Quench Intialisation Transients in ReBCO Coated Conductor Cables

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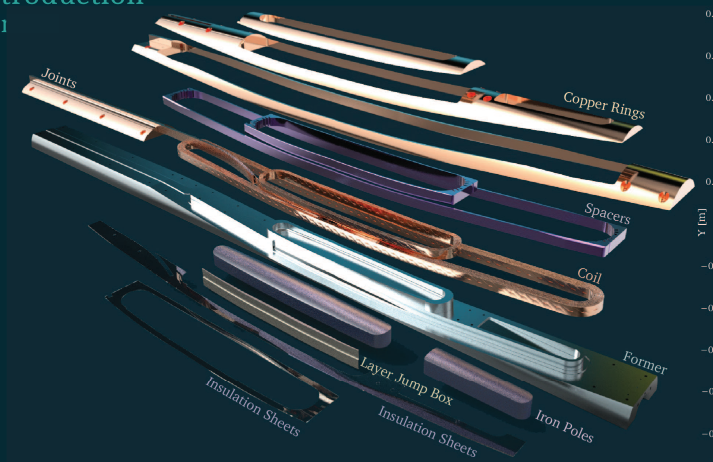
FEATHER-M2
Five Tesla HTS Research Magnet

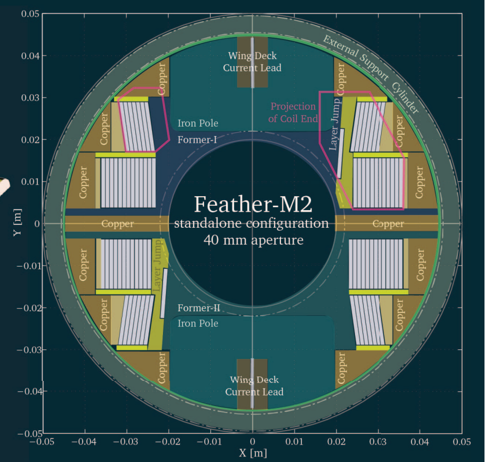
EuCARD²

CERN

Introduction

Co



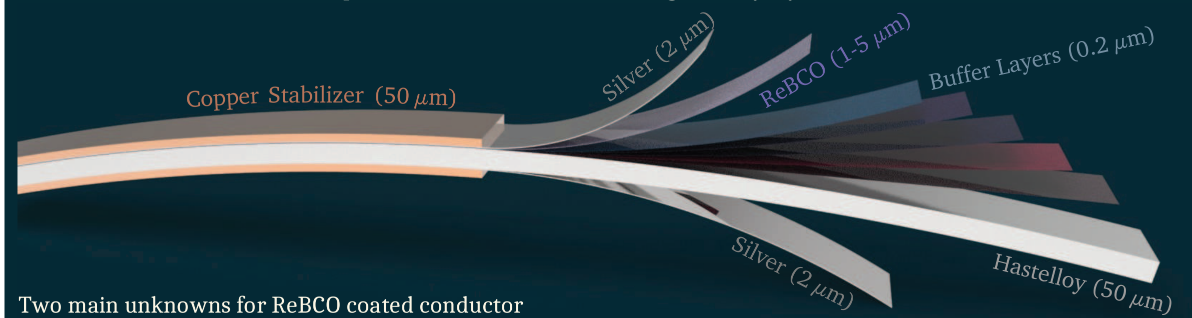


EuCARD2 aims to build a 5 T standalone (in yoke) ReBCO accelerator dipole named Feather-M2, which can later be used as an insert-magnet in the 13 T Fresca2 dipole to produce a total of 17 T (or more)

The crucial design feature of this magnet is that all tapes are aligned with the magnetic field lines in order to exploit the anisotropy and to reduce magnetization effects.

Introduction
ReBCO Coated Conductor

ReBCO comes in the form of a tape (coated conductor) consisting of many layers



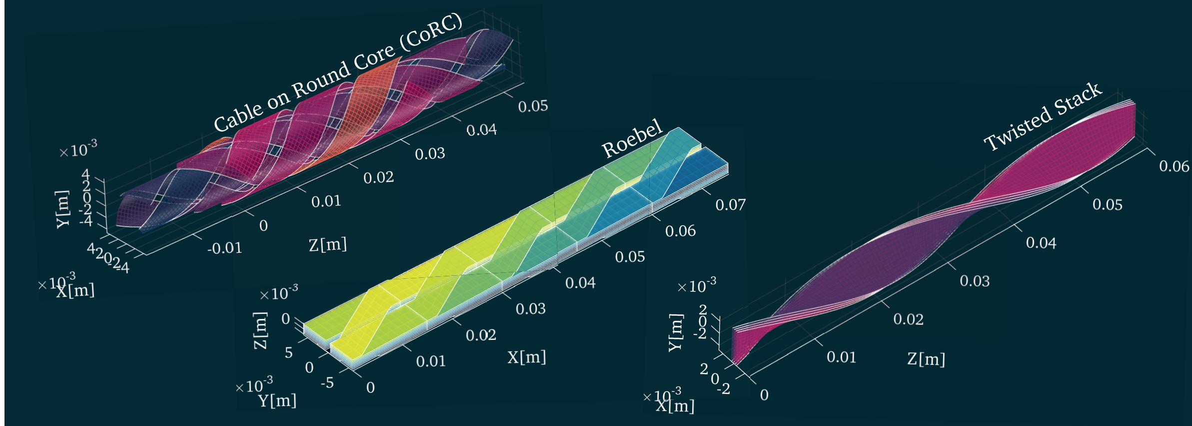
Two main unknowns for ReBCO coated conductor

1. Magnetization - wide tapes act as mono filaments allowing for large screening currents and uncertainty in the position of the current. What is the effect on field quality?
2. Quench - High Minimal Quench Energies (E_{MQE}) but slow Normal Zone propagation velocity (V_{nzp}). Require new methods for detection and protection.

To study these effects an Electrical Network Model capable of simulating ReBCO cables and (small) coils has been developed at CERN as part of my PhD (thesis due November 2016)

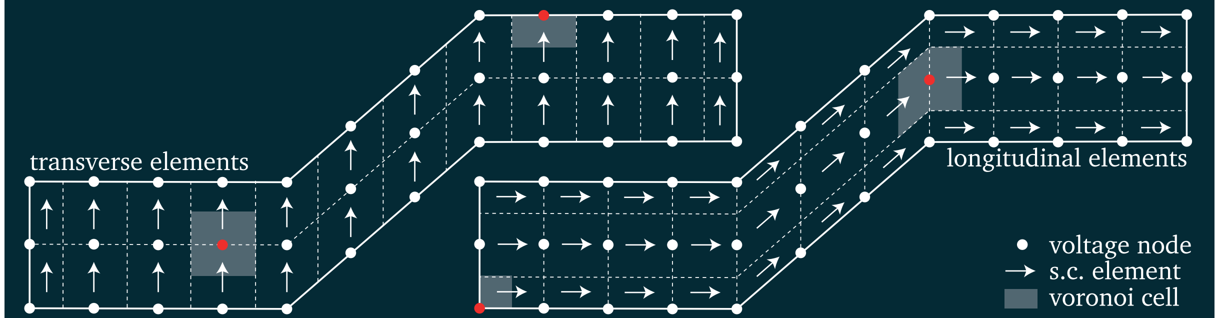
Electrical Network Model
Cable Geometries

There are many ways to make cables out of flat tapes
At present implemented cable geometries are Roebel, CoRC and (twisted) Stack



The superconducting layer of the tapes is modeled as an infinitely thin sheet which is three-dimensional

Electrical Network Model
Discretization



Idea is to use a solver setup similar to JackPot-AC (University of Twente)
Tape surfaces are modelled as a Partial Element Equivalent Circuit (PEEC) network of

- superconducting elements that contain currents I_r
- nodes at which the voltage V_j and temperatures T_p are defined
- contact elements between the tapes (not shown here)

Electrical Network Model

System of equations

The system contains thousands of Differential Algebraic Equations (DAE) which can be written with sparse matrices and several non-linear functions as

$$\underbrace{\begin{bmatrix} G_{ij} & M_{\mathrm{kcl},ir} & 0 \\ M_{\mathrm{kvl},qj} & R_{qr} & 0 \\ 0 & 0 & K_{sp} - K_{\mathrm{cool},sp} \end{bmatrix}}_{\text{time independent linear}} \begin{bmatrix} V_j \\ I_r \\ T_p \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ V_{\mathrm{nl},q}\left(I_r, T_r, \left|\vec{B}_r\right|, \alpha_r\right) \\ P_{\mathrm{nl},s}\left(I_r, T_r, \left|\vec{B}_r\right|, \alpha_r\right) + P_{G,s}\left(V_j\right) + P_{R,s}\left(I_r\right) \end{bmatrix}}_{\text{time independent non-linear}} + \underbrace{\begin{bmatrix} I_{s,i} + I_{\mathrm{bg},i}\left(\frac{\partial \vec{B}}{\partial t}\right) \\ V_{s,q} + V_{\mathrm{bg},q}\left(\frac{\partial \vec{B}}{\partial t}\right) \\ P_{s,s} + K_{\mathrm{cool},ss} T_{\mathrm{bath}} \end{bmatrix}}_{\text{external sources}} + \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & L_{qr} + M_{\mathrm{S2T},qr} & 0 \\ 0 & 0 & -C_{p,sp} \end{bmatrix} \begin{bmatrix} \frac{\partial V_j}{\partial t} \\ \frac{\partial I_r}{\partial t} \\ \frac{\partial T_p}{\partial t} \end{bmatrix} + \begin{bmatrix} 0 \\ V_{\mathrm{mlfmm},q}\left(\frac{\partial I_r}{\partial t}\right) \\ 0 \end{bmatrix}}_{\text{time dependent linear}} = \underbrace{\begin{bmatrix} I_{\mathrm{res},i} \\ V_{\mathrm{res},q} \\ P_{\mathrm{res},s} \end{bmatrix}}_{\text{residual}} \cong 0,$$

The three rows of this equation are

1. Kirchoff's Current law, summing the current at each node
2. Kirchoff's Voltage law, summing the voltage generated on the element
3. The discrete heat equation, summing the power dissipation at each node

The equations needt to be solved every time step for

- The voltages at the nodes V_j and its derivative with respect to time $\frac{\partial V_j}{\partial t}$
- The currents through the elements I_r and its derivative with respect to time $\frac{\partial I_r}{\partial t}$
- The temperatures at the nodes T_p and its derivative with respect to time $\frac{\partial T_p}{\partial t}$

The matrices and functions are explained in the next slides

Electrical Network Model
Connectivity Matrices

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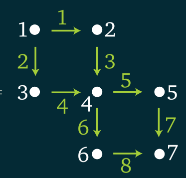
Connectivity matrices defined such that

- $M_{\text{kcl},ir} I_r$ is the current flowing towards each node provided by the elements
- $M_{\text{kvl},qj} V_j$ is the voltage difference between the source and target node of the respective element

Simplified example

$$M_{\text{kcl}} = {M_{\text{kvl}}}^T = \begin{bmatrix} -1 & -1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 1 & \cdot & -1 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & 1 & \cdot & -1 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & 1 & 1 & -1 & -1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 & \cdot & -1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & -1 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & 1 \end{bmatrix} \Leftarrow$$



Electrical Network Model
Contact Elements I

The system contains thousands of Differential Algebraic Equations (DAE) which can be written with sparse matrices and several non-linear functions as

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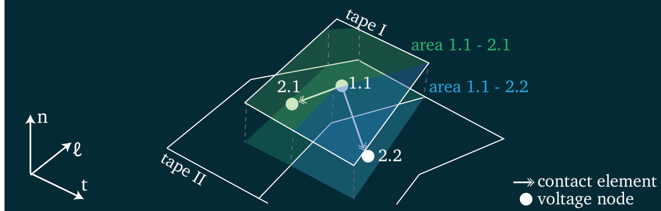
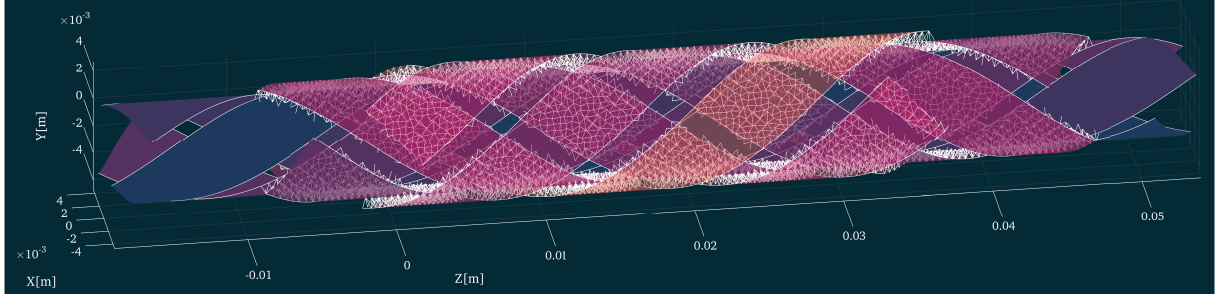
Resistance and Conductance from linearly resistive elements (contact elements)

- Size of the contact area is calculated using the overlap between the Voronoi cells of the nodes
- For Roebel and Stacked in Cartesian and for CoRC in Cylindrical coordinates
- Contact resistances are $0.27\ \mu\mathrm{Ohm.m}^2$ and $100\ \mathrm{W}/(\mathrm{m}^2\mathrm{K})$



Electrical Network Model
Contact Elements II

As an example the contact elements are shown as white lines between the tapes in the CoRC cable geometry



Electrical Network Model
Superconducting Elements

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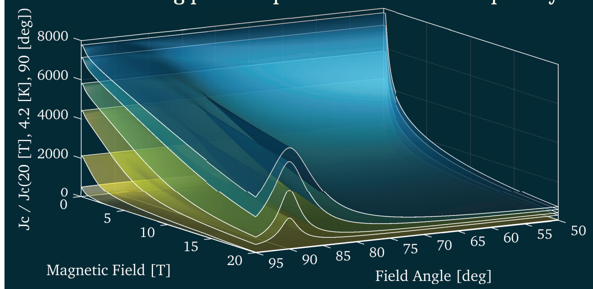
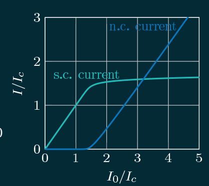
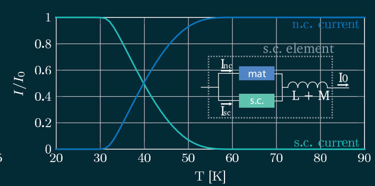
Voltage from non-linear elements (Superconductor)
Calculated using parallel path model solved implicitly for the superconducting current $I_{sc,q}$ with Newton-Raphson



$$I_q - \underbrace{\frac{E_0}{\rho_{tq}\left(\left|B_q\right|, T_q\right)}\left[\frac{I_{sc,q}}{I_{c,q}\left(\left|B_q\right|, T_q, \alpha_q\right)}\right]^{N_{sc}}}_{\text{normal conducting current}} - I_{sc,q} = r \cong 0,$$





Electrical Network Model
Voltage and Current Sources

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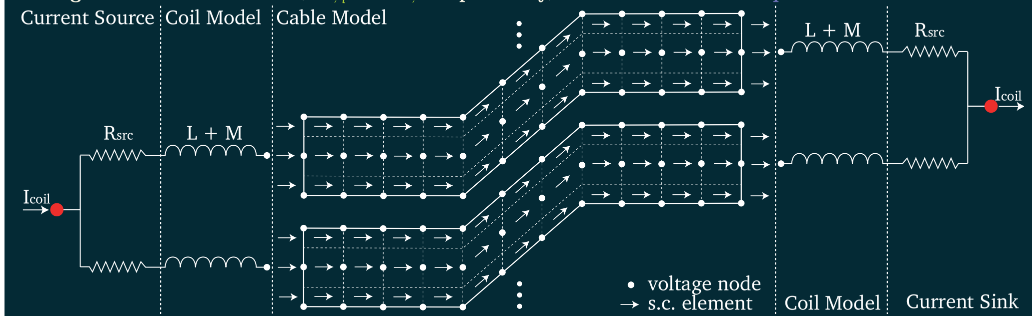
Voltage and current sources, $V_{s,q}$ and $I_{s,i}$ respectively, are used to add transport current to the cable



Electrical Network Model
Coupling with the Background Field

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Coupling with the Background Field

$$V_{\mathrm{bg},q} = \vec{D}_q \cdot \frac{d\vec{A}_q}{dt}.$$

where the time derivative vector potential is given by a perfectly parallel background field.

Electrical Network Model
Self Inductance

The system contains thousands of Differential Algebraic Equations (DAE) which can be written with sparse matrices and several non-linear functions as

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Self inductances are defined such that $L_{qr}\frac{\partial I_r}{\partial t}$ is the self induced voltage

The self inductance of the elements are calculated using analytic equations from literature

- Tape elements are approximated using an equation for Ribbons

$$L_{\mathrm{rib},q} = 200L_{12q}\left[\log\left(\frac{2L_{12q}}{W_{12q}+d_s}\right) + 0.5 + 0.235\left(\frac{W_{12q}+d_s}{L_{12q}}\right)\right],$$

- Contact elements are approximated using equation for Round wires

$$L_{\mathrm{con},q} = 200L_{12q}\left[\log\left(\frac{2L_{12q}}{d_q}\left(1+\sqrt{1+\left[\frac{d_q}{2L_{12q}}\right]^2}\right)\right) - \sqrt{1+\left[\frac{d_q}{2L_{12q}}\right]^2} + \frac{\mu}{4} + \frac{d_q}{2L_{12q}}\right],$$

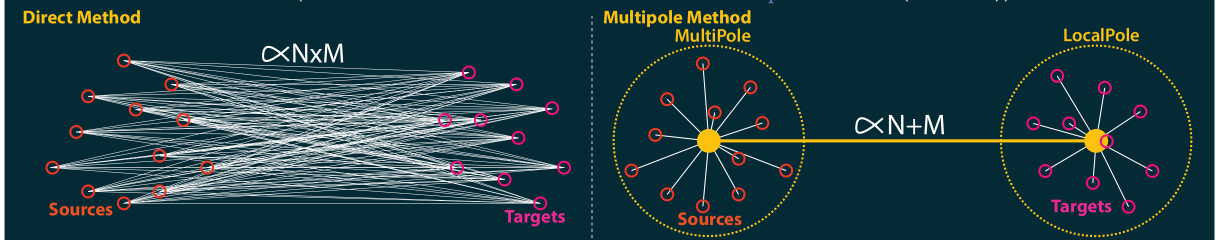
Electrical Network Model
Mutual Inductance and the MLFMM I

The system contains thousands of Differential Algebraic Equations (DAE) which can be written with sparse matrices and several non-linear functions as

$$\overbrace{\begin{bmatrix} G_{ij} & M_{\mathrm{kcl},ir} & 0 \\ M_{\mathrm{kvl},qj} & R_{qr} & 0 \\ 0 & 0 & K_{sp}-K_{\mathrm{cool},sp} \end{bmatrix}\begin{bmatrix} V_j \\ I_r \\ T_p \end{bmatrix}}^{\text{time independent linear}} + \overbrace{\begin{bmatrix} 0 \\ V_{\mathrm{nl},q}\left(I_r, T_r, \left|\vec{B}_r\right|, \alpha_r\right) \\ P_{\mathrm{nl},s}\left(I_r, T_r, \left|\vec{B}_r\right|, \alpha_r\right) + P_{G,s}\left(V_j\right) + P_{R,s}\left(I_r\right) \end{bmatrix}}^{\text{time independent non-linear}} + \overbrace{\begin{bmatrix} I_{s,i} + I_{\mathrm{bg},i}\left(\frac{\partial \vec{B}}{\partial t}\right) \\ V_{s,q} + V_{\mathrm{bg},q}\left(\frac{\partial \vec{B}}{\partial t}\right) \\ P_{s,s} + K_{\mathrm{cool},ss}T_{\mathrm{bath}} \end{bmatrix}}^{\text{external sources}} + \overbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & L_{qr} + M_{\mathrm{SZT},qr} & 0 \\ 0 & 0 & -C_{p,sp} \end{bmatrix}\begin{bmatrix} \frac{\partial V_j}{\partial t} \\ \frac{\partial I_r}{\partial t} \\ \frac{\partial T_p}{\partial t} \end{bmatrix} + \begin{bmatrix} 0 \\ V_{\mathrm{mlfmm},q}\left(\frac{\partial I_r}{\partial t}\right) \\ 0 \end{bmatrix}}^{\text{time dependent linear}} = \overbrace{\begin{bmatrix} I_{\mathrm{res},i} \\ V_{\mathrm{res},q} \\ P_{\mathrm{res},s} \end{bmatrix}}^{\text{residual}} \cong 0,$$

The mutually induced voltages are calculated from the time derivative of the current in each other element

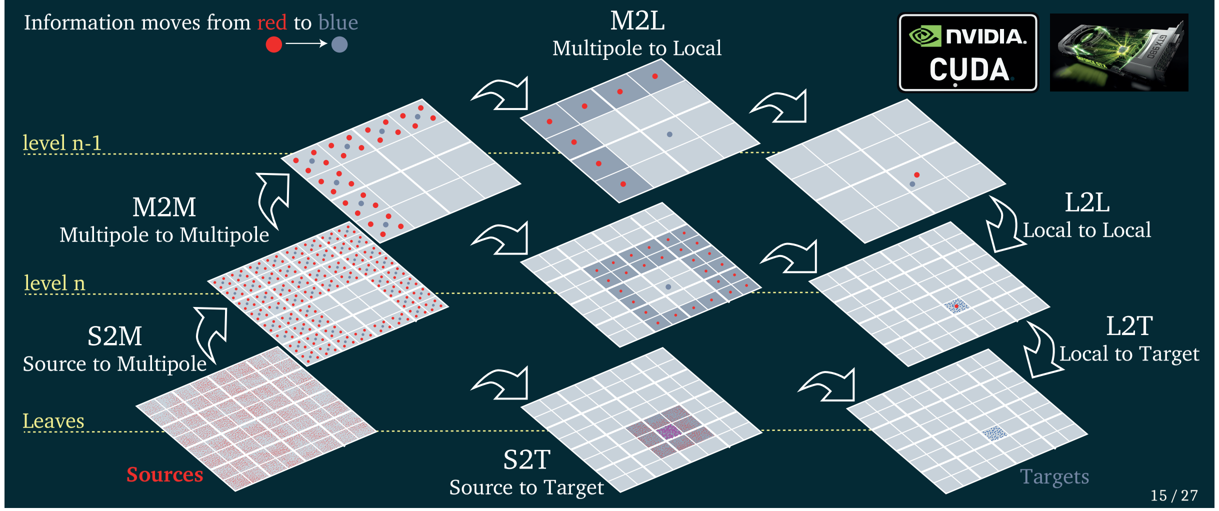
MultiPoles and LocalPoles (series expansions of spherical harmonics) can be used as intermediate steps to reduce the number of calculations (a Method known as the Multi-Level Fast Multipole Method (MLFMM))



Electrical Network Model
Mutual Inductance and the MLFMM II

The MLFMM uses a multi-level three-dimensional octree grid to structure the movement of data

Since these are/were the most compute heavy steps, they are accelerated through GPU processing (NVIDIA CUDA)



Electrical Network Model
Heat Equation

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Last row is standard discrete heat equation

- K_{sp} is the thermal conductivity of each element such that $K_{sp}T_p$ is the heat flux towards each node
- P_{nl} is the non-linear power dissipation, basically $R_r I_{nc,r}^2$
- $P_{G,s}$ and $P_{R,s}$ are the powers at the nodes of the resistive contact elements connected to it
- C_{sp} is the heat capacity of each node (calculated from area of Voronoi cells)
- $P_{s,s}$ is the external power dissipation at each node (heat pulse to start quench)

Electrical Network Model
Solving the System of Equations

The system is now solved, stepping through time with $1/t_{cj}$, using the Sundials IDA solver (by Lawrence Livermore national laboratory), which requires as input

- Pre-Conditioner (linear but with MLFMM), must be solved solved iteratively with GMRES preconditioned with the pre-pre-conditioner matrix

$$\begin{bmatrix} G_{ij} & M_{\mathrm{kcl},ir} & 0 \\ M_{\mathrm{kvl},qj} & R_{qr} + R_{\mathrm{nl},qr} & 0 \\ I_{G,sj} & V_{R,sr} + V_{\mathrm{nl},sr} & K_{sp} - K_{\mathrm{cool},sp} \end{bmatrix} \begin{bmatrix} V_j \\ I_r \\ T_p \end{bmatrix} + t_{\mathrm{cj}} \begin{bmatrix} 0 & 0 & 0 \\ 0 & L_{qr} + M_{\mathrm{S2T},qr} & 0 \\ 0 & 0 & -C_{p,sp} \end{bmatrix} \begin{bmatrix} V_j \\ I_r \\ T_p \end{bmatrix} + \begin{bmatrix} 0 \\ V_{\mathrm{mlfmm},q}\left(t_{\mathrm{cj}} I_r\right) \\ 0 \end{bmatrix} = \begin{bmatrix} I_{\mathrm{res},i} \\ V_{\mathrm{res},q} \\ P_{\mathrm{res},s} \end{bmatrix} = 0,$$

- Pre-Pre-Conditioner given here as a matrix (linear without MLFMM, only near-field), factorized and solved using the MUMPS direct solver

$$A_{\mathrm{tdprepre}} = \begin{bmatrix} G_{ij} & M_{\mathrm{kcl},ir} & 0 \\ M_{\mathrm{kvl},qj} & R_{qr} + R_{\mathrm{nl},qr} & 0 \\ I_{G,sj} & V_{R,sr} + V_{\mathrm{nl},sr} & K_{sp} - K_{\mathrm{cool},sp} \end{bmatrix} + t_{\mathrm{cj}} \begin{bmatrix} 0 & 0 & 0 \\ 0 & L_{qr} + M_{\mathrm{S2T},qr} & 0 \\ 0 & 0 & -C_{p,sp} \end{bmatrix},$$

- Jacobian times Vector Function to guide the non-linear solution

$$J_{td} \begin{bmatrix} V_j \\ I_r \\ T_p \end{bmatrix} = \begin{bmatrix} G_{ij} & M_{\mathrm{kcl},ir} & 0 \\ M_{\mathrm{kvl},qj} & R_{qr} + M_{\mathrm{dVdI},qr} & 0 \\ I_{G,sj} & V_{R,sr} + M_{\mathrm{dPdI},sr} & K_{sp} - K_{\mathrm{cool},sp} + M_{\mathrm{dPdT},sp} \end{bmatrix} \begin{bmatrix} V_j \\ I_r \\ T_p \end{bmatrix} + t_{\mathrm{cj}} \begin{bmatrix} 0 & 0 & 0 \\ 0 & L_{qr} + M_{\mathrm{S2T},qr} & 0 \\ 0 & 0 & -C_{p,sp} \end{bmatrix} \begin{bmatrix} V_j \\ I_r \\ T_p \end{bmatrix} + \begin{bmatrix} 0 \\ V_{\mathrm{mlfmm},q}\left(t_{\mathrm{cj}} I_r\right) \\ 0 \end{bmatrix},$$

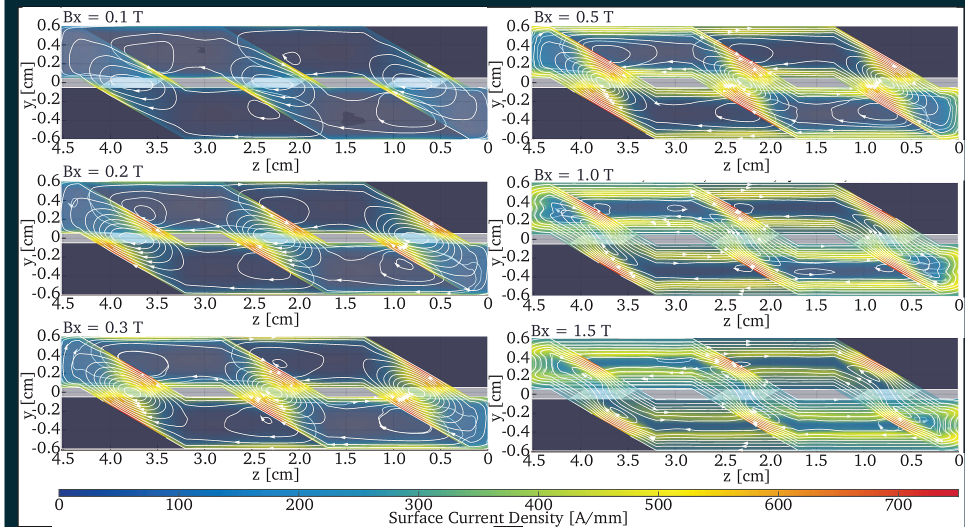
The system is linearized each time step using the matrices in green that are acquired through finite difference

Electrical Network Model
Magnetisation Loops I

To validate the electrical part of the model a Roebel cable in a time varying magnetic background field is modeled
Shown below is a movie showing the time variation of the surface current density for magnetic field amplitude $0.7\,\mathrm{T}$

movie fingers crossed

Electrical Network Model
Magnetization Loops II

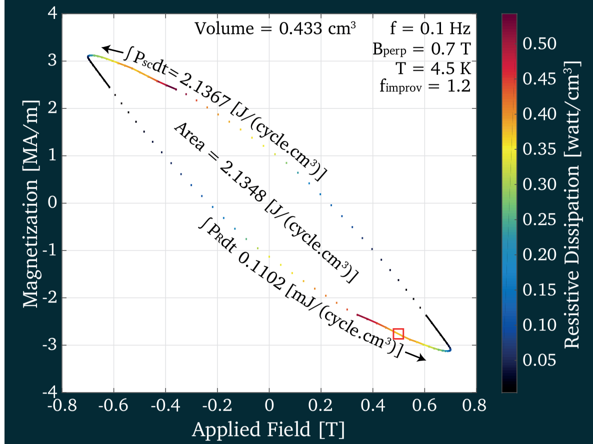


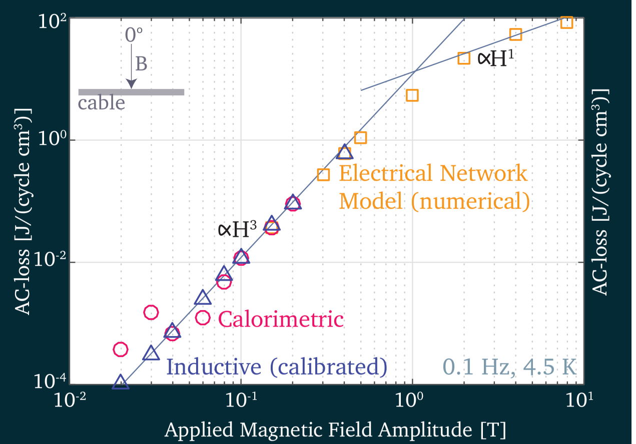
Electrical Network Model

Validation of Electrical Equations

The loop area should match the losses integrated over all elements (check)

Within EuCARD2 a collaboration was started to measure the magnetization losses in Roebel cables in both parallel and perpendicular magnetic fields (perpendicular case shown here)

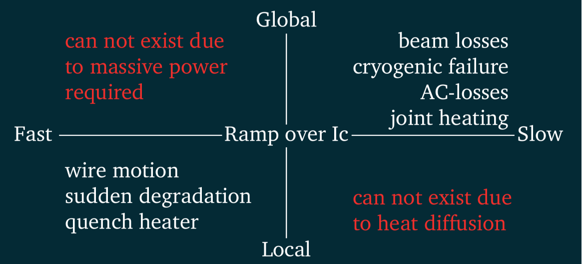




Quench Analysis
Sources of Heat Generation

The origin of a quench is quite unclear for HTS! (might not quench at all)
The cable is connected at to a tape by tape inductance matrix of the Feather-M2 magnet
Must select some sort of heat pulse (duration, energy) and location

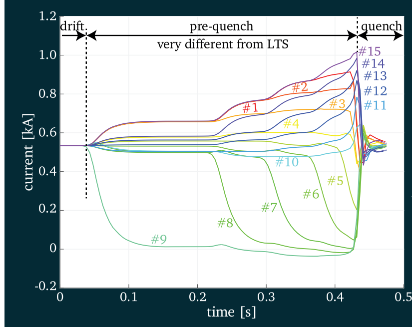
1. Weak point of tape when under longitudinal tension (from mechanical modeling)
2. 50 ms search for MQE found 22 mJ (depends on inductive load)

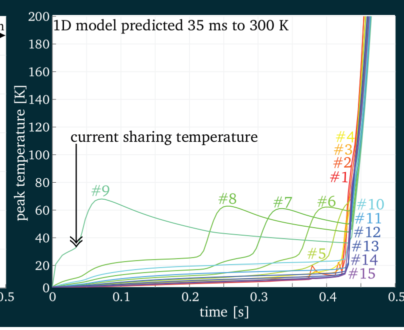


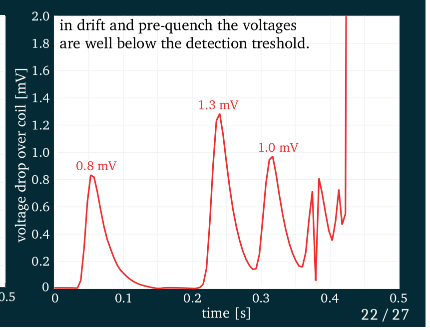
Quench Analysis
Multi-Strand Cable

Quench at 8kA 4.5K 17T 86deg 50ms 25mJ pulse, three phases are distinguishable: Drift, Pre-Quench and Quench

movie fingers crossed

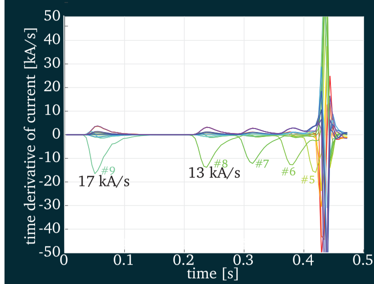


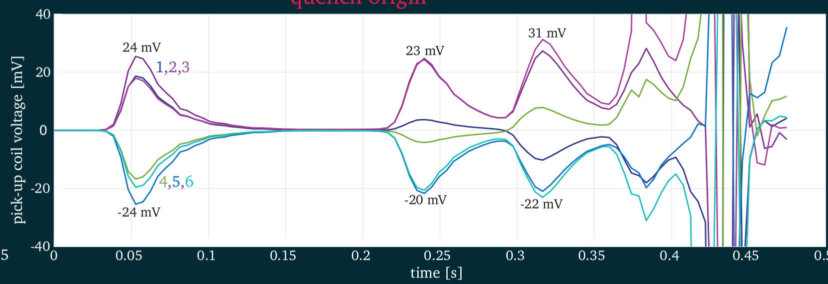




Quench Analysis
PickUp Coils

However the strong redistributions of the current between the tapes can be picked up using pick-up coils located somewhere along the cable



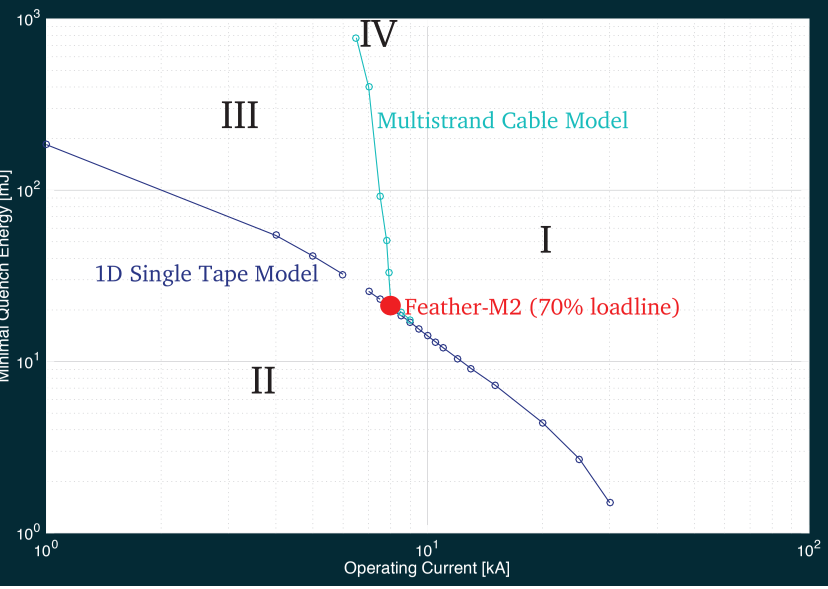


Proposed HTS quench detection strategy (to be tested in Feather-M2)

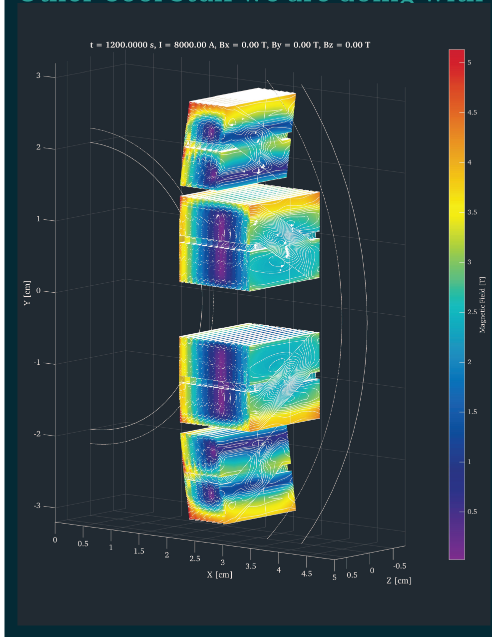
- Fast local quenches are detected during the pre-quench phase using pick-up coils
- Slow global quenches are detected during the drift phase using temperature sensors

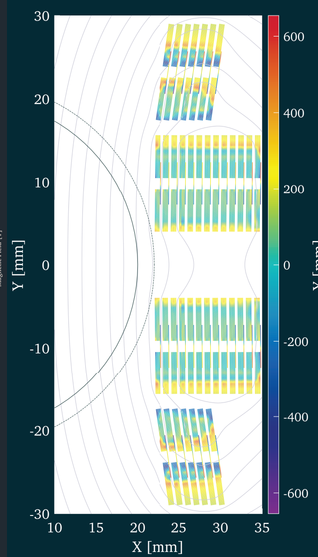
Quench Analysis

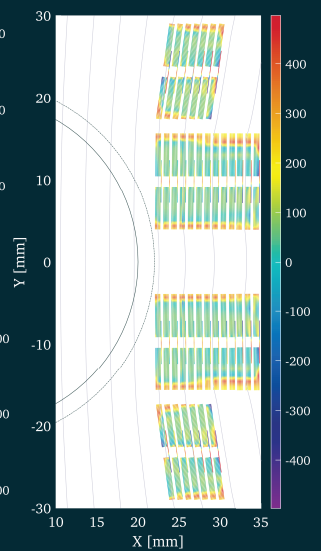
Minimal Quench Energies



Other Cool Stuff we are doing with the Model (ASC 2016)







Conclusion

At CERN an HTS dipole magnet named Feather-M2 is under construction

To understand the conductor in detail an Electrical Network Model that is capable of modelling Electrical and Thermal phenomena in Coated Conductor cables and (small) coils is developed

- The model solves the Currents, Voltages and Temperatures in the network using Sundials IDA, preconditoined using GMRES in turn preconditioned with MUMPS
- The mutual interactions are calculated using the Multi-Level Fast MultiPole Method implemented on GPU processing units

The model includes the current distribution between and inside the tapes making up the cable

Quench analysis on Roebel cables in the Feather-M2 coil configuration has shown that

- The quench consists of three phases: Drift, Pre-Quench, Quench
- The pre-quench should be detectable using pick-up coils (we hope to detect drift with temperature sensors)
- The minimal quench energy has a single strand and multistrand regime (same as for LTS)

The model is also used for dynamic field quality calculations and much more data mining is to be done

Thank you! / Questions?