Quench Characteristics of Power-law Superconductors and Implications to Modelling

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Thanks for my colleagues at Southampton Jorge Pelegrin, Iole Falorio, Edward Young for their experimental and modelling work

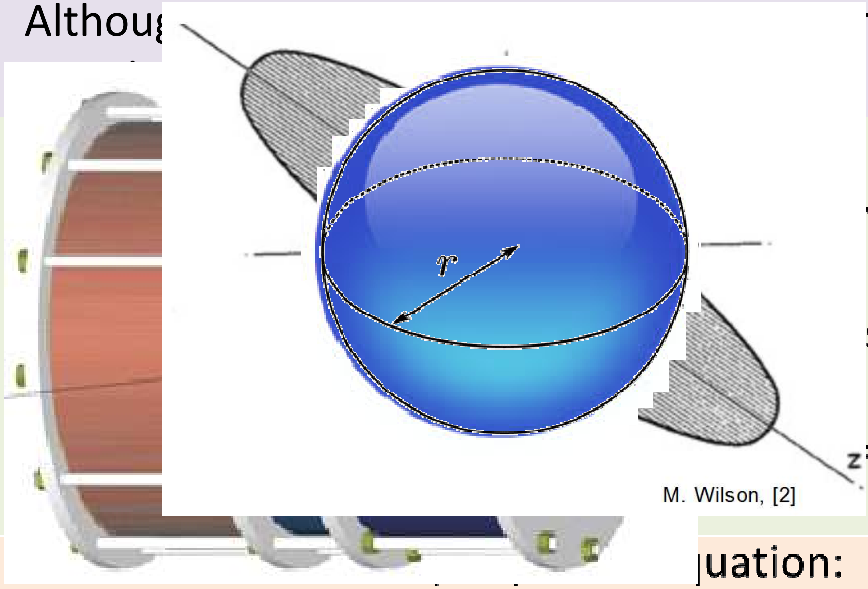
Motivations

1. Growing body of literature on HTS quench, both experimental and modelling
2. Modelling becoming easier and more versatile
3. However our understanding has become more diffused rather than crystalized
   - Complex set of thermal / electrical properties
   - Nonlinear over a much wider temperature range
4. Trying to rationalise the parameters through analysis and streamlined modelling
5. Aim to get some useful scaling for the MQE

Quench Equation in Effective Continuum

Basic Equation: $$C_p(T)\frac{\partial T}{\partial t}=\frac{\partial}{\partial x}\left(k_x\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_y\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k_z\frac{\partial T}{\partial z}\right)+\lambda J\cdot E(T,J)$$

Althou rmed with relative ease, elucidation of underlying



M. Wilson, [2]

nce in t

$\kappa(T),$

s:

$\sqrt{\frac{k_{0,x}}{k_{0,z}}}$

Compromises:
Loss of discrete details of windings
Not suitable for small coils
Restriction on the anisotropy

Advantages
Significant simplification for analysis
Hopefully without loss of generality

uation:

$$C_p(T)\frac{\partial T}{\partial t}=\frac{\partial}{\partial x'}\left(k(T)\frac{\partial T}{\partial x'}\right)+\frac{\partial}{\partial y'}\left(k(T)\frac{\partial T}{\partial y'}\right)+\frac{\partial}{\partial z'}\left(k(T)\frac{\partial T}{\partial z'}\right)+\lambda J\cdot E(T,J)$$

- Reduce to **1d** in a spherical coordinate: (*m*=1: wire, *m*=2: pancake, *m*=3: solenoid)

$$C_p(T)\frac{\partial T}{\partial t}=\frac{1}{r^{m-1}}\frac{\partial}{\partial r}\left(k(T)\,r^{m-1}\frac{\partial T}{\partial r}\right)+\lambda J\cdot E(T,J)$$

Non-dimensionalization of Quench Equation: Reduction of Parameters

- Dimensionless length/radius with l is the "classical" minimum propagation zone length

$$\xi = \frac{r}{l}, \qquad l = l_{MPZ} = \omega_m \sqrt{\frac{k(T_0)(T_C - T_0)}{\lambda \rho_n j\, J_C^2(T_0)}}, \qquad J = j \cdot Jc(T_0)$$

- Dimensionless time (for k and c_p independent of T):

$$\tau = \frac{k\, t}{\rho_d c_p l^2} = \frac{k\, t}{C_p l^2}, \text{(reserve } \rho \text{ exclusively for resistivity hereafter)}$$

- Dimensionless Temperature:

$$\theta = \frac{T - T_0}{T_c - T_0}, \text{and } u = \frac{\theta}{1-j}, \text{(Note } u = 1 \text{ for } T_{cs})$$

Solenoid: $m = 3$ and $\omega_3 = \pi$
Pancake: $m = 2$ and $J_0(\omega_2) = 0$
Wire: $m = 1$ and $\omega_1 = \frac{\pi}{2}$

- Non-dimensional quench equation

$$\frac{\partial u}{\partial \tau} = \frac{1}{\xi^{m-1}} \frac{\partial}{\partial \xi}\left(\xi^{m-1} \frac{\partial u}{\partial \xi}\right) + \omega_m^2 g(u,j) \textbf{ and } g(u,j) = \frac{E(u,j)}{(1-j)\, \rho_n J_c(T_0)}$$

- A rationalised and reduced parameter set:
  - Dimensionless current load j
  - Dimensionless current sharing heat generation $g(u,j)$

MQE for Critical State Superconductors
A simpler case revisited

The current sharing voltage of *critical state* superconductors with a *linear critical current density* $J_C(T)$ is a *linear* function of *relative* temperature rise

$$E(u,j) = \rho_n\left(J - J_c(T)\right) = \rho_n\left(J - J_c(T_0)\cdot\frac{T-T_0}{T_C-T_0}\right) = \rho_n J_c(T_0)(1-j)(u-1)$$

So we have a dimensionless heat generation free of any parame

$$g(u,j) = \frac{E(u,j)}{(1-j)\,\rho_n J_c(T_0)} = u - 1$$

Leading to a quench e

$$\left(\xi^{m-1}\frac{\partial u}{\partial \xi}\right) + \omega_m^2(u-1)$$

Without solving the e stant: $\eta_{MQE} = \text{constant}$

Critical State MQE scales with $(1-j)j^{-\frac{m}{2}}$

MQE expressed in real units: Energy/Temperature Length/volume

$$MQE(j) = \eta_{MQE}\cdot\left(C_p(T_0)(1-j)(T_c-T_0)\right)\left(\frac{k_y}{k_x}^{\frac{1-\delta_{m,1}}{2}}\frac{k_z}{k_x}^{\frac{\delta_{m,3}}{2}}\ l_{MPZ}^m\right)$$

$$= \eta_{MQE}\ \omega_m^m C_p(T_0)\sqrt{\frac{k_x k_y^{1-\delta_{m,1}}k_z^{\delta_{m,3}}}{(\lambda\rho_n J_C(T_0)^2)^m}}\ (T_C-T_0)^{1+\frac{m}{2}}(1-j)j^{-\frac{m}{2}}$$

Stationary Normal Zone of the Critical State and the Existence of a Minimal Energy

Stationary Normal Zone:

- Normal zone length $L(u_0)$ reduces at higher u_0.
- The current sharing length $L_{cs}^m = 1$ at $u = 1$.

$$\frac{1}{\xi^{m-1}}\frac{d}{d\xi}\left(\xi^{m-1}\frac{du}{d\xi}\right) + \omega_m^2(u-1) = 0$$

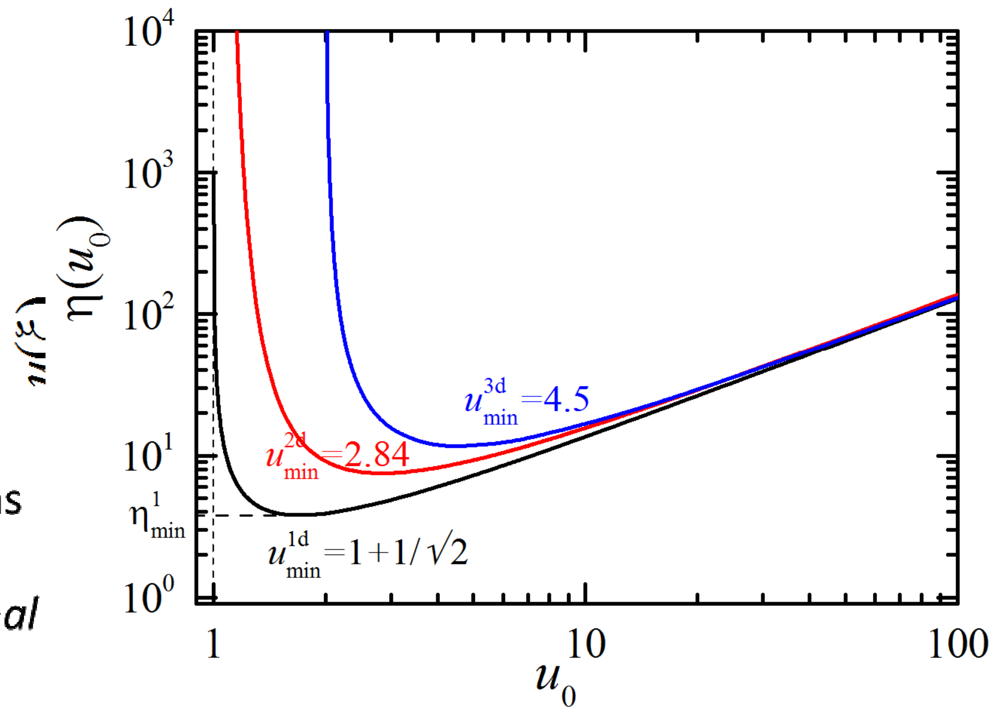
$$u(0) = u_0, u'(0) = 0; (\, u(L) = 0\,)$$

The thermal energy (enthalpy) of the normal zone is the area/volume of $u(\xi)$:

$$\eta(u_0) = \int_0^{L(u_0)} 2^{1+\delta_{m,3}}\pi^{1-\delta_{m,1}}u(\xi)\xi^{m-1}d\xi$$

The existence of a minimum $\eta_{\min}$ is straightforward

- At high temperatures: $\eta(u_0 \to \infty) \to \infty$ (note: $u(1) = 1$ and $L(u_0 \to \infty) \to 1^+$)
- At low temperature: $\eta(u_0 \to 1^+)$ and $L(u_0 \to 1^+) \to \infty$.
- 1D $\eta_{\min}^1 = 2\left(1 + \frac{2\sqrt{2}}{\pi}\right)$at $u_0 = 1 + 1/\sqrt{2}$
- $\eta_{\min}^m$and $u_{\min}^{\mathrm{m}}$increase with dimension m as heat conduction increases.
- Higher heat conduction in 3D leads to *global recovery* of all normal zone with $u_0 \leq 2$.



Stationary Normal Zone And Hot-spot Evolution

How can one get from $\eta_{\min}$ of stationary normal zone to the MQE?

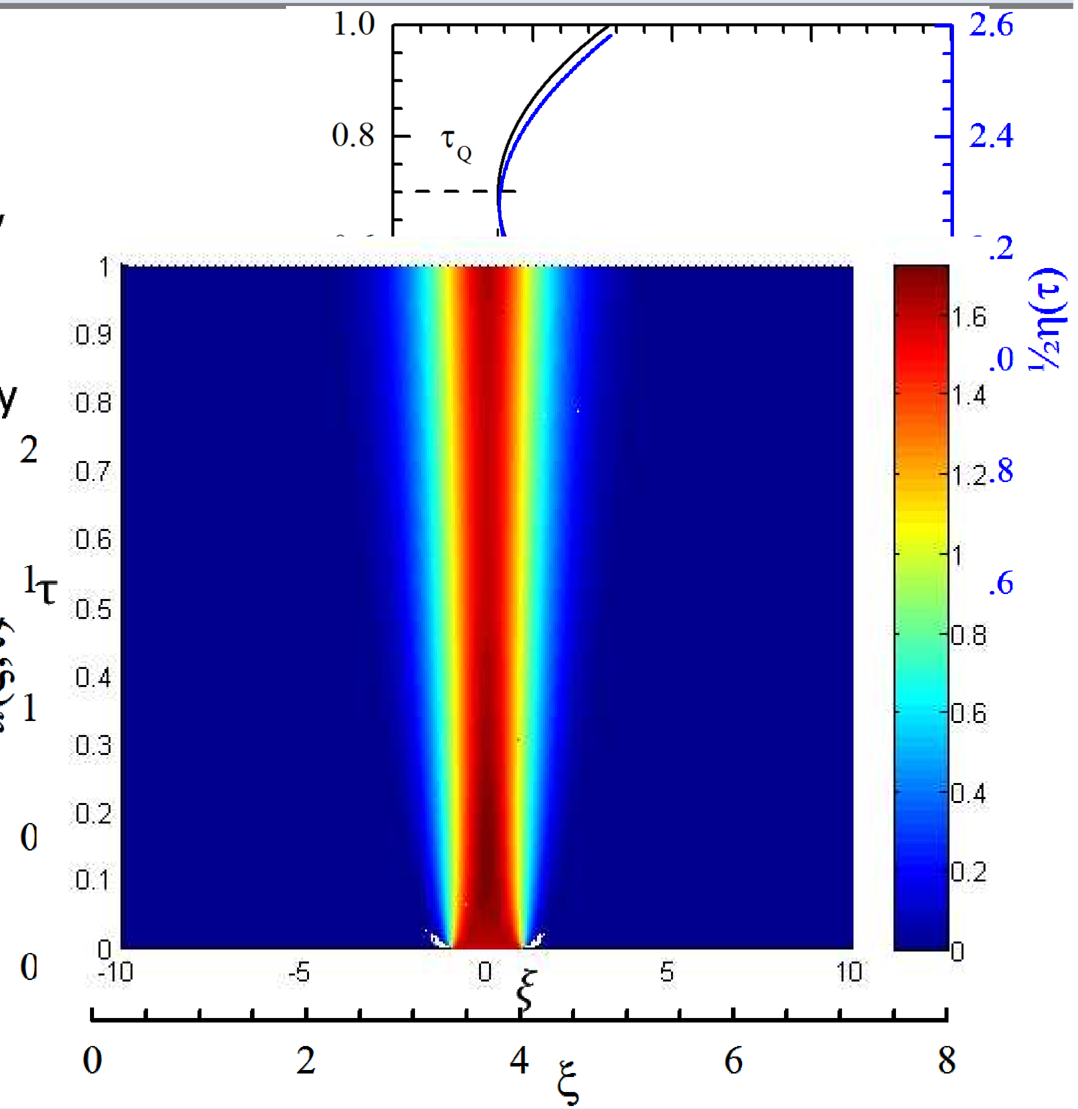
Wilson postulated that they are effectively the same thing…

It's easy to show that hot spot of stationary normal zone at minimal energy $u(\xi,0) = u_{\min}(\xi)$ will lead to quench straight away,

But what about quench from lower energy assisted
by self-heating?

The simplicity of the critical state makes it the easiest case to start:

- No parameters
- Just the initial condition (size and shape of hot-spot)



MQE: From Stationary Normal Zone to Hot-spot Evolution by Numerical Experiments

- Using $u(\xi,0) = u_0 H(s-\xi)$ of an **initial energy** $\eta(u_0,0) = u_0 s$ find minimum of $\eta(u_0,s)$ by varying u_0 and/or s
- With $s=1$, a minimum is found at $1.53873 < u_0 < 1.53874$: $\eta_{\min}^{s=1} \sim 3.07$
- Or with $u_0 = 2$, a minimum is found at $0.694 < s < 0.695$: $\eta_{\min}^{u_0=1} \sim 2.78$

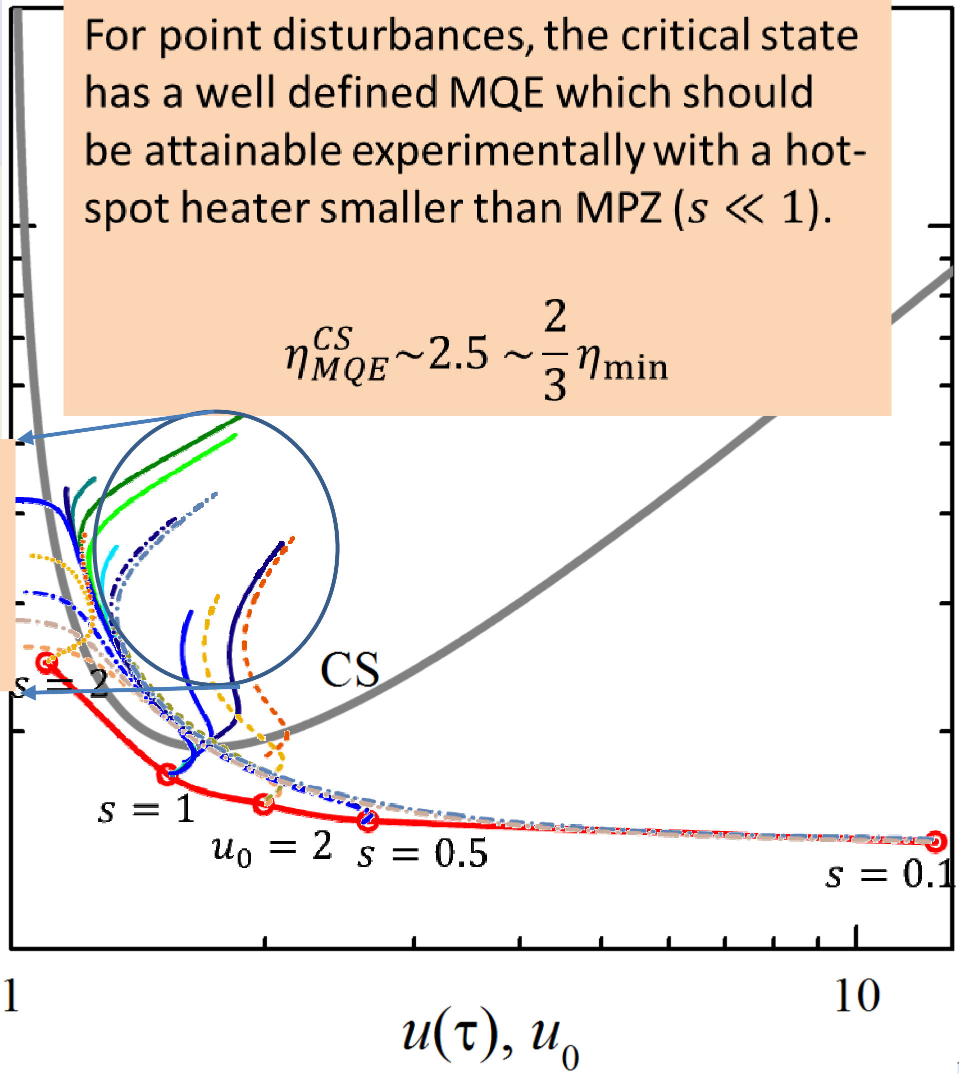
Though less than the stationary minimal energy due to self-heating, MQE is correlated to the stationary energy. Quench only happens when the normal zone evolves to just exceed it.

zone, continue with $s = 0.5$ and $s = 0.1$ leads to $\eta_{\min} = 2.66$ and 2.5 respevtively

- Increasing the normal zone length beyond $s = 1$ leads to sharply increased $\eta_{\min}$, as found in stationary case. Self heating diminishes as $u_0 \to 1$

For point disturbances, the critical state has a well defined MQE which should be attainable experimentally with a hot-spot heater smaller than MPZ ($s \ll 1$).

$$\eta_{MQE}^{CS} \sim 2.5 \sim \frac{2}{3}\eta_{\min}$$



Current Sharing in Power-Law HTS Superconductors

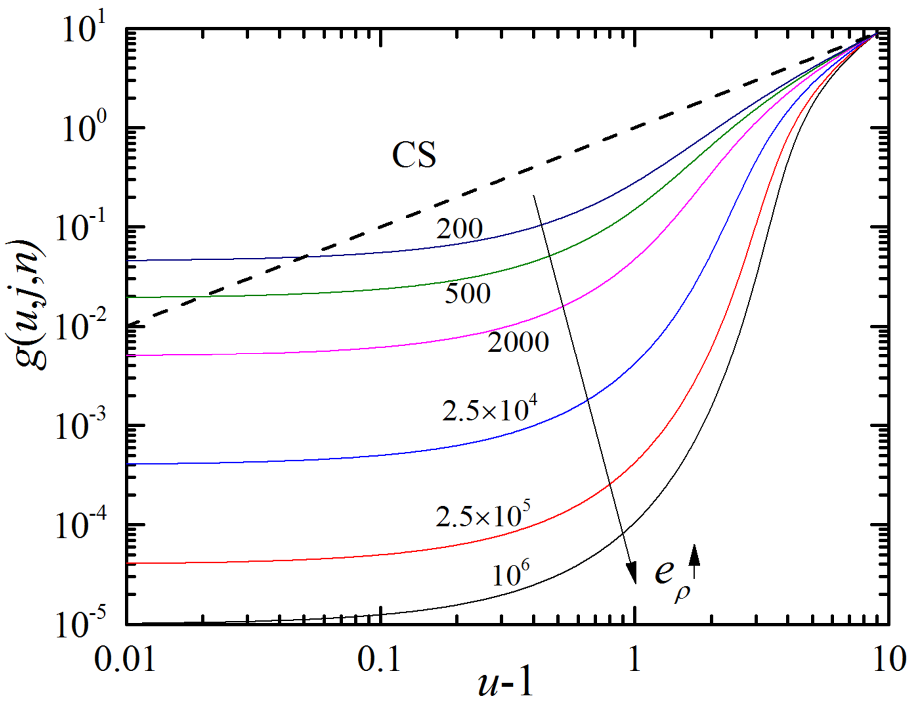
- **Nonlinear current sharing $E(u,j)$:**

$$(1-(1-j)u)\left(\frac{E(u,j)}{E_0}\right)^{\frac{1}{n}}+\frac{E(u,j)}{J_C(T_0)\rho_n}=j$$

- **No close-solution**
- **Additional parameters:** $e_\rho=\frac{J_c(T_0)\rho_n}{E_0}$ and n
- **Nonlinear heat generation $g(u,j)$:**

$$g(u,j,n)=\frac{E(j,u)}{(1-j)J_C(T_0)\rho_n}=\frac{1}{(1-j)e_\rho}\frac{E(u,j)}{E_0}$$

$$=(u-1)-\left(\frac{1}{1-j}-u\right)\left(\left(\frac{E(u,j)}{E_0}\right)^{\frac{1}{n}}-1\right)$$



Less Heat generation in power-law current sharing:

- $g(u,j,n) < g(u,j,\infty) = u-1$ of the critical state;
- Except in the vicinity of T_{cs} i.e. $u_0 = 1^+$

Typical e_ρ values
~ 5000-20000 for 2G HTS
~ 20000-50000 for unstabilized HTS (77K)
~> 1000 for 2212 wire in high field

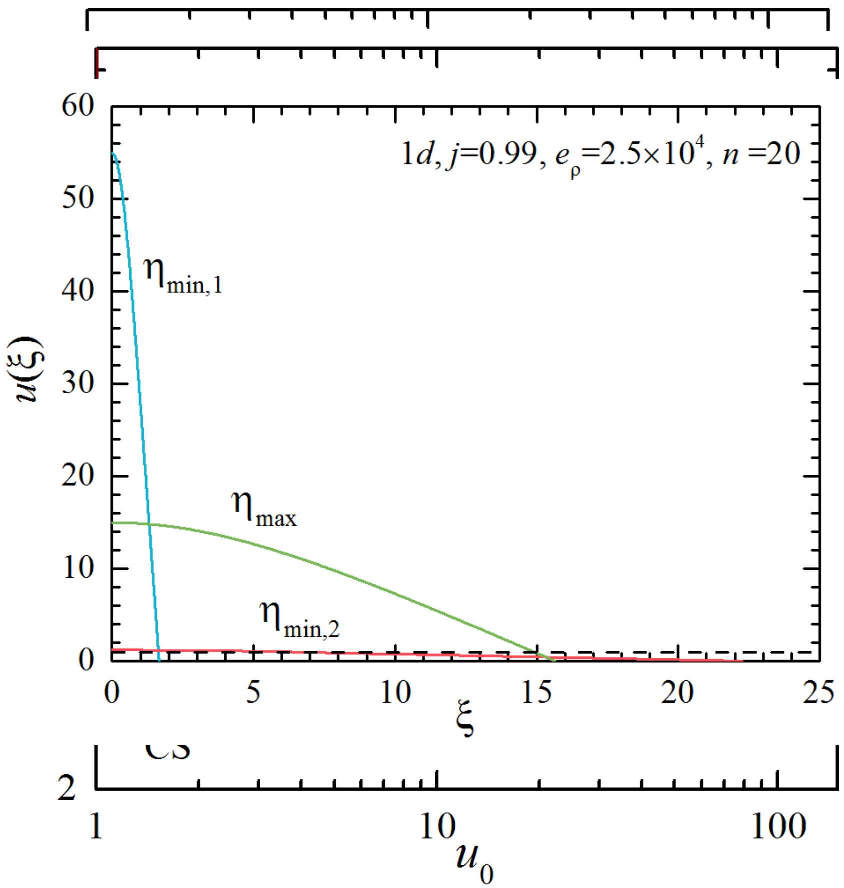
Stationary Normal Zone of Power-Law Superconductors

$$\frac{\partial u}{\partial \tau} = \frac{1}{\xi^{m-1}}\frac{\partial}{\partial \xi}\left(\xi^{m-1}\frac{\partial u}{\partial \xi}\right) + \omega_m^2\left((u-1) - \left(\frac{1}{1-j} - u\right)\left(\left(\frac{E(u,j,\rho)}{E_0}\right)^{\frac{1}{n}} - 1\right)\right)$$

The quench equation now depends on j, n, e_ρ, therefore $\eta_{MQE} \sim \eta_{min}(j, n, e_\rho) \neq$ constant.

<u>The stationary solution shows</u>

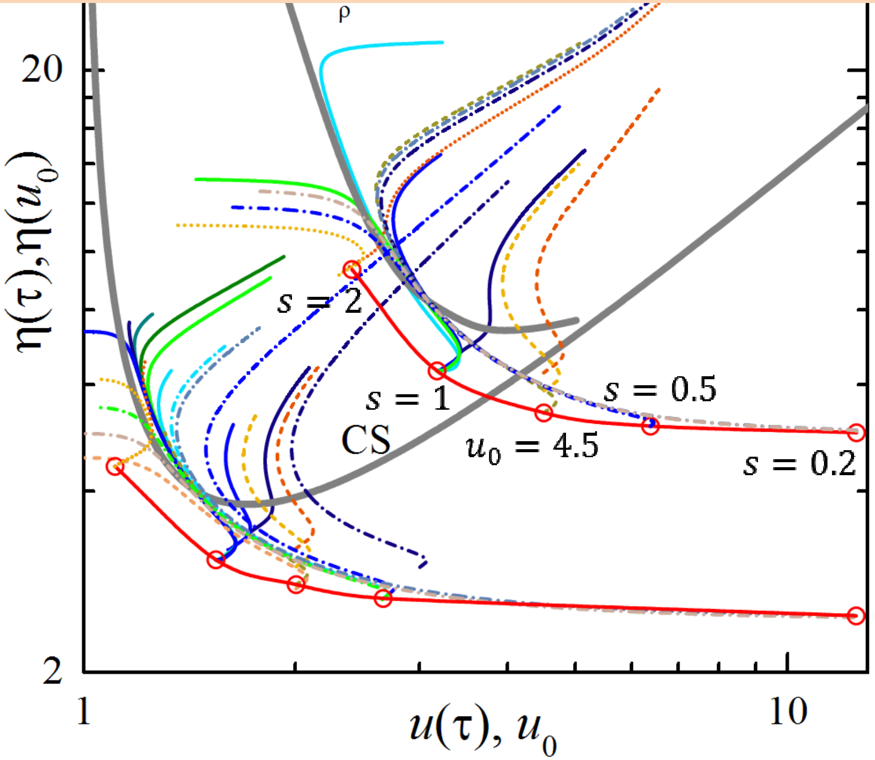
- The current sharing zone depends on u_0 and j.
- $\eta(u_0, j)$ is strongly j dependent, in contrast to the j independent $\eta_{CS}(u_0)$ of the critical state.
- The deviation is substantial at $u < \frac{1}{1-j}$ or $T < T_c$.
- A minimum enthalpy η_{min} still exist:
  - $\eta_{min}(u_{MQE}(j), j)$ and $u_{MQE}(j)$ increases with j, *instead of a constant of the critical state.*
  - An additional minimum near $u \sim 1$ (i.e. $T \sim T_{cs}$) at high j (> 0.9) and eventually becomes lower than the one at higher u.



Power-Law MQE is Correlated to Stationary Minimum Energy

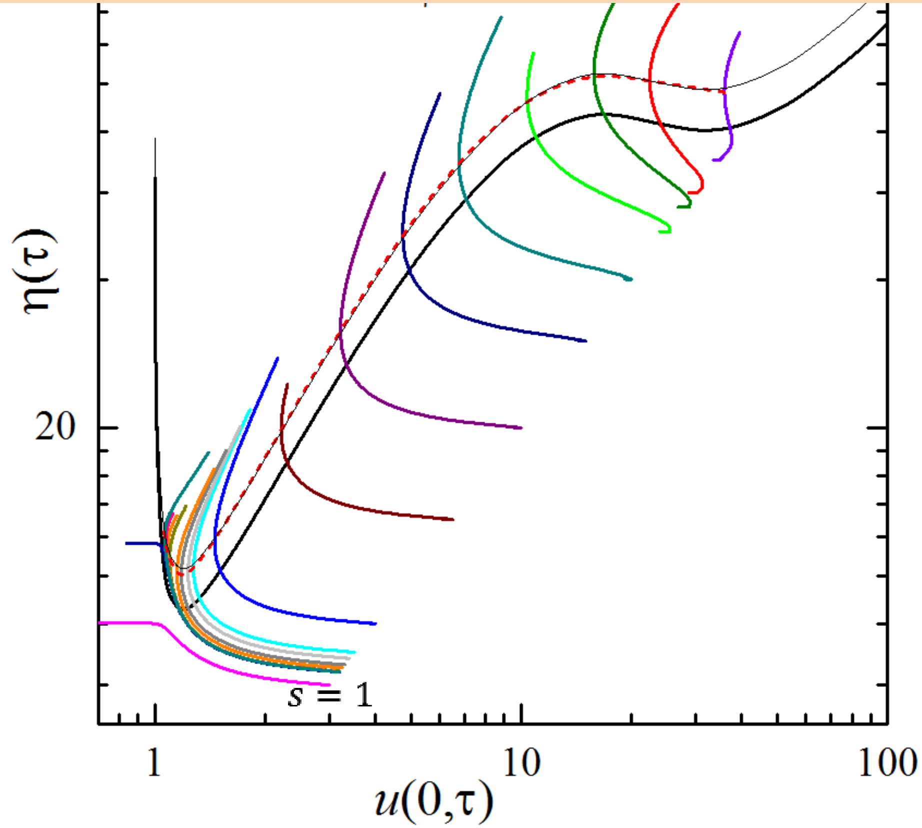
At $j = 0.8, e_\rho = 25000$,
Power-law MQE higher than CS as expected
Similarly correlated to stationary minimum:

$$\eta_{MQE} \sim 25 \sim \frac{3}{5}\eta_{\min}$$

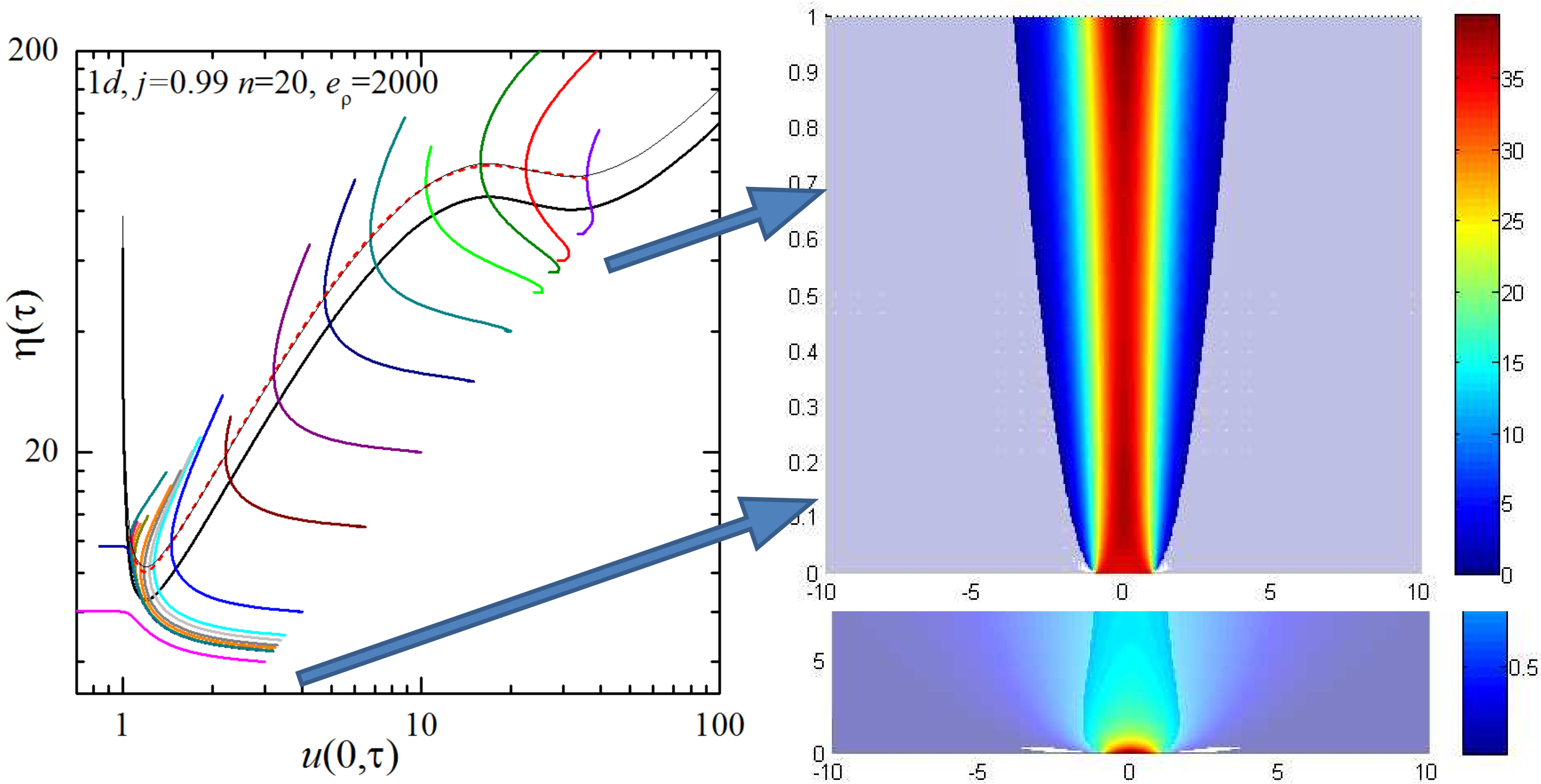


At $j = 0.99, e_\rho = 2000$,
Power-law MQE shifts to lower minimum but remain correlated to stationary minimum:

$$\eta_{MQE} \sim 6.0 \sim 0.7\eta_{\min}$$

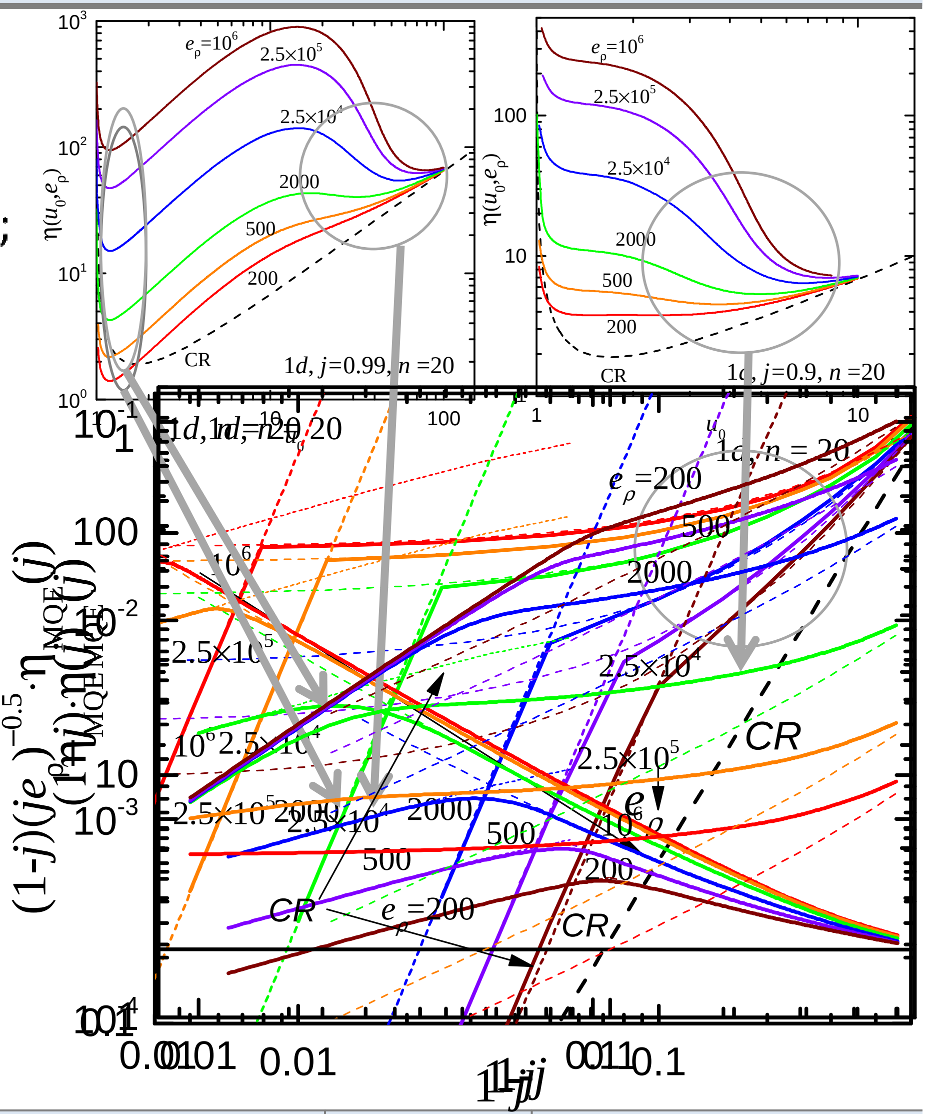


Different Dynamics at High and Low Minima



Power-law MQE Scaling According to Stationary Minimum Energy: Accepting $MQE \propto \eta_{\min}$

- Higher dimensionless η_{MQE} than the critical state constant at $j \leq 0.9$;
- The additional minimum at u_0~1 for $j \gg 0.9$ leads to sharply lower η_{MQE}, esp. with small e_ρ;
- $(1-j)\cdot\eta_{MQE}$ (back to real temperature) shows clearly that MQE does not diminish with $(1-j)$ before the additional minimum takes hold;
- $\text{MQE} \propto (1-j)(je_\rho)^{-m/2}\cdot\eta_{MQE}$ (back to real length): competition of two regimes:
  - The current sharing regime $u \gg 1$, the power-law $MQE \propto \eta_{MQE}(1-j)(je_\rho)^{-0.5}$ is higher than the critical state and does not scale with $(1-j)$.
  - The additional minimum is due to the finite heat generation by power-law at $u=1$ without current sharing, hence independent of e_ρ.

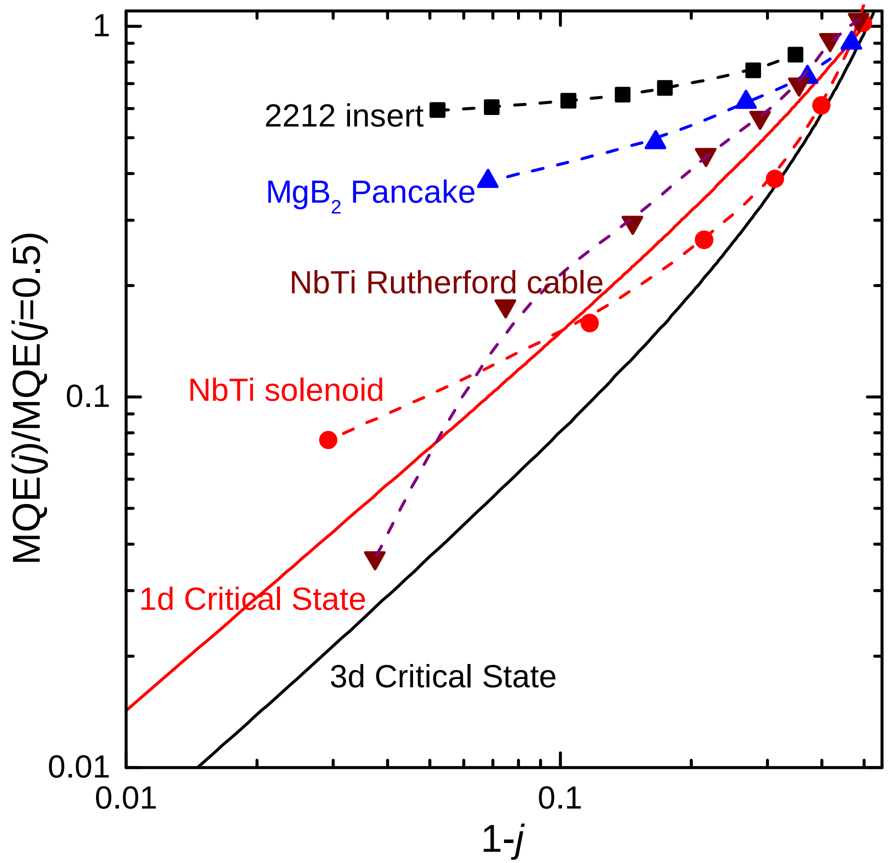


Experimental Evidence of MQE not vanishing with (1-*j*)

Critical state MQE vanishes with $1 - j$

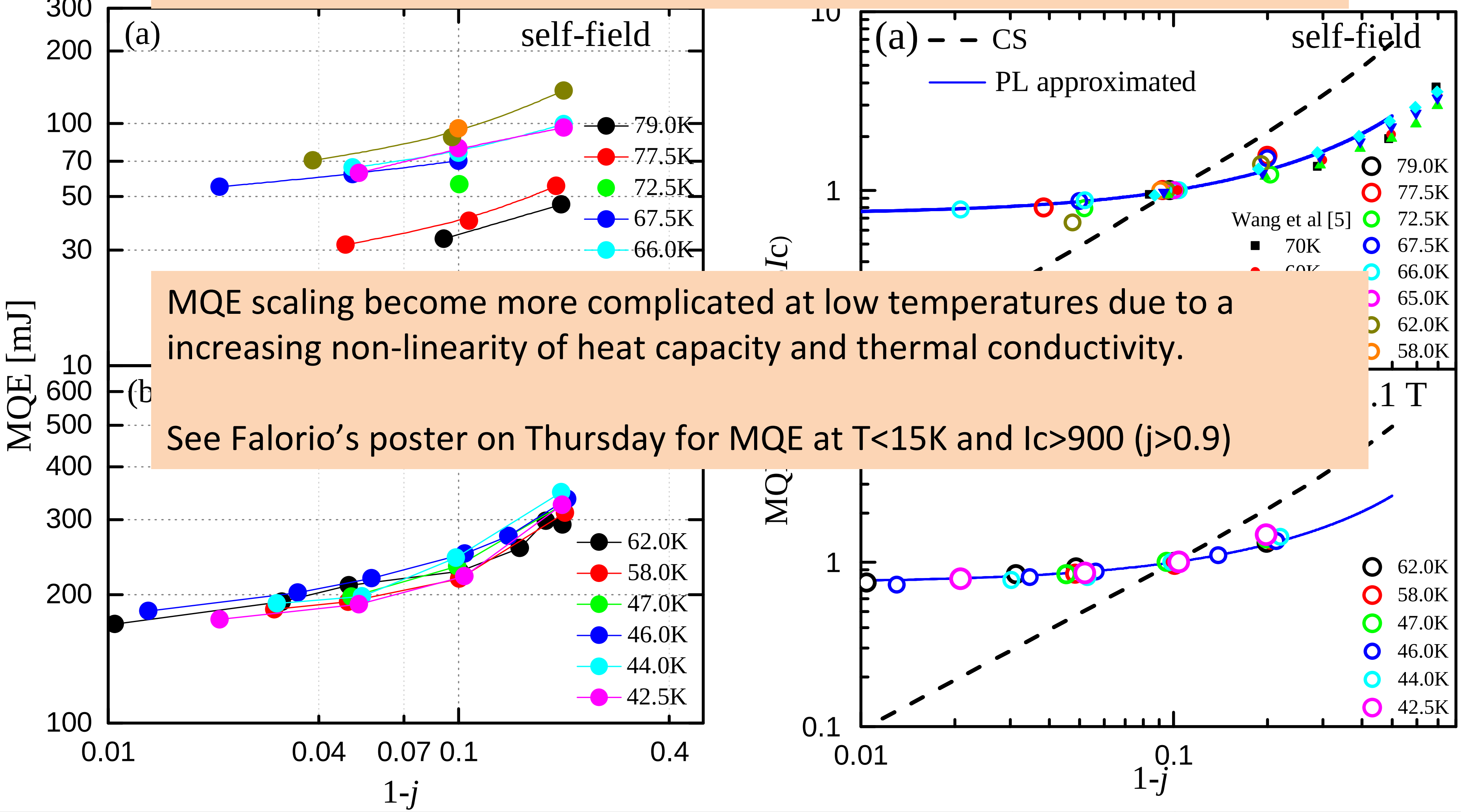
- High current $j > 0.9$ is more sensitive for ascertaining the current scaling of MQE.
- Although MQE measurements at high current are difficult, data do exist and show clearly the experimental MQE deviates from the critical state:
  - Most notably slower reduction at *j*>0.9;
  - In both LTS and HTS;
  - **1d**: Rutherford cable (L Shirshov)
  - **2d**: MgB_2 pancake (J Pelegrin)
  - **3d**: NbTi (Dresner and Scott)
  - **?d**: 2212 (Y Yang) solenoids

Evidence of Power-Law at play?



Power-Law Effect More Prominent at High Temperature, as Expected

Iole Falorio et al (2015) on SuperPower 2G YBCO: Measurement into $j > 0.9$



MQE scaling become more complicated at low temperatures due to a increasing non-linearity of heat capacity and thermal conductivity.

See Falorio's poster on Thursday for MQE at T<15K and Ic>900 (j>0.9)

Approximation of Power-law and explicit current scaling of MQE

- The lack of a closed-form solution for power-current sharing equation makes it difficult to elucidate an *explicit* MQE-j scaling.
- However the nonlinear $E(u,j)$ of the current sharing regime can be approximated with a li one (by a single Newton-Raphson iteration):

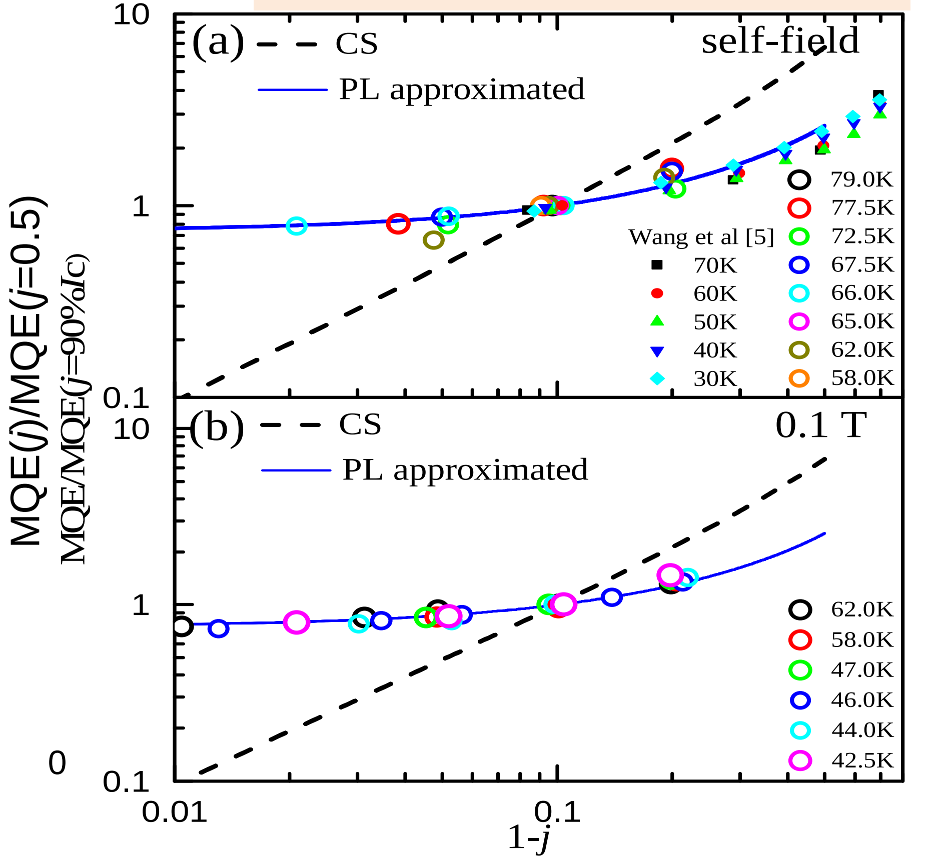
$$\frac{E(u,j)}{E_0} = e_\rho \beta^2 (1-j)\left(u - \frac{1-j\beta^{-2}}{1-j}\right)$$
$$= e_\rho(1-j)\left(\frac{u}{u^*(j,\beta)} - 1\right)\beta^2 u^*(j,\beta)$$

Where $\beta^2 = \frac{(je_\rho)^{\frac{1}{n}}}{1+\frac{1}{n\cdot j}(je_\rho)^{\frac{1}{n}}}$ and $u^*(j,\beta) = \frac{1-j\beta^{-2}}{1-j}$

- Rescale temperature : $v = \frac{u}{u^*(j,\beta)}$ and $\zeta = \beta\xi$
- The corresponding quench equation become identical to the critical state:

$$\frac{\partial v}{\partial \tau} = \frac{1}{\zeta^{m-1}}\frac{\partial}{\partial \zeta}\left(\zeta^{m-1}\frac{\partial v}{\partial \zeta}\right) + \delta_m^2(v-1)$$

Seems to work reasonably well

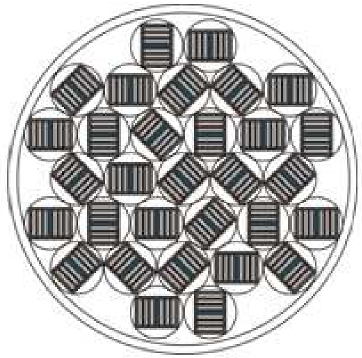


$$MQE(j) \propto (1-j)j^{-\frac{m}{2}}u^*(j,\beta)\beta^m$$
$$= j^{-\frac{m}{2}}(1-j\beta^{-2})\beta^m$$

Quench with Lateral Cooling

1. Liquid cryogen cooling has been the norm for superconducting bus-bar/cables
2. Localised disturbances do not pose a quench risk due to high heat transfer coefficient
3. Gas cooled cables/bus-bars are now seriously considered to take advantage of the wide temperature range found in HTS and MgB_2
4. Heat transfer coefficient by gas cooling is much lower, local disturbance induced quench becomes a risk.

Novel *twisted-pair* cable concept optimized for **tape conductors** (MgB_2, Y-123 and Bi-2223). A. Ballarino " Alternative design concepts for multi-circuit HTS link systems". *IEEE Trans. on Applied Supercond.* **21** pp. 980-984, 2011



Account for lateral cooling (1)

Add the lateral heat transfer term

$$c_p(T(x,t))\frac{\partial T(x,t)}{\partial t} = \frac{\partial}{\partial x}\left(k(T(x,T))\frac{\partial T(x,t)}{\partial x}\right) + J \cdot E(T(x,t),J) - \frac{hP}{A}(T(x,t) - T_0)$$

with $T(x,0) = T_0$ and $T(x \to \pm\infty, t) = T_0$

Maintain the same non-dimensional transformation:

$$\frac{\partial u}{\partial \tau} = \frac{\partial^2 u}{\partial \xi^2} + \left(\frac{\pi}{2}\right)^2 g(u,j) - \frac{hPl_{MPZ}^2}{k(T_0)A}u$$

Hence

$$\frac{\partial u}{\partial \tau} = \frac{\partial^2 u}{\partial \xi^2} + \left(\frac{\pi}{2}\right)^2 (g(u,j) - \mathrm{Cg}\, j^{-1}\, u) \text{ with } \frac{hPl_{MPZ}^2}{k(T_0)A} = \left(\frac{\pi}{2}\right)^2 \frac{\frac{hP}{A}(T_c - T_0)}{J_{C(T_0)}^2 \rho_m} = \left(\frac{\pi}{2}\right)^2 \mathrm{Cg}$$

Account for lateral cooling (2)

Introducing a new dimensionless number:

$$\mathrm{Cg} = \frac{\frac{hP}{A}(T_c - T_0)}{J^2_{C(T_0)}\rho_m}$$

which is the ratio between lateral cooling and current sharing heat generation.

Consider single 2G tape (4mm width):

$$P = 8\mathrm{mm}, A = 0.4\mathrm{mm}^2, \frac{P}{A} = 2\times 10^4\mathrm{m}^{-1}$$

1. In liquid nitrogen pool $T_0 = 77\mathrm{K}$:

$$h = 1 - 3\ \mathrm{Wcm}^{-2}\mathrm{K} \sim 2\times 10^4 \mathrm{Wm}^{-2}\mathrm{K},$$
$$T_C - T_0 \sim 10\mathrm{K}, I_C(T_0) = 100\mathrm{A},$$
$$J_C(T_0) = 2.5\times 10^8\mathrm{Am}^{-2}, \rho_m = 3.2\times 10^{-9}\Omega\mathrm{m}$$
$$\mathrm{Cg} = 2$$

2. Helium gas cooled $T_0 = 20\mathrm{K}$:

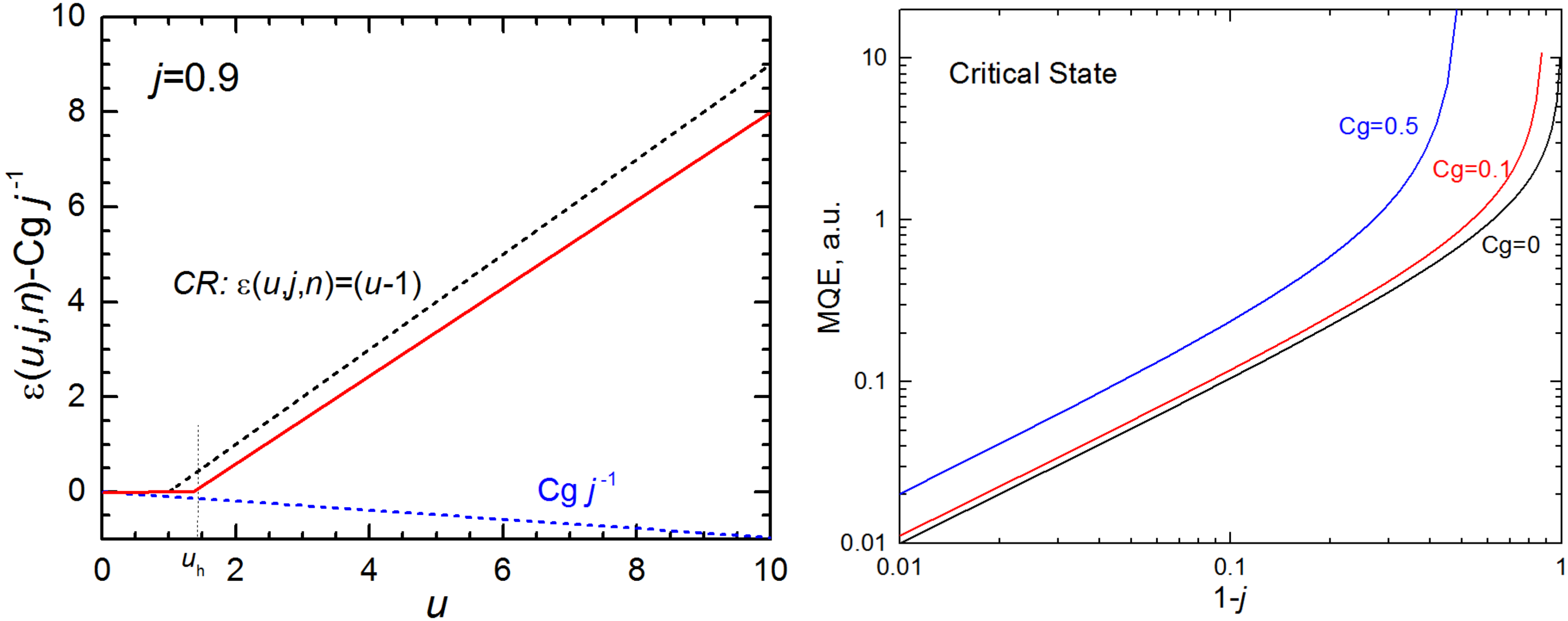
$$h = \frac{\mathrm{Nu}k_{He}}{D} \sim 40\mathrm{Nu}\ \mathrm{Wm}^{-2}\mathrm{K}$$
$$T_C - T_0 \sim 70\mathrm{K}, I_C(T_0) = 800\mathrm{A},$$
$$J_C(T_0) = 2\times 10^9\mathrm{Am}^{-2}, \rho_m = 3.2\times 10^{-10}\Omega\mathrm{m}$$
$$\mathrm{Cg} = 0.1$$

Critical State with lateral cooling

- Approximate transformation to effective critical state

$$\frac{\partial^2 u}{\partial \xi^2} + \left(\frac{\pi}{2}\right)^2 \left(\left(1 - \frac{\mathrm{Cg}}{j}\right) u - 1\right) = 0 \xrightarrow{v = \frac{u}{u_h},\, u_h = \frac{1}{1 - \mathrm{Cg}\, j^{-1}}} \frac{\partial^2 v}{\partial \zeta^2} + \left(\frac{\pi}{2}\right)^2 (v - 1) = 0$$

$$MQE \propto (1-j) j^{-0.5} u_h(j) = \frac{1-j}{\sqrt{(1-\mathrm{Cg}\, j^{-1})^3}} j^{-0.5} = \frac{MQE_{CR}}{\sqrt{(1-\mathrm{Cg}\, j^{-1})^3}}$$

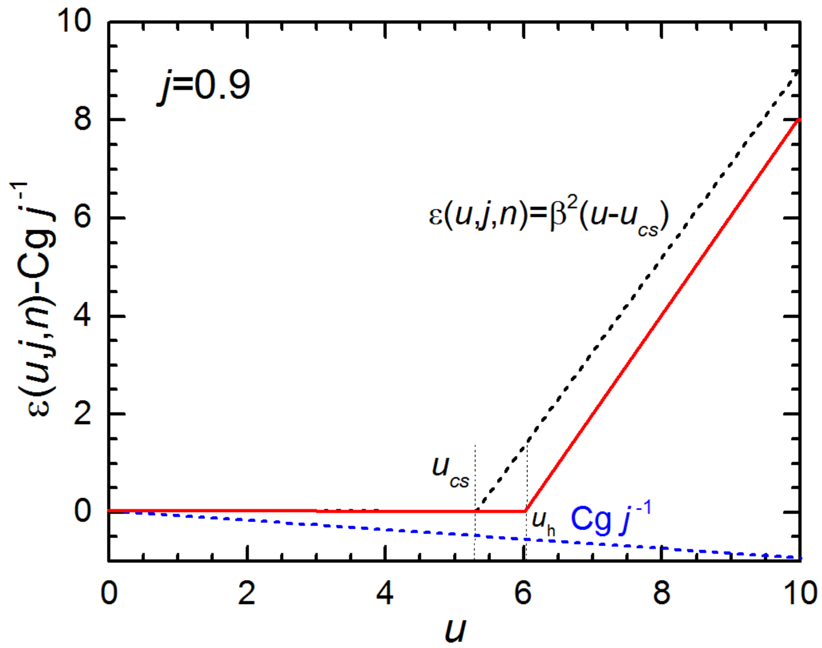


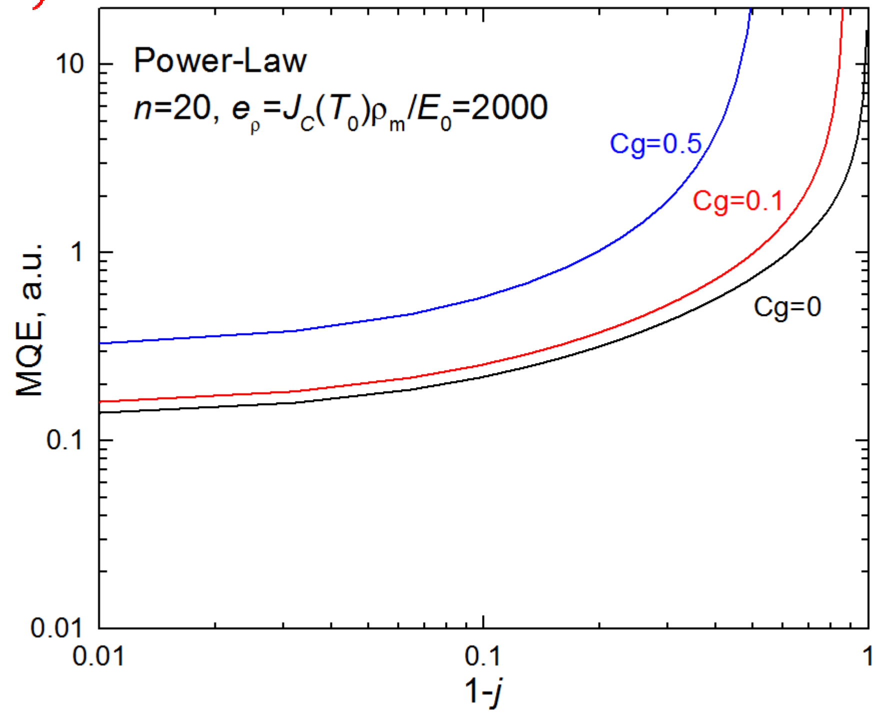
Power-law superconductors with lateral cooling

- Approximate transformation to effective critical state

$$\frac{\partial^2 u}{\partial \xi^2} + \left(\frac{\pi}{2}\right)^2 \left(\beta^2 (u - u_{cs}(j,\beta)) - \frac{\text{Cg}}{j} u\right) = 0 \xrightarrow{v=\frac{u}{u_h}, u_h = \frac{u_{cs}}{1-\text{Cg}\,\beta^{-2} j^{-1}}, \zeta = \sqrt{\beta^2 - \text{Cg}\, j^{-1}}\,\xi} \frac{\partial^2 v}{\partial \zeta^2} + \left(\frac{\pi}{2}\right)^2 (v-1) = 0$$

$$MQE \propto \frac{1-j}{\sqrt{\beta^2 - \text{Cg} j^{-1}}} u_h j^{-0.5} = \frac{1 - j\beta^{-2}}{\beta(1 - \text{Cg}\beta^{-2} j^{-1})^{1.5}} j^{-0.5}$$





Conclusions

1. Critical state MQE vanishes with $1-j$ as the current approaches the critical current I_C;
2. In the vicinity of $I_{C,}$ a higher MQE persists for power-law before disappearing to zero: more room for stable operation near I_C!
3. An approximate power-law $MQE(j)\sim j^{-\frac{m}{2}}(1-j\beta^{-2})\beta^m$ correlates explicitly to power index n and nominal conductor dissipation $e_\rho = J_C(T_0)\rho_n/E_0$: explains the current scaling of experimental MQE;
4. Higher dimensions has higher MQE, but effective continuum breaks down then $L_{MPZ}^{y,z} < d_{wire}$ and/or very large L_{MPZ}^{x}: possibly quench with 1d/2d MQE.
5. Lateral cooling in 1d can be solved using similarity to the critical state. Possibility for global stability, even for the critical state.

Thanks for your attention!