Energy-Based Variational Model for Vector Magnetic Hysteresis

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1. Magnetic levitation systems
2. Coated conductors with soft magnetic substrates.
3. Metamaterials used for:
   - magnetic cloaking;
   - distant invisible transfer of magnetic field;
   - ...

Models for hysteresis in type-II superconductors are better developed.

Modeling magnetic materials in hybrid systems: a constant (finite or infinite) permeability μ_r or a nonlinear function $\mu_r(h)$.

Should the magnetic hysteresis be neglected ?

superconductors AND ferromagnets: modeling fm

Models for hysteresis in type-II superconductors are better developed.
Modeling magnetic materials in hybrid systems: a constant (finite or infinite) permeability μ_r or a nonlinear function $\mu_r(h)$.
Should the magnetic hysteresis be neglected ? Not always.
Example: coated conductors with a Ni substrate.

- The characteristic magnetic field for a sc strip (Brandt and Indenbom, 1993), $H_c = J_c/\pi$, is of the order $10^3 - 10^4$ A/m.
- Coercivity of a Ni substrate is similar, about 6000 A/m (Ijaduola et al. 2004).

Wanted: a model for magnetic hysteresis

Requirements: the model should

- be vectorial:

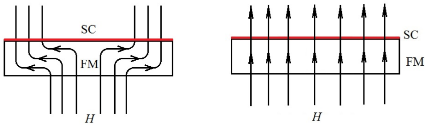

Fig: $\boldsymbol{b}$ lines for a weak (left) and strong (right) external field.

- predict the magnetization losses;
- be able to account for material anisotropy;
- be included into the Maxwell equations as a local constitutive relation with memory.

Models for quasistationary hysteresis

The most popular macroscopic models are

- The Preisach model (1935): a black-box-type method for storing, and using for interpolation, a vast amount of experimental data.

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Other disadvantages:

- The models are scalar; their vectorial versions exist but are not physically justified too.
- Energy loss: can be estimated only for closed loops.

The Bergqvist model

The Bergqvist (1997) model:

- based on consistent energy arguments and an assumption on the dry-friction-like pinning of the domain walls;
- intrinsically vectorial;
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Subsequent works: Bergqvist et al. (97,04,14)
Henrotte, Hameyer, Steentjes, Geuzaine, ... (06, 12, 13, 14).
Weak points: derivation and numerical implementation
An approximation to make the magnetization update explicit.
Should be avoided in the vectorial case.

This talk:

- A simplified energy-based model:
  - derivation;
  - variational structure;
  - numerical algorithm.
- The composite Bergqvist model.
- Identification of the parameters in the model.
- Coupling with the Maxwell equations: an example.

A simplified model: Assumption 1

A1: The density of magnetic field energy in a magnetic material

$$W = \frac{1}{2}\mu_0 h^2 + U(\boldsymbol{m}) \text{ changes as } \dot{W} = \boldsymbol{h} \cdot \dot{\boldsymbol{b}} - |r\dot{\boldsymbol{m}}|,$$

where $\boldsymbol{b} = \mu_0(\boldsymbol{h} + \boldsymbol{m})$,
$\boldsymbol{h} \cdot \dot{\boldsymbol{b}}$ - the rate of magnetic field work, and
$|r\dot{\boldsymbol{m}}|$ - the rate of dissipation due to the irreversible movement of magnetic domain walls.
Here r is a scalar or a symmetric positive-definite matrix.

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This yields

$$\mu_0\boldsymbol{h}\cdot\dot{\boldsymbol{h}} + \nabla U(\boldsymbol{m})\cdot\dot{\boldsymbol{m}} = \mu_0\boldsymbol{h}\cdot(\dot{\boldsymbol{h}}+\dot{\boldsymbol{m}}) - |r\dot{\boldsymbol{m}}|$$

or

$$(\boldsymbol{h} - \boldsymbol{f}(\boldsymbol{m}))\cdot\dot{\boldsymbol{m}} = |k\dot{\boldsymbol{m}}|,$$

where $\boldsymbol{f}(\boldsymbol{m}) = \frac{1}{\mu_0}\nabla U(\boldsymbol{m})$ and $k = \frac{1}{\mu_0}r$.

A simplified model: Assumption 2

Bergqvist presented the magnetic field as a sum,

$$\boldsymbol{h} = \boldsymbol{h}_r + \boldsymbol{h}_i,$$

where

the field $\boldsymbol{h}_r = \boldsymbol{f}(\boldsymbol{m}) = \frac{1}{\mu_0}\nabla U(\boldsymbol{m})$ is called reversible, because the magnetic work it delivers is fully converted into internal energy.

It is assumed $\boldsymbol{h}_r \| \boldsymbol{m}$.

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The energy-based relation $(\boldsymbol{h} - \boldsymbol{f}(\boldsymbol{m})) \cdot \dot{\boldsymbol{m}} = |k\dot{\boldsymbol{m}}|$ becomes

$$\boldsymbol{h}_i \cdot \dot{\boldsymbol{m}} = |k\dot{\boldsymbol{m}}|.$$

A2: For an isotropic material this relation holds if the following "dry-friction law" is postulated:

$|\boldsymbol{h}_i| \le k$;
if $|\boldsymbol{h}_i| < k$ then $\dot{\boldsymbol{m}} = \boldsymbol{0}$;
if $\dot{\boldsymbol{m}} \ne \boldsymbol{0}$ it has the direction of $\boldsymbol{h}_i$.

The variational structure

Equivalent formulation of this law is:

$$\boldsymbol{h}_i \in \widetilde{K} := \left\{\boldsymbol{u} \in \mathbb{R}^3 \ : \ |k^{-1}\boldsymbol{u}| \leq 1\right\} \text{ and}$$
$$\dot{\boldsymbol{m}} \cdot (\boldsymbol{u} - \boldsymbol{h}_i) \leq 0 \text{ for any } \boldsymbol{u} \in \widetilde{K}.$$

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Written in this form, the constitutive relation:

- agrees with the "dry friction law" also in the anisotropic case;
- means that $\dot{\boldsymbol{m}}$ is a subgradient of the indicator function of the set $\widetilde{K}$ at the point $\boldsymbol{h}_i$ (the variational structure);

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- means that $\dot{\boldsymbol{m}}$ is a subgradient of the indicator function of the set $\widetilde{K}$ at the point $\boldsymbol{h}_i$ (the variational structure);
- is similar to the relations between:
  - the rate of plastic deformations and stress in elasto-plasticity with the yield stress k;
  - the electric field and current density in type-II superconductivity models.

Numerical solution: Step 1

S1. Let $\boldsymbol{h}(t)$ be given.
If $\boldsymbol{h}_i \in \widetilde{K}$ then $\boldsymbol{h}_r = \boldsymbol{h}(t) - \boldsymbol{h}_i$ belongs to the set

$$K(t) := \{\boldsymbol{u} \in \mathbb{R}^3 \ : \ |k^{-1}(\boldsymbol{h}(t) - \boldsymbol{u})| \leq 1\}.$$

The model can be reformulated for $\boldsymbol{h}_r$,

$$\begin{gathered}\text{Find } \boldsymbol{h}_r \in K(t) \text{ and } \boldsymbol{m} \text{ such that}\\ \dot{\boldsymbol{m}} \cdot (\boldsymbol{u} - \boldsymbol{h}_r) \geq 0 \text{ for any } \boldsymbol{u} \in K(t),\end{gathered}$$

and discretized in time,

$$\begin{gathered}\text{Find } \boldsymbol{h}_r \in K(t) \text{ and } \boldsymbol{m} \text{ are such that}\\ (\boldsymbol{m} - \widehat{\boldsymbol{m}}) \cdot (\boldsymbol{u} - \boldsymbol{h}_r) \geq 0 \text{ for any } \boldsymbol{u} \in K(t).\end{gathered}$$

Finally, we express $\boldsymbol{m}$ via $\boldsymbol{h}_r$.

Numerical solution: Step 2

S2. Since $\boldsymbol{h}_r \| \boldsymbol{m}$ we set $\boldsymbol{m} = M_{an}(h_r)\frac{\boldsymbol{h}_r}{h_r}$, where M_{an} is a nondecreasing anhysteretic function s.t. $M_{an}(0) = 0$, and define

$$S(\boldsymbol{u}) = \int_0^u M_{an}(s)ds.$$

Then $\boldsymbol{m} = \nabla S(\boldsymbol{h}_r)$.

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$$S(\boldsymbol{u}) = \int_0^u M_{an}(s)ds.$$

Then $\boldsymbol{m} = \nabla S(\boldsymbol{h}_r)$. On each time level the problem becomes

$$\text{Find } \boldsymbol{h}_r \in K(t) \text{ such that}$$
$$(\nabla S(\boldsymbol{h}_r) - \widehat{\boldsymbol{m}}) \cdot (\boldsymbol{u} - \boldsymbol{h}_r) \geq 0 \text{ for any } \boldsymbol{u} \in K(t).$$

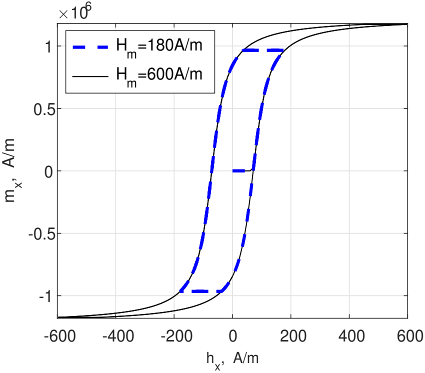
Equivalently, $\boldsymbol{h}_r$ is the unique solution to the optimization problem

$$\min_{\boldsymbol{u} \in K(t)} \{S(\boldsymbol{u}) - \widehat{\boldsymbol{m}} \cdot \boldsymbol{u}\}$$

which we could solve very efficiently (2-3 iterations).
Bergqvist and other authors after him used instead a projection:
$\boldsymbol{h}_r = Proj_{K(t)}\{\widehat{\boldsymbol{h}_r}\}$ (explicit formulae).

The simplified model: Examples

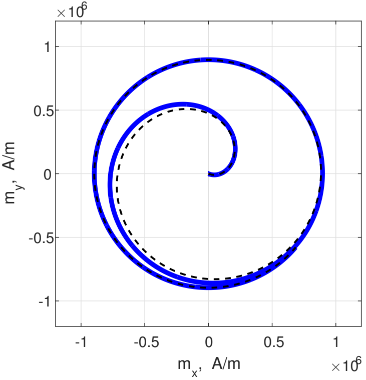
Scalar example: $\boldsymbol{h} = (H_m \sin t, 0)$.
M_{an} and k are chosen to approximate the major hysteresis loop.



The minor loop and the initial magnetization curve are bad.

The simplified model: Examples

Rotating field: $\boldsymbol{h} = H_m(t)(\cos t, \sin t)$. Same M_{an} and k.
The amplitude $H_m(t)$ first grows, then remains constant.

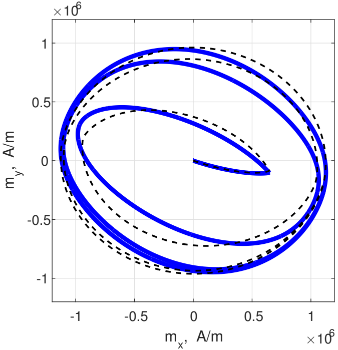


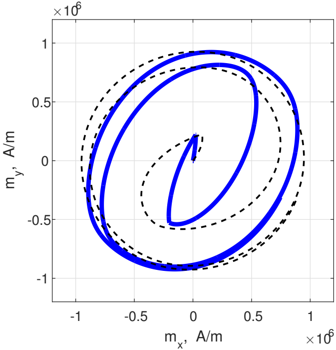
The model solution $\boldsymbol{m} = (m_x, m_y)$ (solid line) and the one based on the explicit approximation (dashed line) are close in this case.

The simplified model: Examples

Left: elliptic field
$\boldsymbol{h} = H_m(t)(3\cos t, \sin t)$;

Right: anisotropic case
$k = \text{diag}(k_0, 0.5k_0)$.





Explicit approximation (dashed line) vs model solution (solid line).

Modification 1: The composite Bergqvist model

One dry friction coefficient is replaced by a distribution of the pinning strength values.

In practice, a mixture of N types of "pseudparticles" is assumed,

$$\boldsymbol{m} = \sum_{l=1}^{N} w^l \boldsymbol{m}^l,$$

where the l-th particle type has the magnetization $\boldsymbol{m}^l$ and volume fraction $w^l \geq 0$; $\sum_{l=1}^{N} w^l = 1$.

Here $\boldsymbol{m}^l = M_{an}(h_r^l)\frac{\boldsymbol{h}_r^l}{h_r^l}$ obeys the dry friction law and $\boldsymbol{h}_r^l$ solves

$$\min_{\boldsymbol{u} \in K^l(t)} \{S(\boldsymbol{u}) - \widehat{\boldsymbol{m}}^l \cdot \boldsymbol{u}\},$$

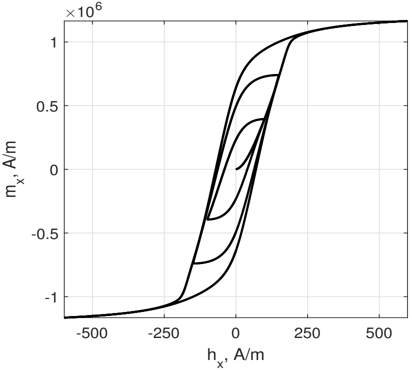
where $K^l(t) := \{\boldsymbol{u} \in \mathbb{R}^3 \; : \; |\{k^l\}^{-1}(\boldsymbol{h}(t) - \boldsymbol{u})| \leq 1\}$.

To account for the partial reversibility of material response set, say,

$$k^1 = 0 \text{ and } K^1(t) := \{\boldsymbol{h}(t)\}.$$

Modification 1: The composite Bergqvist model

Example: $N = 20,\ w^l \equiv 1/N,\ k^l = 140\frac{l-1}{N-1}$ A/m.



Improvement of the initial magnetization curve and minor hysteresis loops.

Modification 2: The interaction term

The idea: magnetic domains do not evolve independently driven by the magnetic field $\boldsymbol{h}(t)$ but interact. The "driving force" is

$$\boldsymbol{h}_{eff} = \boldsymbol{h} + \alpha \boldsymbol{m},$$

where α is a material parameter. Such approach was used also for other models (J-A, Preisach, etc.); an explanation: Della Torre, 99.

Modification 2: The interaction term

With $\boldsymbol{h}_{eff} = \boldsymbol{h} + \alpha \boldsymbol{m}$ we find $\boldsymbol{h}_r^l$ for $l = 1, ..., N$ solving

$$\min_{\boldsymbol{u} \in K^l(t,\boldsymbol{m})} \{S(\boldsymbol{u}) - \widehat{\boldsymbol{m}}^l \cdot \boldsymbol{u}\}$$

with the sets $K^l(t)$ replaced by

$$K^l(t, \boldsymbol{m}) := \{\boldsymbol{u} \in \mathbb{R}^3 \ : \ |(k^l)^{-1}(\boldsymbol{u} - \boldsymbol{h}(t) - \alpha \boldsymbol{m})| \leq 1\}$$

if $k^l \neq 0$ and $K^l(t, \boldsymbol{m}) := \{\boldsymbol{h}(t) + \alpha \boldsymbol{m}\}$ otherwise. Since

$$\boldsymbol{m} = \sum_{l=1}^{N} w^l \boldsymbol{m}^l = \sum_{l=1}^{N} w^l M_{an}(h_r^l) \frac{\boldsymbol{h}_r^l}{h_r^l}.$$

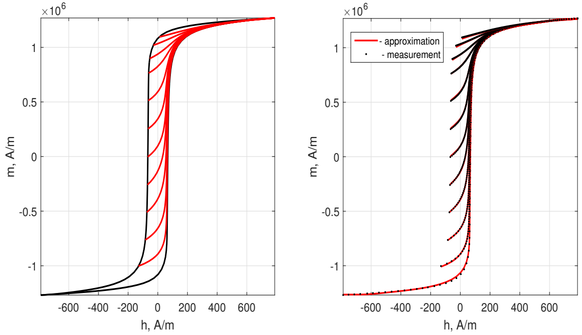
the constraints in the opt. problems depend on the unknown solution: the whole problem becomes quasi-variational.

Identification of parameters

Parameters of the model:

- anhysteretic function M_{an}, a spline;
- weights w^l: assuming $N = 41$ and $k^l = 20 * (l - 1)$ A/m we seek w^l s.t. $w^l \geq 0$, $\sum_{l=1}^{N} w^l = 1$;
- the material parameter α.

A non-oriented steel: the experimental data and fitting results



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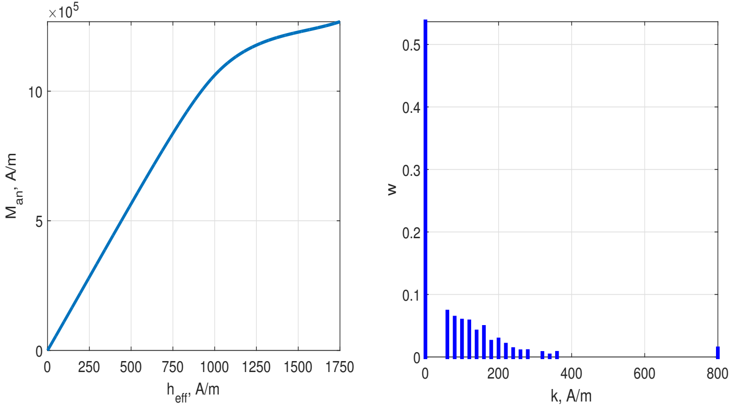


The anhysteretic function M_{an} (left) and weights w^l (right).

The reversible part ($k^1 = 0$) is strong;

only 17 of 41 possible weights are nonzero;

$$\alpha = 8.8 \cdot 10^{-4}$$

2D f.e. simulation: hysteresis + eddy current

- We assumed the material is isotropic and included the hysteresis model with the identified parameters as a constitutive relation with memory,

$$\boldsymbol{m} = \boldsymbol{M}[\boldsymbol{h}_{eff}],$$

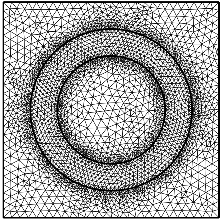
into the Maxwell equations. The local operator $\boldsymbol{M}$ keeps track of the internal variables $\boldsymbol{h}_r^l$ and $\boldsymbol{m}^l$ at each point of a ferromagnet.

- A Newton-like iterations with the numerically approximated derivatives were used to treat this nonlinearity.
- Another constitutive relation is the Ohm law, $\boldsymbol{e} = \rho \boldsymbol{j}$.
- The magnetization and eddy current losses, respectively, are:

$$P_m = \int_0^t \int_\Omega \sum_{l=1}^N |r^l \dot{\boldsymbol{m}}^l|, \quad P_j = \int_0^t \int_\Omega \rho |\boldsymbol{j}|^2.$$

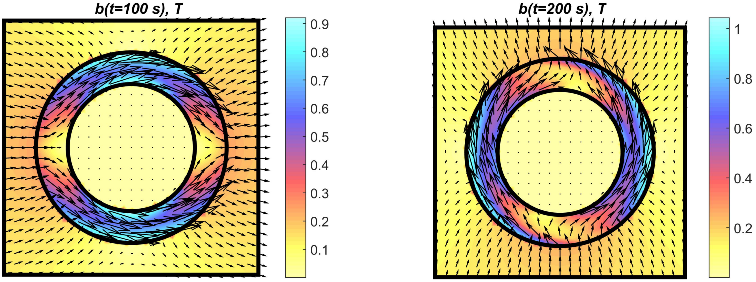
2D f.e. simulation: hysteresis + eddy current

- We solved a 2D eddy current and magnetization problem using an A-V formulation. Hence, the computation was confined to the domain of fm or conducting material.
- The formulation is similar to that proposed in [d'Aquino *et al.* 2013] but employs the Bergqvist model for magnetization.
- The geometrical configuration: hollow cylinder in a perpendicular field.
- The fe mesh: 6424 triangles; 2352 triangles in the fm domain.



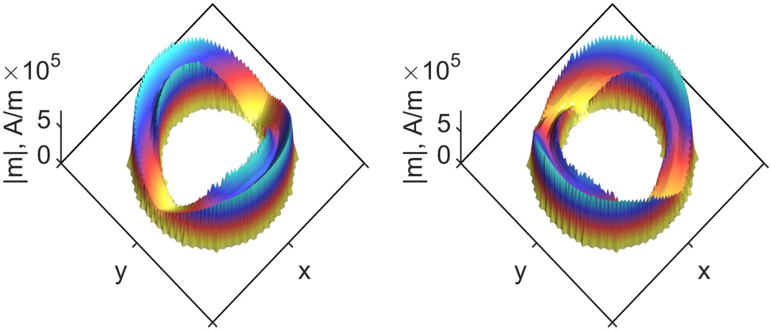
Simulation results

The external field first grows as $\boldsymbol{h}_e(t) = \{10^3 t, 0\}$ A/m for 100 s, then rotates 90^o counter clockwise in the next 100 s.



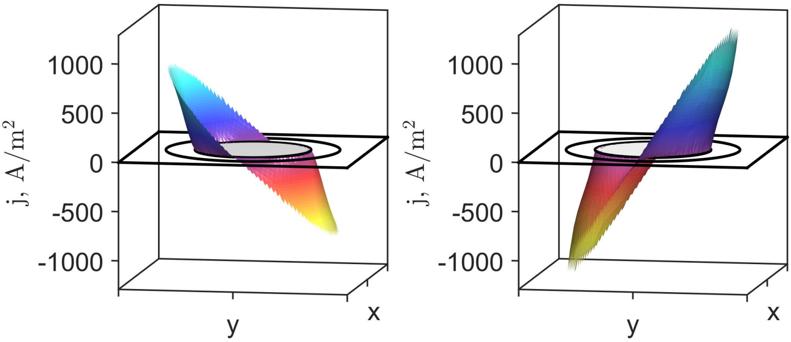
Magnetic induction, $\boldsymbol{b}$.

Simulation results



Magnetization, $|\boldsymbol{m}|$. Left: $t = 100$ s; right: $t = 200$ s.

Simulation results



Eddy current density, $\boldsymbol{j}$. Left: $t = 100$ s; right: $t = 200$ s.

Losses (per unit of length):

magnetization - 2.3 J/m, eddy current - 0.7 J/m.

Conclusions

- The considered model
  - is based on consistent energy arguments and a clear albeit simplified dry friction assumption;
  - is vectorial, has a variational formulation, and can be employed in a f.e.m simulation;
  - has sufficient degrees of freedom to be fitted to hysteretic behaviour of different materials;
  - predicts the dissipation loss at any moment in time.
- The typically employed simplifying approximation can be inaccurate and should be avoided.
- Further comparison to experiments would be desirable.

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