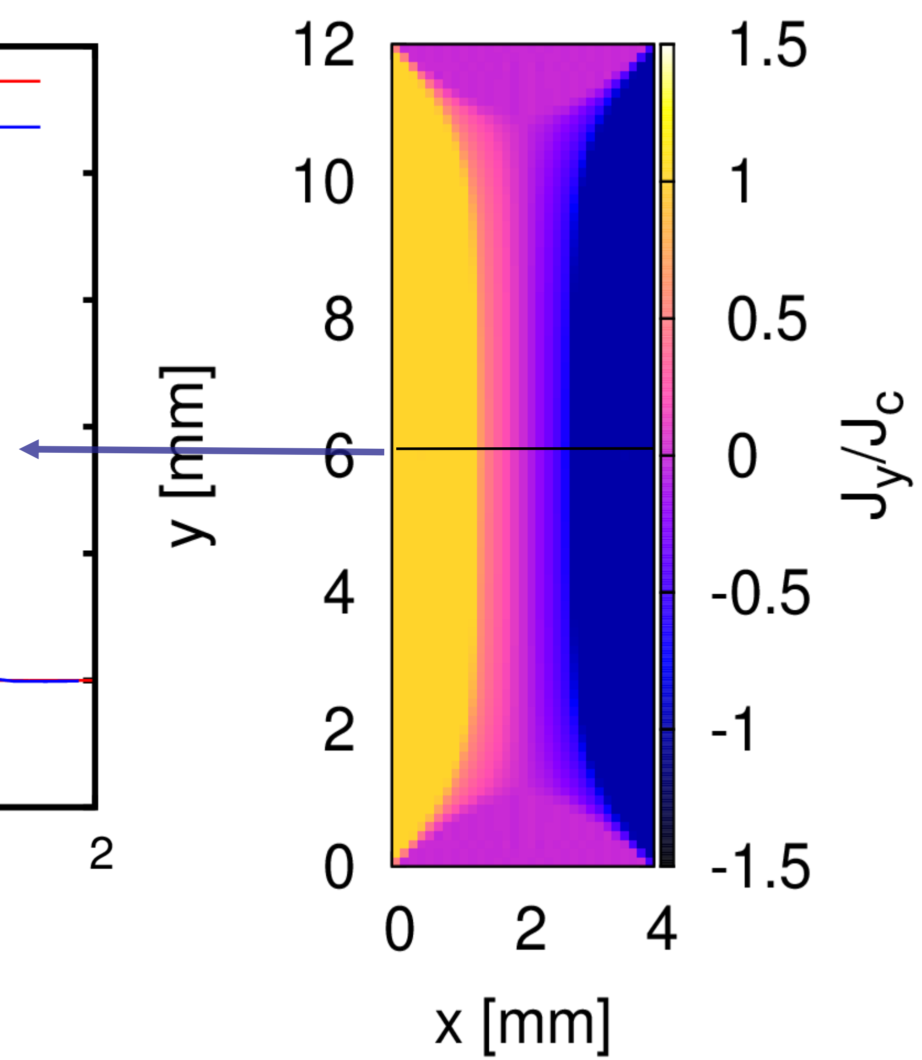
Model agrees with thin film formula

Power-law exponent **1000**

Tape midplane

Applied field: **20 mT**





General variational principle

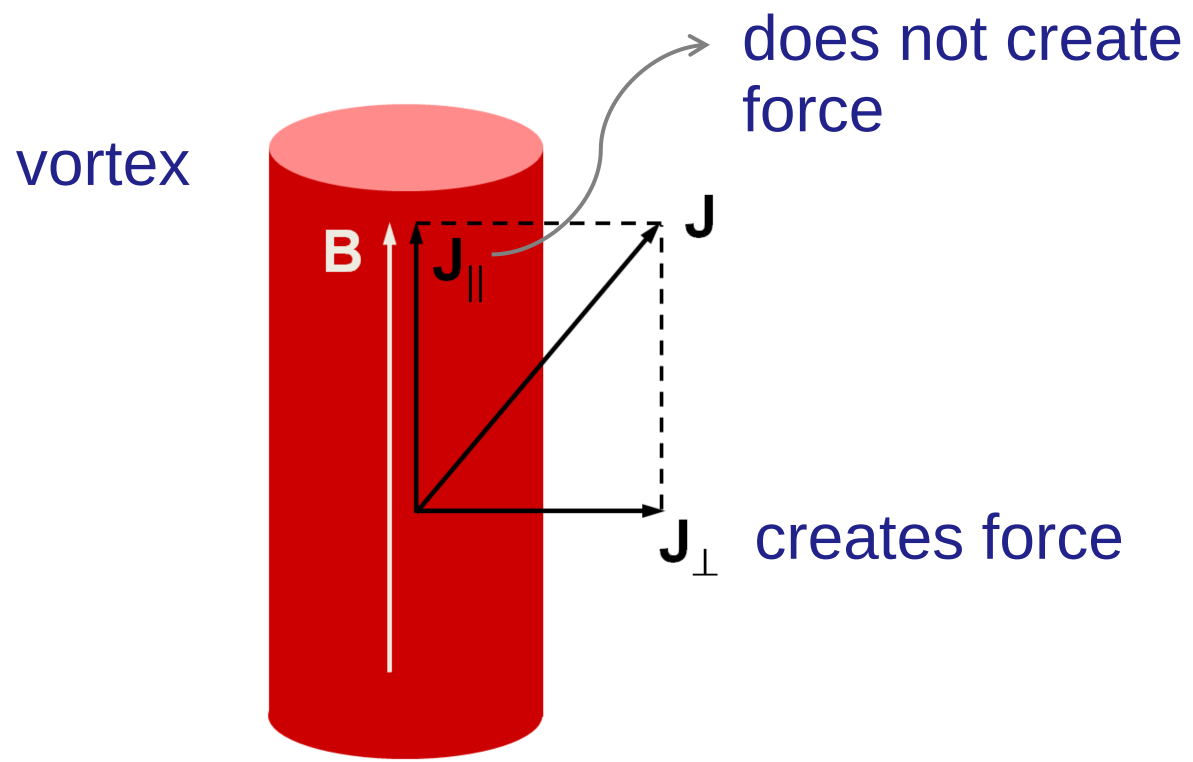
Power applications

3D modelling

- Novel variational principle
- Force-free effects in films
- 3D bulk

Non-linear magnetic materials

Flux-free effects cause anisotropic E(J)



Two critical currents:

$\mathbf{J}_{c\perp}$

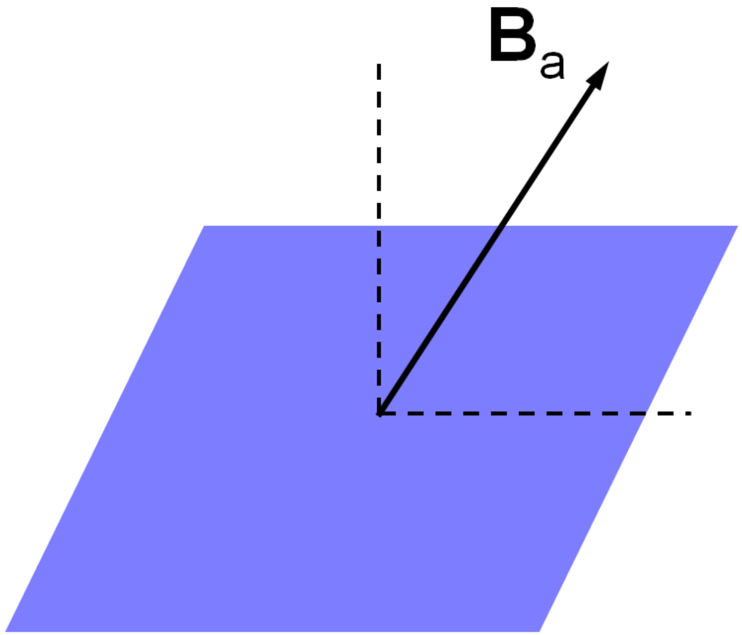
$\mathbf{J}_{c||}$

Anisotropic power law:

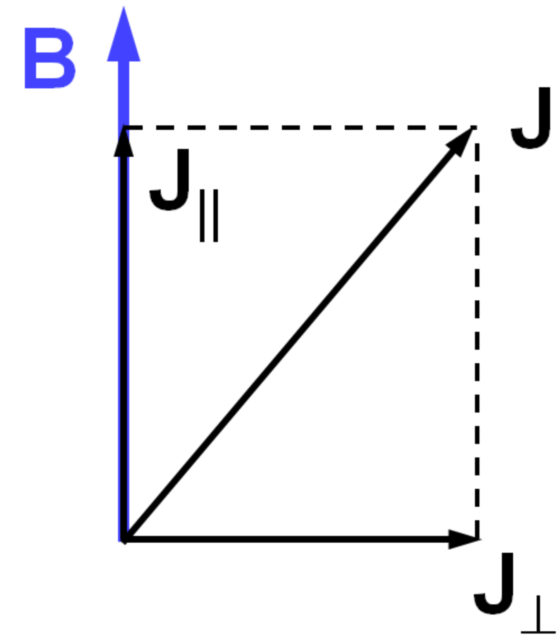
$$\mathbf{E}(\mathbf{J}) = E_c \left[\frac{J_{||}^2}{J_{c||}^2} + \frac{J_{\perp}^2}{J_{c\perp}^2}\right]^{\frac{n-1}{2}} \cdot \left(\frac{J_{||}}{J_{c||}}\frac{J_{\perp}}{J_{c||}}\mathbf{e}_{||} + \frac{J_{\perp}}{J_{c\perp}}\mathbf{e}_{\perp}\right)$$

A Badia, C Lopez DOI: 10.1088/0953-2048/28/2/024003

Flux-free effects in thin films



There is a force-free component



$$J_{c||}=3J_{c\perp}$$

Force-free J_c

Usual depinning J_c

Mishev et al.
DOI: 10.1088/0953-2048/28/10/102001

General variational principle

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3D modelling

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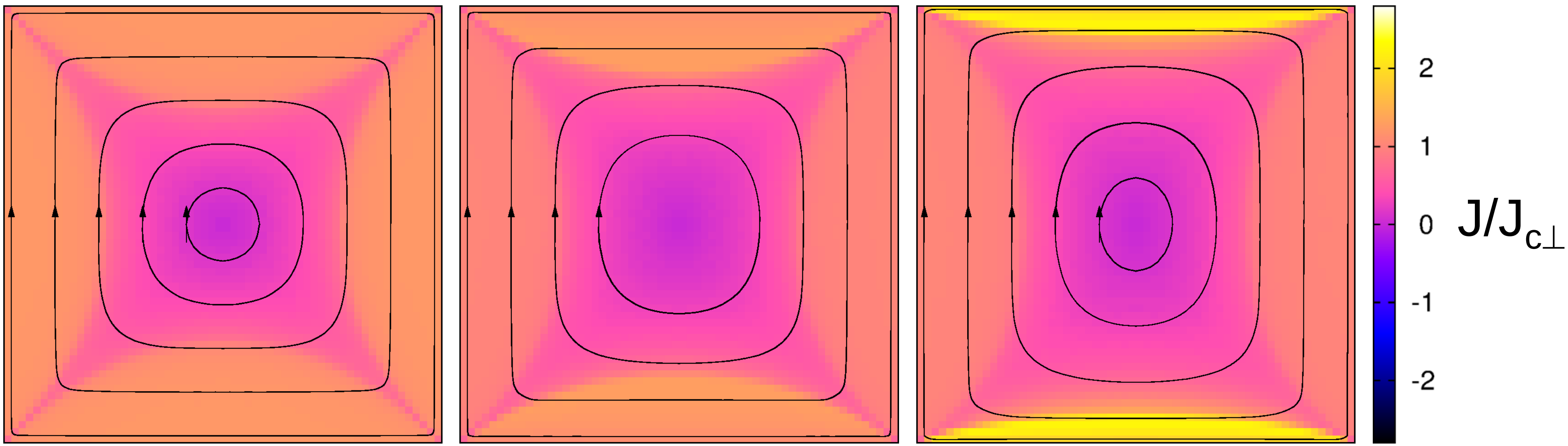
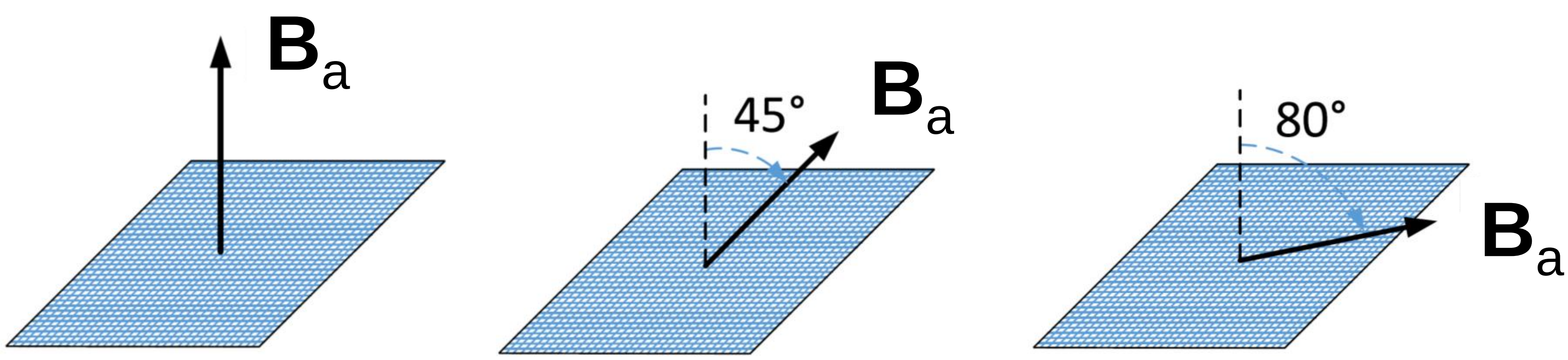
Non-linear magnetic materials

Flux-free effect increases J_c

Perpendicular field component: **23 mT**

Film **top view**

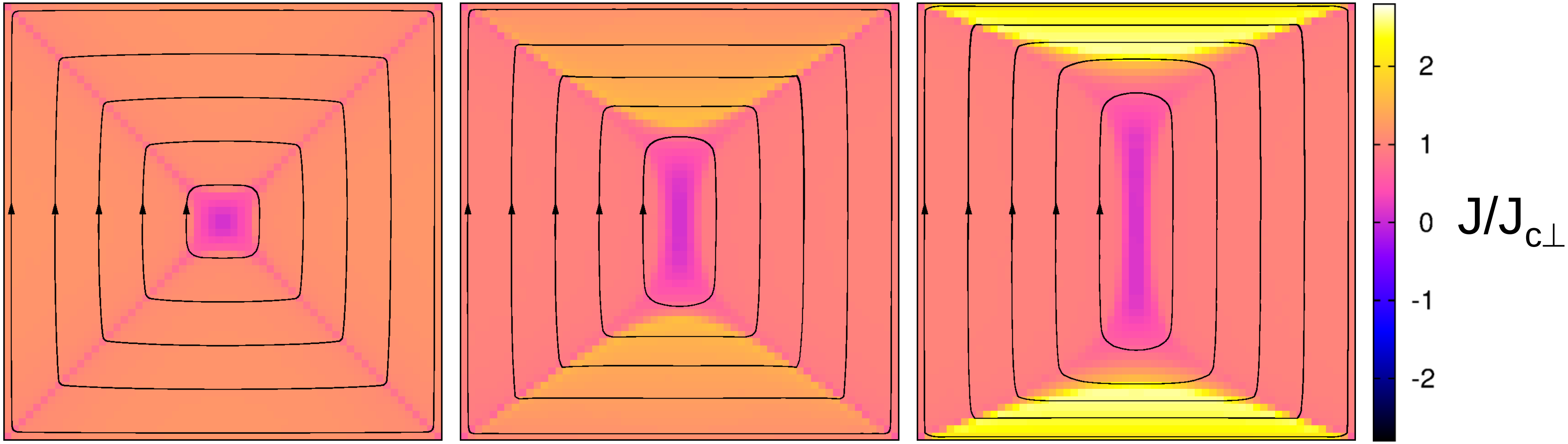


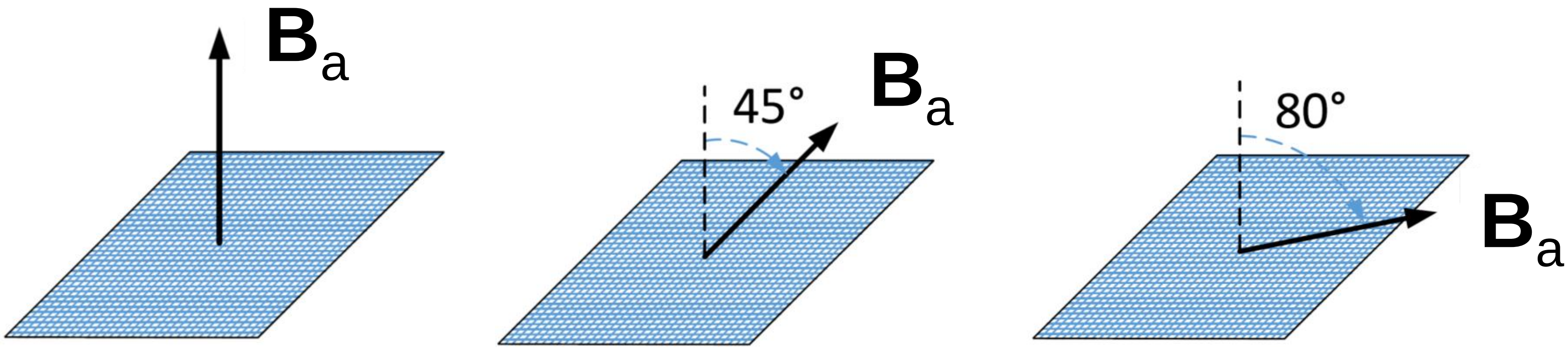


Asymmetric current saturation

Perpendicular field component: **50 mT**

Film **top view**





General variational principle

Power applications

3D modelling

- Novel variational principle
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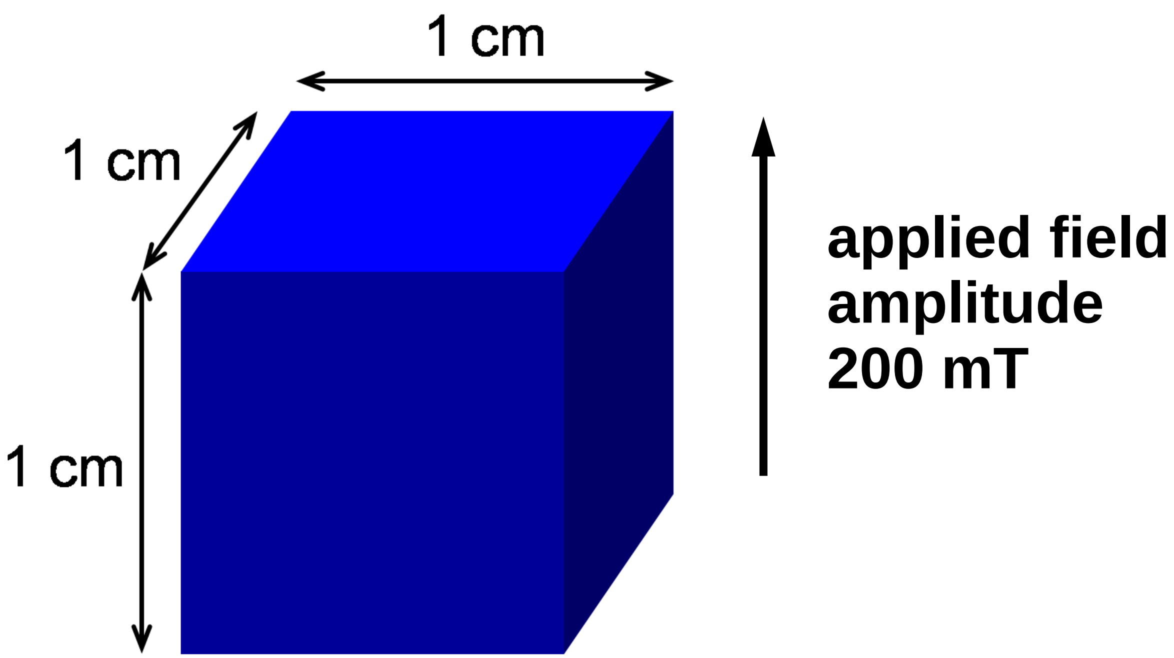
Non-linear magnetic materials

3D bulk

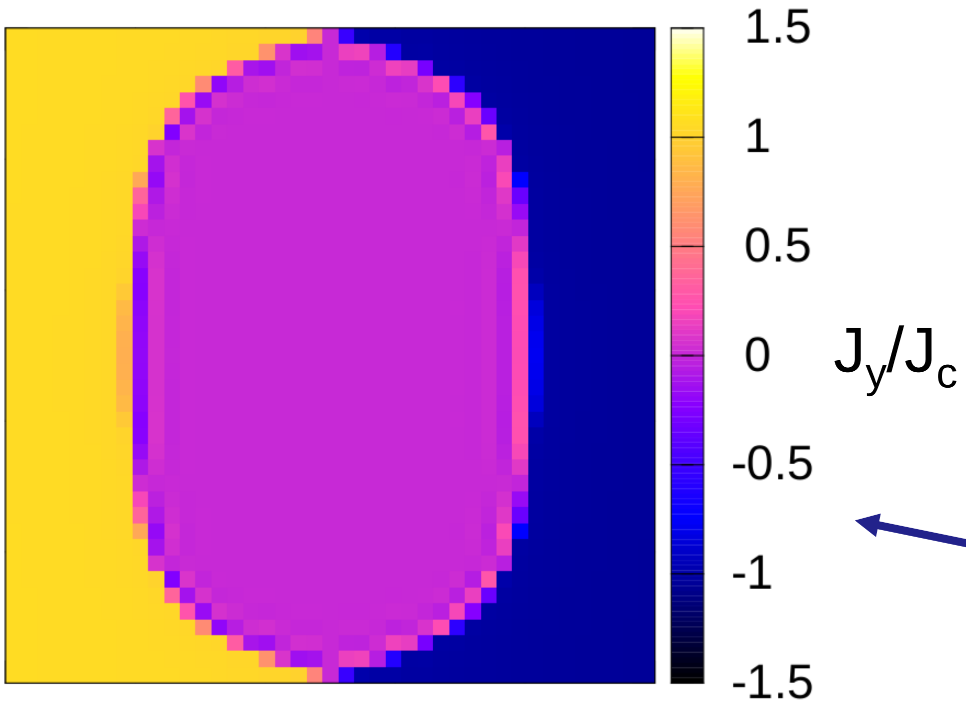
Frequency: **50 Hz** sinusoidal

Power-law exponent: **100**

J_c= $\mathbf{10^8\ A/m^2}$

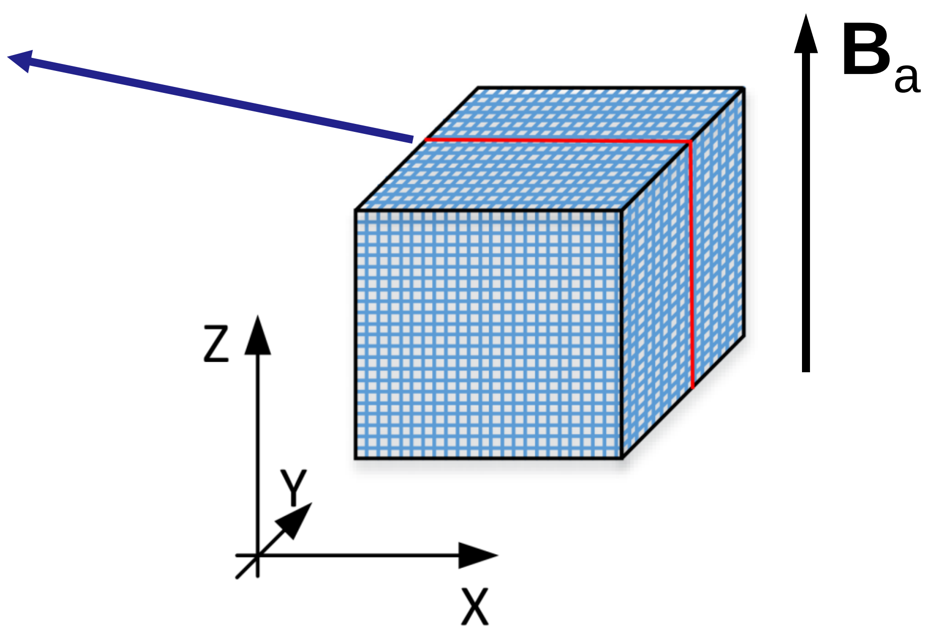


Good resolution for 3D

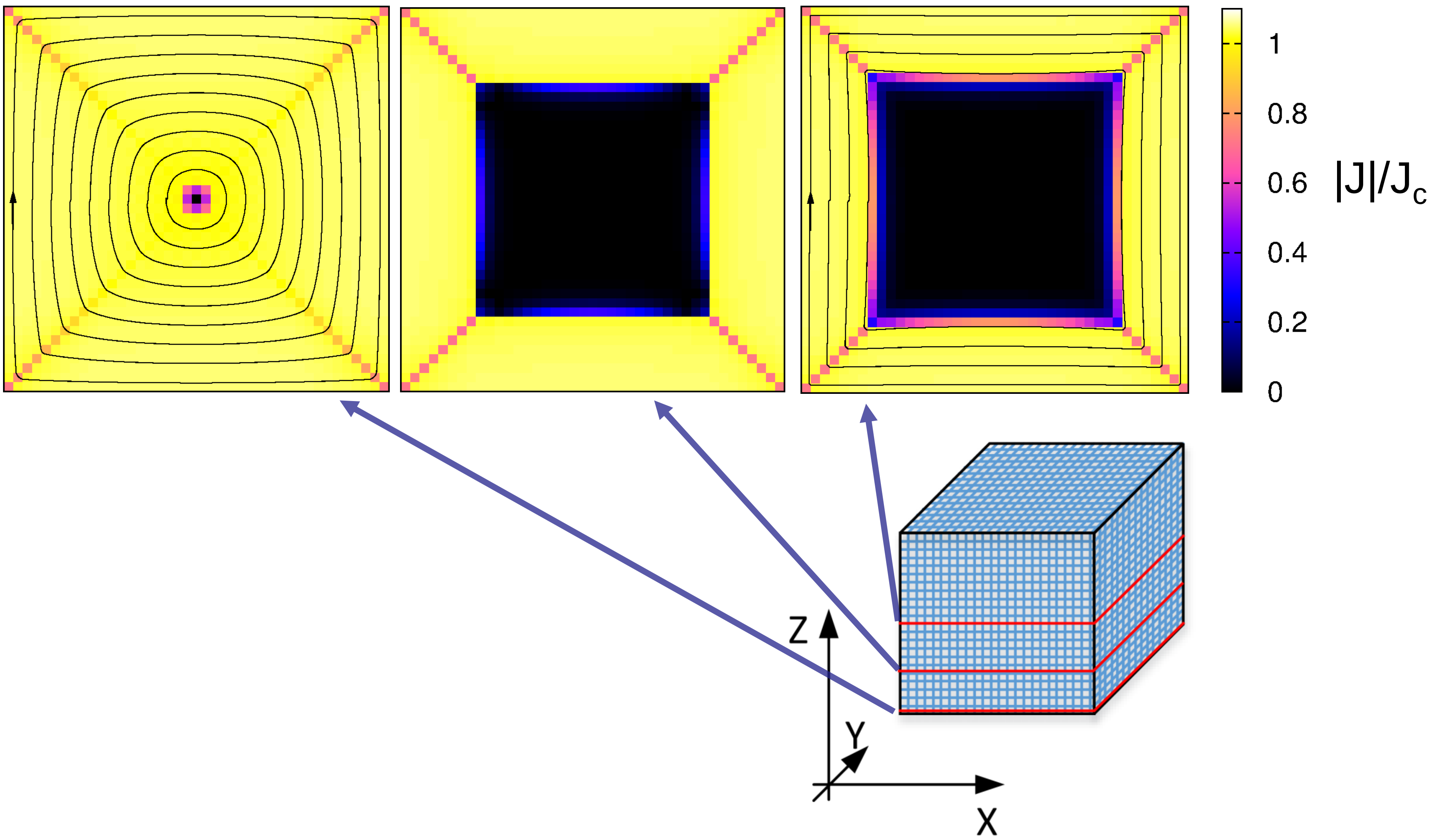


more than **210 000** degrees of freedom **in the superconductor**

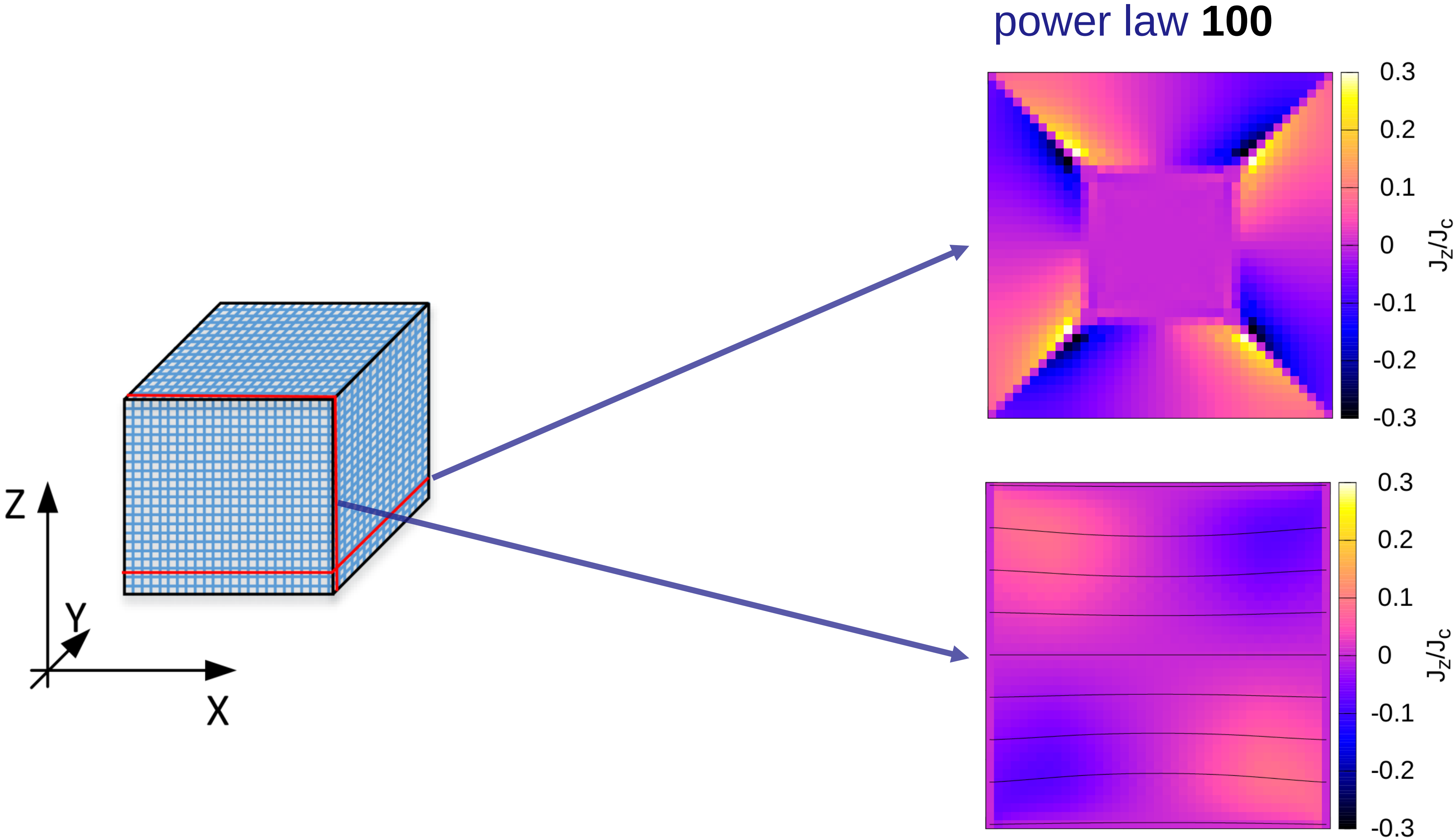
no symmetries taken into account **yet**



3D current flow

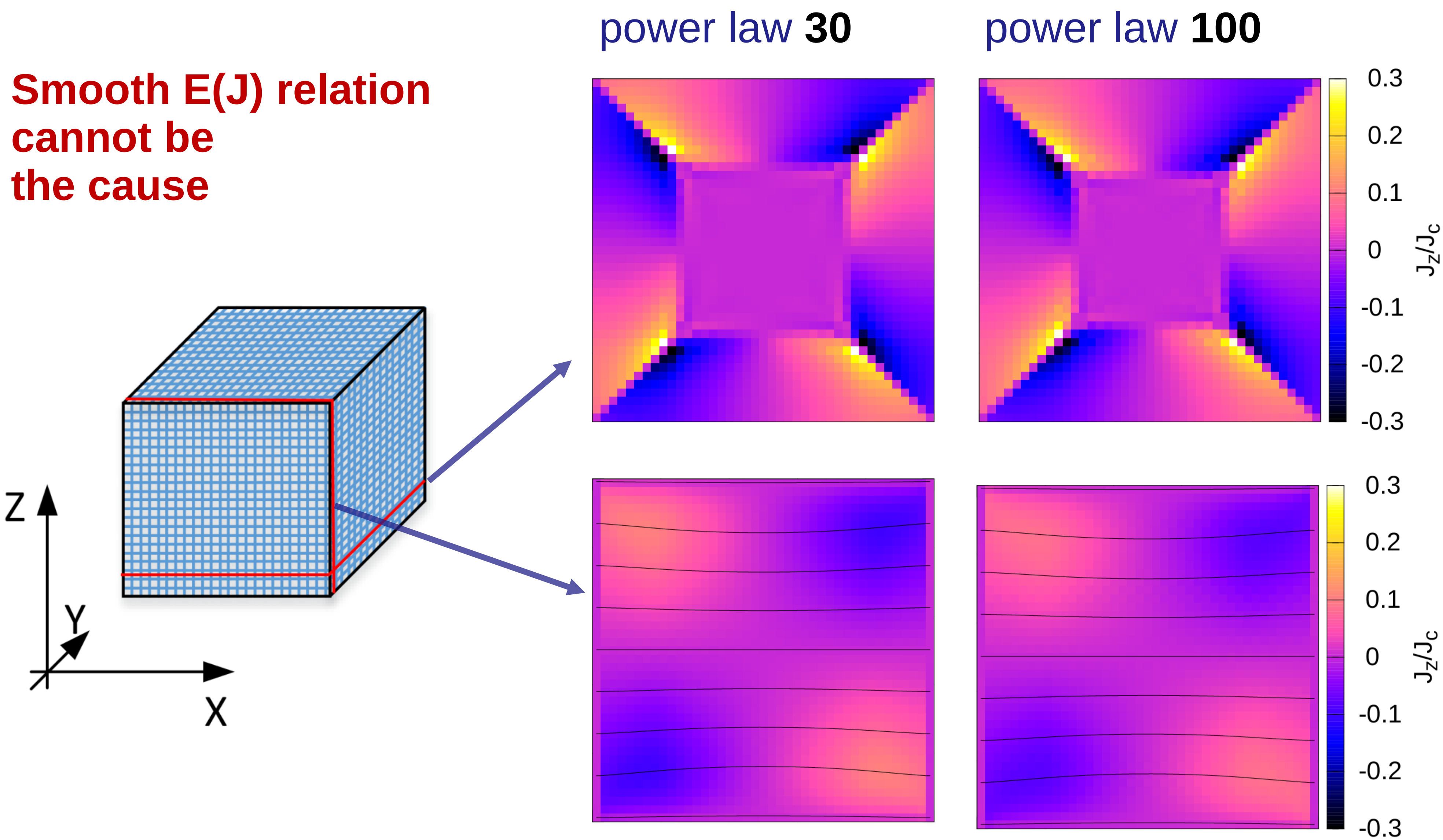


Vertical component is important



Vertical component is important

Smooth E(J) relation cannot be the cause



3D variational principle for the magnetic material

Reversible non-linear materials

Equation

$$\underbrace{\mathbf{B}(\mathbf{M})}_{\text{non-linear relation}} = \overbrace{\mathbf{B}_M}^{\mathbf{B}\text{ created by }\nabla\times\mathbf{M}} + \underbrace{\mathbf{B}_a}_{\text{applied }\mathbf{B}} + \overbrace{\mathbf{B}_J}^{\mathbf{B}\text{ from currents}}$$

is the Euler-Lagrange equation of

$$L_M = \int_V \mathrm{d}V \left[U(\mathbf{M}) - \frac{1}{2}\mathbf{B}_M \cdot \mathbf{M} - \mathbf{B}_a \cdot \mathbf{M} - \mathbf{B}_J \cdot \mathbf{M} \right]$$

$$U(\mathbf{M}) = \int_0^{\mathbf{M}} \mathrm{d}\mathbf{M}' \cdot \mathbf{B}(\mathbf{M}')$$

3D variational principle for the magnetic material

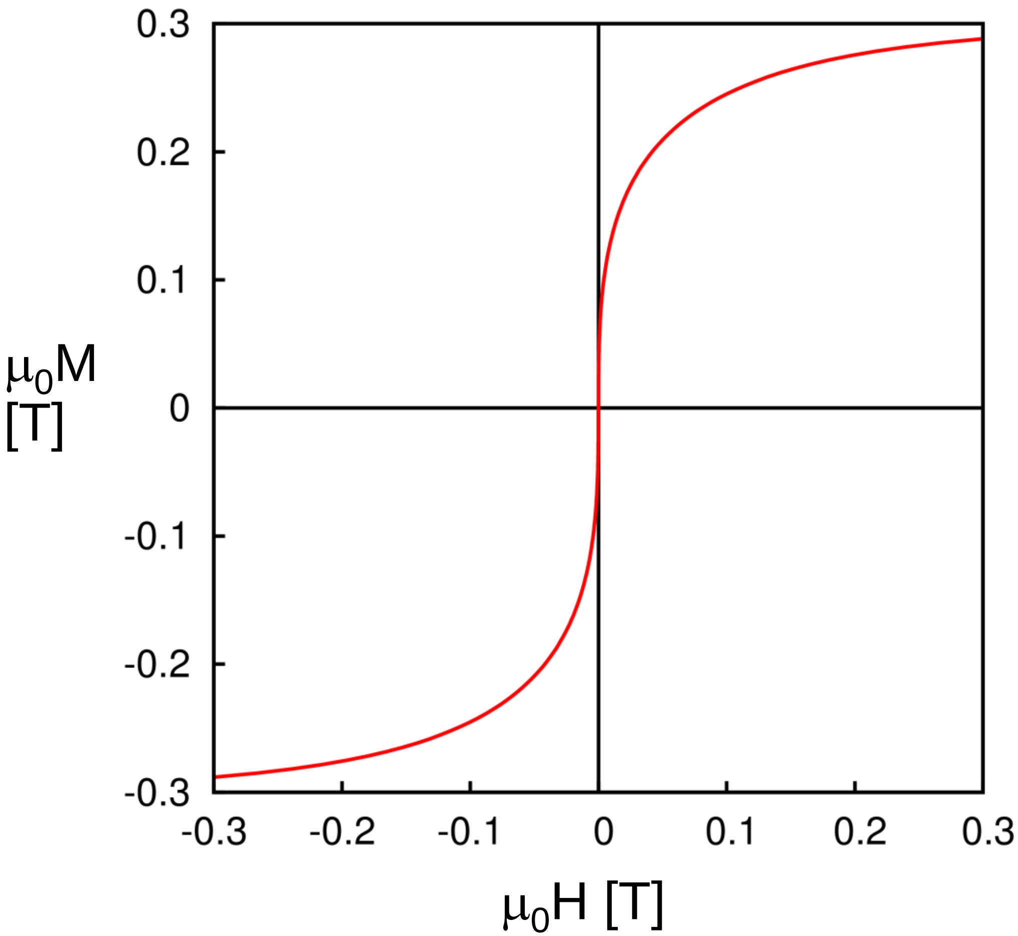
$$L_M = \int_V \mathrm{d}V \left[U(\mathbf{M}) - \frac{1}{2}\mathbf{B}_M \cdot \mathbf{M} - \mathbf{B}_a \cdot \mathbf{M} - \mathbf{B}_J \cdot \mathbf{M} \right]$$

$$U(\mathbf{M}) = \int_0^{\mathbf{M}} \mathrm{d}\mathbf{M}' \cdot \mathbf{B}(\mathbf{M}')$$

Problem restricted to the magnetic material volume

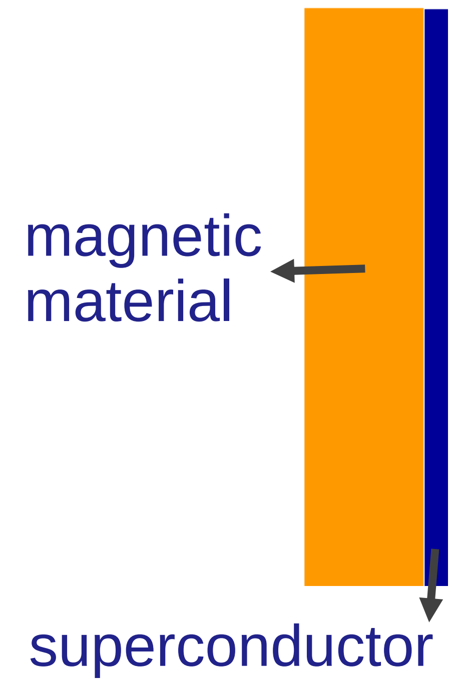
Functionals for magnetic material and superconductor **solved iteratively**

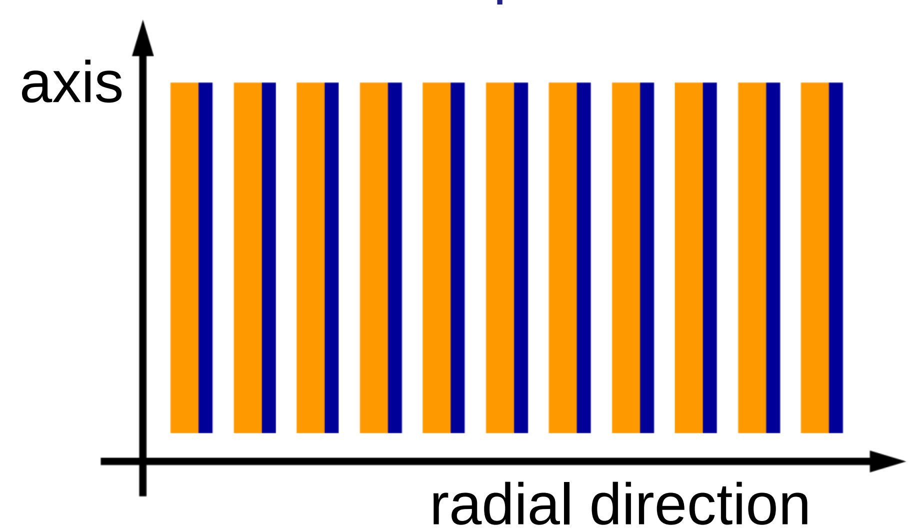
Superconductor with non-linear magnetic substrate



saturation $\mu_0 M$
300 mT

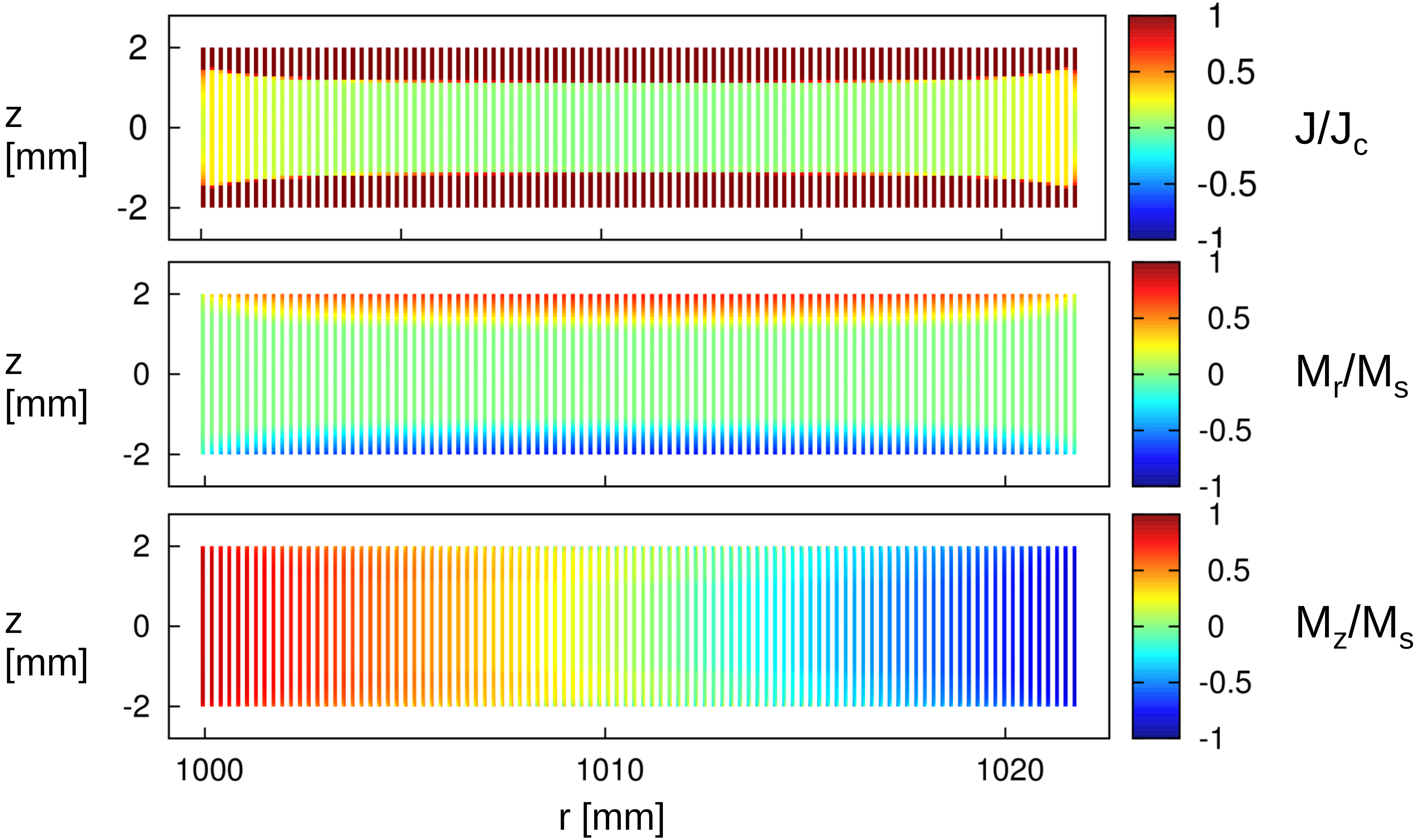
initial susceptibility
500





Magnetic substrate saturates in part of the coil

100 turns, 50% of critical current



Conclusions

Cross-sectional variational method

Results agree with experiments : **transformer**

Models coils with more than

10 000 turns for real cross-section

40 000 turns for continuous approximation

Up to half million elements in the superconductor

High potential for 3D modelling

3D modelling

Novel variational principle in T formulation

Takes force-free effects into account

Thin films:

Tilted magnetic field
changes current patterns

Bulk samples:

3D current penetration

Significant vertical component
due to shape effects

Full power of the method
still not achieved!

Non-linear magnetic material

3D variational principle

Restricts to the material volume

Modelled coils with magnetic substrate for one hundred turns

Variational methods are suitable for power and magnet applications

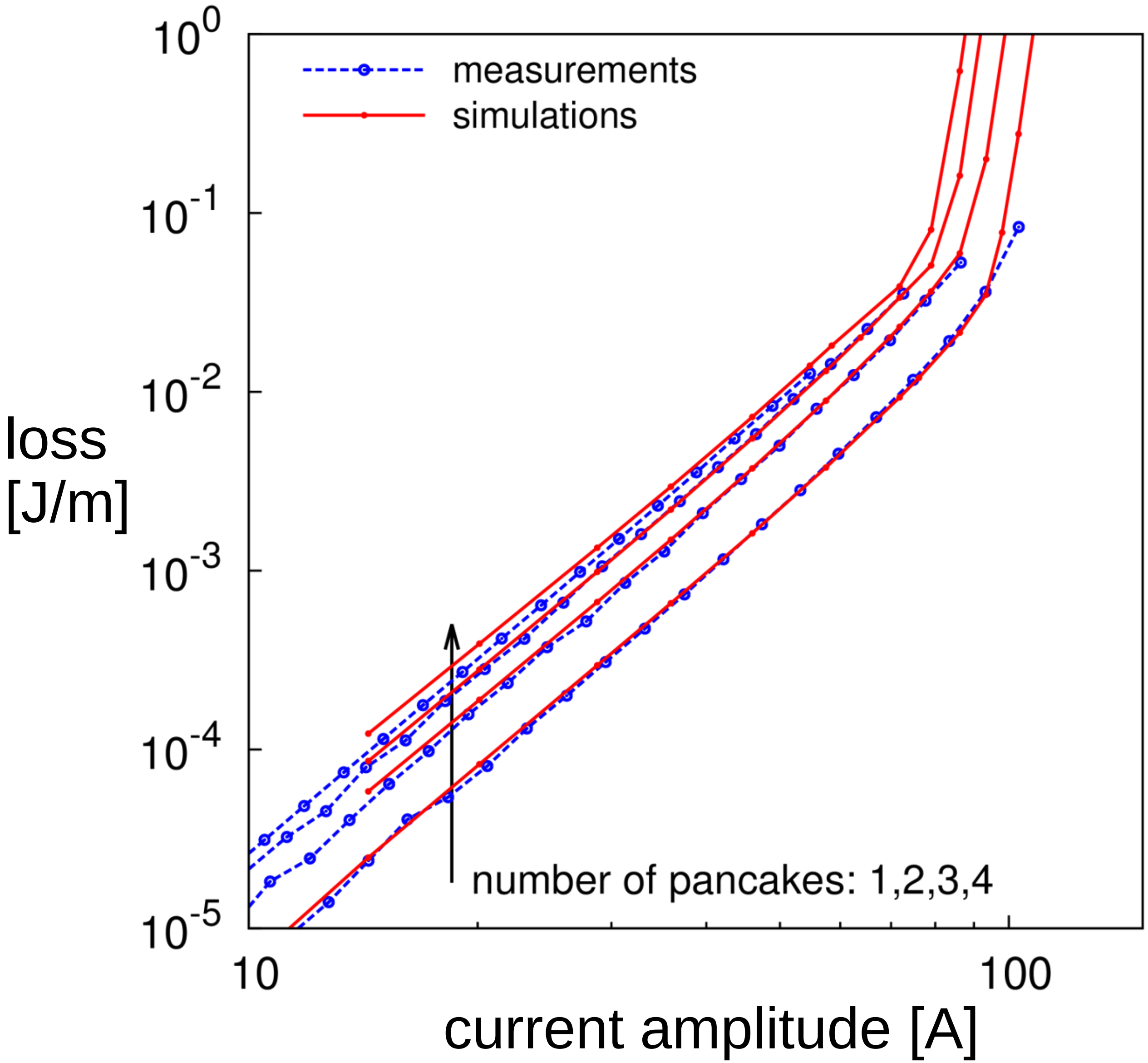
Thank you
for your attention!

Would you like to know more?

Talk available at **zenodo.org**

AC loss in test coils agrees with experiments

E Pardo et al. DOI: 10.1088/0953-2048/28/4/044003

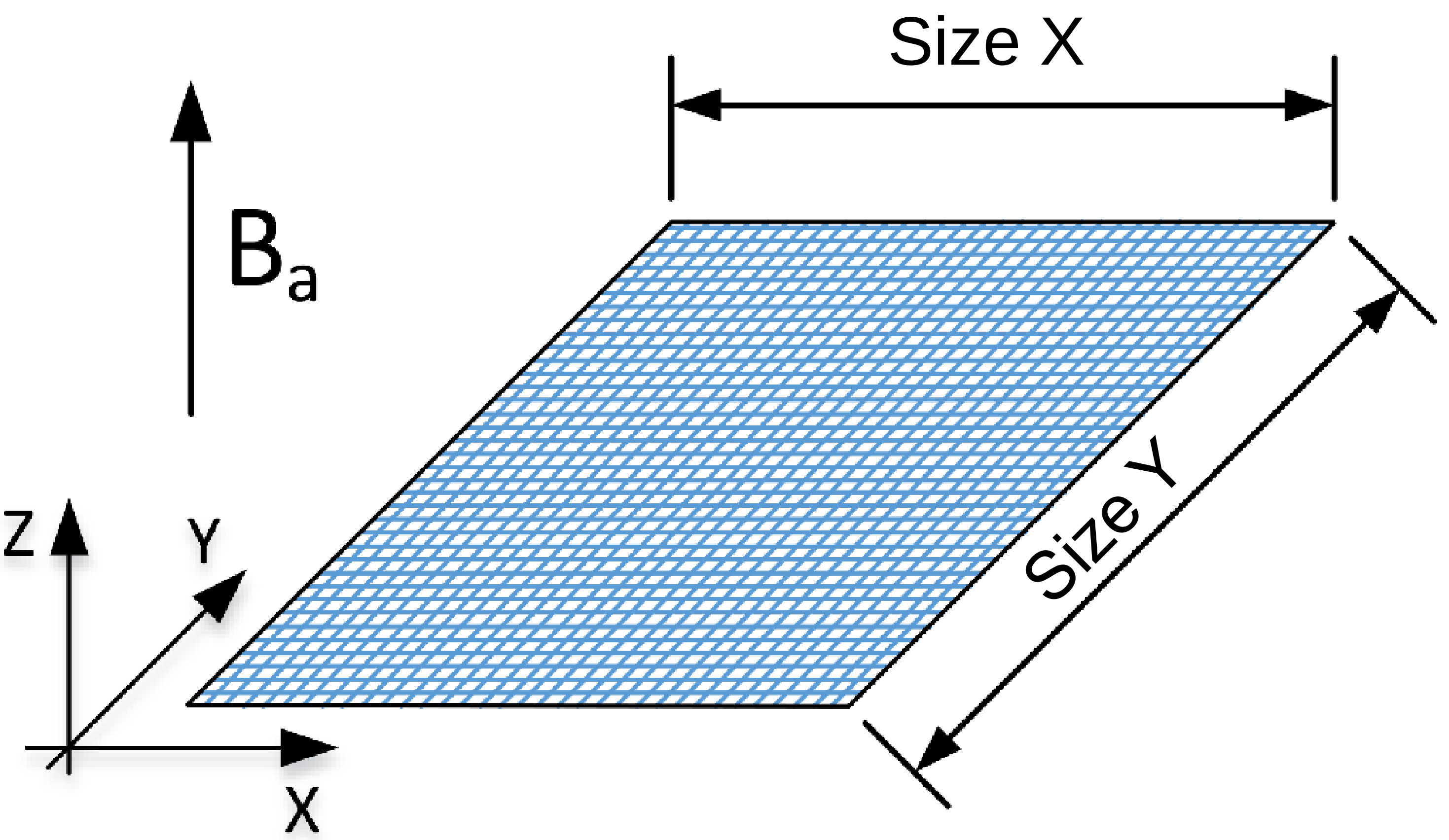


Magnetic field dependent J_c

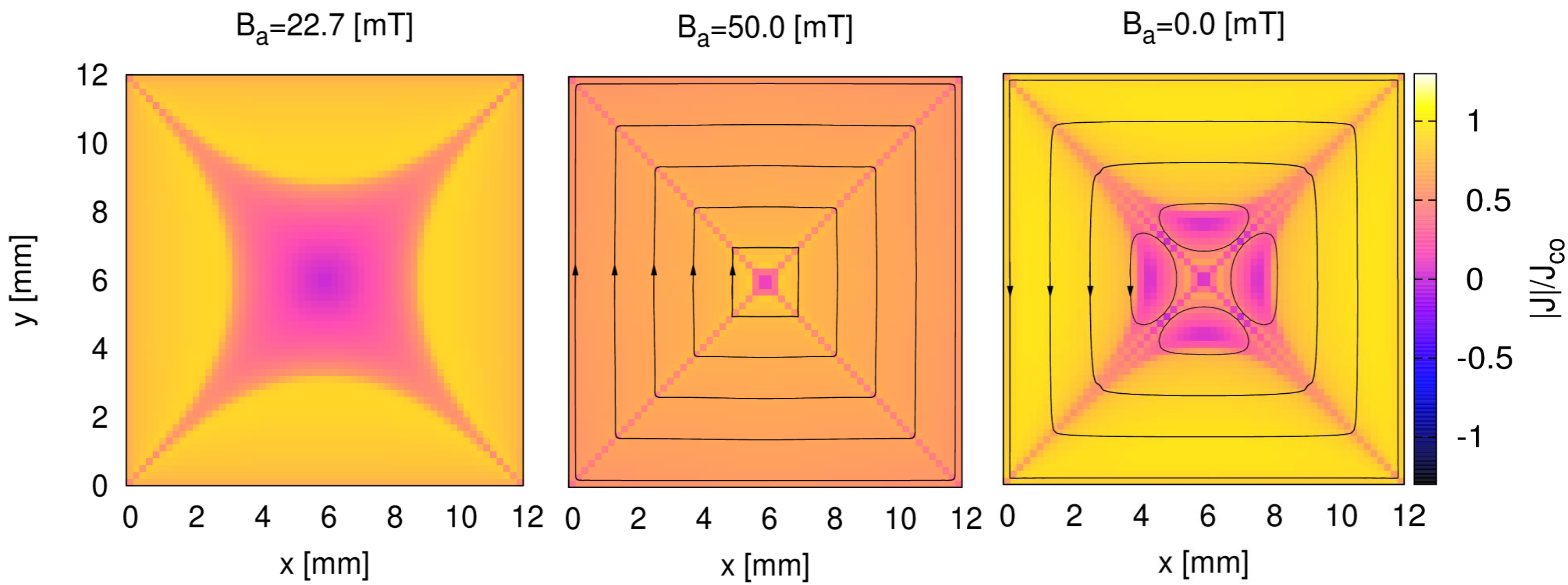
Magnetic field dependent power-law exponent

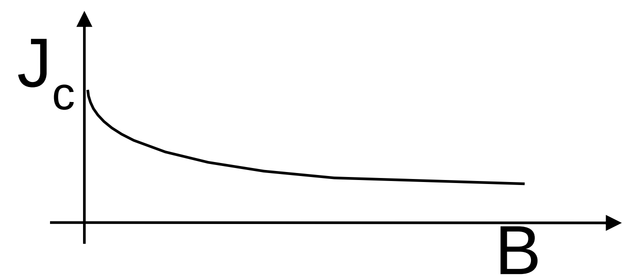
Thin surface

Frequency: **50 Hz** sinusoidal
Power-law exponent: **30**
J_c= $\mathbf{10^{10}\ A/m^2}$



Jc(B) dependence



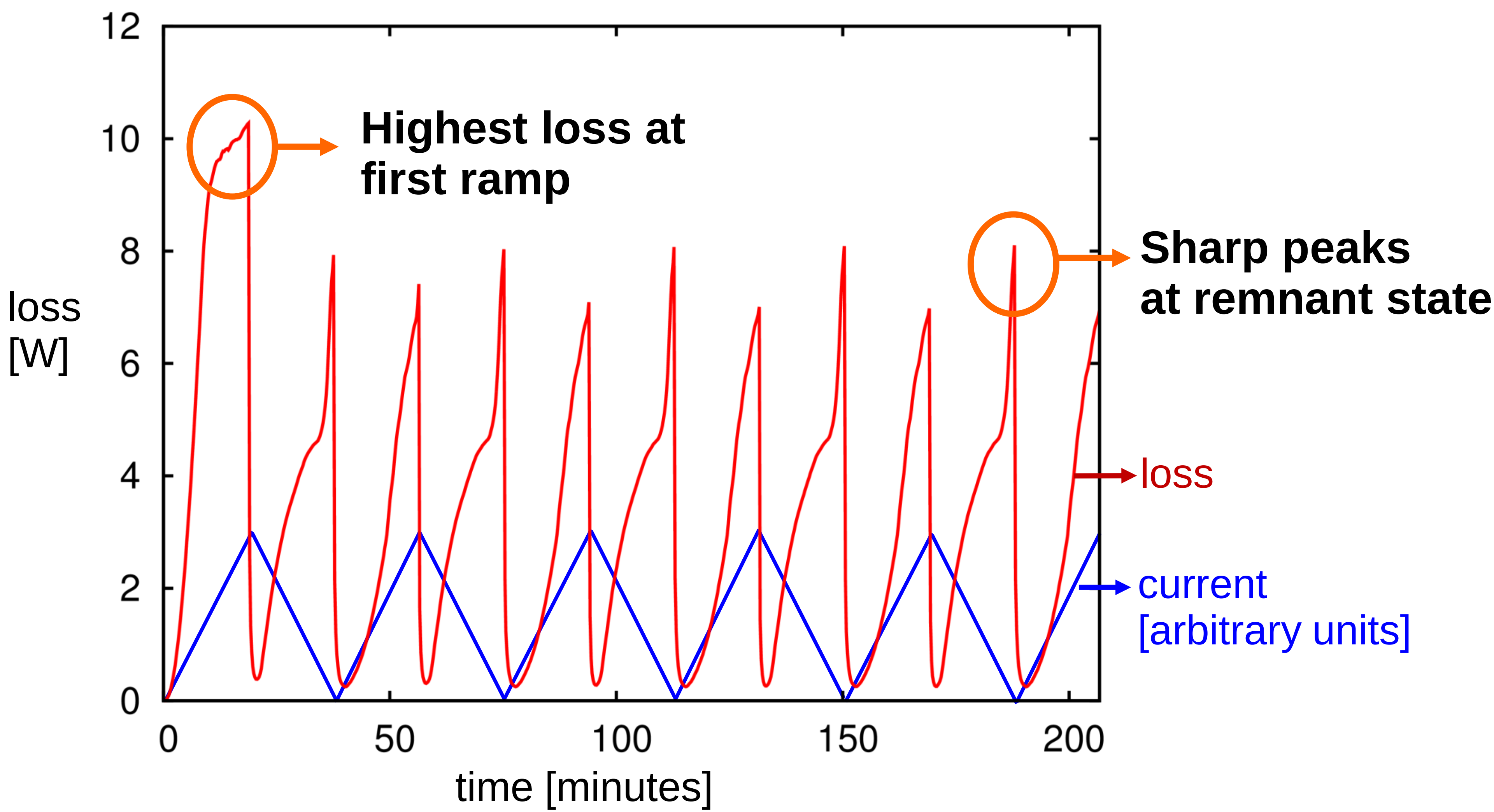


Same penetration of **J** as constant **J$_c$**

J decreased because of **B** (Kim model)

$$J_c(B) = \frac{J_{co}}{\left(1+\frac{B}{B_0}\right)^m}$$

Power loss



Highest loss at
first ramp
Sharp peaks
at remnant state
loss
[W]
loss
current
[arbitrary units]
time [minutes]
0
2
4
6
8
10
12
0
50
100
150
200