The 5th International Workshop on Numerical Modelling of HTS

DC-SQUIDs with topologically trivial and nontrivial barriers: a comparative analysis

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Outline

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1. Introduction

1.1 Majorana fermions and DC - SQUIDs

► Majorana bound states have been predicted to be hosted in Josephson junctions and SQUIDs with topologically nontrivial barriers.

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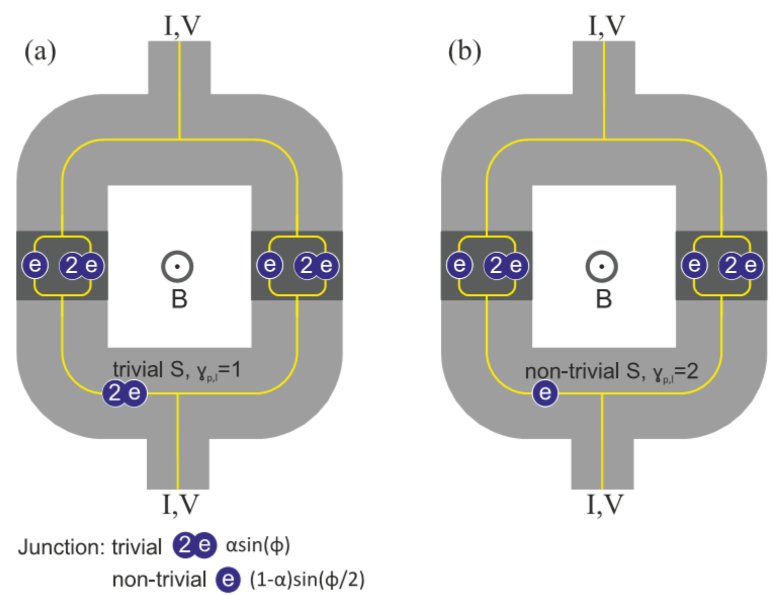
Optimizing the Majorana character of SQUIDs with topologically nontrivial barriers

M. Veldhorst, C. G. Molenaar, C. J. M. Verwijs,* H. Hilgenkamp,† and A. Brinkman

$$\beta_C \frac{d^2\phi_{2,1}}{dt^2} + \frac{d\phi_{2,1}}{dt} + \chi_{2,1} - \frac{1}{2}\frac{I}{I_c} \pm \beta_L^{-1}\left(\phi_2 - \phi_1 - 2\pi\frac{\Phi_e}{\Phi_0}\right) = 0$$

$$\chi\,\chi_0^{-1} = \alpha\,\sin(\phi) + (1-\alpha)\,\sin(\phi/2)$$

$$\beta_L = \frac{2\pi L I_c}{\Phi_0}, \quad \beta_C = \frac{2\pi}{\Phi_0} I_c R^2 C$$



1.1 Majorana fermions and DC - SQUIDs

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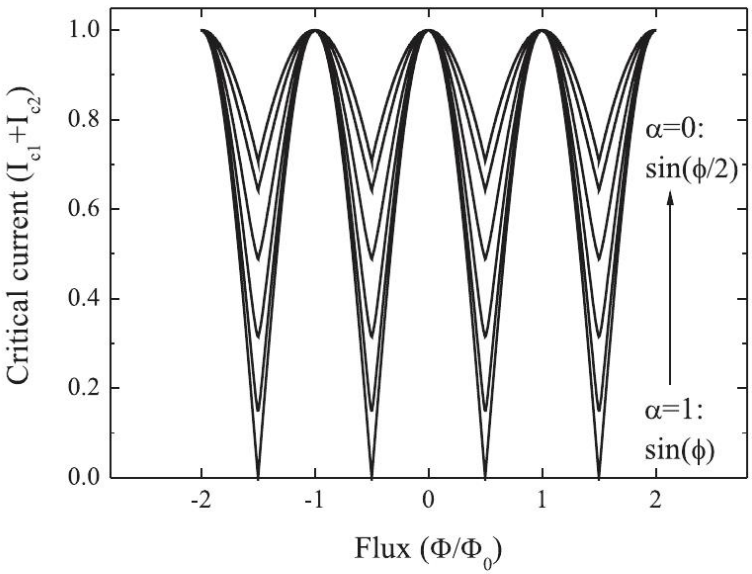
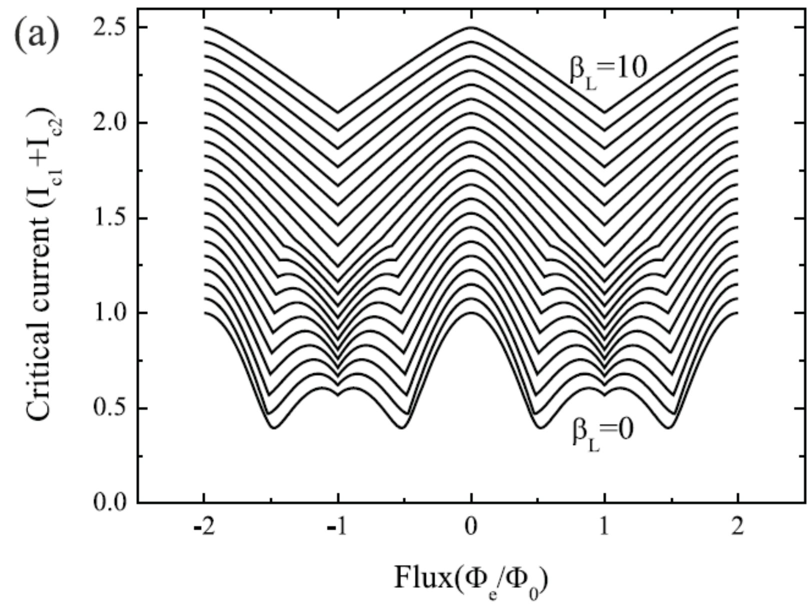




The increase of the $\sin(\phi/2)$ component causes a decrease of the oscillation amplitude without introducing a $2\Phi_0$ component.

Increasing β_L promotes the $2\Phi_0$ period since the effective screening is smaller for Majorana tunneling than Cooper-pair tunneling.

1.2 Motivation of studies

► The investigation of IV – characteristics of the DC-SQUID with the nontrivial barrier is not yet done.

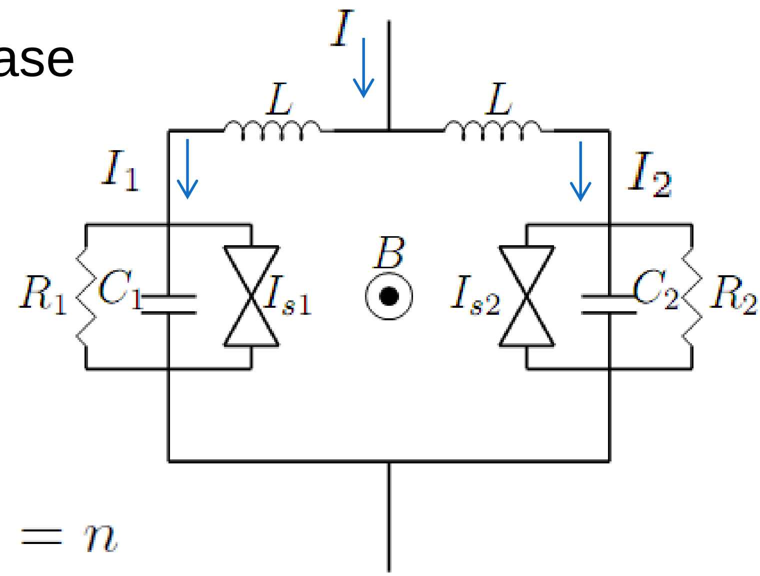
► One of the interesting question is the investigation of resonance behavior of DC-SQUID with the topologically nontrivial barrier.

► How to determine the ratio of Majorana fermions and Cooper pairs?

2. Mathematical model and method of simulation

2.1 System of equations

$2e \to e$ - Trivial case $\quad \varphi \to \varphi/2$ - Nontrivial case



$$\alpha \frac{\hbar}{2e}\frac{d\varphi}{dt} + (1-\alpha)\frac{\hbar}{e}\frac{d(\varphi/2)}{dt} = \frac{\hbar}{2e}\frac{d\varphi}{dt} = V$$

$$\alpha \frac{1}{2\pi}(\varphi_1 - \varphi_2) + (1-\alpha)\frac{1}{2\pi}\frac{\varphi_1 - \varphi_2}{2} + \frac{\Phi_t}{\Phi_0} = n$$

$$I = I_1 + I_2$$

$$\begin{cases} I_1 = \dfrac{C\hbar}{2e}\dfrac{\partial^2 \varphi_1}{\partial t^2} + \dfrac{\hbar}{2eR}\dfrac{\partial \varphi_1}{\partial t} + \alpha I_c \sin\varphi_1 + (1-\alpha) I_c \sin(\dfrac{\varphi_1}{2}) \\ I_2 = \dfrac{C\hbar}{2e}\dfrac{\partial^2 \varphi_2}{\partial t^2} + \dfrac{\hbar}{2eR}\dfrac{\partial \varphi_2}{\partial t} + \alpha I_c \sin\varphi_2 + (1-\alpha) I_c \sin(\dfrac{\varphi_2}{2}) \end{cases}$$

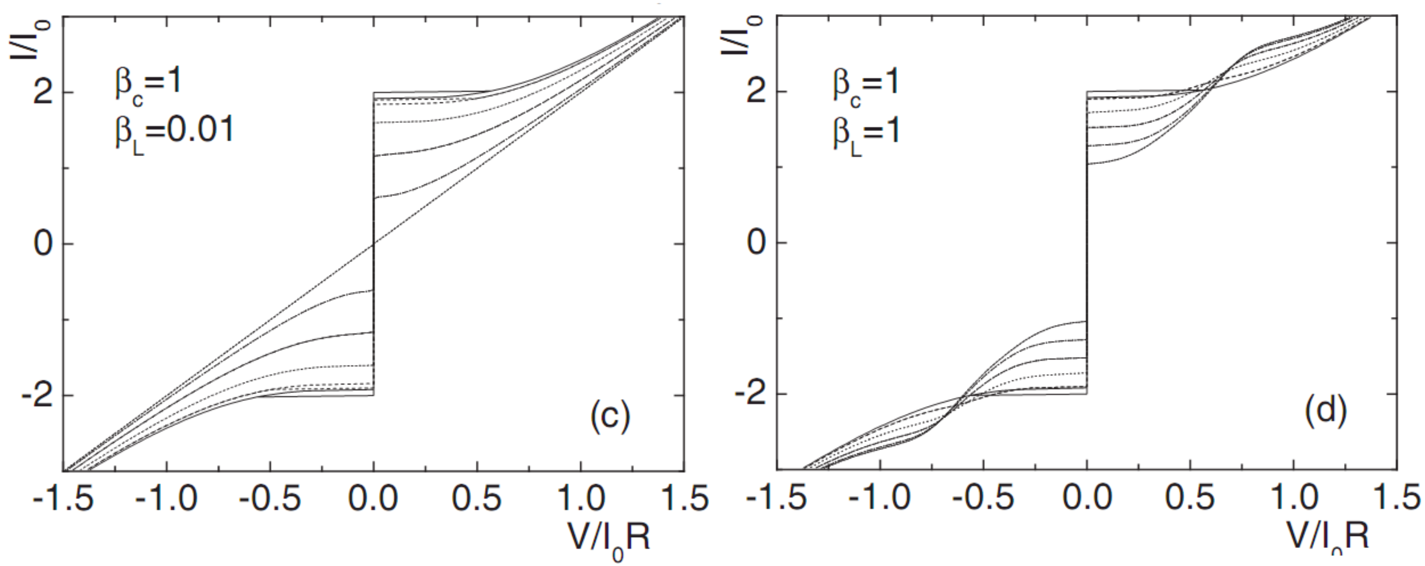
$$\Phi_t = \Phi_{ext} + LI_c\left[\alpha\left(\sin\varphi_1 - \sin\varphi_2\right) + (1-\alpha)\left(\sin(\frac{\varphi_1}{2}) - \sin(\frac{\varphi_2}{2})\right)\right]$$

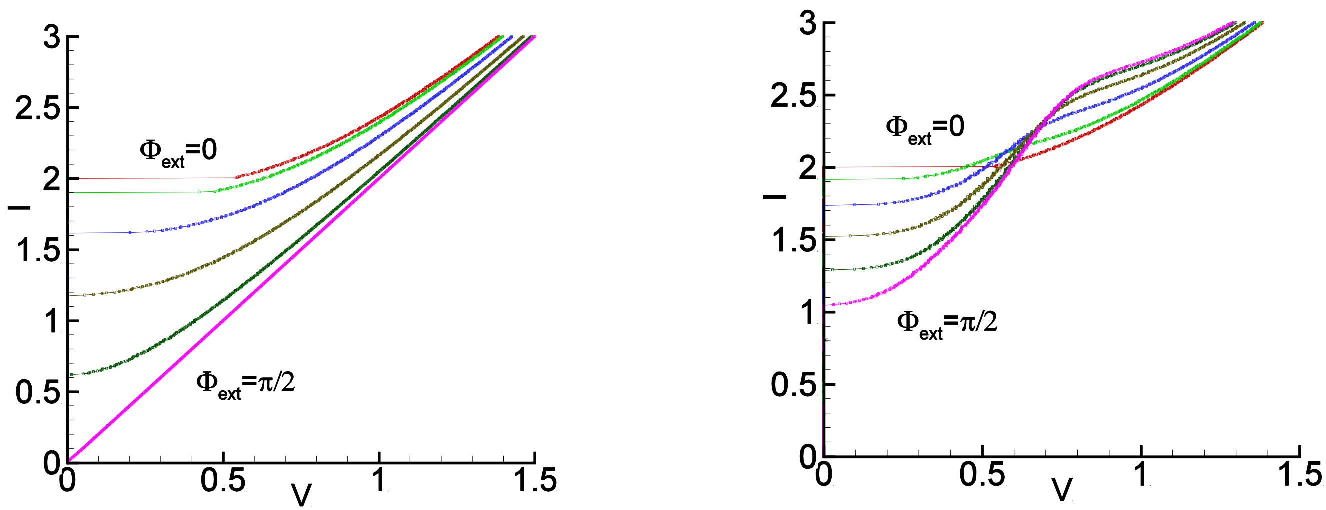
2.1 System of equations

$$
\begin{cases}
\dfrac{\partial \varphi_1}{\partial t} = V_1 \\[2ex]
\dfrac{\partial V_1}{\partial t} = \dfrac{1}{\beta_c}\left\{ \dfrac{I}{2} - V_1 - \left(\alpha \sin \varphi_1 + (1-\alpha) \sin \dfrac{\varphi_1}{2} \right) \right. \\[2ex]
\qquad \left. + \dfrac{1}{2\beta_L}\left[2\pi(n - \varphi_e) - \left(\alpha(\varphi_1 - \varphi_2) + (1-\alpha)\dfrac{\varphi_1 - \varphi_2}{2} \right) \right] \right\} \\[2ex]
\dfrac{\partial \varphi_2}{\partial t} = V_2 \\[2ex]
\dfrac{\partial V_2}{\partial t} = \dfrac{1}{\beta_c}\left\{ \dfrac{I}{2} - V_2 - \left(\alpha \sin \varphi_2 + (1-\alpha) \sin \dfrac{\varphi_2}{2} \right) \right. \\[2ex]
\qquad \left. - \dfrac{1}{2\beta_L}\left[2\pi(n - \varphi_e) - \left(\alpha(\varphi_1 - \varphi_2) + (1-\alpha)\dfrac{\varphi_1 - \varphi_2}{2} \right) \right] \right\}
\end{cases}
$$

$$
\beta_L = 2\pi L I_c / \Phi_0 \qquad \beta_c = 2\pi I_c R^2 C / \Phi_0 \qquad \varphi_e = \frac{\Phi_e}{\Phi_0}
$$

2.1 The main features of DC-SQUID





[J. Clarke, A. I. Braginski, The SQUID Handbook, WILEY-VCH Verlag GmbH & Co. KGaA, Weinheim (2004)]

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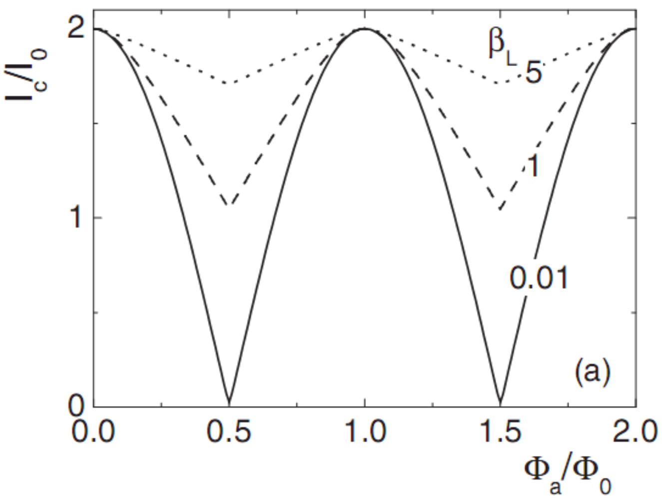

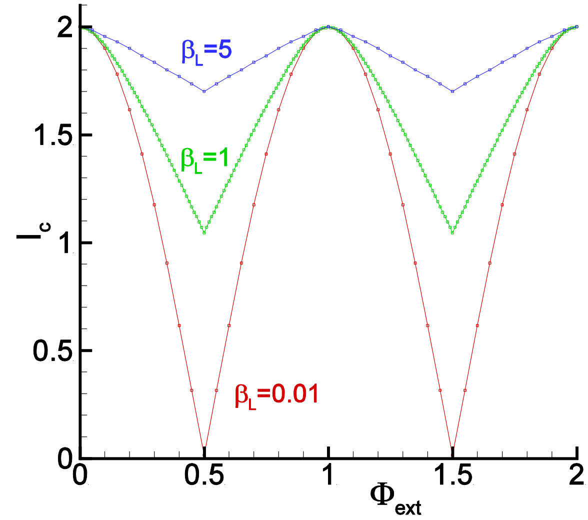

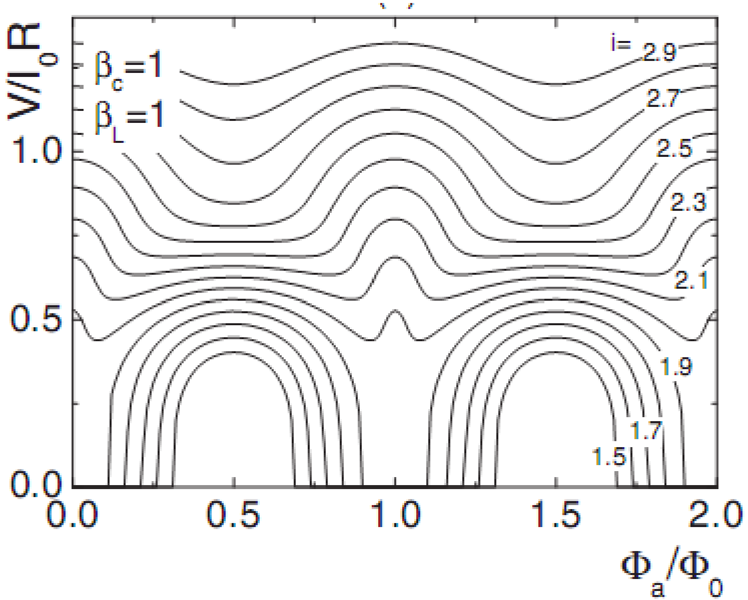

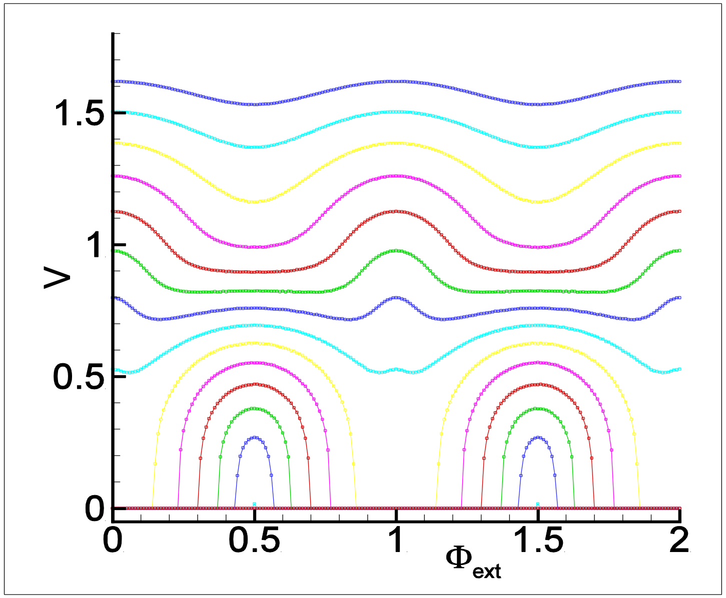

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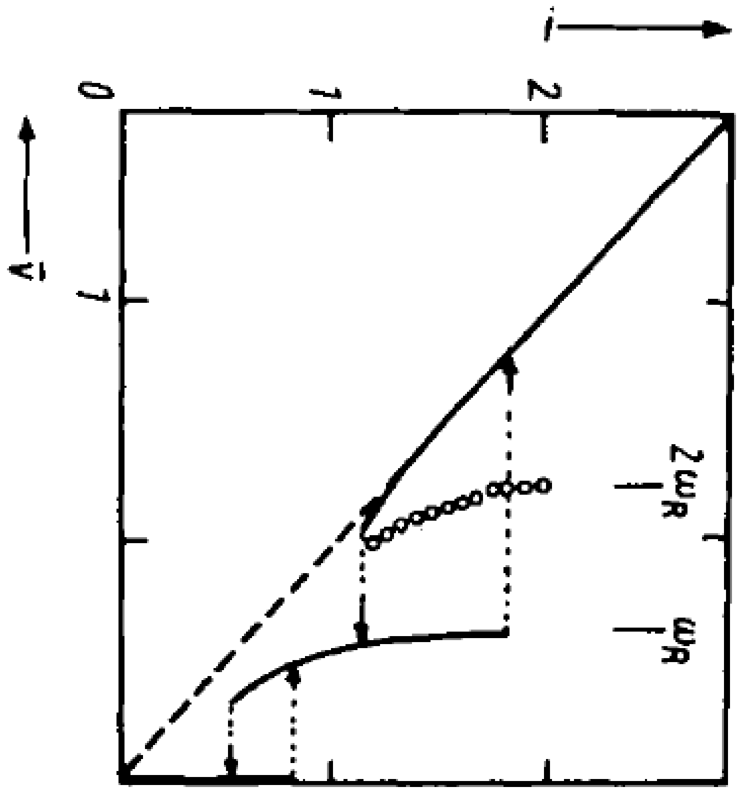
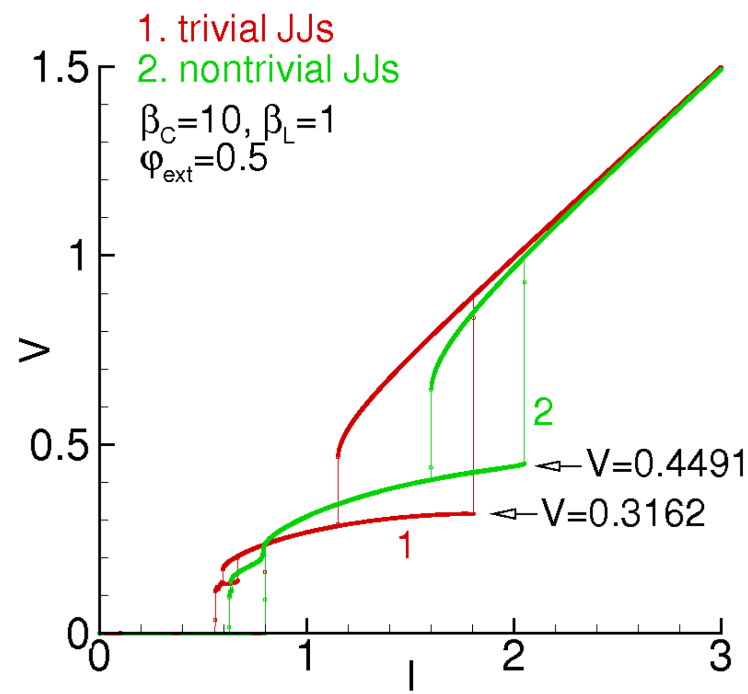
3. Results of simulations

3.1 Comparison analyses of IV - characteristics of the trivial and nontrivial cases





$$\omega_{res} = 1/\sqrt{\beta_c \beta_L}$$

W. D. Schmidt, P. Seidel and S. Heinemann, Phys. Stat. Sol. (a) 91. K155 (1985)

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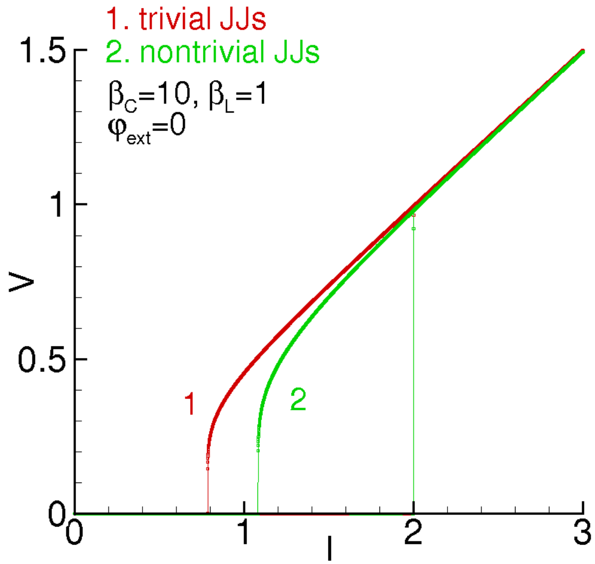
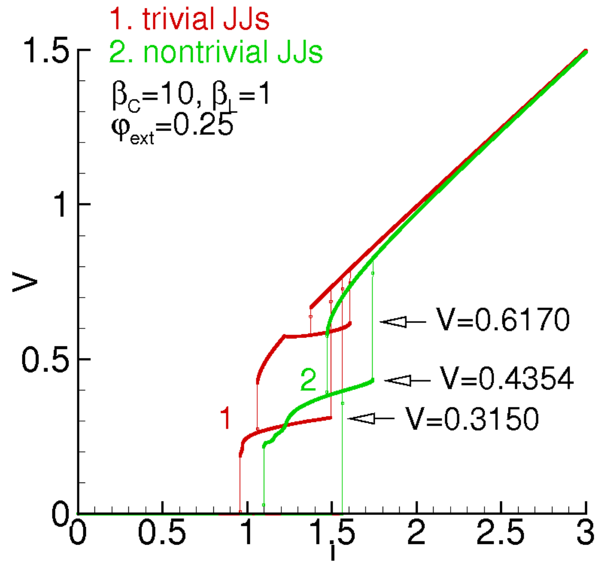
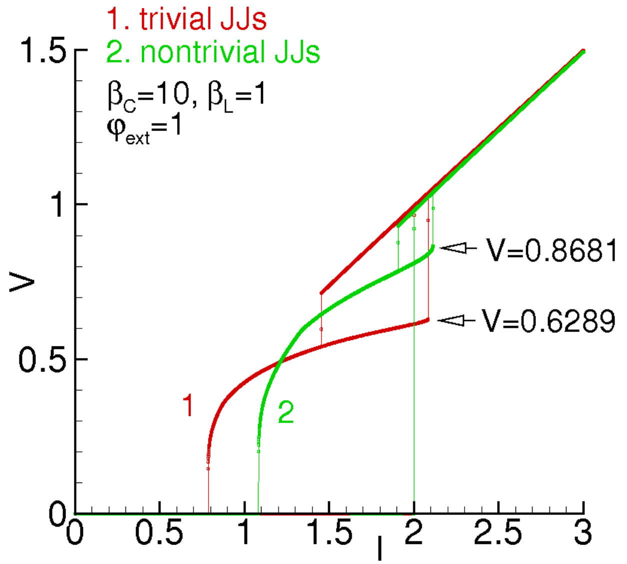
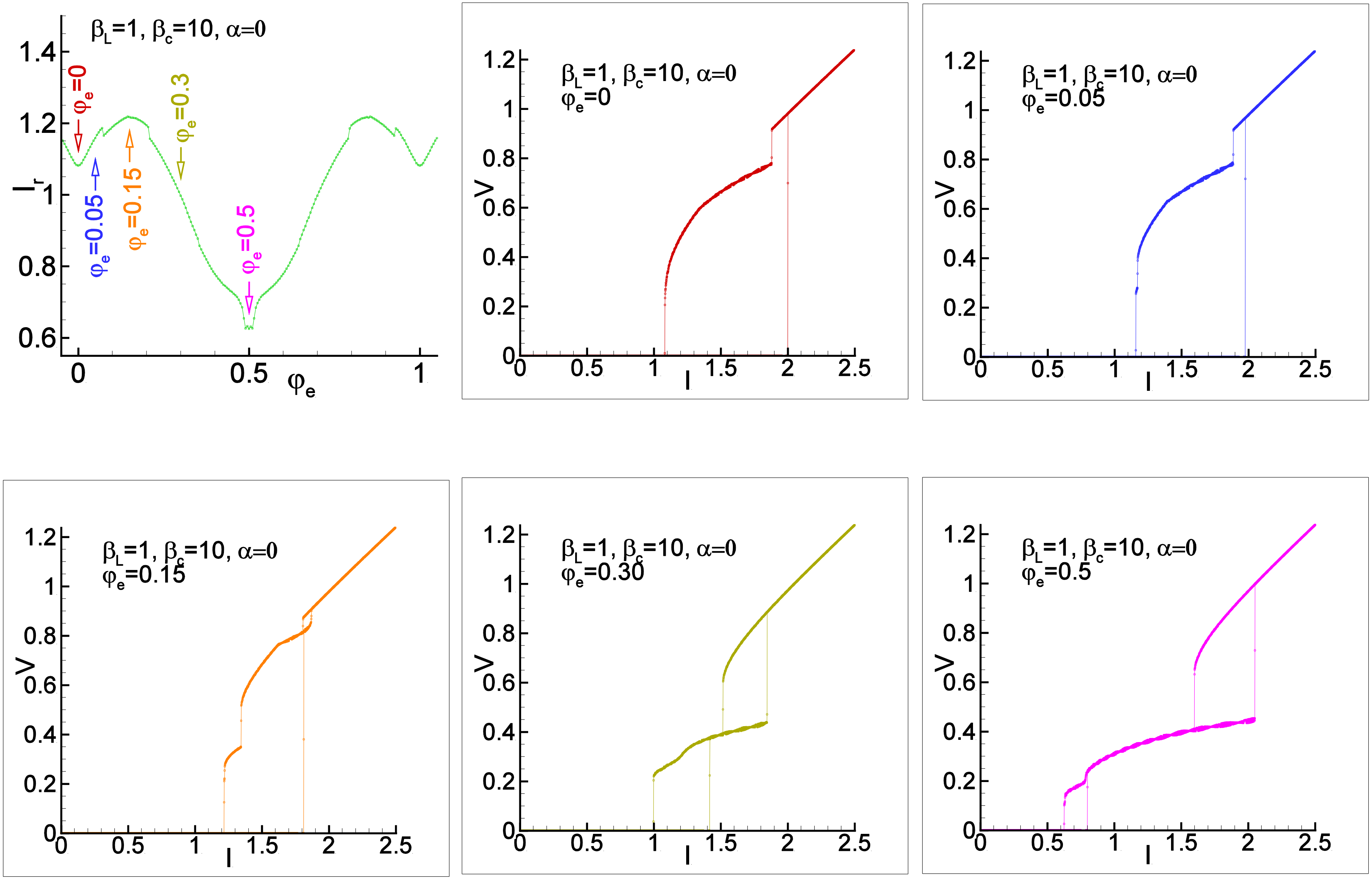






3.2 Magnetic field dependence of critical return current



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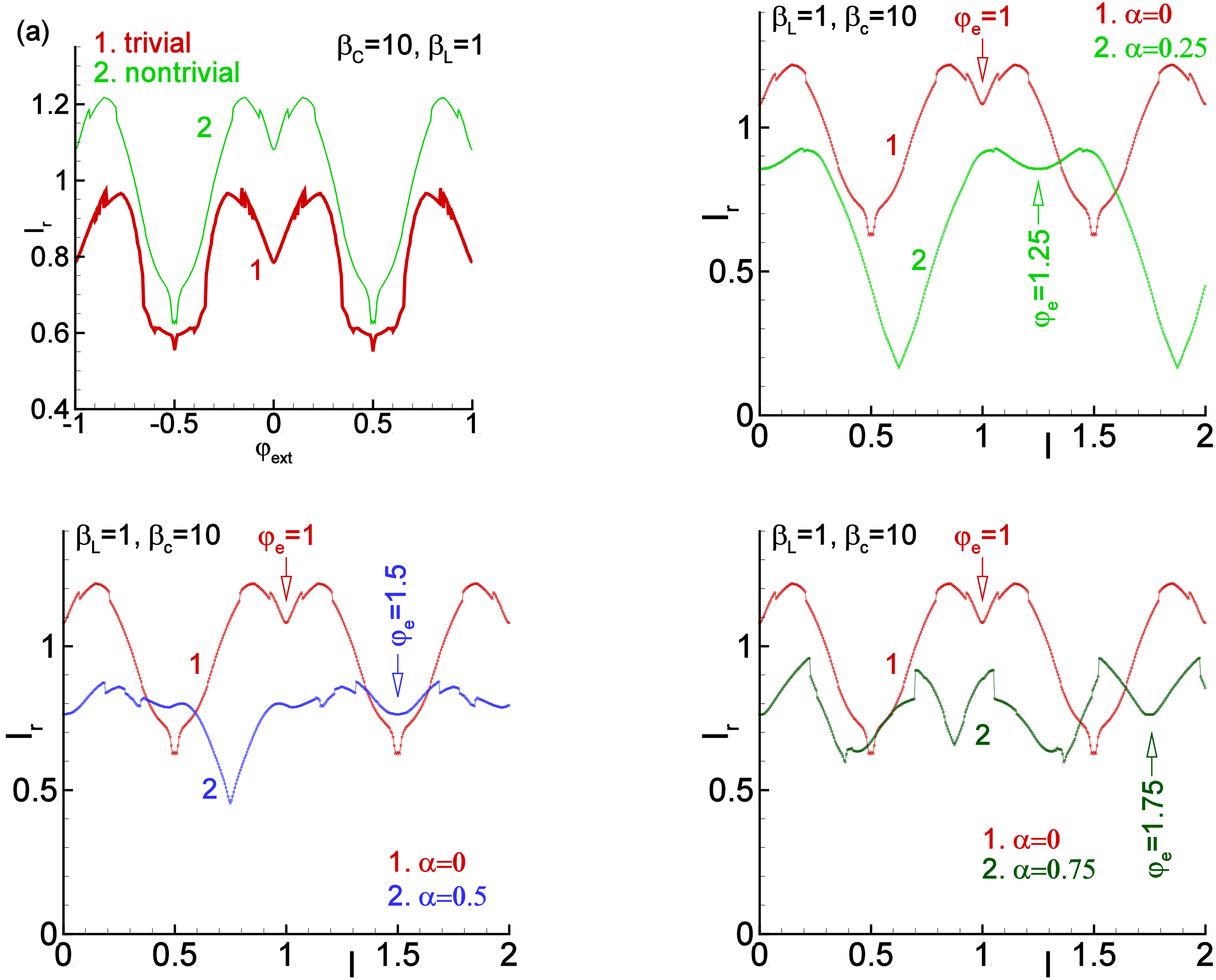

Conclusions

- ► **We demonstrate that in case of nontrivial barrier the resonance branch of IV-characteristic shifts by $\sqrt{2}$.**

- ► **In case of nontrivial barrier the return current is greater than the trivial barrier case by $\sqrt{2}$.**

- ► **The periodicity of field dependence of the return current shifts in ratio coefficient Majorana fermions and cooper pairs.**

- ► **These observed behaviors can be useful for experimental detection of Majorana fermions.**

Thanks for your attention