State of the art and research directions of eddy currents modelling

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Outline

- Statement of the problem
- A historical perspective
- State of the art
- Research directions
  - Focus on fast methods
- An example: fusion devices
- Conclusions & outlook

Disclaimer / caveat

- I will present my personal view of the history, state of the art and research trends of low-frequency computational electromagnetics
  - Others may have different opinions
- I will focus on near-term future developments, basically extrapolating what is currently done
  - No “fiction movie” effort

Statement of the problem

- Something trivial to start with…

Low frequency electromagnetics

- We deal with low-frequency electromagnetics
  - No wave propagation
- Frequency low enough $\omega\tau_{EM} << 1, \quad \tau_{EM} = \frac{L}{c}$
  - For systems with dimensions ≈ 10 m, this means frequencies below around 1 MHz
- At least one of the time derivatives in Maxwell's equation may be neglected
  - Static and quasi-static models

Eddy currents

- Among quasi-static models we deal with eddy currents model

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$
$$\nabla \times \mathbf{H} = \mathbf{J}$$
$$\nabla \cdot \mathbf{B} = 0$$

initial conditions
boundary conditions
continuity conditions
material properties

- Magneto-Quasi-Static
  - Magnetic energy prevailing on electric energy (Magnetic)
  - Low-frequency (Quasi-Static)

Why eddy currents?

- (Obviously...) of interest for HTS community
  - Typical applications fall inside this classification
- Peculiar with respect to full Maxwell model
  - Distinct mathematical properties (parabolic vs. hyperbolic)
  - Need for dedicated solvers and ad hoc analytical and numerical approaches
- Much more difficult than Electro-Quasi-Static model

Why is it difficult? /1

- First problem: formulation
  - Choice of the primary unknown in which the problem must be recast
  - A "first choice" does not exist (not like EQS…)
- Magnetic vs. Electric formulations
  - Warning: electric field outside conductors
- Fields vs. Potentials
  - Warning: gauge conditions
- Differential vs. Integral
  - Fields or sources?

Why is it difficult? /2

- Second problem: the unknown quantities are intrinsically vectors
  - Three components per point
  - Continuity conditions
- Numerical difficulties
  - "Naive" numerical approaches may be inadequate

- Need for specific numerical treatment of eddy currents problem

A historical perspective

- A 40-year long story...

60’s – early 70’s

- First applications of computers to calculation of magnetic fields
- Computer = big calculator

Computer-Aided Design of Magnetic Devices

RALPH E. WAHL, SENIOR MEMBER, IEEE

For years, magnetic devices have been designed by engineers using slide rule and hand calculations. With the advent of computers with large memories and microsecond speeds, most magnetic design engineers have dreamed of the day when these machines would take the drudgery out of their work.

IEEE TRANSACTIONS ON MAGNETICS, VOL. MAG-6, NO. 1, MARCH 1970

Costs of computer design time are still relatively high, 15 dollars per minute, or 900 dollars per hour. In addition, the problems of getting a program written, debugged, and operational are time consuming and thus expensive.

One approach to solving this problem is the use of "conversational programs" with a time-sharing computer doing the mathematical work while the design engineer does the thinking in some areas. For example, the author has written two programs which are now operational. One program is for design of linear coil transformers, using either cut *C* cores, *EI* laminations, or *DU* laminations. The other program is being used to design toroidal transformers

Mid 70's

- Magnetostatic computations, from 2D to 3D

IEEE Transactions on Magnetics, vol. Mag-11, no. 5, September 1975

3-DIMENSIONAL COMPUTATION OF TRANSFORMER LEAKAGE FIELDS AND ASSOCIATED LOSSES

M. Djurović* and C.J. Carpenter**

ABSTRACT

Numerical methods, based on a magnetic scalar potential function, have been used to compute three-dimensional leakage fields and leg-plate losses in large power transformers. Fast convergence is obtained. The currents induced in the leg-plate are calculated by a simple modification of the static scalar-potential formulation. This includes three-dimensional, flux perturbation, and plate edge effects. The results show that two-dimensional approximations are unsatisfactory.

I. INTRODUCTION

The increases in rating of large power transformers have demanded more accurate methods of predicting leakage fields, and the forces and eddy current losses which they cause. Numerical methods are now commonly used, based on finite differences or finite elements[1], and these provide two-dimensional flux maps of any required accuracy, but they depend on the assumption that the magnetic vector potential has only one component in either a cartesian or a cylindrical reference system. This approximation is inadequate when calculating currents induced in the leg plates, and other conducting parts, in which edge effects are important. Moreover, the introduction of horizontal magnetic shunts, and other methods of controlling the leakage flux distribution, may alter it in a way which is difficult to estimate with any confidence from two-dimensional field maps.

wound limb. This region is bounded by the core, the tank, the vertical and horizontal mid-planes, which were assumed to be magnetic equipotentials, and by the shunts. The electrical asymmetry due to the different phase currents round adjacent limbs can be allowed for by super-position when necessary, but was found to have negligible effect near the core.

The complete flux distribution was mapped in a finite difference mesh of 10,500 nodes located by a cylindrical co-ordinate system with a logarithmic radial spacing, giving the greatest node concentration near the core. Iron surfaces intersected obliquely were represented by nodes formed by the intersections. Improved accuracy can be obtained by triangulating the mesh locally in this region, and using the finite element approach to compute the branch components, but the advantages did not warrant the additional complexity. The mesh contained 19 radial planes 5° apart, with approximately uniform node spacing in the different co-ordinate directions close to the core.

The nodes define a magnetic network which is linked by the winding currents - i.e. is mesh-fed - so that some modification is required to define a scalar potential function. The usual vector potential (A) formulation in 2 dimensions is equivalent to replacing the network by its dual and substituting node for mesh sources. In 3 dimensions no simple dual can be derived, and even in two it does not necessarily offer the best strategy because of the boundary conditions associated with A. An alternative is to use a modified

Late 70's – early 80's

- Conferences dedicated to computational electromagnetics
  - COMPUMAG conference from 1976
  - CEFC conference from 1984
  - IGTE conference from 1984


IEEE TRANSACTIONS ON

MAGNETICS

NOVEMBER 1983 VOLUME MAG-19 NUMBER 6 (ISSN 0018-9464)

A PUBLICATION OF THE IEEE MAGNETICS SOCIETY

COMPUMAG

Conference on the Computation of Electromagnetic Fields

GENOA

Italy

May 30–June 2, 1983

Mid 80's

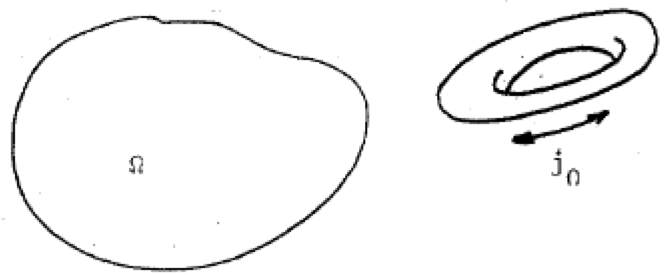
- 3D eddy currents codes

IEEE TRANSACTIONS ON MAGNETICS, VOL. MAG-19, NO. 6, NOVEMBER 1983 2465

THE "TRIFOU" CODE: SOLVING THE 3-D EDDY-CURRENTS PROBLEM BY USING H AS STATE VARIABLE

A. Bossavit and J.C. Vérité

Abstract - In [1] and [2], a variational formulation of the eddy-currents problem using the magnetic field h as state variable was introduced. We describe here two applications of the method: computation of the impedance of a probe in non-destructive testing and prediction of the characteristics of a new design of an iron-free machine. The basic idea is first exposed, leaving aside difficulties like multiple connectedness. These are treated more thoroughly in Part 3, after the description of applications in Part 2.



Late 80’s – early 90’s

- Mathematical and numerical developments
  - Edge elements

74 IEEE TRANSACTIONS ON MAGNETICS, VOL. 24, NO. 1, JANUARY 1988

A RATIONALE FOR "EDGE-ELEMENTS"
IN 3-D FIELDS COMPUTATIONS

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A justification for the use of finite-elements with edge-attached degrees of freedom ("edge-elements") is looked for in some mathematical structures which underly eddy-currents theory. An introduction to so-called "mixed methods" in magnetostatics is given, and two dual methods for eddy-currents computations are shown to derive from these considerations, both making use of edge-elements.

It happens that such a closely similar discrete structure has been invented by H. Whitney (the author of many foundational works in differential geometry), long before the finite elements era /5/. Elements of the above-mentioned finite dimensional vector spaces are known as "Whitney forms", and we shall see that they may serve as finite element bases for magnetic fields problems. In particular, they include edge-elements (along with "node-", "facet-" and "volume-elements").

Late 80's – early 90's

- Mathematical and numerical developments
  - Formulations

16 IEE PROCEEDINGS, Vol. 137, Pt. A, No. 1, JANUARY 1990

SCIENCE

Formulation of the eddy-current problem

R. Albanese, PhD
Prof. G. Rubinacci

Indexing terms: Eddy currents, Electromagnetic theory, Mathematical techniques

Abstract: A critical survey on some of the principal approaches to the solution of the general three-dimensional eddy-current problem is presented. The two main families of formulations, those magnetic and those electric, are discussed, with reference to both mathematical and computational aspects. Particular attention is paid to the vector potential formulations in view of uniqueness, gauge and interface problems.

and suitable initial conditions giving $\nabla \cdot \boldsymbol{B} = 0$.

In addition, the material properties are supposed to be linear and time-independent. The conductivity σ and the permeability μ can, however, be nonhomogeneous in the conducting region V_c.

In the external region $R^3 - V_c$, the field equations are:

$$\nabla \times \boldsymbol{H} = \boldsymbol{J}_s \qquad \text{in } R^3 - V_c \qquad (5)$$

$$\nabla \cdot \boldsymbol{B} = 0 \qquad \text{in } R^3 - V_c \qquad (6)$$

with

Late 80's – early 90's

- The state of the art in 1988

IEEE TRANSACTIONS ON MAGNETICS, VOL. 24, NO. 1, JANUARY 1988 13

ELECTROMAGNETIC COMPUTING: THE WAY AHEAD?

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Abstract

A personal view of Electromagnetic Field Computing is reviewed from 1962 to present day. The major achievments are listed with a summary of the current state of the art. A software computing environment is described that embodies the latest developments with particular emphasis on mesh adaptive solutions controlled by error predictions. It is conjectured that new hardware developments on parallel and vector processing will make practical the computation of moving and coupled systems.

History

Modern Electromagnetic Computing began in the early 1960's with the arrival of the large batch computers - in those days digital machines with a rate of execution of 0.36 MFLOPS (CDC 6600) were available

provided a more democratic environment with the use of high-speed graphics terminals having some real-time capability. Recently, the introduction of the new single user mini-computers (SUM) with raster operations is radically changing methods of working [6,7].

Since 1976 the developments in Electromagnetic Computing have been closely chronicled in the proceedings of the COMPUMAG Conference series [8], some of the major milestones during this period have included the introduction of pre-conditioned conjugate gradient methods in equation solving [9], fully automatic mesh generation, at least for two dimensional geometries, and the tentative beginnings of a posterori error analysis [10,11]. These topics and the impact of CAD techniques in the user interface have brought about many changes in electromagnetic code design.

Table 1: Time Evolution of CODE Development

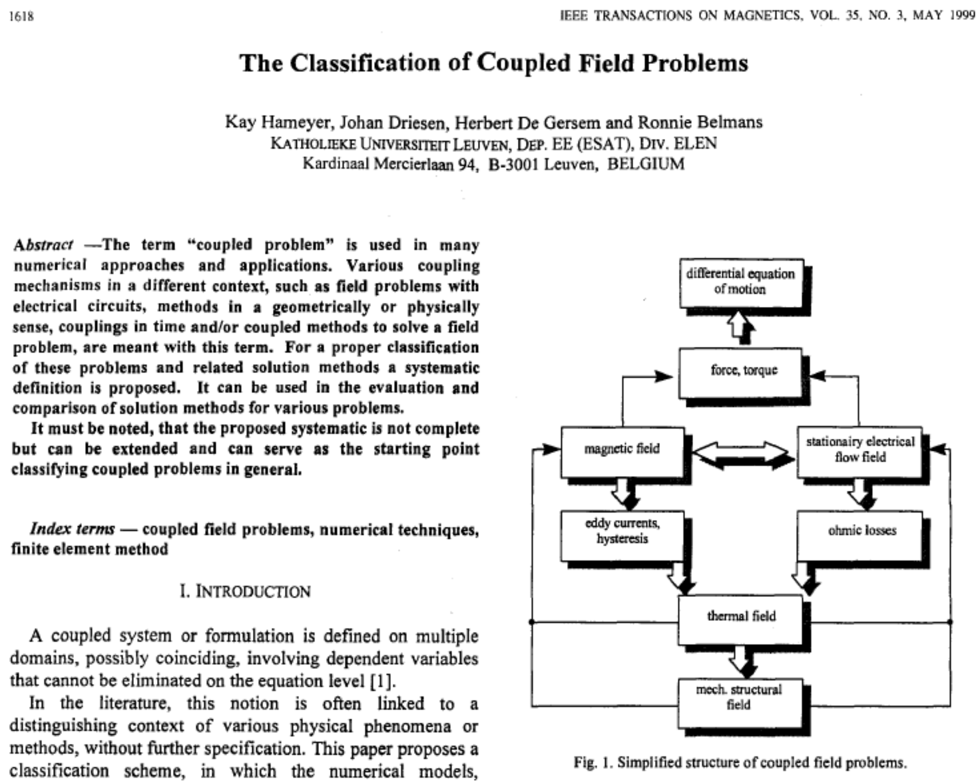
	STATICS	STEADY STATE	TRANSIENT	MOTION
1962	D2D I2D			
1967	I3D			
1972	D2D (NL) I3D (NL) D2D (PM) D3D *	D2D I2D	I2D	
1977	D3D D3D (NL) D2D (A)	D2D (NL) ** D3D	D2D	
1982	D3D (PM) D3D (A)		D2D (NL)	D2D***
1987	D2D (SH) D2D (VH)		D3D	

D2D Differential Two Dimensions
I2D Integral Two Dimensions
D3D Differential Three Dimensions
I3D Integral Three Dimensions
NL Non Linear
PM Permanent Magnets
A Anisotropy
SH Scalar Hysteresis
VH Vector Hysteresis

Notes: * Turbo Generator (special case)
** Approximate model
*** Velocity in one coordinate direction only

Mid – end of 90's

- Coupled problems start to emerge as a challenge to eddy currents community



1618 IEEE TRANSACTIONS ON MAGNETICS, VOL. 35, NO. 3, MAY 1999

The Classification of Coupled Field Problems

Kay Hameyer, Johan Driesen, Herbert De Gersem and Ronnie Belmans
KATHOLIEKE UNIVERSITEIT LEUVEN, DEP. EE (ESAT), DIV. ELEN
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Abstract —The term "coupled problem" is used in many numerical approaches and applications. Various coupling mechanisms in a different context, such as field problems with electrical circuits, methods in a geometrically or physically sense, couplings in time and/or coupled methods to solve a field problem, are meant with this term. For a proper classification of these problems and related solution methods a systematic definition is proposed. It can be used in the evaluation and comparison of solution methods for various problems.

It must be noted, that the proposed systematic is not complete but can be extended and can serve as the starting point classifying coupled problems in general.

***Index terms* — coupled field problems, numerical techniques, finite element method**

I. INTRODUCTION

A coupled system or formulation is defined on multiple domains, possibly coinciding, involving dependent variables that cannot be eliminated on the equation level [1].

In the literature, this notion is often linked to a distinguishing context of various physical phenomena or methods, without further specification. This paper proposes a classification scheme, in which the numerical models,

Fig. 1. Simplified structure of coupled field problems.

Y2k's

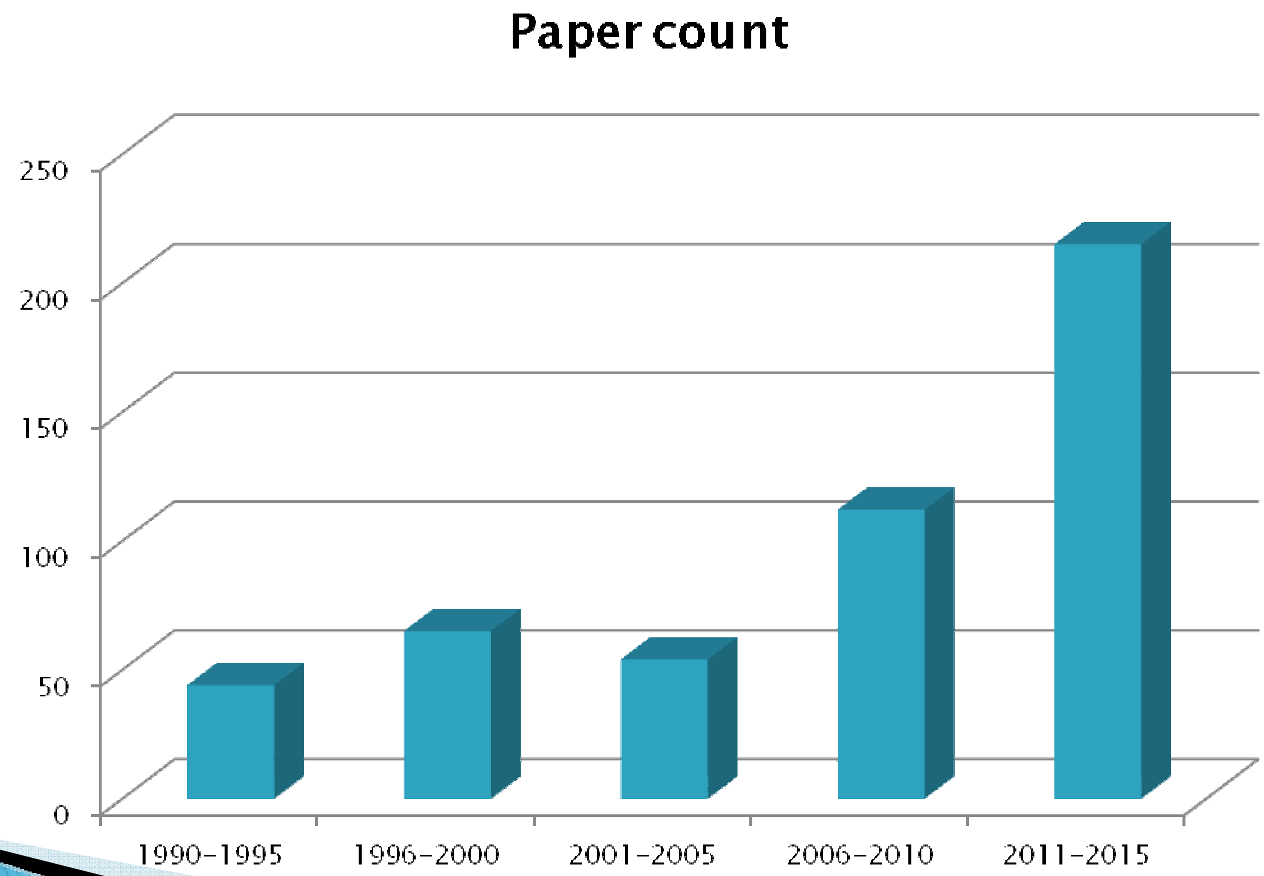
- The advent of commercial codes

	AC/DC Module
	Acoustics Module
2006	RF Module
	Optimization Module
2005	CAD Import Module
	File Import *for* CATIA® V5
	Subsurface Flow Module
	Heat Transfer Module
2004	MEMS Module
2003	
2002	
2001	Chemical Engineering Module
2000	Electromagnetics Module
1999	
1998	COMSOL Multiphysics®
	Structural Mechanics Module

Year	New products introduced
2015	
2015	
	COMSOL Server™,
2014	Design Module
	LiveLink™ *for* Revit®
	Ray Optics Module
	Mixer Module
	Electrochemistry Module
	Molecular Flow Module
2013	Multibody Dynamics Module
	Semiconductor Module
	Wave Optics Module
	ECAD Import Module
	Fatigue Module
	LiveLink™ *for* Excel®
2012	LiveLink™ *for* Solid Edge®
	Corrosion Module
	Nonlinear Structural Materials Module
	Pipe Flow Module
	Particle Tracing Module
	LiveLink™ *for* PTC® Creo® Parametric™
	Electrodeposition Module
2011	Geomechanics Module
	LiveLink™ *for* AutoCAD®
	Microfluidics Module
	Batteries & Fuel Cells Module
	CFD Module
	Chemical Reaction Engineering Module
2010	Plasma Module
	LiveLink™ *for* MATLAB®
	LiveLink™ *for* PTC® Pro/ENGINEER®
	LiveLink™ *for* Inventor®
	LiveLink™ *for* SOLIDWORKS®
2008	
2007	Material Library

Y2k's

- The advent of commercial codes



Nowadays...

- Being commercial codes quite advanced, the interest of the scientific community of low-frequency computational electromagnetics is diverting from “traditional paths”
- Focus on:
  - Specific applications out of reach of commercial codes (e.g. advanced materials modelling)
  - Numerical techniques not (yet?) routinely available on commercial codes (e.g. fast techniques)

State of the art

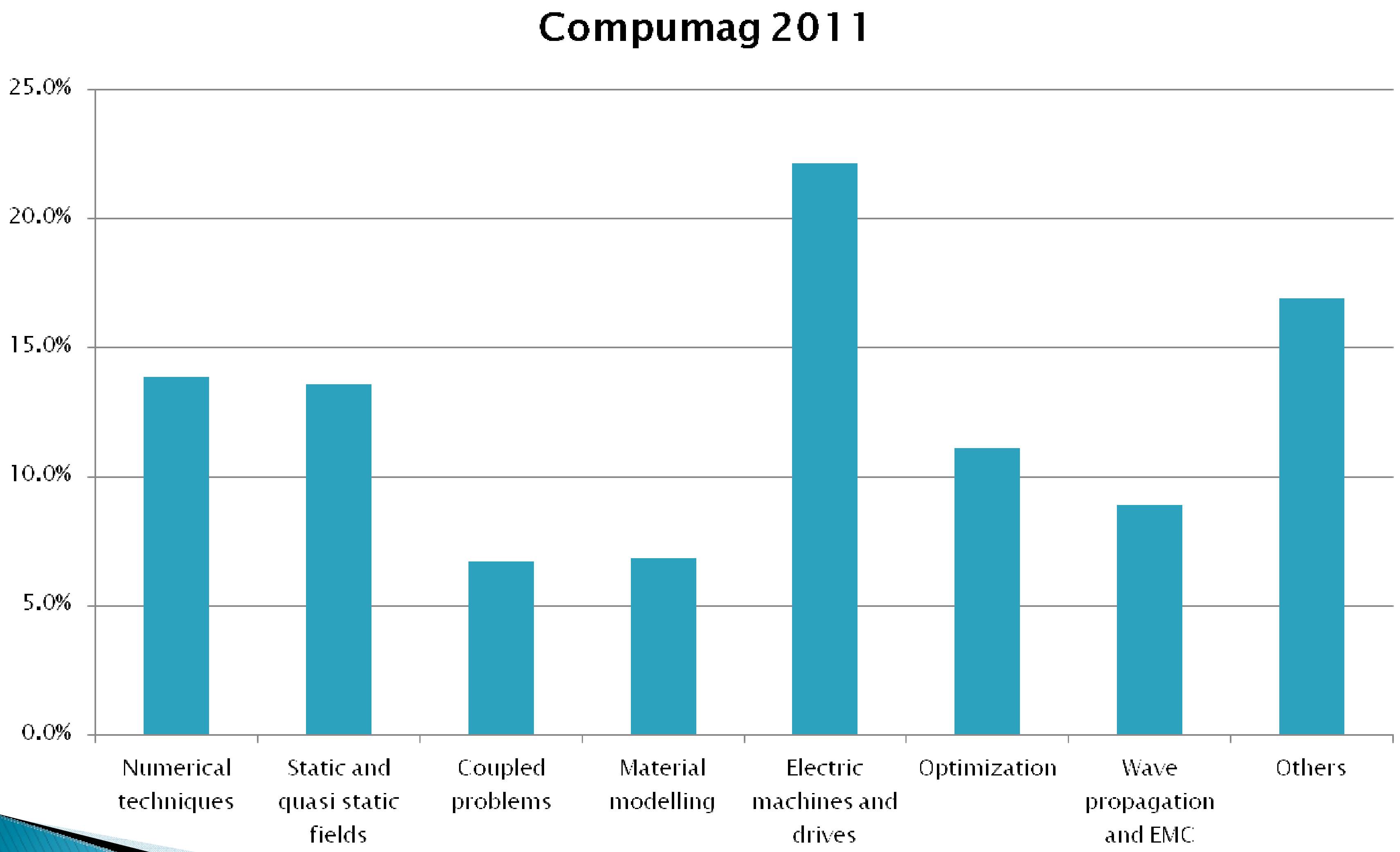
- My personal view

How do we define it?

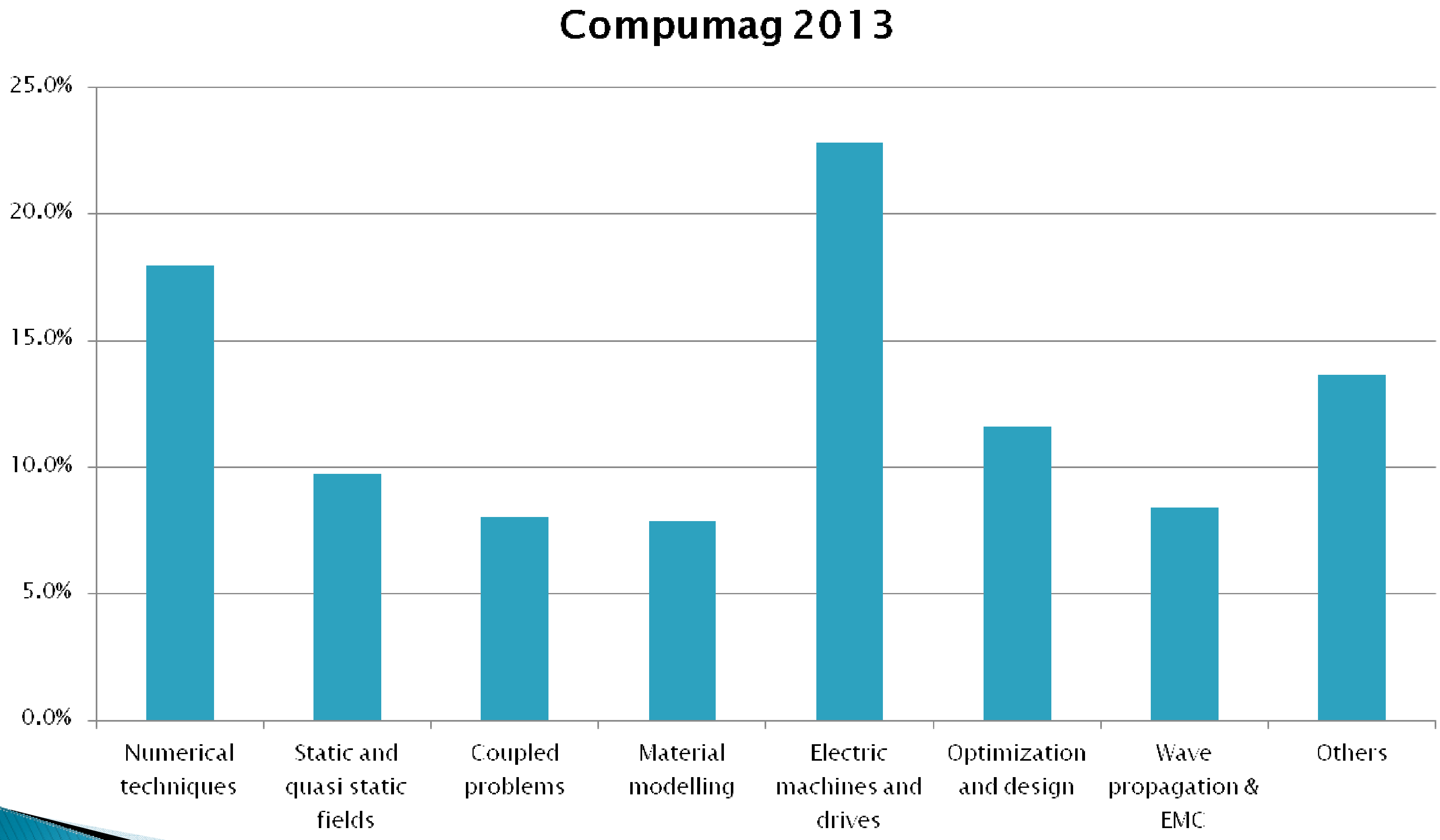
- State of the art: what the community of low-frequency computational electromagnetics is currently doing

- Let us as "proxy" of the state of the art the number of papers presented at the latest COMPUMAG conferences (2011,2013,2015) on each broad topic

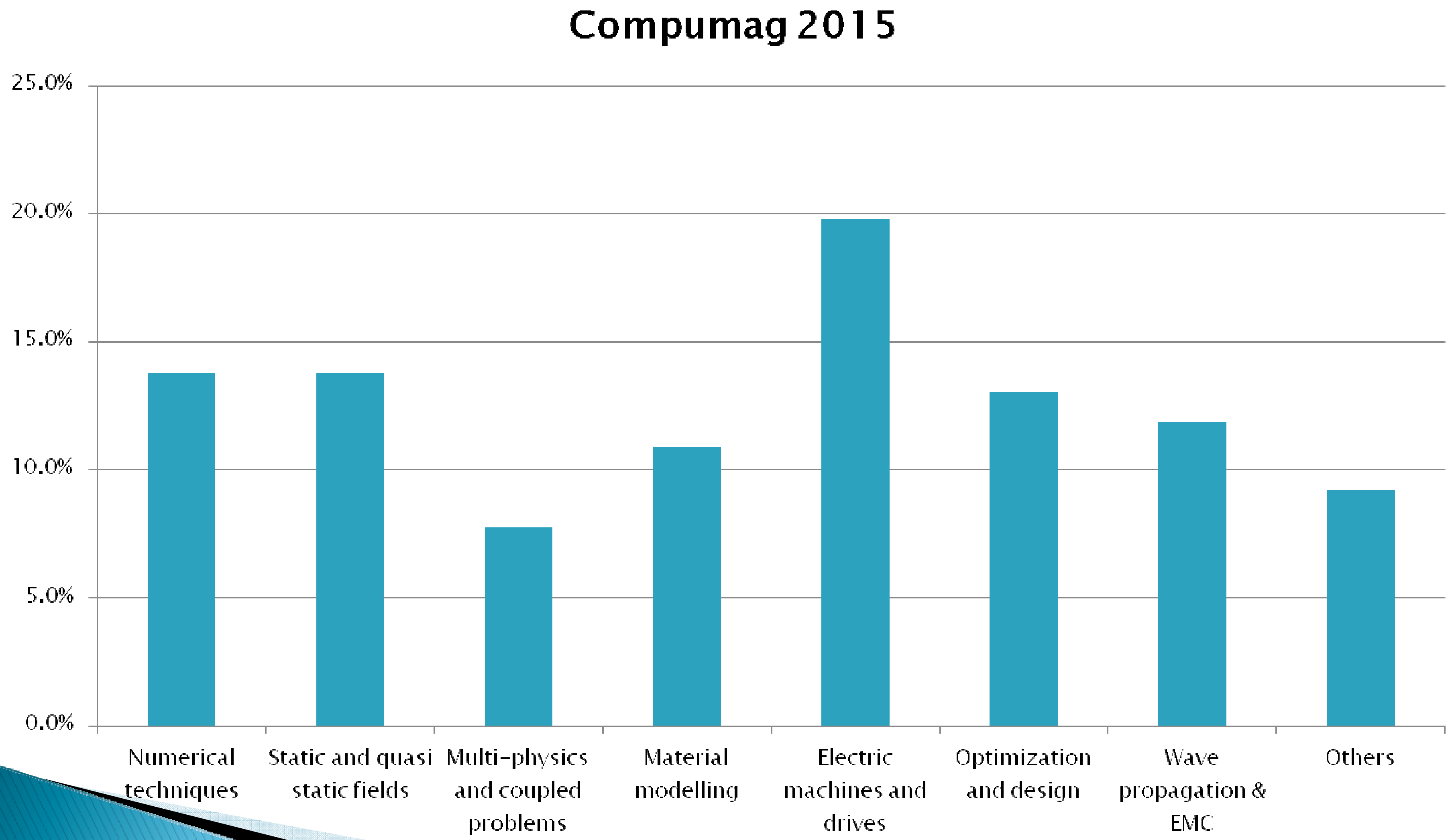
Papers at COMPUMAG



Papers at COMPUMAG



Papers at COMPUMAG



State of the art

- Some general trends emerge
- Not of interest for HTS community ≈ 60%
  - Electric machines ≈ 20%
  - Optimization ≈ 15 %
  - Wave propagation & EMC ≈ 10%
  - Rest of the spectrum ≈ 15 %
- Potential interest for HTS community ≈ 40%
  - Numerical techniques
  - Static & quasi-static fields
  - Coupled problems
  - Material modelling

Research directions

- My personal view

My personal view

- For each of the items of the state of the art of potential interest for HTS community,
  I provide my personal view of the research directions
- Personal selection
  - Most interesting topics (personal preference)
  - Most promising topics for computational electromagnetics
  - (Still) largely out of reach of commercial codes
  - Potential fall-out to HTS community

Numerical techniques

- Fast methods for large-scale problems
  - Domain decomposition
  - Compression
  - Multipoles
  - Model order reduction
  - Parallelization on multiple CPUs
  - GPUs and High Performance Computing
- Numerical advances
  - Meshless method / radial basis functions
  - Discontinuous / moving meshes
  - Gauging/preconditioning/convergence

Static & quasi-static fields

- Numerical analysis of low-frequency systems
  - Inductance and capacitance calculations
  - Shielding
  - New methods/formulations (very few...)

- Force computations

Coupled problems

- Multiphysics problems
  - EM + circuits (interconnections etc.)
  - EM + mechanics (motors, vibrations etc.)
  - EM + thermal (induction heating etc.)
  - EM + fluid (gas, plasmas etc.)
- Commercial codes are catching up...

Material modelling

- Magnetic materials
  - Hysteresis
  - Micromagnetics
- “New” materials
  - Anisotropic / layered materials
  - Metamaterials
  - Graphene et similia
- Homogenization & multiscale
- Superconductors
  - Quench, losses etc.
  - Materials & devices

Focus on: fast methods

- A few examples

Generalities /1

- Integral formulations of the eddy currents problem require the storage of matrices of size scaling as N^2 (N being the number of discrete unknowns), and their inversion needs a computational cost of the order of N^3, if a direct solver is used
  - May get impractically high for detailed meshes
  - Drives towards iterative inversion schemes
  - Need to improve the computational scaling, i.e. the dependence on N of the computational cost

Generalities/2

- Key ingredients (assuming iterative inversion methods)
  - Preconditioning of matrix to be inverted (less iterations needed)
  - Fast matrix-vector product (less computations per iteration)
  - Parallelization

Preconditioning

- Rationale: a lower condition number means less iterations of iterative methods
- Mathematically: $Ax = b \quad \Rightarrow \quad \left(P^{-1}A\right)x = P^{-1}b$
  such that $P^{-1}A$ has a lower condition number than A
- Practically: find an "easy to invert" matrix P (e.g. quasi-diagonal) sufficiently "close" to original matrix so that $P^{-1}A$ gets "close to identity"

Fast matrix-vector product /1

- Matrix – vector product often has a clear physical meaning
  - Magnetic field produced by a given source
- Fast Fourier Transform (FFT)
  - Equivalent sources on suitable regular grids
  - Matrix-vector product can be accelerated by means of a fast convolution product
- Fast Multipole Method (FMM)
  - If the field point is far enough, the electromagnetic field source can be characterized by few parameters (multipoles)
  - The sources are expanded in spherical harmonics and the field computation takes into account a limited number of such harmonics

Fast matrix-vector product /2

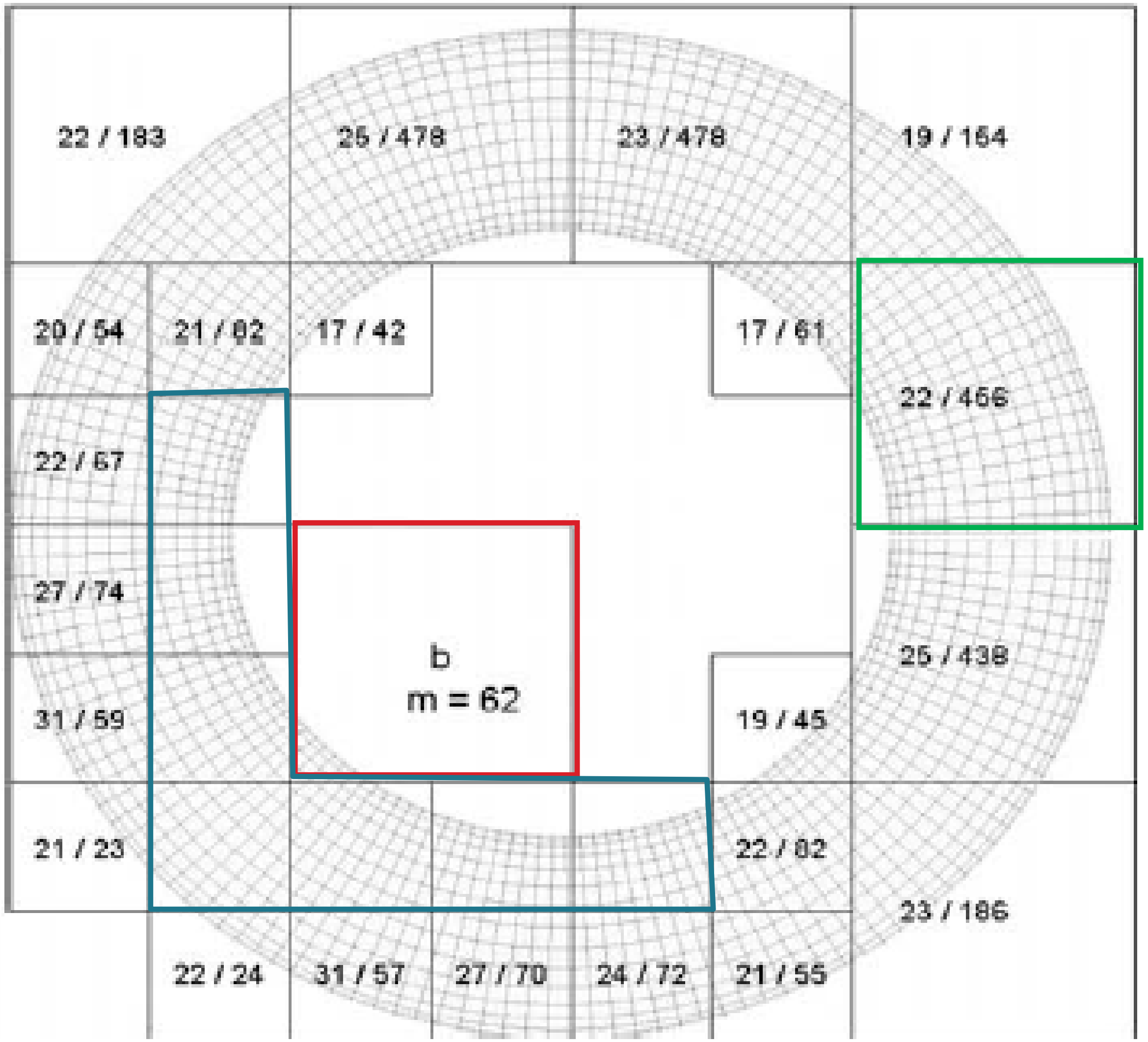
- Singular Value Decomposition (SVD)
  - Physically: the magnetic field produced by a set of sources grouped in a given region VS, when evaluated in a different region VE, can be described through a linear operator having a rank r decreasing as the relative separation between VS and VE is increased.
  - Mathematically: low rank QR factorization of original matrix (e.g. through Gram-Schmidt)

$$A x \approx QR x = Q y, \quad y = R x$$

$$A : N \times N, \quad Q : N \times r, \quad R : r \times N$$

$$r << N$$

Fast matrix-vector product / 3



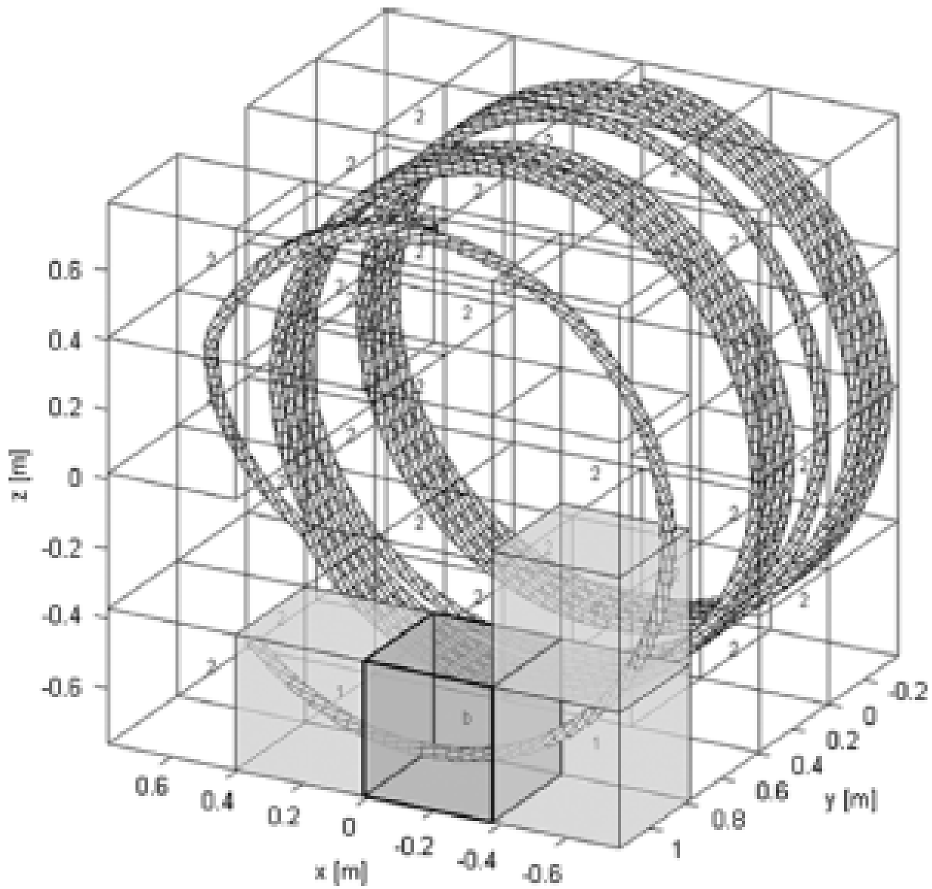
Location of magnetic field sources

Close region (no approximation)

Far regions (low-rank approximation)

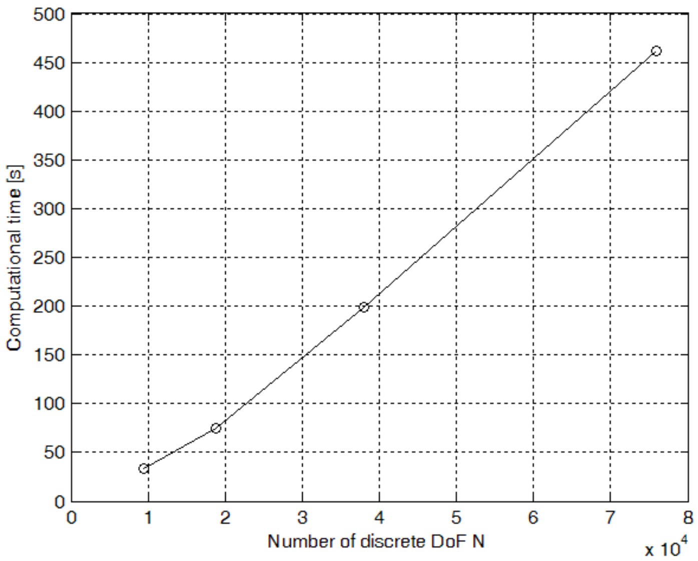
Fast matrix-vector product /4

- A more complicated example...

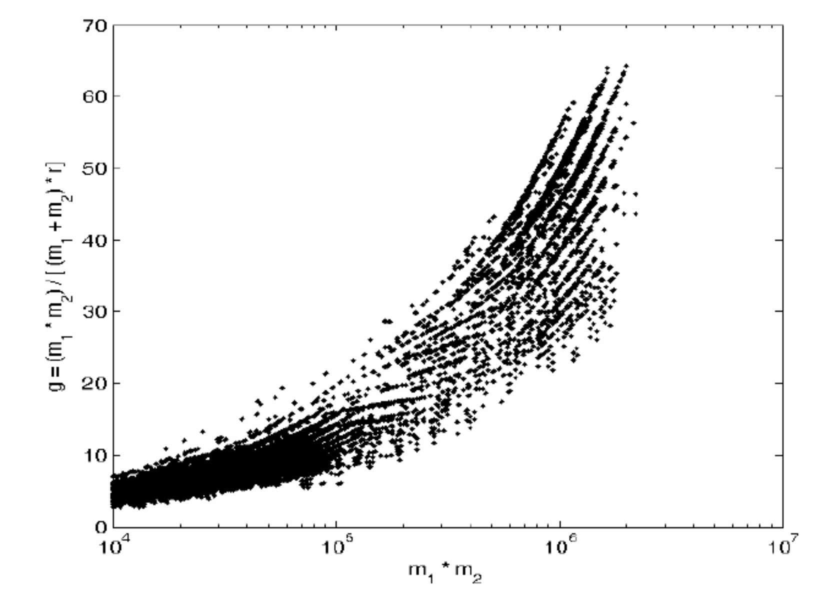


To sum up..

- The overall performance can be quite satisfactory



Computational time scaling almost linearly with *N*



Speed-up with respect to standard calculations

Parallelization /1

- Multiple CPUs
  - Standard libraries support parallelization on CPUs
- Which are the factors to be taken into account?
  - Assembly balancing: the computational times for the matrix assembly should be balanced among the processor
  - Memory balancing: local memory required to store each part of the matrix should be equally distributed among processors
  - Computational balancing: computational time to build matrix-vector product should be balanced among processors
- Solving this optimal allocation problem has an exponential complexity
  - sub-optimal algorithms required

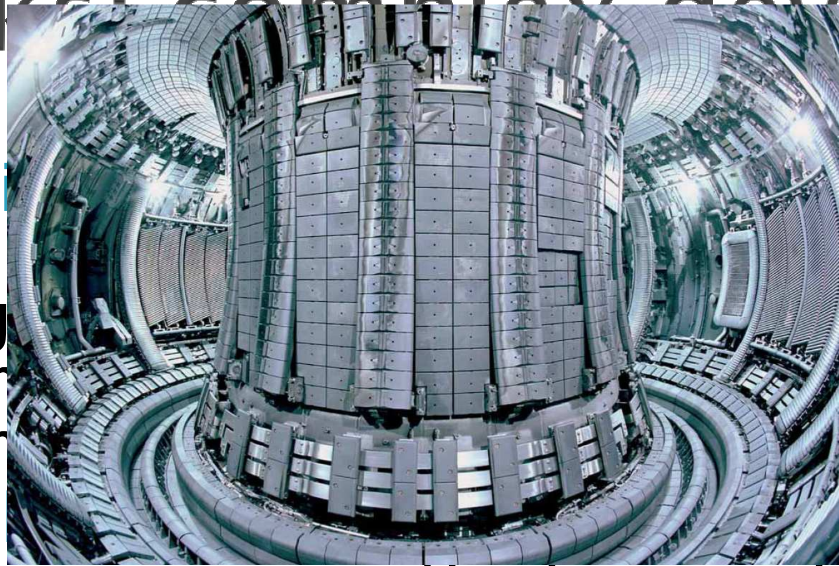
Parallelization /2

- GPUs
  - Design philosophy tailored on the inherently parallel nature of graphics rendering
  - Large amount of cores in order to execute a large number of execution threads at the same time
  - Massive multithreading (up to thousands cores), small cache memory with very simple control unit
  - Each computational thread performs roughly same task onto different partitions of data
  - The code needs to be split into the sequential parts (on the CPU) and the numerically intensive (on the GPUs)
    - GPUs complement CPU execution
    - Reprogramming of codes needed

An example: fusion devices

- My favourite topic…

Tokamak… …ices



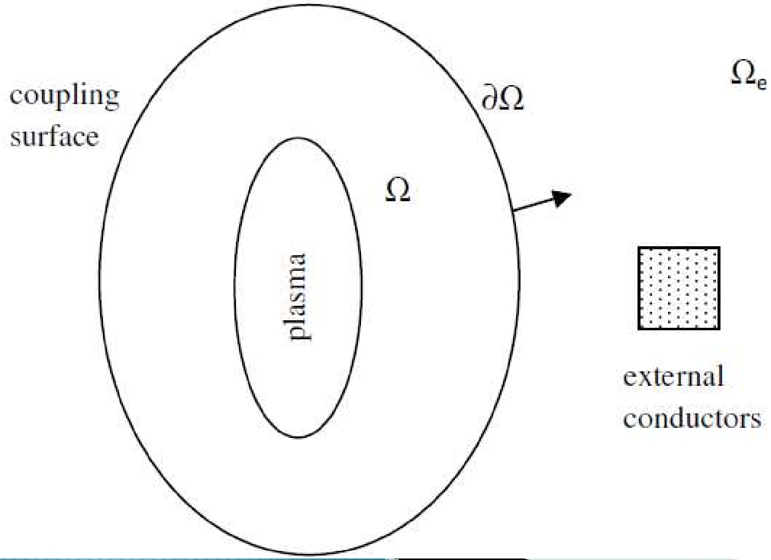
- Nuclear fus… …ble option for future e… …the fusion of nuclei of lig… …s), to give heavier eler… …net energy thanks to m…
- Thermonuc… …epulsion is overcome by increasing the temperature of the

Challenges

- Multiphysics
  - Electromagnetic interaction plasma-conductors
- Nonlinear
  - Plasma behaviour
- Large scale
  - Large devices with fine geometrical details
- Free boundary
  - Plasma/vacuum interface not defined a priori
- Force computation
  - Currents-fields interactions on plasma and on structures
- Superconducting coils
  - Not treated in the following...

Main idea

- Coupling surface to describe the electromagnetic interaction between the plasma and the conductors
- Different formulations in each domain
  - the best choice in each region
  - Can be generalized to other multiphysics problems



Inside Ω: Grad-Shafranov equations (elliptic nonlinear problem)

Outside Ω: eddy currents in 3D structures (parabolic linear problem)

On ∂Ω: coupling conditions

Formulation: inside Ω /1

- Inside Ω: Magneto-Hydro-Dynamics (MHD) eqns.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \qquad \textit{mass balance}$$

$$\frac{\partial (\rho \mathbf{v})}{\partial t} + \nabla \cdot (\rho \mathbf{v}\mathbf{v}) = \mathbf{J} \times \mathbf{B} - \nabla p \qquad \textit{momentum balance}$$

$$\frac{\partial p}{\partial t} + \mathbf{v} \cdot \nabla p + \Gamma p \nabla \cdot \mathbf{v} = 0 \qquad \textit{energy balance (adiabatic)}$$

- If the time scale is slow enough: neglect plasma mass
  - Plasma mass plays a role only on μs time scale
  - Plasma evolves instantaneously through equilibrium states (evolutionary equilibrium)
- If the plasma evolution is around an equilibrium point: linearization

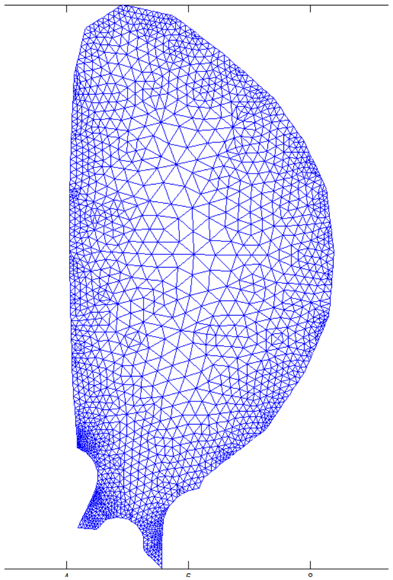
Formulation: inside Ω /2

- Grad-Shafranov nonlinear elliptic equation

$$\nabla_{pol} \cdot \left(\frac{1}{r} \nabla \psi \right) = j_\varphi(\psi) \text{ in } \Omega$$
$$\psi|_{\partial\Omega} = \hat{\psi}$$

ψ : poloidal magnetic flux
$j_\varphi(\psi)$: plasma current density (nonlin. funct. of ψ)
boundary value: coupling conditions

- Differential formulation in weak form
- 2nd order triangular finite elements



$$-\int_\Omega \frac{1}{r} \nabla \psi \cdot \nabla w \, d\Omega + \int_{\partial\Omega} \frac{1}{r} \frac{\partial \psi}{\partial n} w \, dS = \int_\Omega \mu_0 j_\varphi(\psi) w \, d\Omega$$

$$\psi = \sum_{i \in N_i} \psi_i \lambda_i + \sum_{j \in N_b} \hat{\psi}_j \lambda_j$$

- Overall system: $\underline{\underline{A}}\,\underline{\psi} = \underline{f}(\underline{\psi}) - \underline{\underline{\hat{A}}}\,\underline{\hat{\psi}}$

$$A_{i,j} = -\int_\Omega \frac{1}{r} \nabla \lambda_i \cdot \nabla \lambda_j \, d\Omega \quad , \quad i, j \in N_i \qquad f_j(\underline{\psi}) = \int_\Omega \mu_0 j_\varphi \left(\sum_{i \in N_i} \psi_i \lambda_i \right) \lambda_j \, d\Omega \quad , \quad j \in N_i$$

Formulation: outside Ω /1

- Outside Ω: eddy currents (linear parabolic)

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \nabla \times \mathbf{H} = \mathbf{J} \quad \text{in } V_e \qquad \nabla \cdot \mathbf{B} = 0$$

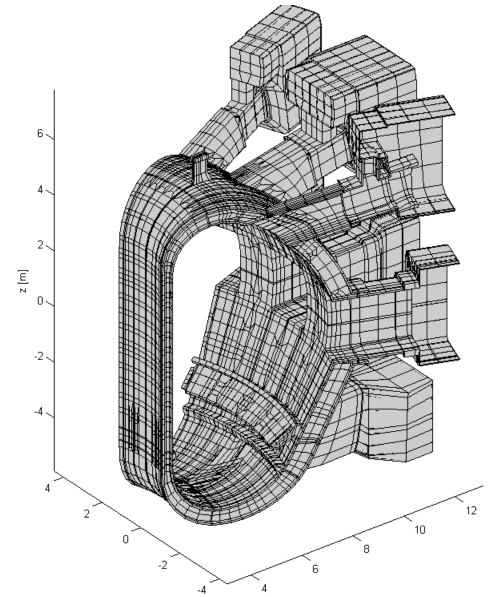
$$\mathbf{E} = \underline{\underline{\eta}}\, \mathbf{J} \qquad \mathbf{B} = \mu_0 \mathbf{H} \quad \text{in } V_e$$

Linear nonmagnetic conductors

- Integral formulation in terms of J in weak form

$$\int_{V_c} \underline{\underline{\eta}} \cdot \mathbf{J} \cdot \mathbf{w}\, dV + \frac{\partial}{\partial t}\int_{V_c} \mathbf{A} \cdot \mathbf{w}\, dV + \int_{V_c} \nabla \varphi \cdot \mathbf{w}\, dV = 0$$

- Electric vector potential with two-component gauge
- Volumetric finite elements (hexa, tetra,…)
- Edge elements: $\mathbf{T} = \sum_k I_k \mathbf{N}_k \;\Rightarrow\; \mathbf{J} = \sum_k I_k \nabla \times \mathbf{N}_k$



Formulation: outside Ω /2

- Dynamical eqns solved with implicit time stepping

$$\underline{\underline{L}}\frac{d\underline{I}}{dt}+\underline{\underline{R}}\,\underline{I}+\frac{d\underline{U}}{dt}=\underline{\underline{D}}\,\underline{V}$$

$$L_{i,j}=\frac{\mu_0}{4\pi}\int_{V_c}\int_{V_c}\frac{\nabla\times\mathbf{N}_i(\mathbf{r})\cdot\nabla\times\mathbf{N}_j(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}dV\,dV'$$

$$R_{i,j}=\int_{V_c}\nabla\times\mathbf{N}_i\cdot\eta\cdot\nabla\times\mathbf{N}_j dV$$

$$U_i=\int_{V_c}\nabla\times\mathbf{N}_i\cdot\mathbf{A}_p dV$$

Flux induced by plasma current on 3D structures

- Equivalent currents located on the coupling surface, producing the same magnetic field as plasma outside the coupling surface
  - Proportional to plasma current density $\underline{I}_{eq}=\underline{\underline{S}}\,\underline{f}(\underline{\psi})$
  - Coupled to 3D structures via mutual inductance $\underline{U}=\underline{\underline{M}}\,\underline{I}_{eq}$
- Overall discrete equations: $(\underline{\underline{L}}+\Delta t\,\underline{\underline{R}})\underline{I}+\underline{\underline{M}}\,\underline{\underline{S}}\,\underline{f}(\underline{\psi})=\Delta t\,\underline{\underline{D}}\underline{V}+\underline{c}$

Formulation: coupling on $\partial\Omega$

- Coupling condition on poloidal flux $\hat{\psi} = \hat{\psi}_p + \hat{\psi}_e$
- Plasma contribution: $\underline{\hat{\psi}}_p = \underline{\underline{K}}\,\underline{f}(\underline{\psi})$
  - Proportional to plasma current density
- External contribution: $\underline{\hat{\psi}}_e = \underline{\underline{Q}}\,\underline{I}$
  - Proportional to 3D currents (Biot-Savart)
- Combining everything, at each time step we have:

$$\underline{\underline{A}}\,\underline{\psi} = \underline{f}(\underline{\psi}) - \underline{\underline{\hat{A}}}\,\underline{\underline{K}}\,\underline{f}(\underline{\psi}) - \underline{\underline{\hat{A}}}\,\underline{\hat{\psi}}_e$$

$$\left(\underline{\underline{L}} + \Delta t\,\underline{\underline{R}}\right)\underline{I} + \underline{\underline{M}}\,\underline{\underline{S}}\,\underline{f}(\underline{\psi}) = \Delta t\,\underline{\underline{D}}\,\underline{V} + \underline{c}$$

$$\underline{\hat{\psi}}_e = \underline{\underline{Q}}\,\underline{I}$$

$$\boxed{\underline{F}(\underline{\psi}) = 0}$$

Nonlinear set of N_i equations (as many as nodes in 2D triangular mesh inside Ω), solved with Newton-Raphson method

Application to ITER /1

- “ITER is … that
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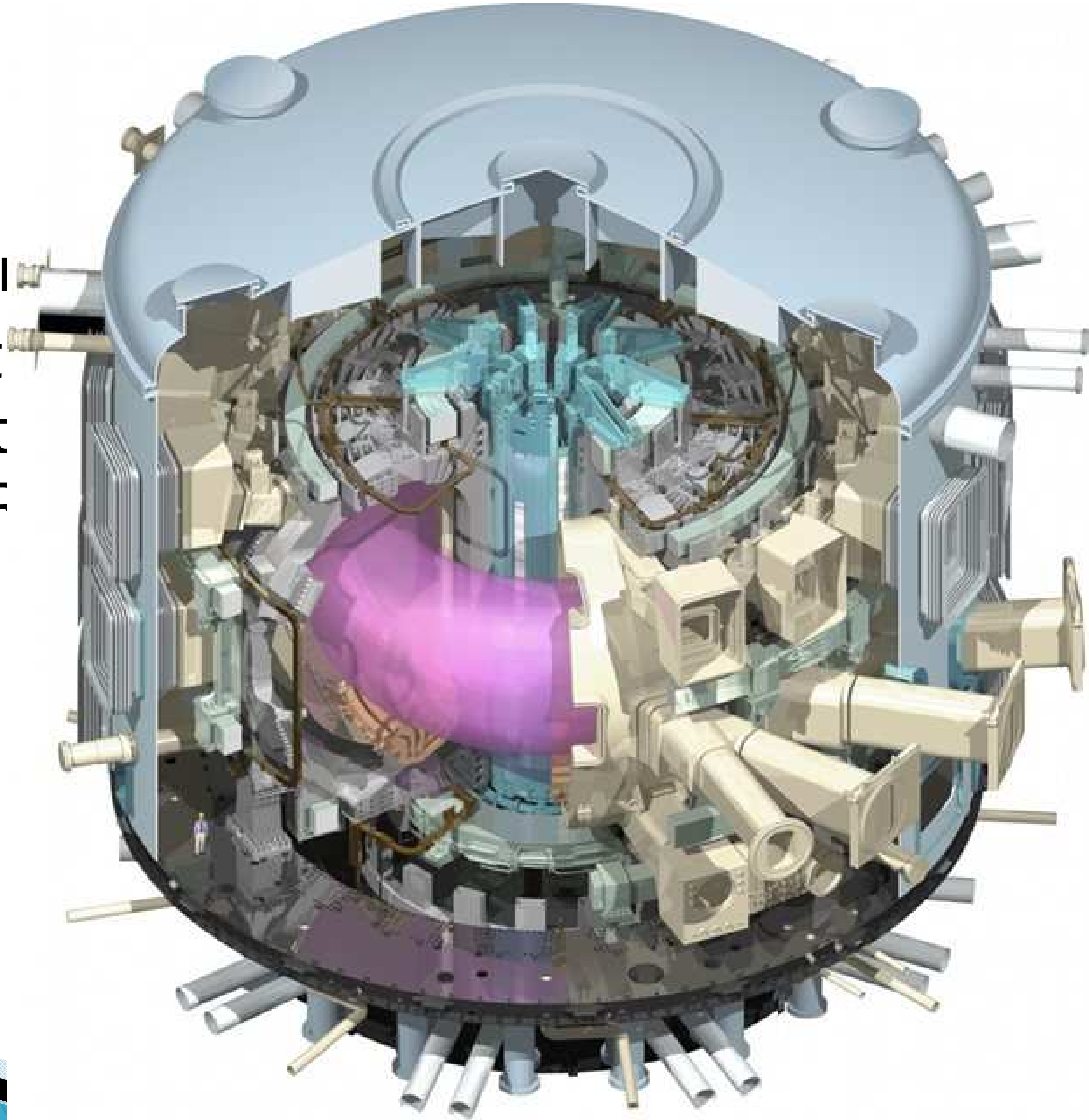
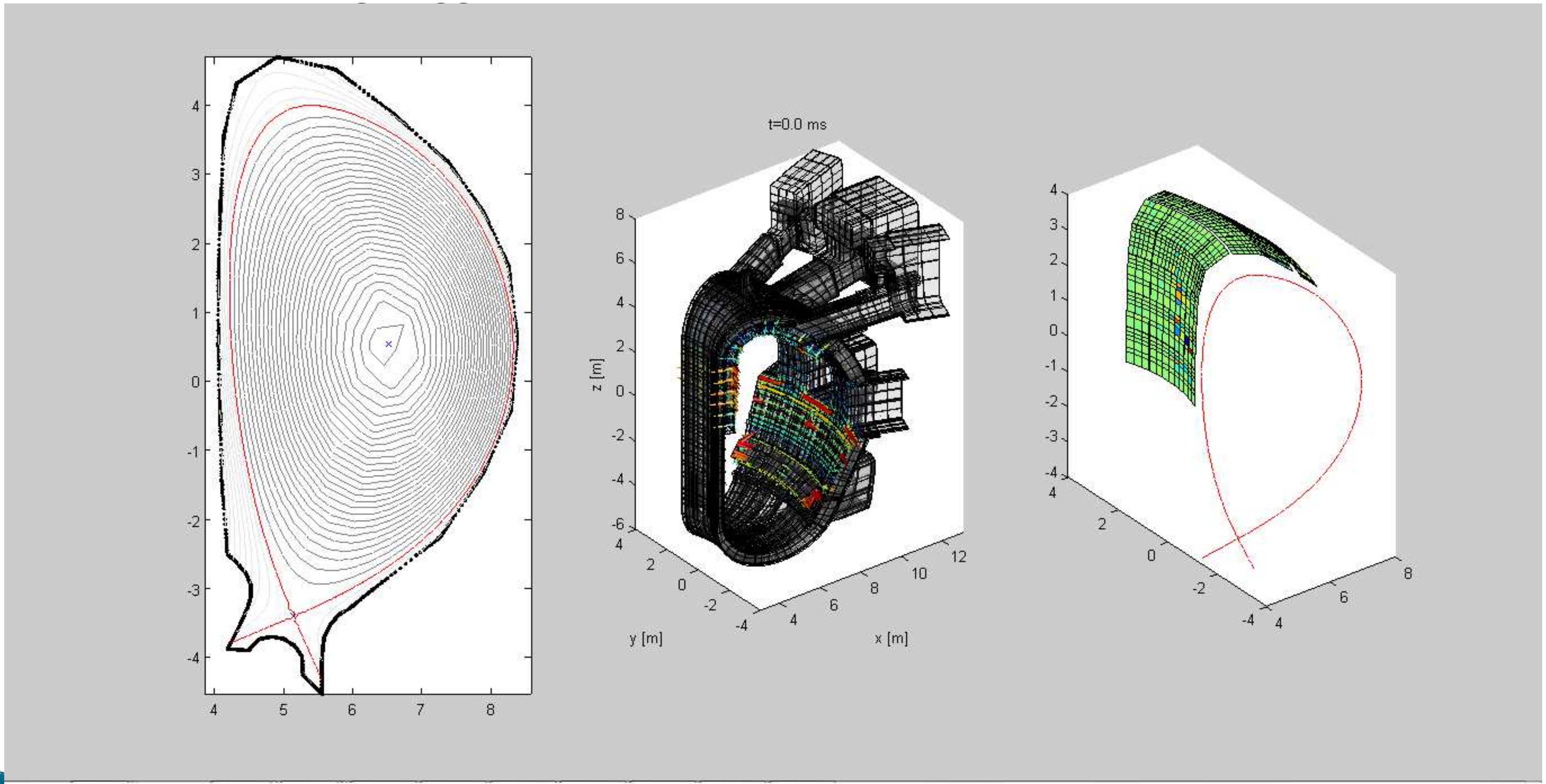
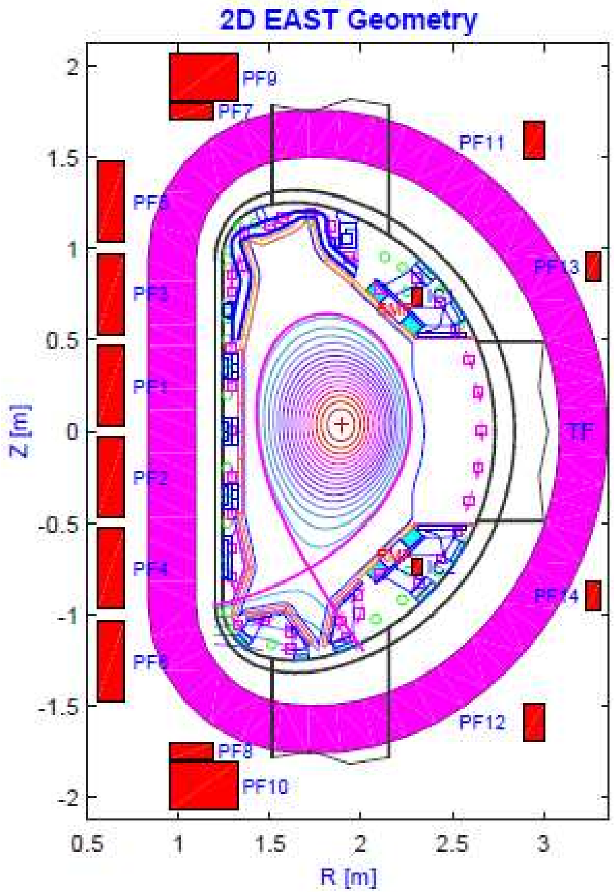
  commer…
- Current…
  internat… a, India,
  Korea, F…
- Multi-b…

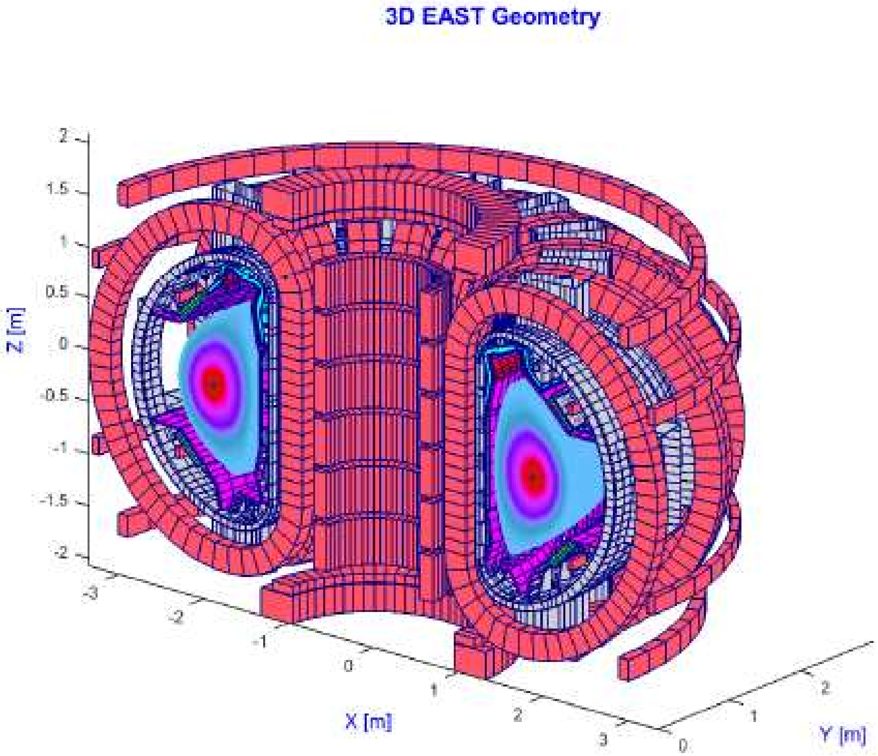


Application to ITER /2



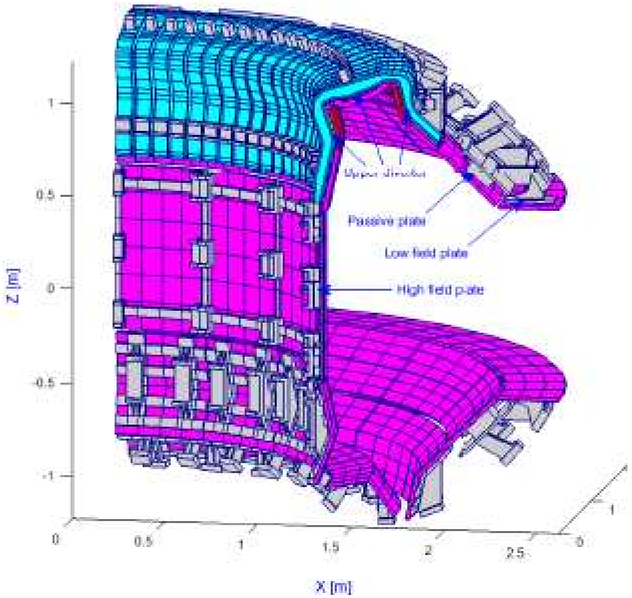
Application to EAST /1

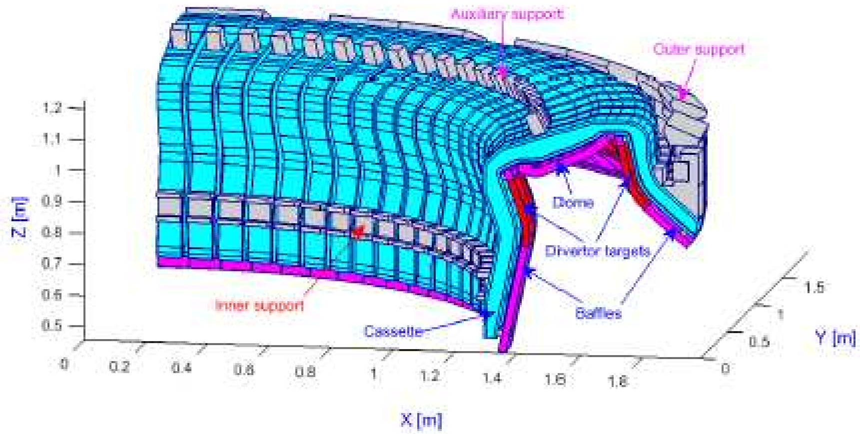




Application to EAST /2

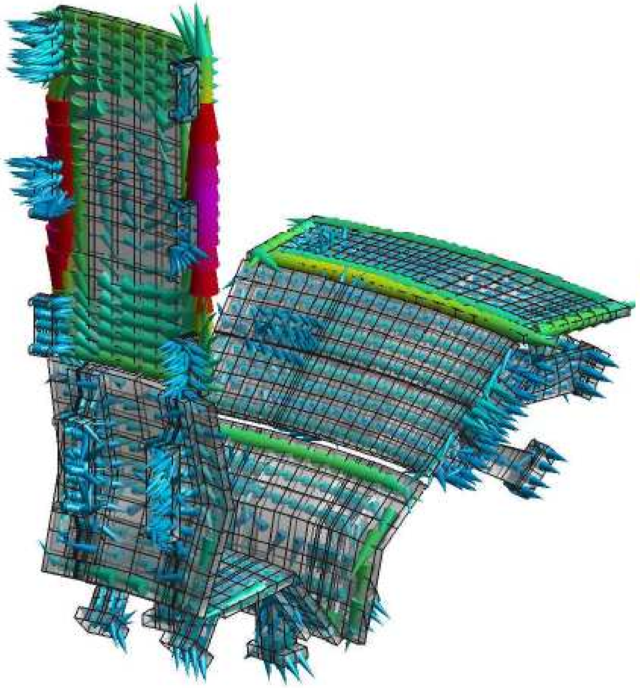
- EAST is an intrinsic 3D device
  - The axisymmetric conducting structures are “too far” from plasma
  - Need for a detailed 3D modelling

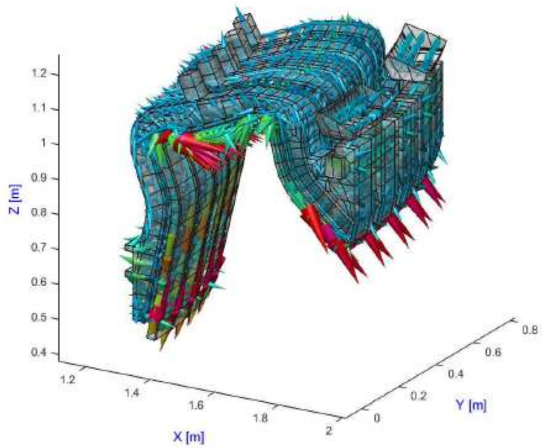


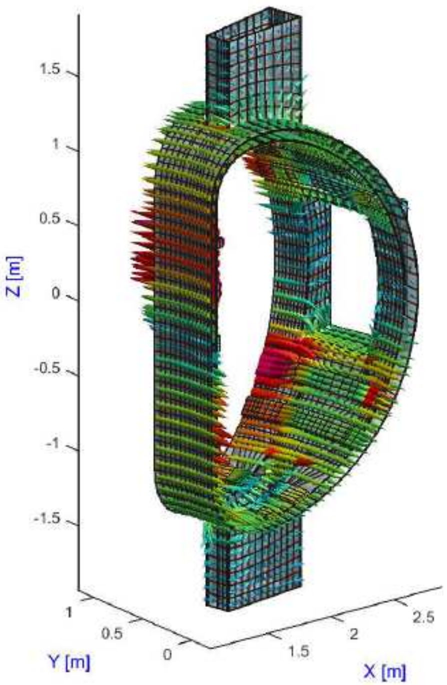


Application to EAST /3

- Complicated 3D eddy current density patterns induced in conducting structures

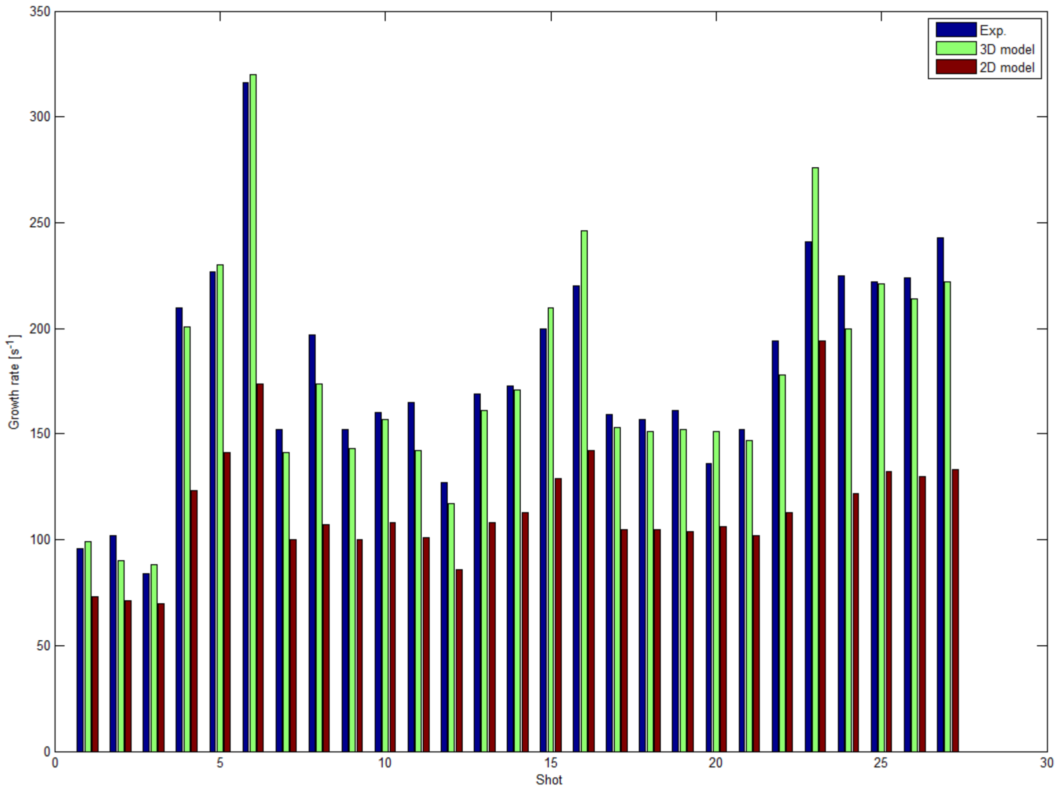






Application to EAST /4

- 3D effects are fundamental in providing a correct prediction of experimental results



Conclusions

- Finally at the end...

Conclusions & outlook /1

- Low-frequency computational electromagnetics: a 40-year long story
- Tumultuous and fast advances for the first 20 years or so
- Now it can be considered a more “mature” sector
  - Advent of commercial codes, which can treat routinely “standard” applications
  - The interests of the scientific community are diverting
  - WARNING: need for awareness of use of commercial codes!!

Conclusions & outlook /2

- New research trends are emerging
  - Focus on topics/techniques still out of reach of commercial codes...
  - ... shifting to commercial applications in the near future?
- Several new research directions may be of interest for the HTS community
  - Fast methods, force computations, multiscale, multiphysics etc.

Thank you for your attention

"Παντα ρει, και ουδεν μενει"

"*Everything flows, nothing stands still*"

Heraclitus, around 500 BC