Surrogate Modelling for Optimal Design of a HTS Insert for Solenoid Magnets

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Outline

- Introduction & Scope
- The ENEA HTS CICC
- Finite Element Model
- Direct Optimization
- Surrogate Optimization
- Results & Conclusions
Introduction

- Recently, a HTS CICC cable comprised of 2nd generation ReBaCuO coated conductors has been designed and manufactured by ENEA.

- With the availability of 2G HTS, high field magnets are now being considered.

The ENEA HTS conductor is considered to be inserted into the bore of an existing high field magnet.
Scope of this work

- to minimize total conductor length needed for an HTS insert magnet to reach a peak magnetic field (based on a background field), guaranteeing structural integrity

- length minimization means costs minimization
The ENEA slotted core CICC

10 kA - class cable: 150 2G-wires (5 stacks x 30 wires)

Fundamental Design driver: industrial process feasibility
Finite Element Model description

Axial Symmetry axis

Bore radius

Creating the background field

Creating the background field

Sym
Finite element modelling

- Parametric approach taking advantage of ANSYS parametric design language (APDL)
  - 2D axial symmetric
  - Magneto-static analysis, using the magnetic vector potential (MVP), with:
    - Background field 12 T
    - Current inside the bore 22.4 KA
  - Magneto-structural analysis with loads:
    - Lorentz forces, from magnetic analysis results
  - Same mesh (no interpolation needed), switching from magnetic (PLANE13) to thermo-mechanical elements (PLANE42)
  - Temperature-dependent material properties
### Standard Trial-and-Error design approach

<table>
<thead>
<tr>
<th>Number of turns</th>
<th>Number of layers</th>
<th>Total conductor Length [m]</th>
<th>Max Field B [T]</th>
<th>Bore Diameter [m]</th>
<th>Max Von Mises [MPa]</th>
<th>B variability in axial direction [%]</th>
<th>B variability in radial direction [%]</th>
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<tbody>
<tr>
<td>26</td>
<td>3</td>
<td>252</td>
<td>13.7</td>
<td>0.97</td>
<td>218</td>
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<td>0.77</td>
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<td>212</td>
<td>0.69</td>
<td>1.52</td>
</tr>
</tbody>
</table>

- An **optimization methodology** is adopted to minimize the needed HTS cable length (**HTS material costs minimization**) to achieve a **peak field** of **17 T**, withstanding the relative Lorentz forces.
Mathematical definition of optimization

Optimization is a mathematical process:

Find $\mathbf{X}$ to minimize (or maximize)

$$F(\mathbf{X}) \text{ objective}$$

where:

$$\mathbf{X} = \{x_1, x_2, \ldots, x_n\} \text{ design variables}$$
Numerical approach to optimization

- **Numerical Optimization** aimed to:
  - minimize the total conductor length (cost reduction) of the high field HTS insert demonstration magnet, with:
    - achieving $B_{\text{max}} \geq 17$ T (background field: 12 T)
    - Failure criterion:
      - Von Mises stress $< 200$ MPa (yield stress=300 Mpa with a 1.5 safety factor) ([*])
      - Internal bore diameter $\geq 30$ cm (strain tolerance of $J_c$ [**])
  - Design variables:
    - jacket width $L$ [25 ÷ 40 mm]
    - Number of turns and layers

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Direct Optimization

Optimization Problem Setup
Design variables, Objective, Constraints

Optimization Process

Optimum found

Significant Calculation Time & Resources

Finite Element Simulations
Direct Optimization loop

Simulation time about 220 sec (*)

- Geometry def., magnetic properties & BC implemented
- Magnetic analysis: calculates Lorentz forces
- Magnetic forces applied to the structural mesh (as body loads)
- Thermo-structural properties and BC implemented
- Induced magneto-thermal stress evaluated

(*) Intel® Xeon® CPU E5645 @ 2.40 GHz RAM 24 GB
FE Direct Optimization results

Optimal conductor Length = 360 m
Number of turns = 16
Number of layers = 12
Jacket width L = 35.4 mm
Max B ≈ 17.2 T
Max Von Mises stress = 198 MPa

Computational Effort:
Total time for optimization: 4 days
Total FE simulations: 1498
Surrogate Optimization

Optimization Problem Setup
Design variables, Objective, Constraints

Numerical Sampling Process

Surrogate model creation & Surrogate Optimization

Approximate Optimum found

Finite Element Simulations

Local Direct Optimization

Optimum found

Reduced Number of Calculations
Surrogate Optimization

- CPU-intensive calculations for constraints and/or objective functions are replaced by approximations
- Approximations are then used to find **approximated optima**
- Starting from approximated optima, a direct FE optimization is performed locally
From Statistics: **Response Surface Methodology (RSM)**

With RSM, a simple polynomial model is fitted to a set of data collected at the points of a **sampling set**

Since nonlinearity is expected in the surface shape, the model also considers **cross-product terms** and / or **pure quadratic terms**
Response Surface Methodology /2

- With RSM:
  - an approximated relationship between y and $x_1, x_2, \ldots, x_k$ that can be used to predict response values for any given set of the control variables

$$y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i<j} \beta_{ij} x_i x_j + \sum_{i=1}^{k} \beta_{ii} x_i^2 + \epsilon$$

- to do this, a series of n numerical experiments should first be carried out (sampling), in each of which the response y is measured for specified settings of the control variables
Response Surface Methodology /3

\[ y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i<j} \beta_{ij} x_i x_j + \sum_{i=1}^{k} \beta_{ii} x_i^2 + \epsilon \]

- Model coefficients \textit{beta} are calculated using the least square criterion on a set of “numerical experiments”

- Industrial process are always smooth in a limited factor range, so RSM can be trusted to approximate the FE model

- Any continuous and differential function can be arbitrarily well approximated by a Taylor series in a given interval
Sampling Strategy

- A set of “numerical experiments” is used to tune the surrogate model to replace the FE model.

- The set of “numerical experiments” should supply a relationship between input factors and output responses, with best precision and least computational cost.

- With Design of Experiments (DOE), an estimation of interaction and even quadratic effects is achieved.
Response Surface Designs: CCD

- To calibrate quadratic models, Central composite designs (CCDs) are much more efficient than full factorial designs, using three or five levels for each factor, but not using all combinations of levels.
- Each CCDs design consists of a factorial design (the corners of a cube) together with center and star points that allow for estimation of second-order effects.
Direct vs. Surrogate Optimization

Direct FE Optimization:

\[
\begin{align*}
\text{Turns}_\text{number} & \\
\text{Layers}_\text{number} & \\
\text{Jacket}_\text{width} &
\end{align*}
\Rightarrow \text{ANSYS} \Rightarrow \begin{align*}
\text{Max}_\text{Von}_\text{Mises} \\
\text{Peak}_\text{Magn}_\text{Field}
\end{align*}
\]

\[220 \text{ sec}\]

RSM Surrogate Optimization:

\[
\begin{align*}
\text{Turns}_\text{number} & \\
\text{Layers}_\text{number} & \\
\text{Jacket}_\text{width} &
\end{align*}
\Rightarrow \text{RSM approx. Surrogate Model} \Rightarrow \begin{align*}
\text{Approx}_\text{Max}_\text{Von}_\text{Mises} \\
\text{Approx}_\text{Peak}_\text{Magn}_\text{Field}
\end{align*}
\]

\[< 1 \text{ sec}\]
RSM Surrogate Model Regression Plots

Peak Magnetic field regression plot

Max Von Mises Stress Regression Plot

RSM predicted Peak Magnetic Field vs FE Peak Magnetic Field

RSM predicted Max Von Mises Stress vs FE Max Von Mises Stress
Results: solution cost comparison

![Graph showing solution cost comparison]

- Approximated Optimum
- Optimum
- Direct Optimization
- CCD + RSM Optimization

- CCD sampling & RSM Opt. 15 FE simulations
- Local Opt. 419 FE simulations
- Direct Opt. 1498 FE simulations

1 day
4 days
Conclusions

- A new design approach was proposed, which guarantees HTS material costs minimization, avoiding the standard trial-and-fail design approach, which does not

- By means of the **direct** optimization, an optimal 360 m total conductor length, achieving 17 T, was determined in terms of *jacket width* and *number of turns and layers*, that ensures structural integrity, with a solution cost of **1498 FE simulations** (4 days of calculations)

- With **surrogate RSM** optimization, the same configuration is determined with **434 FE simulations** (about 1 day of calculations), taking full advantage of statistics derived from numerical sampling

<table>
<thead>
<tr>
<th>Direct FE Optimization</th>
<th>Surrogate RSM Optimization</th>
</tr>
</thead>
<tbody>
<tr>
<td>1498 FE Simulations</td>
<td>434 FE Simulations</td>
</tr>
<tr>
<td>4 days of optimization time</td>
<td>1 day of optimization time</td>
</tr>
</tbody>
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Thanks for your kind attention!